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ANALYSIS OF UNBALANCES FORCES USING METHODS OF IDENTIFICATION AND FINITE ELEMENTS

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Abstract. *The study of damage in rotating machineries is of fundamental interest in the field of machine and structure design. A rotating system supported by bearings and under some dynamic conditions can generate a variety of problems that are encountered in many different types of rotating machines. One of these problems is the unbalance due to non-homogeneous mass distribution along the shaft. Although several methods for the identification of unbalance excitation force are available in the literature, none of them can be considered unrestricted to be applied for all rotating systems. In this study, two methodologies to identify unknown excitations, such as unbalance, have been proposed. This method is based on Fourier series and Legendre polynomials. In order to investigate the effectiveness of this methodology, a mass-spring-damper system with three degrees of freedom (DOF), and a rotating system considering its foundations effects have been simulated and investigated. Moreover, experimental measurements were carried out to show the agreement between the theory and practice.*

Keywords: *Unbalances Forces, Finite Element, Fourier Series, Rotate Machines.*

1. INTRODUCTION

Most industries operate with rotary machines, being of great interest to monitor the same work safely and not occur abrupt stops. Unbalance, misalignment and bearing failure in bearings are primarily responsible for vibration equipment, such that depending on the vibration intensity can cause damage. One solution to avoid such problems is the constant monitoring of machines so if any abnormality, it can intervene even before more serious damage occurs. One of the techniques that are now widespread, is the identification of parameters and excitation forces (COOPER; DESFORGES, 1996), (KUDVA; VISWANADHAM; RAMAKRISHNA, 1980), (GERTLER, 1988), (MELO, 1998), which seeks to determine the unknown values for handling the input and output of the system with the help of state variables.

Several methods were developed for the identification of parameters of dynamic systems using orthogonal functions in recent years (TRENDAFILOVA et al. 2000), although none of them can be considered universally suitable for all situations (RICH; HEYDT, 2000). Knowing the strength of excitation systems, can be accompanied by monitoring and identification techniques the evolution of possible variations of these parameters. For the development of these methods, there is the need for construction of mathematical models able to represent the mechanical behavior of various types of systems (MORAIS, 2006).

Several orthogonal functions are applied to the identification of unknown parameters, Walsh, Block-Pulse Fourier Chebyshev polynomial Jacobi and Legendre (CHEN; PATTON, 1996) (CHEN; HSIAO, 1975) (CHEN; TSAY; WU 1977) (CHOW; WILLSKY, 1984). In this study we applied the techniques of identifying parameters through the Fourier series and Legendre polynomials in rotating systems with several degrees of freedom, where such techniques can use any kind of response on time, and in relation to displacement and velocity.

2. IDENTIFICATION OF FORCES BY ORTHOGONAL FUNCTIONS

If the system is subjected to mechanical excitation forces is possible to identify arrays of structural parameters, allowing the model generation system (SANTOS, 2004).

The Equation of motion of a mechanical system subjected excitation degrees of freedom is given by Equation below:

$$[M]\{\ddot{x}(t)\}+\left([C]+[G]\omega\right)\{\dot{x}(t)\}+[K]\{x(t)\}=\{F\} \tag{1}$$

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in which:

$[M]$, $[C]$, $[K]$ e $[G]$: global matrices of mass, damping, stiffness and gyroscopic effect, respectively;

$\{F\}$: vector that contains external excitation unbalanced forces;

ω : engine rotate;

$\{\ddot{x}\},\{\dot{x}\},\{x\}$: acceleration, velocity and displacement vector.

After some mathematical manipulations applying property for integration of orthogonal basis was obtained by following Equation.

$$\begin{bmatrix}[M] & -[M]\{x(0)\} & \{-[M]\{\dot{x}(0)\}-[C+G(\omega)]\{x(0)\}\} & [C+G(\omega)] & [K]\end{bmatrix}\begin{bmatrix}[X]\\ \{e\}^T\\ \{e\}^T[P]\\ [X][P]\\ [X][P]^2\end{bmatrix}=[F][P]^2 \tag{2}$$

in wich:

$[X]$: displacement matrix with the displacement of the all elements;

$\{x(0)\}$: initial displacement vector;

$\{\dot{x}(0)\}$: initial velocity vector;

$[P]$: integration matrix of the each method;

Equation (2) can be named by Equation below:

$$[E]=[H][J] \tag{3}$$

in which their portion are given by following Equations:

$$[H]=\begin{bmatrix}[M] & -[M]\{x(0)\} & \{-[M]\{\dot{x}(0)\}-[C+G(\omega)]\{x(0)\}\} & [C+G(\omega)] & [K]\end{bmatrix} \tag{4}$$

$$[J]=\begin{bmatrix}[X]\\ \{e\}^T\\ \{e\}^T[P]\\ [X][P]\\ [X][P]^2\end{bmatrix} \tag{5}$$

$$[E]=[F][P]^2 \tag{6}$$

So the force coefficients are calculated isolating the Equation (6), how showed below:

$$[F]=[E][P]^{-2}=[H][J][P]^{-2} \tag{7}$$

The Fourier Integration matrix is given below, with dimension $(r=2s+1)$, being s the number of sine terms:

$$[P]=\begin{bmatrix}\frac{T}{2} & \{0\}_{1xs} & -\frac{T}{\pi}\{\tilde{e}\}_s^T\\ \{0\}_{sx1} & [0]_{sxs} & \frac{T}{2\pi}[\tilde{I}]_{sxs}\\ \frac{T}{2\pi}\{\tilde{e}\}_s & -\frac{T}{2\pi}[\tilde{I}]_{sxs} & [0]_{sxs}\end{bmatrix}_{rxr} \tag{8}$$

where:

T: difference between final time and initial time

$$\{e\}_s = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{3} \\ \vdots \\ \frac{1}{s} \end{bmatrix} \tag{9}$$

$$[I]_{sxs} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{s} \end{bmatrix} \tag{10}$$

The Legendre Integration matrix is given by follow Equation, with dimension r , being r the number of terms:

$$[P] = \frac{t_f}{2}\begin{bmatrix} 1 & 1 & 0 & 0 & \ldots & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 & \ldots & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & \frac{-1}{2r-3} & 0 & \frac{1}{2r-3} \\ 0 & 0 & \ldots & 0 & \frac{-1}{2r-1} & 0 \end{bmatrix} \tag{11}$$

3. MATHEMATICAL MODELING SYSTEMS ROTARY

In this methodology, there is a division of the continuous system in a finite number of elements, which are connected by nodes. Using the standard finite element formulation, the nodal displacement vector for an element is defined by following Equation and showed, which include the displacements δu and δw, corresponding to the motion on directions X and Z respectively, and showed by Figure 1.

$$\delta = [u_1, w_1, \theta_1, \psi_1, u_2, w_2, \theta_2, \psi_2]' \tag{12}$$

Figure 1. Degrees of freedom of the model F.E.M

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The each element matrix of $\left[M_{ET}\right],\left[M_{ER}\right],\left[G_E\right]$ e $\left[K_E\right]$ are given by following Equations (NELSON; MCVAUGH, 1976).

$$\left[M_{ET}\right]=\frac{\rho AL}{420}\begin{bmatrix} 156 & 0 & 0 & -22L & 54 & 0 & 0 & 13L \\ 0 & 156 & 22L & 0 & 0 & 54 & -13L & 0 \\ 0 & 22L & 4L^2 & 0 & 0 & 13L & -3L^2 & 0 \\ -22L & 0 & 0 & 4L^2 & -13L & 0 & 0 & -3L^2 \\ 54 & 0 & 0 & -13L & 156 & 0 & 0 & 22L \\ 0 & 54 & 13L & 0 & 0 & 156 & -22L & 0 \\ 0 & -13L & -3L^2 & 0 & 0 & -22L & 4L^2 & 0 \\ 13L & 0 & 0 & -3L^2 & 22L & 0 & 0 & 4L^2 \end{bmatrix} \tag{13}$$

$$\left[M_{ER}\right]=\frac{\rho d^2}{480L}\begin{bmatrix} 36 & 0 & 0 & -3L & -36 & 0 & 0 & -3L \\ 0 & 36 & 3L & 0 & 0 & -36 & 3L & 0 \\ 0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\ -3L & 0 & 0 & 4L^2 & 3L & 0 & 0 & -L^2 \\ -36 & 0 & 0 & 3L & 36 & 0 & 0 & 3L \\ 0 & -36 & -3L & 0 & 0 & 36 & -3L & 0 \\ 0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0 \\ -3L & 0 & 0 & -L^2 & 3L & 0 & 0 & 4L^2 \end{bmatrix} \tag{14}$$

$$\left[G_E\right]=\frac{\rho Ad^2}{240L}\begin{bmatrix} 0 & -36 & -3L & 0 & 0 & 36 & 3L & 0 \\ 36 & 0 & 0 & -3L & -36 & 0 & 0 & -3L \\ 3L & 0 & 0 & -4L^2 & -3L & 0 & 0 & L^2 \\ 0 & 3L & 4L^2 & 0 & 0 & -3L & -L^2 & 0 \\ 0 & 36 & 3L & 0 & 0 & -36 & -3L & 0 \\ -36 & 0 & 0 & 3L & 36 & 0 & 0 & 3L \\ 3L & 0 & 0 & L^2 & -3L & 0 & 0 & -4L^2 \\ 0 & 3L & -L^2 & 0 & 0 & -3L & 4L^2 & 0 \end{bmatrix} \tag{15}$$

$$\left[K_E\right]=\frac{EI}{L^3}\begin{bmatrix} 12 & 0 & 0 & -6L & -12 & 0 & 0 & -6L \\ 0 & 12 & 6L & 0 & 0 & -12 & 6L & 0 \\ 0 & 6L & 4L^2 & 0 & 0 & -6L & 2L & 0 \\ -6L & 0 & 0 & 4L^2 & 6L & 0 & 0 & 2L^2 \\ -12 & 0 & 0 & 6L & 12 & 0 & 0 & 6L \\ 0 & -12 & -6L & 0 & 0 & 12 & -6L & 0 \\ 0 & 6L & 2L^2 & 0 & 0 & -6L & 4L^2 & 0 \\ -6L & 0 & 0 & 2L^2 & 6L & 0 & 0 & 4L^2 \end{bmatrix} \tag{16}$$

being:

- $\left[M_{ET}\right]$: mass matrix of each element;
- $\left[M_{ER}\right]$: mass matrix due to the effect of the rotational axis of each element;
- $\left[G_E\right]$: gyroscopic matrix of each element;
- $\left[K_E\right]$: stiffness matrix of each element;
- L : length of the element;
- A: cross-sectional area of the element;
- ρ : density of the material;
- d : diameter of the element;
- E : modulus of elasticity of the material;

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4. RESULTS

A mechanical rotating rotor-bearing nine degrees of freedom of a structure of concentrated parameters whose physical model is shown in Figure 2.

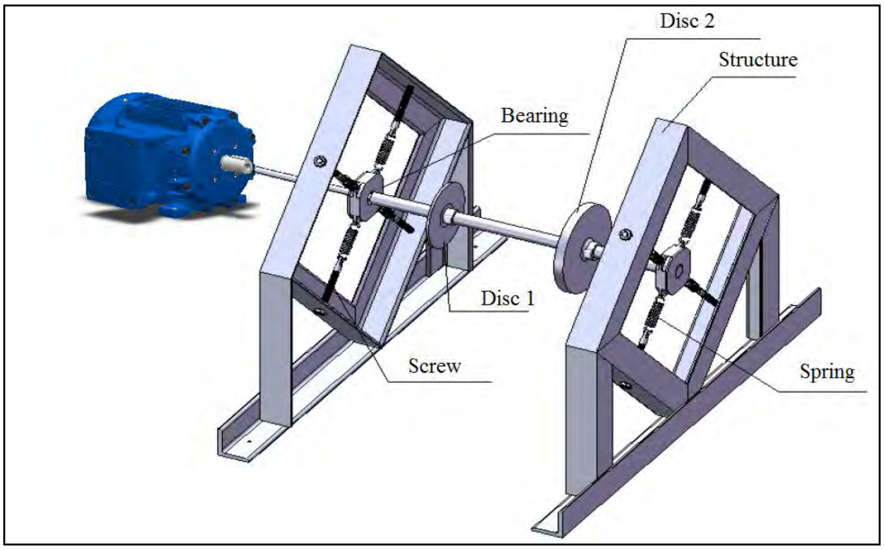

Figure 2. Solidworks model of the experiment used.

It was discretized to research a model of rotating shaft that has two disks in the rotor unbalance and two bearings at its ends. The analysis was performed only in a plane perpendicular to the rotor (XZ), and the axis is represented by eight elements represented by Figure 3.

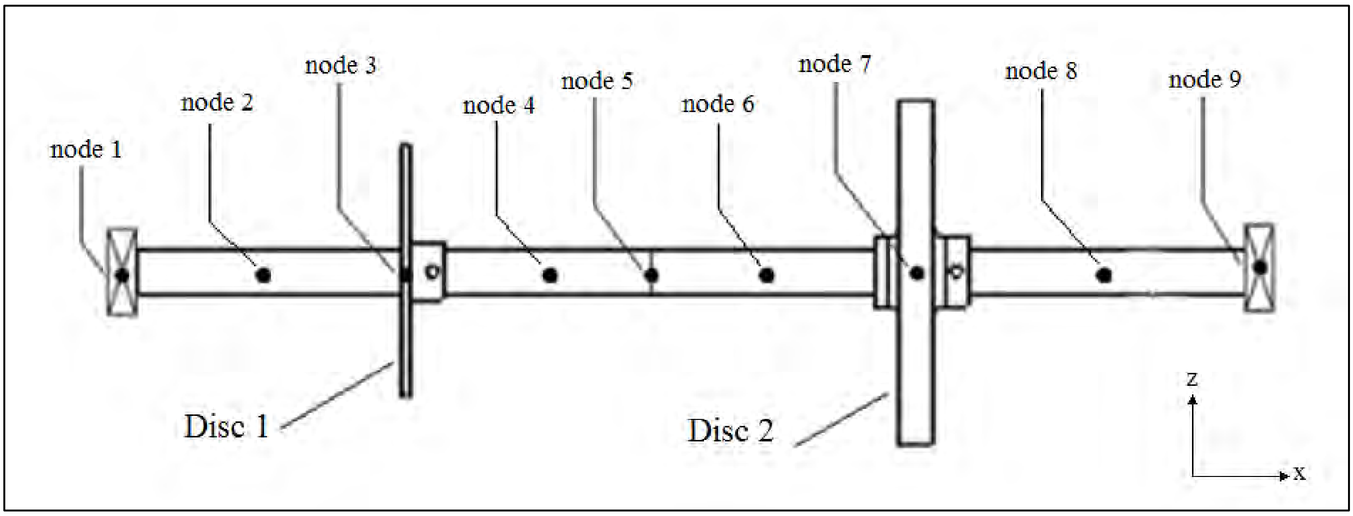

Figure 3. Modeling rotor in finite elements

Constituted a fundamental part of the design, construction, assembly and characterization of a workbench designed for the analysis of unbalanced forces present in a set-axis rotor with proper operation of the system data acquisition, signal conditioners, drivers and fixtures sensors, as shown in Figure 4.

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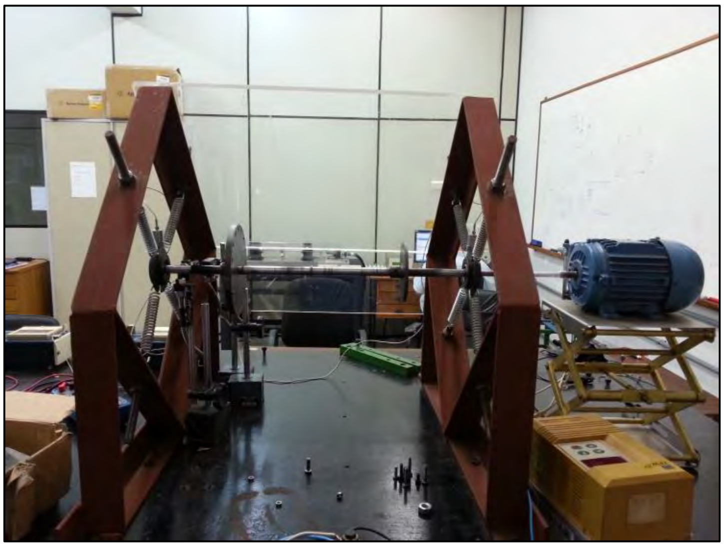

Figure 4. Mounting the test bench

The stiffness K1 and K2 and the damping C1 and C2 of the bearings were obtained through an optimization function using the Quasi-Newton method, based on an input impulse in the system. Was constructed an objective function in which the input values are the parameters to be determined (stiffness and damping) and the output values is the error of the experimental system in relation to the simulated system. Table 1 shows the parameters for the discretization as well as the properties of other elements (disc, bearings and shaft).

Table 1. Parameters of the Experiment

Density of the shaft material	ρ=7900 kg/m^3
Modulus of elasticity	E = 205 GPa
Poisson's ratio	ν = 0.3
Diameter of the elements 1,2	$D_{1,2}$ = 0.0145 m
Length of elements 1,2	$L_{1,2}$ = 0.065 m
Diameter of the elements 3,4,5,6	$D_{3,4,5,6}$ = 0.0145 m
Length of elements 3,4	$L_{3,4}$ = 0.06m
Length of elements 5,6	$L_{5,6}$ = 0.0755m
Diameter of the elements 7,8	$D_{7,8}$ = 0.01395 m
Length of elements 7,8	$L_{7,8}$ = 0.0565m
Diameter of the element 9	D_9 = 0.01085 m
Length of elements 9	L_9 = 0.093m
Mass of Disc 1	M_{d1} = 0.287 kg
Mass of Disc 2	M_{d2} = 1.9 kg
Mass of bearing 1	M_{m1} = 0.4 kg
Mass of bearing 2	M_{m2} = 0.43 kg
Inertia of Disc 1	I_y = 0.004843 kg.m^2 $I_x = I_z$ = 0.002495 kg.m^2
Inertia of Disc 2	I_y = 0.0004801 kg.m^2 $I_x = I_z$ = 0.0003411 kg.m^2

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The system is coupled to an electric motor, rotating at a constant speed of 900 *rpm* (15 *Hz* ou 94.25 *rad/s*). Initially, was carried out rotor balancing in two planes. Was then added to the disc 2 a mass of about 0.006 *kg* at a distance from the center of 0.0065 *m*, creating an unbalanced force on the system known from approximately 3.48 *N*, calculated by the Equation below:

$$F_z = mr\omega^2 sen(\omega t) = 0.006*0.0065*94.25sen(94.25t) = 3.464sen(99.25t)\ N \quad (17)$$

$$F_x = mr\omega^2 cos(\omega t) = 0.006*0.0065*94.25cos(94.25t) = 3.464cos(99.25t)\ N \quad (18)$$

in which:

m: unbalanced mass *[kg]*
r: distance between the center of the shaft and the hole to cause the disc unbalance *[m]*
ω: angular velocity in radians per second *[rad/s]*
F_x: unbalance force on the direction x *[N]*
F_z: unbalance force on the direction z *[N]*

Due to the presence of many noises, the speed signal was processed through a band-pass filter that filtered the frequencies between 13 and 17 *Hz*.

Thus, with the matrices global mass, damper (included the matrix due to the gyroscopic effect and the matrix due to structural damping axis) and stiffness matrix were obtained the dynamic matrix of the system determining the modal parameters of the system resolving the eigenvalues and eigenvectors (the natural frequencies factors, damping and mode shapes), also was able obtain the response time (displacement and velocity) considering an input signal u (t), defined by the user.

With the data obtained of the simulation by state space, displacement and velocity, was possible to identify the unbalance force on disc 2, by the Equation (7), by Fourier method and Legendre method, presented in section 2 of this work, and compare them with the real force on disc 2, according the Equation (18) and Equation (19). To display the *rms* value of the real forces and identified forces are shown in the table below and are plotted in Figure 5.

Table 2. Rms values of the forces real and identified force

	Rms value in the x direction	**Rms value in the z direction**
Real force experimental	2.041460993629443	2.041460993629443
Force identified by Fourier	2.041297469210325	2.119438085990136
Force identified by Legendre	2.043895315583009	2.412447565583048

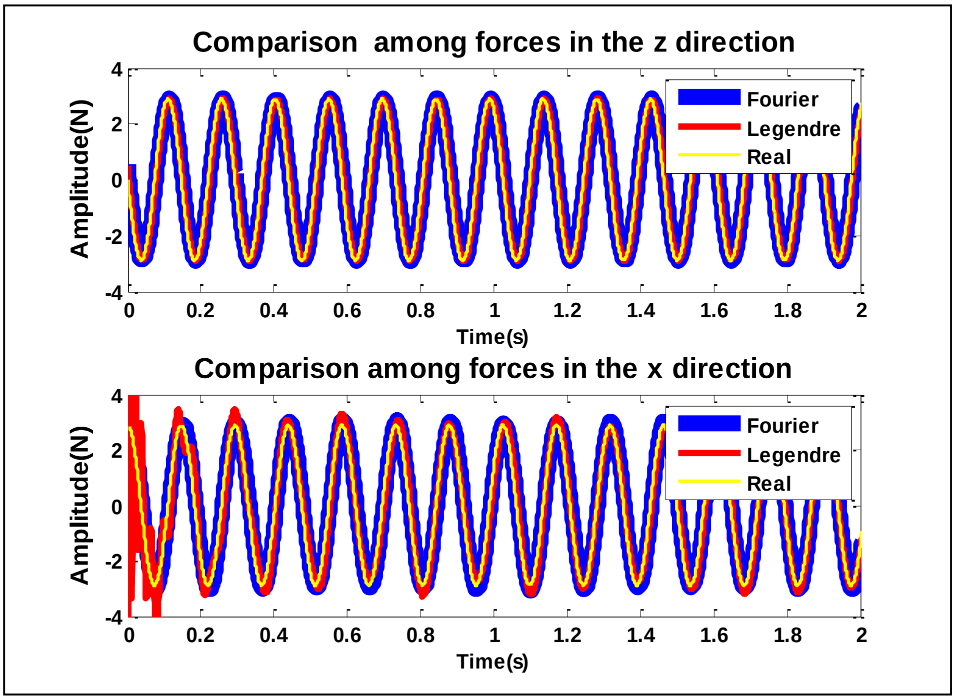

Figure 5. Comparison of the real force and identified force

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5. FINAL REMARKS

This research project focused on one of the identification techniques unbalance forces operating in the time domain. Studied, in particular, the method based on orthogonal functions in terms of Fourier series and Legendre polynomials.

The study involved the development of the formulations of the method for two types of response function, displacement and velocity, being in this work have been presented only for the displacement. For experimental validation, shown in item 4 of this work and tests were conducted at the Laboratory of Vibrations of Mechanical Engineering Department of the Faculdade de Engenharia de Ilha Solteira.

The real power system represented by Equations (18) and (19) and the yellow curve plotted in Figure 5, were compared with forces from each identified method, it is possible to identify 36 words from the series Fourier, blue curve, and 120 terms of Legendre polynomials, red curve.

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