# A PROSTHESIS MODELLING METHOD BASED ON SELF-ADJUSTED ELLIPSES DESCRIPTORS OF SKULL SHAPE 

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Abstract. The complexity of the geometrical modelling of the natural shapes is the main problem in order to build the biomedical models. This work addresses the design requirements in order to build an anatomical skull prosthesis piece in the CAD systems based on self-adjusted ellipses descriptors. A novel methodology based on the Ellipse Adjustment Algorithm (EAA) has been investigated in order to define the manufacture parameters. In geometric terms an ellipse seems with the bone's border shape in a Computed Tomography (CT) slice. The arc that fills the correspondent failure in the bone border is extracted from the respective adjusted ellipse to each CT slice and the set of those extracted arcs can be superimposed to define the stack of images to build a $3 D$ CAD model. The Harmony Search (HS) in the evolutionary algorithms field were also applied to improve the quality of data generated. A prototype was implemented by an open source java based tool (ImageJ/Fiji) in order to create synthetic failures in the $3 D$ skull image and the resulting image of the piece of prosthesis can be compared with the original image. In context of product development this approach brings an essential integration between design and manufacture engineering to reduce the elapse time among the medical procedure, modelling and machining.

Keywords: CAD, 3D image, Computed Tomography, Ellipse Adjustment, Prosthesis Modelling

## 1. INTRODUCTION

A piece of prosthesis in terms of engineering requirements can be thought as a complex product and an essential phase is the modelling. Skull prosthesis has been receiving special attention because deals with esthetical and functional problems and these issues define the level of possible restrictions in machining.

Nowadays the literature demonstrates a gradual evolution of methods to bone modelling and anatomic prosthesis conception. Different methods have been experimented as (Francesconi, 2008; You et al, 2009; Lee et al, 2009; Chen et al, 2010; Canciglieri Jr. et al, 2011; Aquino et al, 2011; Huang et al, 2011; Saldarriaga et al, 2011; Jin et al, 2013; Rudek et al, 2013) where the objective is to develop a tool to aid the manual development of prosthesis due the difficulty level and imposed limitations in the artisanal production as exposed by (Rudek et al, 2010)

The prosthesis is a customized product for each individual problem, and there is a particular difficulty with production requirements and processes' organization in its production. A facilitation is been proposed by Greboge (2013) based on the concept the ellipses adjustments on the skull slices through a method called Ellipse Adjustment Algorithm (EAA). An ellipse can be found by intuitive and natural way using the superformula equation, as described in Gielis (2003) in his study with leaves shapes in plants. This concept was adapted to skull modelling by (Rudek et al, 2013). The skull bone curvature registered in the Computed Tomography (CT), has a circular form with different variations among all tomography slices. In this approach, the basic concept is adjusting ellipses to each bone border in all CTs with different parameters. The adjusted ellipse is a facilitator in the process, because it is possible to find a shape descriptor based on the ellipse parameters. This shape descriptor can be exported to CAD/CAM Computer-Aided Design/Computer-Aided Manufacturing) environment as a previous step before machining,

The main problem in this approach is that several ellipses can be created with similar shape of a skull bone border for the same CT slice, differing themselves only by slight displacement in their parameters. A rule of selection is necessary as a decision factor, and the existent techniques based on evolutionary algorithms have been demonstrated promisor in the generation of descriptors of transversal sections of skull as Genetic Algorithms (GA), Particle Swarm Optimization (PSO) and Harmony Search (HS), discussed and exemplified previously in (Greboge et al, 2011)

Also is true that some skull problems could be solved by symmetry, where a side of skull (good side) might be mirrored to cover the failure in opposite side using image processing techniques, but for instance failures in frontal
region cannot be solved by mirroring. The proposal here overtakes this limitation and produces a general case application.

A previous method proposed by (Canciglieri Jr. et al, 2011) had described some essential stages in prosthesis modelling, and the conceptual model is suggested as in the figure 1.


Figure 1. Overview about context of the modelling process, adapted from [5];
As shown in the context overview in figure 1, the method proposes three main parts, as:

Step 1: Acquisition of Tomography Images from DICOM files.
From computed tomography device it is possible to apply computing techniques to map in images the patient's body structure. The images are transferred and stored in DICOM (2013) format and this standard defines a medical communication protocol and a file format including defined metadata. It is used for medical intercommunication inside and outside the clinical environment. The aim of this model is to maintain the information standardized among manufacturers, as storage images format, data compression, information about exams procedures, and parameters of resolution between images and the respective aspect ratio of physical information. The images and respective dimensional information are extracted to be used as imputing data to virtual model creation.

Step 2: Generation of the Virtual Prosthesis Model in 3D.
The second part in the process is to build the 3D model of piece of prosthesis. This phase has three distinct steps:
i.The selection of defective CT slices: In this step, the proposed method identifies and separates the group of defective slices. Those are having an interrupted bone contour are selected as imputing data to next stage;
ii.The application of Ellipse Adjustment Algorithm (EAA): This is the kernel of the proposed method and deals with the representation of bone border shape by an elliptical descriptor.
iii.The generation of 3D model of prosthesis piece: Deals about the exporting data and 3D reconstruction in CAD.

Step 3: Machining of 3D Model
The last step is the machining. All data prepared in previous step must be prepared to build the customized prosthesis as a real product. This phase involves various medical requirements and they are not addressed in this moment.

The generation of the virtual prosthesis model (step 2) is the main objective of this research and the second phase steps are detailed following in the text.

## 2. PROPOSED METHOD

### 2.1. Proposition of problem

A hypothetic condition as presented in figure 2 can be used to simulate a problem in the skull. An application developed in the Java based tool called ImageJ (Schindelin et al, 2012) permits the creation of a synthetic failure straight in the 3D visualization. This is an important condition because we might use this previous known information to evaluate the method and also we might to create different cases to study in skull repairing.

The main question is to find a way to fill the defective region by an automatic way. The design strategy adopted is to decompound the 3D view in 2D representations of the defective area. The better way is to use the same original CT slice information to fill each separated slices contours, and after rebuild the complete 3D information to prosthesis modelling. This strategy intends to be linked with manufacturing procedures during engineering phases.

The adopted approach is based on adjustment of ellipses as proposed by (Greboge, 2013) because the method was thought to provide the linkage with 3D modelling and machining steps.


Figure 2. ImageJ interface to 3D definition of the failure position in original image, and respective resulting image after application of the EAA method.

### 2.2. Proposition of the Ellipse Adjustment Algorithm (EAA)

The objective of the Ellipse Adjustment Algorithm (EAA), proposed by Greboge (2013), is to find one ellipse capable to perform a self-adjustment on skull border. How discussed in (Rudek, 2013) some CT slices at middle of skull seem as an ellipse and in the most of the cases this method can be applied. The EAA uses this principle in order to create a synthetic contour with similar shape of the internal and external skull border for each CT slice that contains a failure. The EAA have two main parts. First, the failure position identification, and second the ellipse's parameters estimation based on an optimization method.

### 2.2.1. Failure Position Identification

For each CT slice, a fundamental step is finding the failure position. The Region of Interest (ROI) is delimited by the discontinuity of the skull border. The position where the skull edge is interrupted must be identified in order to define the limits of the space of solution. The space solution is the arc that fills those uncompleted region in CT. In figure 3.a is representing a polar mapping of bone border coordinates and respective dashed arc-solution.

In this example it is shown the inner edge of the skull. Each point in this edge can be mapped by a radius $(r)$ and its respective coordinate $(x, y)$ on the edge. In convenient form, the initial angle $\theta=0 \mathrm{rad}$ is defined wright down from centre straight to base of skull edge. In the figure, this position is the starting position denoted by $c p_{1}$ (contour point 1 ), and all others contour points are named with $c p_{\mathrm{i}}$, with $\mathrm{i}=1, . . n$, where $n$ is total number of pixels in the edge. The sequence in $c p_{\mathrm{i}}$ values might be interrupted if occurs a gap in border. If exists an interruption in border continuity, we have a failure in this respective skull slice, and the $r$ value in those positions tends to infinite ( $r \rightarrow \infty$ ). The set of points along region of missing pixels are denoted as $d p_{\mathrm{i}}$, (disconnected points) with $i=1, \ldots, m$, where $m$ depends of size of
failure. Applying the variation of $\theta$ with a step length (for example 0.001 rad ) in the range [ $0_{\mathrm{rad}}<\theta<2 \pi_{\mathrm{rad}}$ ] it is created a set of $c p_{i}$ points with another subgroup inside that contain the $d p_{i}$ points. Without lose the generality, the $d p_{\mathrm{i}}$ values can assume $r=0$ because in those respective $\theta$ positions no pixels borders exists and the radius is null.

Figure 3.b shows a scheme in histogram format of the radius ( $r$ ) mapping and respective failure position identification (ROI - Region of Interest). With this approach is possible to gather the exact coordinates (respective $\theta$ ) of the break edge points.

This process must be repeated for all CT slices. If the $d p_{\mathrm{i}}$ set is empty, is because there is no interruption (failure) in the respective slice. Using this approach, we can select only CTs with defective slices among all others CTs of exam.


Figure 3. (a) Edge points position by polar coordinate system. (b) Representative scheme in 'histogram' format of radius ( $r$ ) mapping and failure position identification (ROI - Region of Interest) of inner skull edge.

### 2.2.2. Ellipse's parameters estimation

Figure 4 shows an example of a CT slice with an uncompleted bone region, and respective parameters that can be found by the EAA. The parameters to be finding are the minor ' $a$ ' and major ' $b$ ' axis length, and the centre point coordinate $\mathrm{P}\left(\mathrm{x}_{0}, \mathrm{y}_{0}\right)$ for both bone borders. Those parameters are corresponding values to the inner $\left(\mathrm{E}_{1}\right)$ and external ( $\mathrm{E}_{2}$ ) ellipses.


Figure 4. Identification of parameters used in the Ellipse Adjustment Algorithm (EAA).

Also in figure 4, it is defined a limited range to restrict the set of values of the lower and upper bounds. This range limits the values of the coordinates of ellipses centres $(P)$ and the respective sizes of $\mathrm{a}_{+/-}$and $\mathrm{b}_{+/-}$lengths. This initial setup is applied in order to reduce the processing time. The conditions are defined in equation 1 to the ellipse's radius $r_{k}$
and in equation 2 to centre $P_{k}$, to $k$ possible ellipses with $k=1, \ldots, \max$, with $\max$ is the quantity of desired iterations. These values represent the limits for internal $E_{1}$ and external $E_{2}$ sets of parameters.

$$
\begin{align*}
& r_{k}=\left\{\begin{array}{l}
\left(a_{-}-\delta_{x}\right) \leq a \leq\left(a_{+}+\delta_{x}\right) \\
\left(b_{-}-\delta_{y}\right) \leq b \leq\left(b_{+}+\delta_{y}\right)
\end{array}\right.  \tag{1}\\
& P_{k}=\left\{\begin{array}{l}
\left(x_{0}-\varepsilon_{x}\right) \leq x_{0} \leq\left(x_{0}+\varepsilon_{x}\right) \\
\left(y_{0}-\varepsilon_{y}\right) \leq y_{0} \leq\left(y_{0}+\varepsilon_{y}\right)
\end{array}\right. \tag{2}
\end{align*}
$$

Based on several simulations, the lower and a higher value of range can be defined among some limits as for example ( $\delta_{x}=10, \delta_{y}=10$ ) to axis size, and ( $\varepsilon_{x}=15, \varepsilon_{y}=15$ ) to the variation of central point position, in both $x$ and $y$ directions. Through experimentations, the conditions are that the ' $a$ ' value is between 120 and 200, and ' $b$ ' value is between 90 and 200. The coordinates of the centre of the image will be between 240 and 275 . The unit of measurement is in pixels. If necessary, the values of $\delta$ and $\varepsilon$ can be changed according size modifications of image or particular cases (for example, in the slices more on top of skull).

Slight changes in the values of these parameters generate a lot of possibilities to create arcs and the more feasible answer must be found. The main objective is to find the best ellipse that fits the skull border for each CT slice. To perform this, we need to find the best values of $a, b, x_{0}, y_{0}$ in order to obtain the both ellipses $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ as similar as possible with original inner and external border edge. The equation 3 shows the polar formulation to create an ellipse.

$$
\begin{equation*}
r=\frac{a b}{\sqrt{a^{2} \sin ^{2} \theta+b^{2} \cos ^{2} \theta}} \tag{3}
\end{equation*}
$$

By changing the values of $a, b, x_{0}, y_{0}$ (as represented in figure 4) we can create an ellipse to be superimposed on the original contour. The problem is to find the best combination of those parameters. An objective function is proposed to evaluation of both $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ ellipses. The equation 4 shows the fitness function $F$ whose objective is minimizing this distance between the original edge and the respective created ellipse. This is a reformulated equation based on Greboge (2013).This improved approach permits to measure the distance between a pixel of the generated ellipse $E$ and its corresponding pixel on the original contour. The $F$ value can be evaluated for each possibility and the best one permits to identify the corresponding values of $\mathrm{a}, b, x_{0}, y_{0}$ that are closer than original edge.

$$
\begin{equation*}
F(K)=\sum_{i=1}^{n} \sqrt{\left(\left(x_{E}(i)-x_{C}(i)\right)^{2}+\left(y_{E}(i)-y_{C}(i)\right)^{2}\right)} \tag{4}
\end{equation*}
$$

The $F$ must be evaluated for each ellipse $E$ (internal and external ellipses) in each $K$ slice, using the information about the pixels positions. In this definition we have $x_{\mathrm{E}}$ and $y_{\mathrm{E}}$ as pixels coordinates of generated ellipse $E_{1}$ or $E_{2}$, and $x_{\mathrm{C}}, y_{\mathrm{C}}$ are the coordinates of pixels on the respective original CT contour. The evaluation is performed to both internal and external ellipses and contours. An important parameter to define the quality of adjustment is the $n$ value that represents the amount of border pixels used. If $n$ assumes the total number of pixels of the contour, the cost of processing time is high. By experimentations it is not necessary to use all border pixels and the $n$ can assume less values to avoid long processing time. Also by observation some "more interesting" points can be selected from the contour. For example, the points nearest to the ROI have more influence in the result. So, the coordinates of selected contour pixels produces the same ellipse solution, and it can be chosen based on histogram position (figure 3.b).

Due to the many possibilities of values assumed by the ellipse parameters, some optimization method is necessary to estimate the best of them. The investigation of Greboge (2011), shows that Genetic Algorithms (GA), Particle Swarm Optimization (PSO) and Harmony Search (HS) could be applied in this matter. A generic formulation to the optimization algorithm in order to evaluate the value of $F$ has the general guidelines:

1. Initializing the parameters $a, b, x_{0}, y_{0}$ and also the length $(n)$ of vector solution ;
2. Execute the analysis of initial values set evaluating the fitness F;
3. Update the possible solution following the rules of optimization method to generation of new set of testing values;
4. Evaluate the new fitness value. Update the best stored if the new value is better than the previously stored;
5. If maximum number of interactions not achieved, return to step 3.

With the best $F$ found, the corresponding values of $a, b, x_{0}, y_{0}$ can be stored for each evaluated CT slice. These values define the more appropriated ellipse that subscribes the real bone edge. After all ellipses found, these values can be exported to the CAD system and the ellipses can be created in that environment.

## 3. APPLICATION OF THE METHOD AND DISCUSSION

The example about the problem in skull addressed in figure 2 is used for demonstration. A total of 24 CT slices were extracted from DICOM file, and the $k=20$ intermediate slices were used because those are the slices closest from failure position. The figure 5 .a shows a sampled group of some CT slices from the skull failure.


Figure 5. a) The CT samples of slices from testing skull. b) Example of ellipse adjustment in the CT slice $k=10$.

The CT slices with respective open border were processed by the EAA with Harmony Search (HS) optimization strategy (Greboge, 2011), and the values of each $k_{3, \ldots, 22}$ ellipses' fitness values are presented in table 1.

Table 1. The fitness values for each adjusted inner and external ellipse.

| $\boldsymbol{k}$ | $\mathbf{F}_{\text {Eint }}$ | $\mathbf{F}_{\text {Eext }}$ | $\boldsymbol{k}$ | $\mathbf{F}_{\text {Eint }}$ | $\mathbf{F}_{\text {Eext }}$ | $\boldsymbol{k}$ | $\mathbf{F}_{\text {Eint }}$ | $\mathbf{F}_{\text {Eext }}$ | $\boldsymbol{k}$ | $\mathbf{F}_{\text {Eint }}$ | $\mathbf{F}_{\text {Eext }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | - | - | 7 | 56 | 59 | 13 | 43 | 65 | 19 | 57 | 82 |
| 2 | - | - | 8 | 44 | 65 | 14 | 51 | 61 | 20 | 51 | 60 |
| 3 | 63 | 61 | 9 | 37 | 64 | 15 | 66 | 58 | 21 | 56 | 48 |
| 4 | 61 | 44 | 10 | 35 | 69 | 16 | 73 | 51 | 22 | 65 | 46 |
| 5 | 59 | 42 | 11 | 43 | 63 | 17 | 65 | 67 | 23 | - | - |
| 6 | 56 | 46 | 12 | 45 | 64 | 18 | 61 | 45 | 24 | - | - |

From table 1, the $F_{\text {Eint }}$ values are the fitness results to each $k$ slice applied to internal edge $E_{1}$, and the $F_{\text {Eext }}$ values are the fitness results in each edge $E_{2}$. As the objective of $F$ is measure the minority distance difference between the generated ellipse and the bone border, the small values of $F$ represents the best adjustment. In table 1, for internal edge, the slice $k=10$ have the best adjustment $(F=35)$ and the worst are in the slices $k=16(F=73)$. Note that $k$ represents the number of tested slices. For convention we use ' $k$ ' to represent the sequence of slices and ' $z$ ' as the range coordinate that carries the distance (in mm ) between slices. The transformations $k \rightarrow z$ depends of relationship pixel $\times m m$ defined in DICOM file.

Figure 5.b shows a representation of generated ellipses after application of EAA and the images presents the influence of edge thickness. For instance, the figure 5.b shows the similarity between $\lambda_{\text {left }}$ and $\lambda_{\text {right }}$ thickness. A good proportion of the thickness measure performs a better adjustment to ellipse creation, as proved by the fitness value $F=35$. Even existing differences of information generate in relation to the original characteristic of surface, the individual ellipses can be superimposed to build the 3D representation of the volumetric model, as shown in figure 6 .


Figure 6. (a) Top view of generated ellipses superimposed. (b) Corresponding arcs extracted of each ellipse.
The figure $6 . a$ presents all ellipses obtained to each CT slice from image of figure 2 . The respective centres are denoted by $\mathrm{P}=\left\{\left(x_{1}, y_{1}, z_{1}\right),\left(x_{2}, y_{2}, z_{2}\right), \ldots\left(x_{\mathrm{n}}, y_{\mathrm{n}}, z_{\mathrm{n}}\right)\right\}$ where $n=20 \mathrm{CT}$ slices tested. The symmetry line $(L)$ defines two separated hemispheres to identify the cut positions in ellipses. The cut positions in ellipses are the same of the interruptions positions in original bone edge extracted by $\min \left(d p_{\mathrm{i}}\right)$ and $\max \left(d p_{\mathrm{i}}\right)$. These $d p_{\mathrm{i}}$ coordinates' values are obtained from "histogram of position" (as in figure 3.b) and define the arcs of ROI. The complementary part of ellipse out of ROI is discarded. Figure $6 . b$ shows the extracted arcs from ellipse $\mathrm{E}^{\prime}{ }_{1}$ to $\mathrm{E}^{\prime}{ }_{20}$ and respective centres $\mathrm{C}^{\prime}{ }_{1}$ to $\mathrm{C}^{\prime}{ }_{20}$. These arcs are the solutions that represent the missing region on the skull as an example of the inner borders.

The same procedure is performed to both edges (internal and external) and all arcs can be superimposed to build a 3D view of prosthesis in CAD as figure 7.


Figure 7. (a) Superimposed solution found after ellipses' adjustment and (b) its corresponding 3D surface.
The figure 7.a shows the superimposed arcs obtained from each best ellipse generated. With ellipses' parameters, a geometric model can be built in CAD system to 3D visualization as in figure 7.b. The 3D model is an important premachining step, because the piece can be evaluated together with all medical requirements. Depending of resulting 3D model, a new configuration of parameters can be submitted to algorithm to improve the shape of piece.

A geometric model can be built in a CAD system to perform the visualization of skull shape and prosthesis piece as well.

A file in ASCII coding can be used to export the ellipse values to CAD system, as in figure 8. The pattern of data stored in ".txt" files have a commonly accepted format, where data can be organized in a table form using a known character as separator. This avoids incompatibilities problems among different software and versions. This file might be operated through intervention of the user, or handled by a script of instructions in a plug-in added into the CAD menu commands.


Figure 8. Cloud of points stored in the ASCII format.


Figure 9. a) Reconstructed defective Skull and respective hole filled by prosthesis. b) The deviation comparing between moulded piece and original region information.

In the figure 9.a it is shown the 3D defective skull reconstructed in CAD. There is a large failure region to be filled, which occupies lateral and frontal sides and the piece of prosthesis was superimposed on this region.

The quality of adjust is better to small failures. In large failures as in the presented example, some junction problems are more visible, as the junction difference present between skull and prosthesis around the entire piece border. This is caused by the arc points are totally flat, and do not contains the surface details. And also, the light and shadow position enhances the discontinuity.

The coloured scale presented in Figure 9.b illustrates the surface deviation. Due it is a case study with a synthetic failure, it is possible to perform a comparison with the original surface. Analysing the deviations presented by surface analysis software (Geomagic Studio, 2013), it obtained an average deviation of approximately 2 mm with standard deviation of 1.726 mm . In other regions near to the deformation the deviation was around 1 mm . By analysing images and parameters, some specific new improvements are necessary and it will be provide in next steps of this research.

## 4. CONCLUSION

In this paper was presented the improvement to the new technique for skull border modelling. The approach is based on ellipse adjustment algorithm (EAA), and the method demonstrates that it is possible to build a virtual model of prosthesis using the CT slices straight from medical exam file. The presented example shows that the method is feasible to large defective areas in skull, and in particular when the failure does not be solved by symmetry. When the problem in skull is asymmetric there are no enough information to build the failure region and the EAA algorithm provide a way to give the missing information. The link between the problem identification and the early steps before machining is well defined in method. A CAD system is a necessary tool in the middle of process to perform the connexion between the bone shape characterisation and machining parameters definition. Also as shown in the example, there are some open points to be solved, becoming a large field of research to new improvements. In a gradual way, the method is still in evolution and the next step is to experiment an interpolation method between extreme points of arc and skull breaks position to smooth the discontinuity. After this, the prosthesis piece might be better evaluated by specialized doctor before implementation of an automatic manufacturing procedure.

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