

There are two main characteristics about thermal barrier. They have high thermal inertia and thermal resistance. Those characteristics are possible because of the low thermal conductivity of the insulating used. Further, the water boiling from the reservoir that leaves the thermal barrier are responsible for an expressive quantity of energy that left the system and keep the temperature of the logger's chamber under 100 °C. Researches like Mei *et al.* (2012), reach the high thermal inertia by Zircon oxide ceramic coating surface.

From the geometry and the design of the thermal barrier is simple to identify that while there is water in the reservoir the logger's chamber is under the critical temperature. The validation of the mathematical model was done by comparing the quantity of water left in the water reservoir with the model's result.

1.1 Thermal barrier

Figure 1 shows the thermal barrier in a schematic view where is possible identify most of the components.

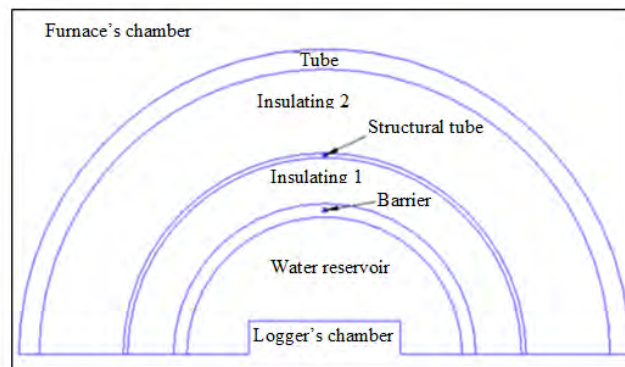


Figure 1. Schematic view of thermal barrier

Figure 2 shows the barrier in a longitudinal, section and cross-section view. There are a couple details to observe, first, the logger's chamber size is not the same as barrier's length. Second, the pipe that connect the reservoir's chamber to the external, it means that does not have considerable increase of pressure into the reservoir's chamber, besides it makes possible the water steam leaves the reservoir's chamber.

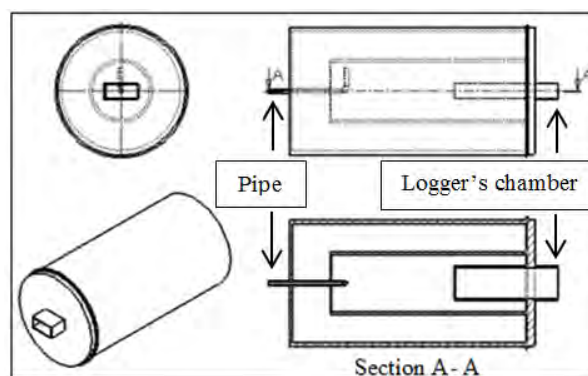


Figure 2. Longitudinal and section view

1.2 Goals

During the process of obtain the material's thermal profile there is a question that is the inspiration of the paper. The question is: How long the temperature logger can be into the industrial furnace? It is the main goal of this paper.

This paper brings a mathematical model capable to answer how long the temperature logger can be into the industrial furnace. In our case of thermal barrier the availability time is about 305 (three hundred and five) minutes or 5h: 5min (five hours and five minutes).

There are two specific goals based on the mathematical model. First, optimize the barrier's radius and length. Second, analyze the barrier's availability time with respect to the one of the material's properties, the thermal conductivity.

2. METHODOLOGY

The mathematical model has been created by energy and mass balances in the control volume. The software EES – Engineering Equation Solver – has been used to solve the equations and get the material's thermal and physical properties.

Aim making the mathematical model simpler some assumptions has been done:

- Axisymmetric problem.
- One dimensional problem, radius direction.
- Exhaust gases flow reach the thermal barrier at a right angle.
- Factor view equal to one.
- Exhaust gases average velocity equal to 1 m/s.
- Furnace's wall temperatures are constant and equal to exhaust gases flow.
- Have been inserted on the software the both insulating's thermal conductivity from the manufacture table.
- The material's properties are updating every single interaction.

2.1. Mass Balance

When the process runs it starts with the water reservoir completely full. As long the thermal barrier is exposed to the furnace's chamber that vary from 500 up 900 °C as more water steam leaves the water reservoir. This situation has been well solved by the equations presented on the book Introduction to Fluid Mechanics, Fox *et al.* (2001). Equation (1), in the first term, shows the mass variation with respect to time and, in the second term, the mass rate that cross the control surface. Equation (2) associate the water steam rate that cross the control surface to the energy rate that reach the water into the reservoir.

$$\frac{\partial}{\partial t} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} d\vec{A} = 0 \quad (1)$$

$$q = \dot{m}_{eb} * h_{fg} \quad (2)$$

Where:

ρ : Specific mass, kg/m³

V : Volume, m³

V : Velocity, m/s

A : Area, m²

q : Energy rate that reaches to the reservoir, W

\dot{m}_{eb} : Water steam rate, kg/s

h_{fg} : Heat of vaporization, kJ/kg

2.2. Energy Balance

This section is based on the Incropera *et al.* (2011).

For the energy balance we have all the three ways of heat transfer, conduction, convection and radiation, besides the change of phase, ebullition, when the water becomes water steam and leaves the water reservoir. Heat conduction occurs by heat flux, on radius direction, that starts from the control surface to the water reservoir. Heat convection occurs by the exhaust gases that reach the thermal barrier at right angle and heat transfer by radiation occurs between the thermal barrier's external wall and furnace's internal wall.

For heat transfer by conduction we model by Fourier's law as show in the Eq. (3).

$$q = A * q_x'' = k * A * \frac{dT}{dx} \quad (3)$$

Where:

q : Energy rate, W

q_x'' : Energy flux, W/m²

k : Thermal conductivity, W/m/K

T : Temperature, K

x : Thickness, m

For heat transfer by forced convection we model by Newton's law of cooling as show in the Eq. (4). For forced convection is essential to evaluate the Reynolds, Prandtl and Nusselt numbers, all three numbers are dimensionless

numbers, by the Eq. (5), Eq. (6) and Eq. (7) respectively. It is necessary for evaluate the heat transfer coefficient as showing in the Eq (8).

$$q'' = h_{conv} * (T - T_{\infty}) \quad (4)$$

$$Re = \frac{\rho * V * D}{\mu} \quad (5)$$

$$Pr = \frac{\nu}{\alpha} \quad (6)$$

$$Nu = \frac{h_{conv} * D}{k} \quad (7)$$

$$Nu = 0,664 * Re^{\frac{1}{2}} * Pr^{\frac{1}{3}} \quad (8)$$

Where:

h_{conv} : Heat transfer coefficient by convective, W/m²/K

T_{∞} : Exhaust gases temperature, K

μ : Dynamic viscosity, kg/m/s.

ν : Kinematic viscosity, m²/s

α : Absorptivity, m²/s

D : Diameter, m

For heat transfer by radiation we model by Stefan Boltzmann's law rearranged in an appropriated format as showing in Eq. (9). Equation (10) evaluates the heat flux by radiation.

$$q'' = h_{rad} * (T^4 - T_{wall}^4) \quad (9)$$

$$h_{rad} = \epsilon * \sigma * (T + T_{wall}) * (T^2 + T_{wall}^2) \quad (10)$$

Where:

h_{rad} : Heat transfer coefficient by radiation, W/m²/K

ϵ : Emissivity.

σ : Stefan Boltzmann constant, W/m²/K⁴

T_{wall} : Furnace's wall temperature, K

For change of phase, ebullition, first it is rearranged in an appropriated format as showing in the Eq. (11). For evaluating the energy flux by ebullition we model by equation developed by Rohsenow, Eq. (12).

$$q'' = h_{ebu} * (T - T_{sat}) \quad (11)$$

$$q'' = \mu * h_{fg} \left[\frac{g * (\rho_l - \rho_v)}{\sigma_s} \right]^{\frac{1}{2}} * \left(\frac{C_{p,l} * (T - T_{sat})}{C_{s,f} * h_{fg} * Pr_l^n} \right)^3 \quad (12)$$

Where:

h_{ebu} : Heat transfer coefficient by change of phase, W/m²/K

T_{sat} : Saturation temperature of water, K

g : Gravity acceleration, m/s²

ρ_l : Specific mass of water, kg/m³

ρ_v : Specific mass of water steam, kg/m³

σ_s : Surface tension, N/m

$C_{p,l}$: Water specific heat, J/kg/K

$C_{s,f}$ and n : Coefficients for solid-liquid interface. For water – stainless steel interface $C_{s,f} = 0,0132$ and $n = 1$.

2.3. Equivalent electrical circuit

Usually, an equivalent electrical circuit is done with purpose of modeling a thermal situation. It makes easier the solution and the overview of the problem. The analogy or the equivalence of the method is that the heat flux is evaluated like current and the temperature at each surface is evaluated like tension at each node, Fig. 3 shows the analogy. The equation for evaluate the heat flux, equivalent to the current, follow as Eq. (13).

The thermal resistance for each way of heat transfer is showed at table 1. Operations with resistance follow unaltered.

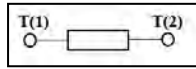


Figure 3. Equivalent electrical circuit

$$q = \frac{T_2 - T_1}{R} \tag{13}$$

Where:

R: Resistance, K/W

T₁: Temperature at node 1, K

T₂: Temperature at node 2, K

Table 1. Thermal resistance for heat transfer

	Equation	Unit
Conduction	$R = \frac{\ln\left(\frac{r_i}{r_o}\right)}{2 * \pi * L * k}$	K/W
Convection	$R = \frac{1}{A * h_{conv}}$	K/W
Radiation	$R = \frac{1}{A * h_{rad}}$	K/W
Ebullition	$R = \frac{1}{A * h_{ebu}}$	K/W

Where:

r_{int}: Internal radius, m

r_{ext}: External radius, m

L: Tube's length, m

For solve each node we have to follow the Kirchoff's law that implies that at any node in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.

Figure 4 show the result of the equivalent electrical circuit.

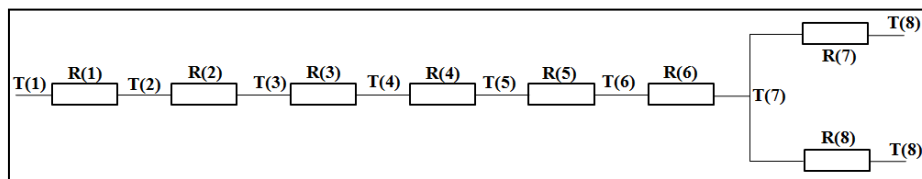


Figure 4. Equivalent electrical circuit of the mathematical model

Where:

R(1) – Resistance by ebullition, K/W

R(2) – Resistance by conduction, part of barrier, K/W

R(3) – Resistance by conduction, part of insulation 1, K/W

R(4) – Resistance by conduction, part of structural tube 1, K/W

R(5) – Resistance by conduction, part of insulation 2, K/W

R(6) – Resistance by conduction, part of tube, K/W

R(7) – Resistance by convection, exhaust gases flow with tube's surface, K/W

R(8) – Resistance by radiation, furnace's wall with tube's surface, K/W

T(1) – Logger's temperature, K

T(2) – Water's temperature, K

T(3) – Barrier's temperature, K

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- T(4) – Insulation 1's temperature, K
- T(5) – Structural tube's temperature, K
- T(6) – Insulation 2's temperature, K
- T(7) – Tube's external Temperature, K
- T(8) – Furnace's chamber temperature, K

2.4. Control Volume

Figure 5 presents a schematic control volume. It is possible to observe the energy and mass flux that cross the control surface.

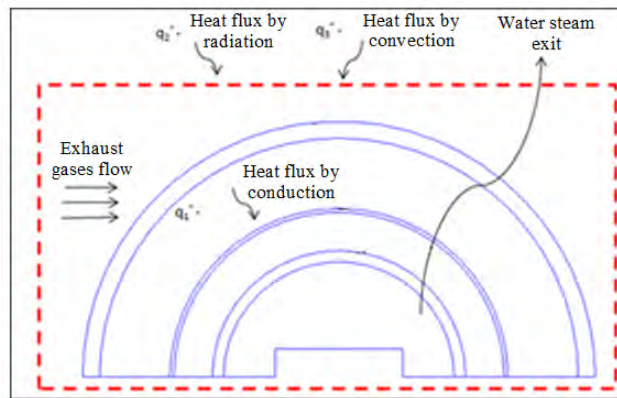


Figure 5. Control volume of the mathematical model

All these equations were been written as a command line for the software EES. Figure 6 shows the main screen of the program. To operate the program we need some information that depend of the material's instruction such as maximum temperature, soaking time, set point temperature for the four zones and the time that the material have been exposed into each one of the four zones.

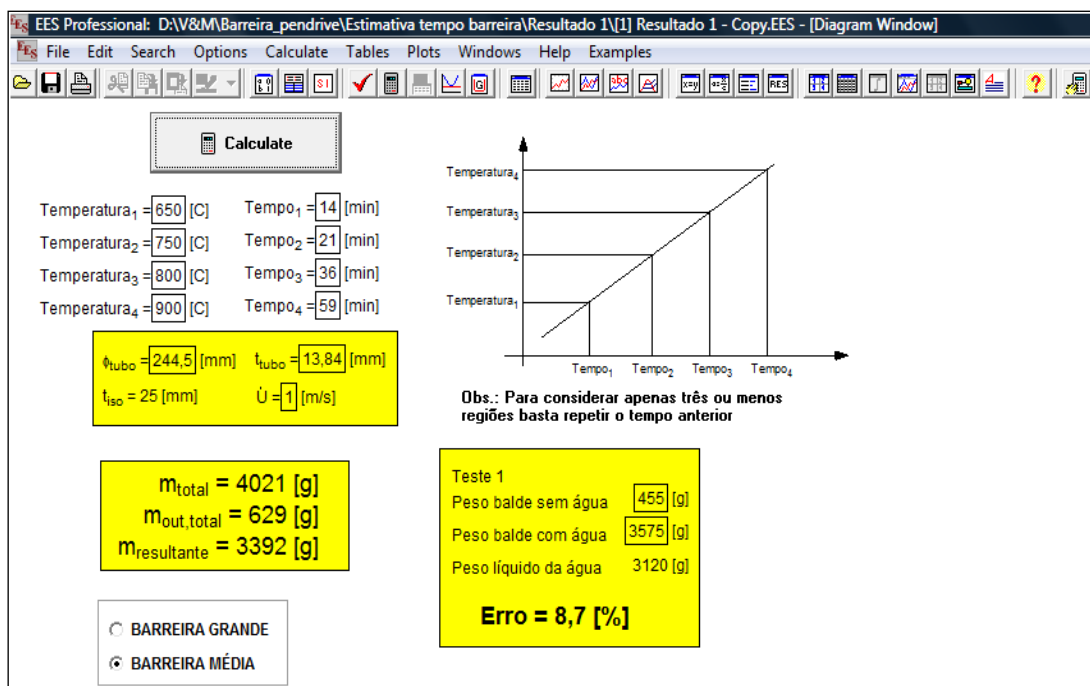


Figure 6. Main screen of the EES

2.5. Experimental

The experimental or the process to obtain the material's thermal profile is based on place a group of thermocouple to the material (the object of study of this paper is a tube), those thermocouple are connected to the temperature logger. The logger is placed into a thermal protection apparatus (thermal barrier) and inserted into the material as shown in Fig. 7, then both, material and logger, are heated until the soaking temperature and keep on this temperature during the soaking time. We evaluated our model by comparing the model's results and the experiment's results. The parameter to evaluate is the quantity of water remaining in the water reservoir.

The mathematical model will be evaluated to a Vallourec do Brasil's thermal barrier and then its parameter as barrier's radius and insulation 2 will be variation with respect to availability time.



Figure 7. Placing the thermal barrier into the material (tube)

3. RESULTS

The results consist in compare the mathematical model's results with experiment's results and then provide improvements by optimizing the parameter, barrier's radius.

3.1. Validation of the mathematical model

Aim validate the mathematical model, table 2 shows the comparative table between an experiment's results and the mathematical model's results.

Table 2. Validation results.

-	Experimental Remaining water in the reservoir (g)	Model Remaining water in the reservoir (g)	Error (%)
Result 1	3.120,0	3.392,0	8,7
Result 2	3.369,0	2.987,0	11,3
Result 3	3.347,0	3.360,0	0,4
Average	3.278,7	3.246,3	6,6
Standard deviation	137,8	225,2	5,6
Indication result	$3278,7 \pm 137,8$	$3246,3 \pm 225,2$	$6,6 \pm 5,6$

3.2. Barrier's radius

Figure 7 shows the behavior of the thermal protection apparatus with respect to variation of the barrier's radius. All the others parameters have no changes except the insulation's thickness. Because, when increase the barrier's radius it means that the insulation's thickness has to reduce and vice versa.

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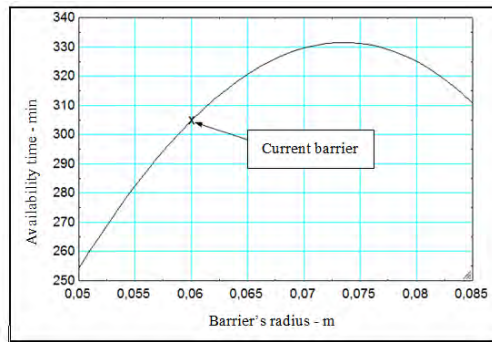


Figure 8. Behavior of the thermal protection apparatus with respect to variation of the barrier's radius

Figure 8 shows that the barrier's radius optimal value is 74 mm. Thus the insulating 2's thickness (external insulation) is 12 mm.

3.3. Barrier's length

Figure 9 shows the behavior of the thermal protection apparatus with respect to the barrier's length. The barrier's radius keeps the original value.

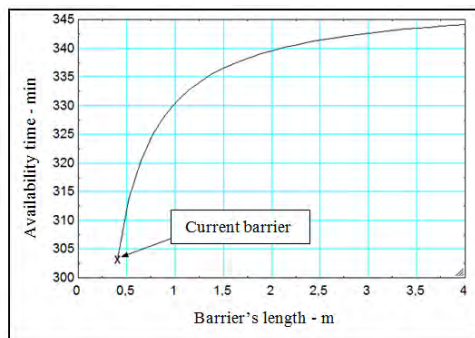


Figure 9. Behavior of the thermal protection apparatus with respect to the barrier's length

The longer is the barrier's length the longer will be the availability time for the temperature logger. Therefore, one of the limiters of the barrier's length is how difficult is to place the thermal barrier into the material (tube), as show on Fig. 7.

3.4. Thermal propriety analysis

Figure 10 shows the behavior of the thermal protection apparatus with respect to the insulating's thermal conductivity. The values used to create the Fig. 9 were considering the thermal conductivity independent of temperature and equal a constant. The model shows that the thermal conductivity is an important parameter, since the variation of this property results in significant changes in the availability time.

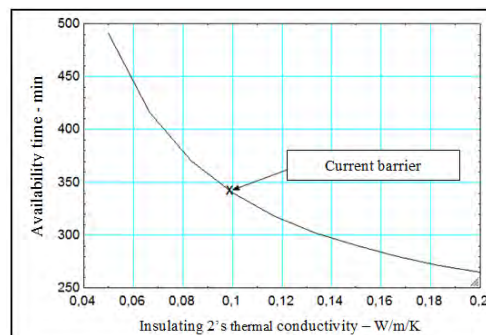


Figure 10. Behavior of the thermal protection apparatus with respect to the insulating 2's thermal conductivity

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4. CONCLUSION

For the variation of the availability time with respect to the thermal conductivity has showed the most important factor for increasing the availability time. Obtain, get or discovering a new material with lower thermal conductivity has showed the best way to improve the thermal protection apparatus.

The mathematical model has been validated by the experimental results. The percentage error between them is $6,6 \pm 5,6$. Based on this result the mathematical model is trustful. Thus the mathematical model is able to identify the barrier's availability time into the industrial furnace.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

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7. RESPONSIBILITY NOTICE

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