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COMPARISONS BETWEEN TRADITIONAL SAFETY-FACTOR DESIGNS AND RELIABILITY-BASED DESIGNS USING HEURISTIC OPTIMIZATION

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Abstract. *Reliability may be interpreted as the probability that a structure will endure performing its function as it was designed. In Reliability-Based Design Optimizations (RBDO) one should take into account the randomness and uncertainties of the variables of the problem, such as material proprieties, loads, modeling errors, etc. Usually, from the engineering point of view, these uncertainties are unknown and some safety factors may be used to simplify the problem moving away the design to a safe region. In fact, this approach neither predicts how far is the design from hazardous situations nor how much cost has been added to it. In this paper, a simple comparison is made on both optimization approaches using structural examples. The particle swarm optimization (PSO) is the heuristic method used as optimization engine. The reliability study is limited to the First Order Reliability Method (FORM) and to RIA (Reliability Index Approach). However the conclusions can be extended to other reliability methods, since the RBDO approach can be treated as a decoupled double-loop optimization: an outer structural optimization problem and an inner reliability analysis. Computational costs and accuracy issues are also commented.*

Keywords: *reliability, optimization, PSO, FORM*

1. INTRODUCTION

It is known that real life problems in many different fields have some degree of uncertainty in its design variables or parameters. This variability should be taken into account somehow when systems are built, but that quantification is a hard problem to be defined. Usually designers and standard codes add safety factors to give some confidence for their design. In practical optimization studies, reliability based techniques are becoming popular, due its capability to deal with that uncertainty.

In deterministic optimizations, it is usual to find the global optimum in one or more constraints intersections. If there are some random variables in the problem with a probabilistic distribution, the solution might be unfeasible. In reliability-based design optimization (RBDO), a cost-effective solution can be reached ensuring a target confidence (reliability index).

On structural reliability theory, different methods have being developed in the last decades. The First Order Reliability Method (FORM), is the one used in this study. This method and its variations are largely used in projects giving its precision and computational cost when compared to other methods, for instance Monte Carlo Simulation. Complementarily, the Reliability Index Approach (RIA), that describes the probabilistic constraint as a reliability index, will be adopted as well.

In respect of the optimization engine, the particle swarm optimization (PSO) was the chosen one in this paper. The PSO is a heuristic method based on populations where the individuals are called particles. The particle swarm is modeled as individuals in a multidimensional space with a position and velocity. The particles best known positions are updated to all particles and it is expected that all of them move toward the optimal solution.

In the last few years some papers have started to develop the RBDO with heuristic techniques, such as Elhami *et al.*, 2011, or Deb *et al.*, 2007. There are also studies (Youn and Choi, 2004), comparing different probabilistic design approaches, like the RIA used in this study and the performance measure approach (PMA). In the case of the well-known safety factor, there are guidelines proposed by Ullman (2009), followed in this paper.

Trusses are examples widely used in structural optimizations problems to benchmark and compare results. In this paper, the classical 10-bar truss and the 25-bar truss are used and described. As a third example, it is compared the results on the design of a coil spring from Arora (2004).

2. THE SWARM OPTIMIZATION ALGORITHM

The particle swarm optimization (PSO) has been inspired by the observation of social behavior of beings, such as fish schooling, insects swarming and birds flocking. This method has been used to search global optimum in a wide

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variety of engineering problems. It was introduced by Kennedy and Everhart (1995). The basis for the method relies on the social influence and social learning which enable persons to maintain cognitive consistency, so the exchange of ideas and interactions between individuals may lead them to solve problems. The particle swarm simulates this social plot. As stated by Li *et al.(*2007), it involves a number of particles, which have a defined position and velocity, and they are initialized randomly in a multidimensional search space of a cost function. Each particle represents a potential solution for the problem and the measure of this potentiality is its cost function. The set of particles is generally referred as "swarm". These particles "fly" through the multidimensional space and have two essential reasoning capabilities: the memory of their own best position and knowledge of the global or their neighborhoods best position.

In a minimization problem, "best" simply means the position of the particle ($\mathbf{x}_i$) with the smallest objective value (minimum of ($\mathbf{x}_i$)**)**. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on this information of good neighborhood positions. Thus, related to each particle, there is a set of design variables ($\mathbf{x}_i$) and the respective velocities ($\mathbf{v}_i$) that represent the potential solution of the optimization problem. The basic swarm parameters position and velocity are updated during iterations by Eq. (1) and Eq. (2):

$$v_{i,j}^{k+1} = \chi\left[\varpi v_i^{k+1} + \lambda_1 r_1\left(xlbest_{i,j}^k - x_{i,j}^k\right) + \lambda_2 r_2\left(xgbest_j^k - x_{i,j}^k\right)\right] \quad (1)$$

$$x_{i,j}^{k+1} = x_{i,j}^k + v_{i,j}^{k+1} \quad (2)$$

where ϖ is the velocity inertia weight (previously set between *0* and *1*, in this paper it was set as 0,8), $x_{i,j}^k$ is the current value (k) of design variable j and particle i, $v_{i,j}^k$ is the updated velocity of design variable j and particle i, $xlbest_{i,j}^k$ is the best design variable j ever found by particle i, $xgbest_j^k$ is the best design variable j ever found by the swarm, r_1 and r_2 are uniform random numbers in the range $[0,1]$, λ_1 means the cognitive component (particle self-confidence), λ_2 means the social component (swarm confidence). It is usually set $\lambda_1=\lambda_2=2.0$. Both constants indicate how each particle is directed towards good positions taking into account personal and global best information.

The role of the inertia weight ϖ is crucial for the PSO convergence. It is employed in order to control the impact of previous velocities on the current particle velocity. A general rule of thumb indicates to set a large value initially to make the algorithm explore the search space and then gradually reduce it in order to get refined solutions. In this paper, it was initially set at $\varpi = 0.8$ and updated based on the coefficient of variation (CV=σ/μ) of the swarm cost function according to $\varpi=0.4[1+min\ (\text{CV}, 0.6)]$. The χ parameter is used to avoid divergence behavior in the algorithm, and it was proposed by Bergh and Engelbrecht (2006).

3. RELIABILITY BASED DESIGN OPTIMISATION PROBLEM

3.1 Deterministic Design Optimization

In deterministic Based Design Optimization, safety factors are previously assigned to the uncertain parameters depending on its role in the design equations (safety factors greater than 1 if Load and less than 1 if Resistance), then the optimization is carried out using any of the optimization method appropriate to the problem.

$$\begin{aligned} &Find\ \mathbf{d}\ to \min f(\mathbf{d}) \\ &subject\ to\ \mathbf{d}^L < \mathbf{d} < \mathbf{d}^U \quad with\ \mathbf{d} = \{\mathbf{d}_1, \dots, \mathbf{d}_2\}^T \\ &\qquad g_i(\mathbf{d}) < 0 \qquad with\ i = 1, \dots, m \\ &\qquad h_j(\mathbf{d}) = 0 \qquad with\ j = 1, \dots, o \end{aligned} \quad (3)$$

where **d** is the deterministic design variables vector, g are the inequalities constraints, h are the equality constraints. Safety factors can be added multiplying the **d** vector, as well as the constraint functions.

3.2 Reliability Based Design Optimisation

The Reliability Based Design Optimization generally can be stated as:

$$\begin{aligned} &Find\ \mathbf{u}, \mathbf{d}\ to \min f(\mathbf{u}, \mathbf{d}) \\ &subject\ to\ \mathbf{d}^L < \mathbf{d} < \mathbf{d}^U \\ &\qquad g_i(\mathbf{u}, \mathbf{d}) < 0 \qquad with\ i = 1, \dots, m \\ &\qquad h_j(\mathbf{u}, \mathbf{d}) = 0 \qquad with\ j = 1, \dots, m \\ &\qquad P(g_k(\mathbf{u}, \mathbf{d}) > 0) < P_f^{target} \quad with\ k = 1, \dots, p \end{aligned} \quad (4)$$

where P means probability, **d** is the deterministic vector of design variables, **u** is the probabilistic vector of random variables. In a general sense, both vectors can be optimized concurrently. P_f^{target} is a target probability of failure for a

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specific mode of failure that can be one from the deterministic constraints or other diverse from this set. Figure 1 shows an illustrative sketch of what a reliability optimization might look like.

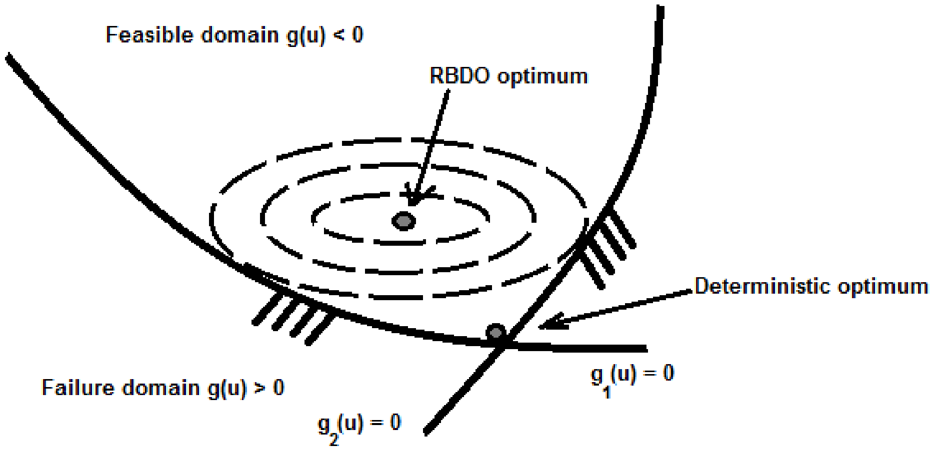

Figure 1. Sketch of RBDO problem.

3.3 FORM

Within FORM framework, the random variables **u** are transformed in standard normal and independent variables, **x**. The most probable point of failure (MPP) is calculated by finding the nearest distance (β) between the linearized failure surface, g(**x**) = 0, and the origin. Figure 2 exemplifies the FORM method in a two-dimensional case.

The failure probability can be calculated by Eq. (5)

$$P_f = P(g(\boldsymbol{x}) < 0) = \int_{g(x)<0} f(\boldsymbol{x}) d\boldsymbol{x} \cong \Phi(-\beta) \tag{5}$$

where Φ is the standard normal cumulative distribution function (CDF).

As can be observed, there is a minimization problem with one equality constraint in order to define the MPP

Minimize ||**x**||, subject to g(**x**) = 0 (6)

One of the most used algorithms for structural reliability analysis to find ||**x**|| is that created by Hasofer and Lind (1974) and improved by Rackwitz and Fiessler (1978). Eq (7) defines the used formulation:

$$x^{k+1} = \frac{1}{\|\nabla g(x^k)\|^2} [\nabla g(x^k)^{\mathrm{T}} x^k - g(x^k)] \nabla g(x^k) \tag{7}$$

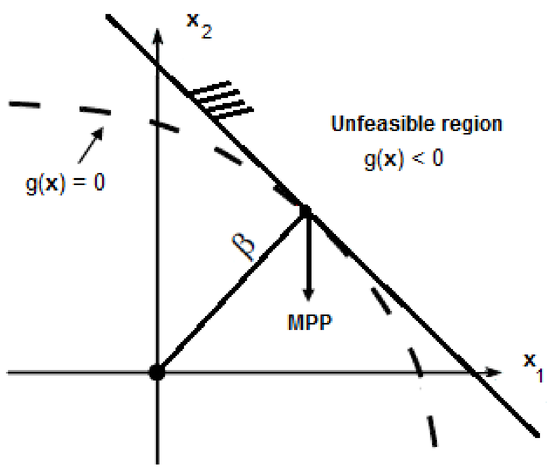

Figure 2. The FORM approach.

3.4 Probabilistic Design Approaches

Generally speaking, the Reliability Based Design Optimization problem can be treated by two different ways:

3.4.1 Reliability Index Approach (RIA)

In this approach, the probabilistic constraint is treated as an extra constraint formulated in the uncorrelated Gaussian space given by the reliability index β. So, the following can be written:

$$\begin{aligned}&Find\ \mathbf{d}\ to\ \min f(\mathbf{u},\mathbf{d})\\&subject\ to\ \mathbf{d}^L < \mathbf{d} < \mathbf{d}^U\\&\qquad g_i(\mathrm{E}[\mathbf{u},\mathbf{d}]) < 0 \qquad with\ i = 1,\dots,m\\&\qquad h_j(\mathrm{E}[\mathbf{u},\mathbf{d}]) = 0 \qquad with\ j = 1,\dots,n\\&\qquad \beta_{target} - \beta(g_k(\mathbf{u},\mathbf{d}) = 0) < 0 \quad with\ k = 1,\dots,p\end{aligned} \tag{8}$$

with **u** being the uncorrelated standard Gaussian random variables vector and g_k and h_k equality and inequality limit state functions.

3.4.2 Performance Measure Approach (PMA)

This approach is formulated as the inverse of the reliability analysis in RIA, so it can be formulated as:

$$\begin{aligned}&Find\ \mathbf{d}\ to\ \min f(\mathbf{u},\mathbf{d})\\&subject\ to\ \mathbf{d}^L < \mathbf{d} < \mathbf{d}^U\\&\qquad g_i(\mathrm{E}[\mathbf{u},\mathbf{d}]) < 0 \qquad with\ i = 1,\dots,m\\&\qquad h_j(\mathrm{E}[\mathbf{u},\mathbf{d}]) = 0 \qquad with\ j = 1,\dots,n\\&\qquad g_k(\|\mathbf{u}\| = \beta_{target},\mathbf{d}) - g_k(\mathbf{u},\mathbf{d}) < 0 \quad with\ k = 1,\dots,p\end{aligned} \tag{9}$$

with **u** being the uncorrelated standard Gaussian random variables vector and g_k and h_k equality and inequality limit state functions.

Related to the advantages and disadvantages in using one or other method the reader is referred to Tu and Choi (1999).

4. GUIDELINES FOR THE SAFETY FACTOR

The safety factor (SF) is a surplus given to the design process of a problem, since in the real world many aspects of the design have some degree of uncertainty, or an uncontrollable noise. Ullman (2009) proposes a guideline to evaluate that factor. There are several possible ways to apply safety factors, like reducing the allowable strength, increasing the applied stress or a ratio of the allowable strength to the applied stress such as in the following formula:

$$SF = S_{al}/\sigma_{ap} \tag{10}$$

where S_{al} is the allowable strength (defined as percentiles of a given CDF for a given confidence) and σ_{ap} the applied stress and SF the safety factor. Common values for SF can be estimated associating values greater than 1 for five measures (based on previous statistical knowledge):

- Material proprieties: ranging from 1.0 to 1.4 depending on the knowledge of the material, if its values obtained from tests or handbooks;
- Load stress: ranging from 1.0 to 1.7, varying if it is well defined as static or fluctuating, the presence of overloads, if that load is defined in an average manner and on the stress analysis accuracy.
- Geometry: ranging from 1.0 to 1.2, it is basically linked with the manufacturing tolerance.
- Failure analysis: ranging from 1.0 to 1.5, depends on the failure analysis development, if it is an extension of known theories or not well researched ones.
- Reliability: 1.1 form values less than 90%, around 1.2 and 1.3 if the reliability is between 92% and 98%, and 1.4 to 1.6 if the reliability must be greater than 99%.

The product of those five quantities can define the final safety factor:

$$SF = SF_{material} \cdot SF_{stress} \cdot SF_{geometry} \cdot SF_{failure\ analysis} \cdot SF_{reliability} \tag{11}$$

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5. DESIGN EXAMPLES

Three examples, often found in the optimization literature, have been used for the proposed comparisons. The first two examples are related to the design optimization a 10-bar truss and a 25-bar truss. Then, the third example is the design optimization of a coil spring, found in numerous practical applications (Arora, 2004, Sedeghati, 2005).

5.1 The 10-bar truss example

The classical 10-bar truss, Fig. 3, is a well known non-convex optimization problem. The objective is to reach the minimum structural weight allowing each of the 10 member cross-sectional areas varying starting from 64.516 mm^2. The allowable stress for each member is 172.37 MPa for both tension and compression. Vertical downward loads of 444.82 kN are applied at nodes 2 and 4. The material's density is 2768 kg/m^3, Young's modulus is 68.95 GPa. The L dimension is 9.144 m.

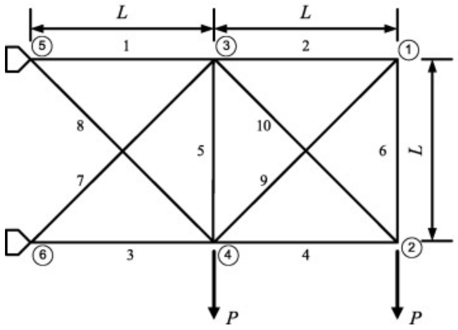

Figure 3. 10-bar truss (adapted from Behdinan and Perez, 2007).

The traditional design optimization without safety factors or reliability constraints, using just design variables constraints (64.516 $mm^2 < A_i$), stress constraints (±172,37 MPa on all members) and displacements constraints (±50.8 mm on nodes 1, 2, 3 and 4) gives results (third column of Table 1) very close to those presented by Sedaghati (2005) using the Displacement Method (DM) (second column of Table 1). Now, it is imposed a target reliability for the ultimate limit state is 99.9% and for the serviceability limit state is 99.0%.

Table 1 – Comparisons between Safety Factor designs and RBDO with RIA method. (10-bar truss)

	Sedaghati (2005) DM	Present Study PSO	Present Study (SF=1.694) PSO	Present Study RBDO PSO
Constraint for Reliability Index for ultimate LSF			-	3.0902 (0.999)
Constraint for Reliability Index for serviceability LSF			-	2.3263 (0.99)
Design variable	Area (mm^2)	Area (mm^2)	Area (mm^2)	Area (mm^2)
1	19691.44	19632.53	33300.37	25489.60
2	64.52	64.52	64.52	274.24
3	14967.65	15135.67	25764.92	21868.94
4	9821.21	9818.90	16602.99	14463.04
5	64.52	64.52	64.52	64.52
6	355.74	332.75	428.56	64.52
7	4811.09	4811.85	8268.04	7507.37
8	13571.84	136.47	23065.10	17197.86
9	13889.26	13760.39	22853.82	19201.73
10	64.52	64.52	64.52	436.58
Mass (kg)	2295.56	2295.77	3871.3	3162.20
Obtained Reliability Index for ultimate LSF	-	-	3.4827 (0.999752)	3.3882 (0.99965)
Obtained Reliability Index for serviceability LSF	-	-	3.5869 (0.999833)	2.3268 (0.99001)

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For the deterministic design optimization using the safety factor, Eq. 10 is used, dividing the limit stress on each bar by the SF as well as the allowable displacements. That will be the only parameters of the design that will gain a surplus for the safety. Since the material properties are being taken from handbook tables, it will be considered $SF_{material} = 1.1$; it is also assumed that there would be a slight fluctuation on the applied load, so $SF_{stress} = 1.1$; and finally the contribution for reliability is $SF_{reliability} = 1.4$. Equation 11 is used to find the final safety factor of about 1.694. This safety factor was used for both stress and displacement constraints as reducing factor for the limit stress and limit displacements. This gives the results in the fourth column of Table 1.

For the RBDO the cross-sectional areas are assumed to have a CV = 0.05, material proprieties such as Young's modulus a CV = 0.10, and finally for the applied loads CV = 0.05. Since material proprieties have the same variability for all bars (assumed uncorrelated), this problem has 13 random variables (10 cross-sectional areas, 1 material propriety and 2 loads) being 10 of them also design variables of the optimization problem. All random variables have a standard Gaussian distribution as CDF. The results of fifth column of Table 1 shows reliability indexes for both ultimate and serviceability limit states functions (LSF) that are not violated and a total mass that is even lower than the total mass using a safety factor. The use of safety factor results in high reliability indexes for both ultimate and serviceability LSF. This shows that the RBDO presents a more rational design.

It should the noticed that using the Safety Factor method the notion of reliability is loss for the designer point of view, since the final reliability is only possible to be assessed after being assumed *a priori* Safety Factor. Moreover, the obtained reliability for Ultimate and Serviceability are quite different and independent, a not desirable situation, since ultimate LSF should be treated with higher reliability indexes than serviceability LSF ones. Looking for the obtained reliability index in Table 1, it can be noticed that in this case the Serviceability LSF prevails over the Ultimate LSF, since this last constraint is relatively far from the initial constraint but the Serviceability is almost active.

5.2 The 25-bar truss

The second example in this paper is related to the weight design optimization of a spatial 25 bar truss, presented in Fig. 4. The design variables are the cross-sectional areas divided into 8 groups as shown in the Table 2. The loads are presented in Table 3. It is imposed a minimum cross-sectional area of 64.516 mm^2 for the bars and the displacement at each node cannot be greater than ±8.89 mm. The allowable stress for tension or compression is 275.79 MPa. In this problem, it is settled a target reliability of 97% for the serviceability LSF constraint and 99% for the ultimate LSF.

Table 2. Truss members groups

Group	Truss members
1	1
2	2 to 5
3	6 to 9
4	10 and 11
5	12 and 13
6	14 to 17
7	18 to 21
8	22 to 25

Table 3. Loads (kN)

Nodes	F_x	F_y	F_z
1	4.448	-44.482	-44.482
2	0	-44.482	-44.482
3	2.224	0	0
6	2.669	0	0

For the deterministic optimization, it is changed the $SF_{reliability} = 1.2$, keeping the other two safety factors of the previous example. This result in $SF = 1.452$. For the used random variables in the RBDO it will be kept the same coefficient of variation for material proprieties, loads and the design variable (areas).

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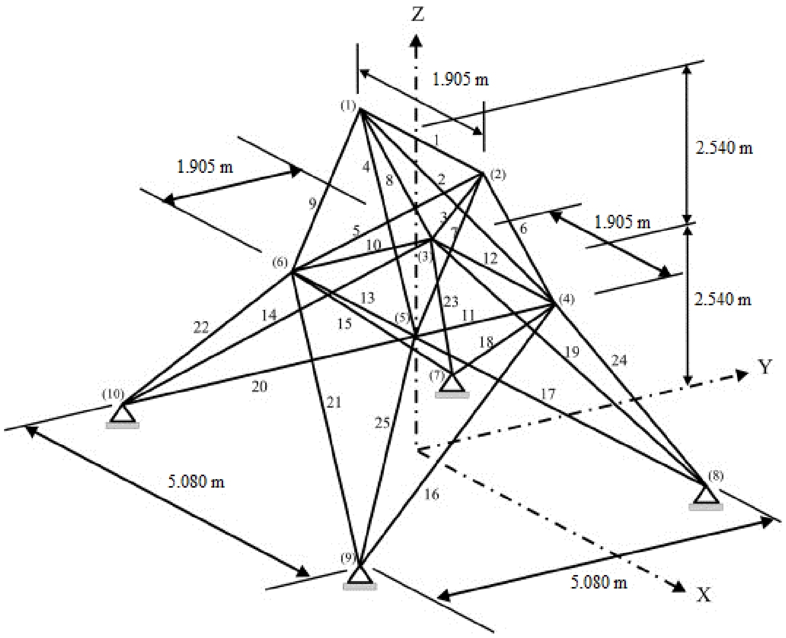

Figure 4. The 25-bar truss (adapted from Behdinan and Perez, 2007).

This example is similar with the previous one, where we have a more rational analysis if it is known the randomness of the problem and the RBDO method is utilized. In this case, the serviceability constraint is more restrictive, since the ultimate LSF is not even reached and the index was kept in a higher value (bigger than 8). Again, as indicated by Table 4, the serviceability LSF prevails over the ultimate LSF as the reliability index for serviceability LSF are attained while the reliability index for ultimate LSF was not violated but presented a high value.

Table 4 – Comparisons between Safety Factor designs and RBDO with RIA method. (25-bar truss)

	Behdinan and Perez (2007) PSO	Present Study PSO	Present Study (SF=1.425) PSO	Present Study RBDO PSO
Constraint for Reliability Index for ultimate LSF			-	2.3263 (0.99)
Constraint for Reliability Index for serviceability LSF			-	1.8808 (0.97)
Design variable	Area (mm^2)	Area (mm^2)	Area (mm^2)	Area (mm^2)
1	64.52	64.52	64.52	64.52
2	294.52	64.52	64.52	64.52
3	2193.54	2337.78	3364.16	2849.99
4	64.52	64.52	64.52	64.52
5	1249.61	1303.86	1706.82	1346.88
6	622.39	501.08	718.99	692.48
7	285.35	88.68	154.77	418.80
8	2193.54	2520.97	3585.73	3079.10
Mass (kg)	219.47	212.02	301.53	275.37
Obtained Reliability Index for ultimate LSF	-	-	> 8 (1.0000)	> 8 (1.0000)
Obtained Reliability Index for serviceability LSF	-	-	2.1245 (0.9832)	1.8812 (0.97003)

5.3 Design of a coil spring

In the original problem (Arora, 2004) the objective is to minimize the mass of the coil shown in Fig. 5, varying the wire diameter, d between 1.27 mm and 17.78 mm, the mean coil diameter, D between 1.27 mm and 35.56 mm, and the number of active coils, N between 1 and 8. This coil spring is subjected to an axial load and must overcome material failure and two serviceability constraints: the spring must deflect at least $\Delta \geq 12.7$ mm and the natural frequency must not be less than $\omega_0 \geq 100$ Hz. The outer diameter of the spring should not be greater than $D_0 \leq 38.1$ mm. Finally we have the following optimization problem:

$$\begin{aligned}
&Find\ \mathbf{d},\mathbf{D},\mathbf{N}\ to\ \min Mass = \frac{1}{4}(N+Q)\pi^2 D d^2 \rho \\
&subject\ to\ deflection\ constraint: \quad \frac{8NPD^3}{d^4 G\Delta} - 1.0 \geq 0 \\
&\quad natural\ frequency: \quad \frac{d}{2\pi N D^2 \omega_0}\sqrt{\frac{G}{2\rho}} - 1.0 \geq 0 \\
&\quad material\ failure\ (shear\ stress): 1.0 - \frac{8PD}{\pi d^3 \tau_a}\left(\frac{4D-d}{4D-4d} + \frac{0.615d}{D}\right) \geq 0 \\
&\quad diameter\ constraint: \quad 1.0 - \frac{D+d}{D_0} \geq 0 \\
&\quad lateral\ design\ variable\ constraints: \quad \mathrm{d}^{min} < \mathrm{d} < \mathrm{d}^{max} \\
&\quad \quad \mathrm{D}^{min} < \mathrm{D} < \mathrm{D}^{max} \\
&\quad \quad \mathrm{N}^{min} < \mathrm{N} < \mathrm{N}^{max}
\end{aligned} \tag{12}$$

where Q is the number of inactive coils, in that case is 2; ρ is the material density of 7890 kg/m^3; G is the shear modulus of 7.929 GPa; τ_a is the allowable shear stress of 551,58 MPa; the load P is 44,482 N.

The coefficient of variation for d, D and N are 0.5%, 5% and 10% respectively, for the RBDO analysis. The reliability for the deflection and frequency constraints are defined as 95% and for the shear and total diameter as 98%. In the case of safety factor analysis it is defined a SF of 1.500. It is applied directly on the constants D_0, τ_a, ω_0 and Δ.

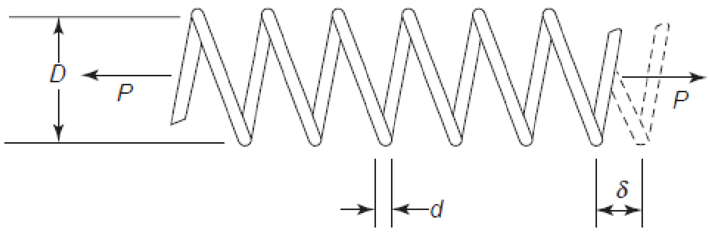

Figure 5. A coil spring.

Table 5 – Comparisons between Safety Factor designs and RBDO with RIA method.

	Present Study PSO	Present Study (SF=1.500) PSO	Present Study RBDO PSO
Constraint for Reliability Index for stress and diameter LSF			2.0537 (0.98)
Constraint for Reliability Index for deflection and frequency LSF			1.6449 (0.95)
d (mm)	1.469	1.880	1.783
D (mm)	13.275	18.800	22.589
N	5.77430	7.96130	3.39840
Mass (kg)	1.122E-05	3.341E-05	1.955E-05
Obtained Reliability Index for ultimate LSF		5.6806 (1.0000)	2.0637 (0.9805)
Obtained Reliability Index for serviceability LSF		2.0741 (0.9809)	1.6688 (0.9524)

6. CONCLUSIONS

In this paper a comparison between the RBDO methodology and the deterministic optimization with a safety factor was performed through three different applications. The safety factors are the easiest and fastest way to guarantee a surplus in the problem design. Those factors are found in handbooks guidelines, manuals and standards. The RBDO is a rational way to tackle the design optimization problem with control over the reliability at each limit state function. As

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expected in the implementation of the RBDO in this paper, the methodology presented computationally more expensive, since a two loop framework was adopted when performing the comparisons with the deterministic optimization using SF. With the examples presented in this paper it is possible to see the importance of knowing the variability intrinsic to a project. Even a conservative guess of some design variables distribution will result in a more understandable cost of the problem using the reliability method. Finally, the used PSO algorithm showed to be a flexible and useful tool to be used together with the reliability analysis. Future works will focus on the development of a parallelized version of the RBDO foreseeing time savings in problems with several design and random variables.

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