ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil


# A 2DOF LOW COST CONTROL WORKSTATION FOR CONTROL TECHNIQUES APPLICATION

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***Abstract.*** *The analysis of systems with n-degree of freedom can be motivated from real common problems as in aerospace processes and robotic. Within this context, this paper proposes a 2DOF (two-degrees-of-freedom) low cost control workstation and presents some discussions about its dynamics. The project of such workstation is based on user guides of commercial didactic processes and previous publications, as well in examples found in control and system identification references. In this paper, the proposed plant simulates some behavior aspects of a helicopter, namely, its pitch and yaw. Phenomenological methods is used to obtain and study its dynamics. The distinctiveness approached in this work is that the structure involves the utilization of reused materials, resulting in a low cost improvement, which can be applied in several studies, from control research projects to practical classes in engineering graduation.*

***Keywords:*** *low cost control workstation; 2 DOF systems; phenomenological modeling; engineering education*

## 1. INTRODUCTION

There is a common sense that the teaching in engineering is presented, typically, in a extremely theoretical way. The lack of practical workload during the graduation can reduce the potential of the graduated engineer. Such need can also be found in research projects in levels of graduation and master's degrees. These practices, when existing, are mostly confined in computation simulations, which do not necessarily have abidance with the dynamic representation of real systems.

The usage of didactic workstations is suggested as a possible solution to the presented problem. These workstations are able to behave such as equipaments found in the engineering field, approaching non linearities, multi variables and physical constraints.

In such context, this paper will present the project, building and analysis of a low cost didactic workstation. The goal of this tool is to apply it both in graduation subjects as in research projects, with the intent to upgrade the graduation course and improve the pedagogical approach in control subjects.

This paper is divided in a section that comprehends the general concepts that will permeate through it; a section describing the planning and building of the workstation studied in this work; a section dedicated to describe the analyzing processes of the dynamics of the workstation; a section attempting to discuss about the results that have been achieved and a final section approaching general concluding remarks.

## 2. GENERAL CONCEPTS

It is important to introduce, at first, the concepts involved in the area of control and systems dynamics that will permeate through all the development of this paper.

### 2.1 CONTROL

Controling systems is an intrinsic practice to the human being, either manually or automatically. Such practice consists in imposing a behavior to a determined process.

The majority of control techniques are based in a comparison between the measured result and an expected value. In an engineering language, this measurement can be named as feedback, and, the expected value, as the reference. When these values are different, a control action is required in way to obtain the expected result.

## 2.2 PROCESS MATH MODELING

Summarizing, modeling a system or a process refers to obtain a representative scheme of it, in a way that such could suit as basis to describe, study or simulate its behavior and characteristics, without the requirement of possessing the object of study. Photographies, flowcharts and molds are some model examples.

Into the engineering context, the concept of mathematical modeling is predominant. This one aims to present the referred model through numeric language by means of differential equations, rational equations, matrices, difference equations and other ones. In this way, real processes and systems are translated into numbers, which are easily manipulated by computational systems as well as by man himself.

Mathematical modeling allows a large part of the internal or external agents of the modeled system to be represented by a coefficient associated to its contribution to the studied dynamic. Therefore, its major benefit is to enable an in-depth study of the system, allowing the user to monitor, control and foresee its behavior against several situations.

### 2.2.1 Variables

At this point, two concepts applied to systems control in the context of mathematical modeling are presented: manipulated variables and controlled variables. Ogata (2010) explains: manipulated variables are those that the user has access and is able to change its value; they are also known as the inputs of a system. The controlled variable, or output of a system, is the one that the user wants to impose some especific behavior, tipically according to a taken reference.

### 2.2.2 Classification of Models

The mathematical models have a wide classification related to their different aspects. In this paper, only the most relevant features to describe the studied workstation will be presented, as seen in Aguirre (2007).

- Regarding to time dependence: static models do not present time dependence; they are tipically represented by algebric equations, whereas dynamic models, which present time dependence, are represented by equations as a function of time.
- Regarding to model description: continuous models are described continuously in time, while discrete models are sampled in a determined period with discretized values.
- Regarding to the amount of variables: monovariable models are called SISO (Single Input, Single Output) models. Moreover, regarding to multivariable models, those are subdivided in SIMO (Single Input, Multiple Outputs), MISO (Multiple Inputs, Single Output) and MIMO (Multiple Inputs, Multiple Outputs).
- Regarding to the consideration of uncertainty: the models that consider uncertainty due to an output noise are called stochastics. Those that disconsider it receive the name of deterministic.

### 2.2.3 Types of representation

Due to the range of system types shown previously, the representation of a certain process must be adapted to its characteristics, mainly refering to the description (continuous or discret) and the data collecting.

Among the continuous representations, the transfer function and the state space are more used this context. Formally, the transfer function is defined as the response of the system to the unit impulse, assuming all the initial conditions as null. It shows the relation between output and input, explicited by a proper equation in the frequency domain.

The state space refers to the representation in time domain, through a system of equations involving vectors and matrices. Its main feature is reducing a nth-order diferential equation to a set of n first-order diferential equations.

### 2.2.4 Linearity and time invariance

The linearity of a system is verified when principles of superposition and homogeneity are satisfied, i.e., if $F(x_1) = y_1$ and $F(x_2) = y_2$:

- $aF(x_1) = ay_1, \forall\, a \in \Re$
- $F(x_1 + x_2) = y_1 + y_2$

In real cases, linearity is rarely viewed in a system as a whole. As a non linear system requires a complex and detailed study, usually its division in several ranges whose caracteristics can be approximate by linear models is done.

The time invariance makes reference to the non flutuation of the system according to time. If a system replies to a determined input in a time $t_1$, it must reply in the same way to the same input in a different time $t_2$. Algebrically, if an input $u(t)$ generates a $y(t)$, an input $u(t - t_0)$ generates $y(t - t_0)$.

ISSN 2176-5480



### 2.3 PHENOMENOLOGICAL MODELING

The system modeling approach can be based in different concepts and, nevertheless, result in similar models. There are basically two main kinds of modeling, namely, white box and black box.

The white box is characterized by the need of having a depth knowledge about the system and the mathematical relations that describe its behavior. In oher words, it is a modeling made by the knowledge of the physics and the nature of the process, called as phenomenological or conceptual modeling.

On the other hand, in black box modeling, or empirical, a priori, it is not necessary to have a previous knowledge of the system, because the system model is obtained based on series of tests, in which are observed the behavior of the controlled variables due to several excitements in the manipulated variables.

A balance between these two kinds of modeling methods can be called as gray box, in which some of the involved phenomenon are modelled without deeply knowing the system dynamics.

#### 2.3.1 Stages of system modeling

Regarding to the system identification process, some steps for achieving a satisfactory model are presented in this subsection, based on Aguirre (2007).

- Physical describing of the system: at first, the system must be mathematically modeled, based on physical concepts. Also, one should test the workstation in way to observe its behavior and have an idea about the limitations of the system.
- Choice of the mathematical representation: at this point, the mathematical representation form is chosen. This choice must be done based in the way the workstation has been characterized.
- Determination of the model structure: as the representation has been chosen, its complexity must be defined. For example, in a transfer function, the amount of poles and zeros that will compose it. As more complex the model is, the more reliable it becomes, but the costs for dealing with it are bigger, demanding harder computation and more difficulties to control.
- Parameters estimation: This step is realized with computational algorithms, which adept the mathematical coefficients so that this estimated system presents a similar behavior to the obtained inputs in the initial period. It is important to notice that, in black box modeling, these coefficients do not necessarily represent physical aspects (damping or dilation coefficients, electric resistance, among others), differently from white box modeling.
- Model validation: this stage depends on the application. The model validation occurs according to the desired precision to control the workstation. Each system has its own peculiarities, so, it is necessary to ascertain if the obtained model presents significative errors against the expected result.

## 3. WORKSTATION - PLANNING AND BUILDING

One of the main desired characteristics in the prototype is to make it applicable both in graduation courses and research projects, covering simple and complex dynamics. Furthermore, since the workstation must be built, it is desirable to use low cost materials in the project.

There are, nowadays, several commercial didactic workstations, such as magnectic levitators, tracking controlers and object positioners. Among the manufacturers, Quanser®, Festo® and FeedBack® feature as the most popular. In this paper, a workstation that simulates the behavior of a hover is presented as object of study.

A hover can have three DOFs (degrees of freedom). This freedom of movement allows dynamics in lateral, horizontal and vertical ways by means of the pitch, yaw and roll axis, as shown in Fig. 1.

The hover is an inherently unstable system with a complex dynamic. To build the workstation, the model involving a main rotor and a tail rotor has been chosen. The first one is responsible for supporting the whole system, as the other one acts in the opposing way of the first one's torque, stabilizing the structure.

Resorting a lock feature, the 2 DOF steady hover developed allows the system to behave as SISO or MIMO. The propellers used have a fixed attack angle, not allowing the roll movement.

The formulation of the mechanical structure was based in works that involve similar platforms, as seen in Quanser Inc. (2006). The main building features are:

- two rollers fixed in wooden boards, which serve as a support for a vertical stem and mark the yaw DOF;
- a machined piece posted perpendicularly to the previous structure, in which a horizontal axis is linked, marking the pitch DOF.

ISSN 2176-5480



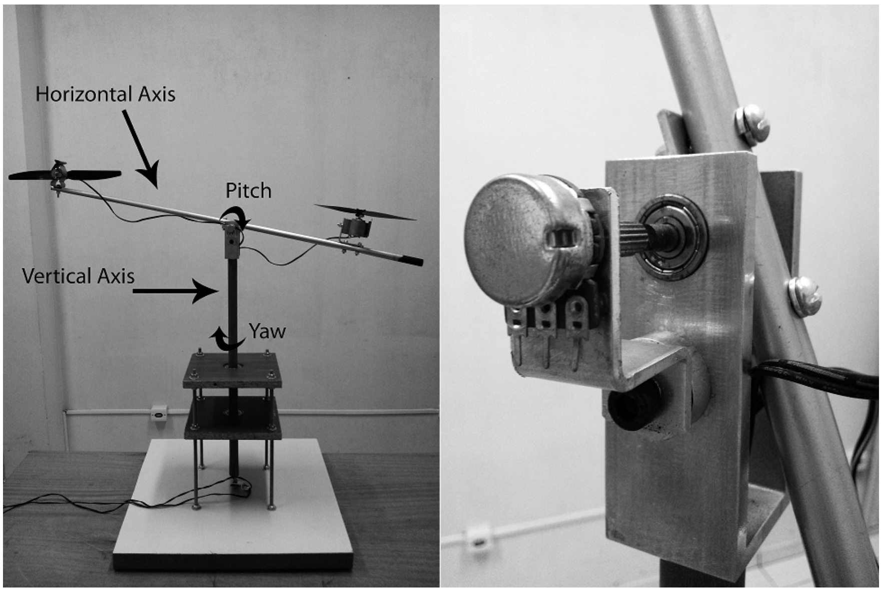


Figure 1. The workstation planned

- two DC motors screwed to the horizontal axis, which simulate the rotors of the hover.

All these features are in a solid wooden base, as seen in Fig. 1. Also in Fig. 1 is seen the sensoring of the workstation, which is based on linear common potentiometers attached to the stems of each degree of freedom. Preliminary tests showed that the imprecision of the measurements acquired with these devices were irrelevant in the data acquisition process. This characteristic and the low cost of the component were responsible for its choice.

## 4. ANALYSIS OF DYNAMICS AND BEHAVIOR

After the workstation was built, the process of the dynamic analysis was made through practices and process modelling involving the transfer of energy by the motors and they contribution to the pitch and yaw axis. Hereafter is shown the procedures involved in two methods that can be used to realize this analysis, namely, phenomenological and parametric modelling.

### 4.1 PHENOMENOLOGICAL MODELING

In this section the workstation is phenomenologically modeled. The system structure is represented by the Fig. 2, in which there are two motors that contribute to the pitch and yaw movement, by the thrust forces $F_p$ e $F_y$, respectively.

This modelling is based mainly in Quanser Inc. (2006), “Quanser 2 DOF Helicopter: User and Control Manual”. Moreover, basic physical concepts and analytical mechanics are used, referenced mainly in Nussenzveig (1996) and Cederwall (2009), respectively.

#### 4.1.1 Movement Analysis

As can be seen in the Fig. 2, the workstation has two DOFs: a rotation around the Z axis (yaw), represented by the angle $\psi$, and another one around the Y axis (pitch), represented by the angle $\theta$. The rotation angles are considered positive when exist a movement in direction to the rise of the helicopter and in a clockwise direction, respectively. This condition assumed to the angles can be better visualized in the Fig. 3, that represent the side and top view. The motor are localized at distances $R_y$ and $R_p$ of the center. This measure is important to determine the center of mass of the system.

To model the system it is considered that the center of mass is deslocated from the central axis. Therefore, it is necessary to define its transformation matrix that considers this factor combined with the 2 DOF. Thus, it is defined the translation and rotation matrices as:

- $T_\psi$: Rotation Matrix in Z axis
- $T_\theta$: Rotation Matrix in Y axis
- $T_{cm}$: Translation Matrix related to the $l_{cm}$

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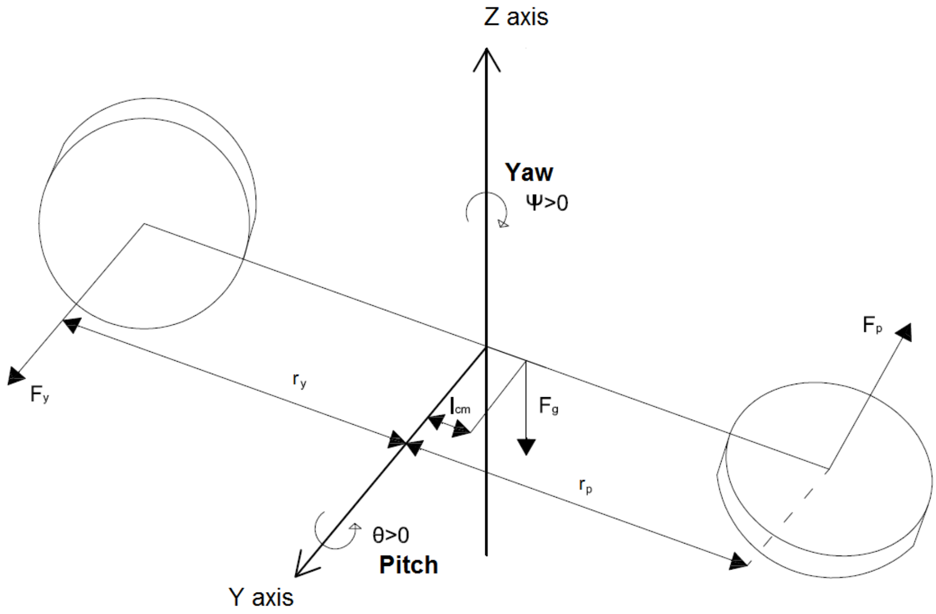


Figure 2. The workstation planned based in Quanser Inc. (2006)

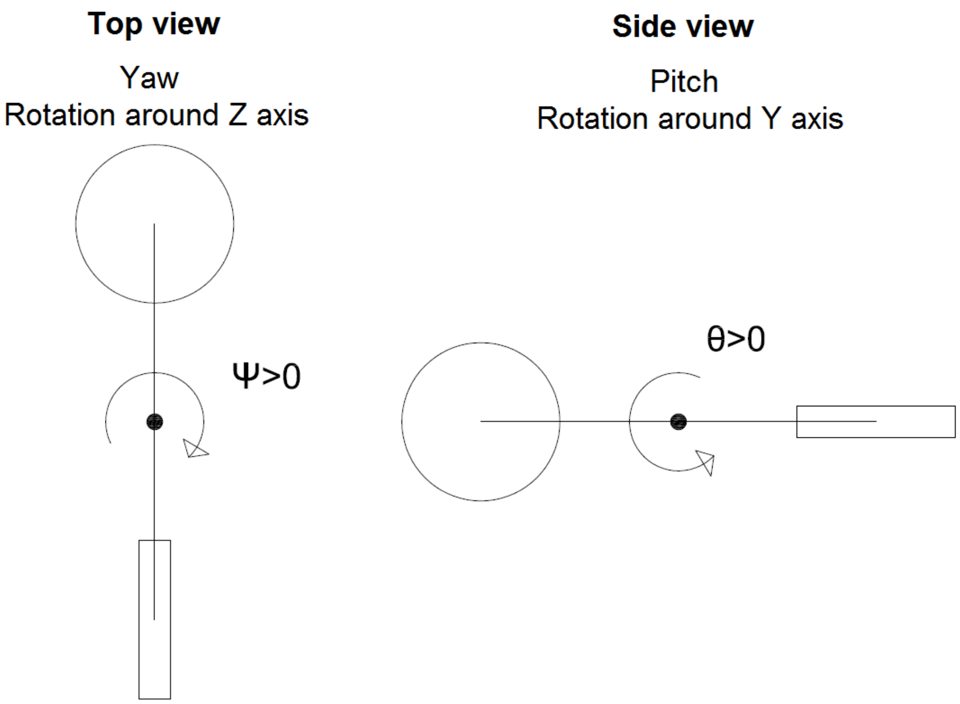


Figure 3. Superior and lateral views of the workstation, based in Quanser Inc. (2006)

The resulting matrix $T_0$ is result of this three matrices multiplied $T_0 = T_\psi T_\theta T_{cm}$.

$$T_0 = \begin{bmatrix} cos(-\psi) & -sin(-\psi) & 0 & 0 \\ sin(-\psi) & cos(-\psi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} cos(-\theta) & 0 & sin(-\theta) & 0 \\ 0 & 1 & 1 & 0 \\ -sin(-\theta) & 0 & cos(-\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & l_{cm} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$T_0 = \begin{bmatrix} cos(\psi)cos(\theta) & sin(\psi) & -cos(\psi)sin(\theta) & l_{cm}cos(\psi)cos(\theta) \\ -sin(\psi)cos(\theta) & cos(\psi) & sin(\psi)sin(\theta) & -l_{cm}sin(\psi)cos(\theta) \\ sin(\theta) & 0 & cos(\theta) & l_{cm}sin(\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

The center of mass $l_{cm}$ of a system, according to Nussenzveig (1996), can be determined as discrete distributions when there is mass concentration in specified positions. Applying this concept in the workstation analysis, there is the mass of the motor summed to the mass of the supports of the motors, that are concentrated at distances $r_p$ and $r_y$. In this way, the $l_{cm}$ is given by the equation above.

$$l_{cm} = \frac{\sum_{i=1}^{N} \vec{r_i} \cdot m_i}{\sum_{i=1}^{N} m_i} \tag{3}$$

$$l_{cm} = \frac{(m_{m,p} + m_{e,p}) \cdot \vec{r_p} + (m_{m,y} + m_{e,y}) \cdot \vec{r_y}}{m_{m,p} + m_{m,y} + 2 \cdot m_{shield}} \tag{4}$$

### 4.1.2 System Energy Analysis

The involved energy in the system consists basically in potential and kinetic energy. Nussenzveig (1996) define the potential energy as an energy that is stored in the potential means, when an object is suspended in a $Z$ height, and that can be converted in kinetic energy. As the position $z_{cm}$ was found previously, the potential energy $V$ due to the gravity is equated as:

$$V = m_{heli} g z_{cm} \tag{5}$$

$$V = m_{heli} g l_{cm} sin\theta \tag{6}$$

The kinetic energy of the sistem is a sum of three energies, two of rotation, $T_{r,p}$, due do the pitch, and $T_{r,y}$, due to the yaw, and one energy of translation $T_t$, due to the movement of the center of mass.

$$T = T_{r,p} + T_{r,y} + T_t \tag{7}$$

The rotational energy is the kinetic energy related to an angular displacement in a fixed axis. It is dependent of the inertial moment equivalent in the pitch $J_{eq,p}$ and yaw axis $J_{eq,y}$, as given in Eqs. 8 and 9.

$$T_{r,p} = \frac{1}{2} J_{eq,p} \dot{\theta}^2 \tag{8}$$

$$T_{r,y} = \frac{1}{2} J_{eq,y} \dot{\psi}^2 \tag{9}$$

The translational energy is related to the movement of the center of mass from a point $A$ to $B$. Consequently, its value is dependent of the mass and velocity of the center of mass, as:

$$T_t = \frac{1}{2} m_{heli} [(-l_{cm}\dot{\psi} sin\psi cos\theta - l_{cm}\dot{\theta} cos\psi sin\theta)^2 + (-l_{cm}\dot{\psi} sin\psi cos\theta + l_{cm}\dot{\theta} sin\psi sin\theta)^2 + (l_{cm} cos\theta)^2] \tag{10}$$

Working in Eq. 10, it is possible to simplify some terms and obtain the translational energy, as shown in Eq. 11. It is made in order to make it easier to obtain the non linear movement equations, that will be explained in the next section, therefore

$$T_t = \frac{1}{2} m_{heli} l_{cm}^2 [\dot{\theta}^2 + \dot{\psi}^2 cos^2\theta] \tag{11}$$

The resulting equation to the kinetic energy is:

$$T = \frac{1}{2} J_{eq,p} \dot{\theta}^2 + \frac{1}{2} J_{eq,y} \dot{\psi}^2 + \frac{1}{2} m_{heli} l_{cm}^2 [\dot{\theta}^2 + \dot{\psi}^2 cos^2\theta] \tag{12}$$

where the equivalent inertial moments in pitch ($J_{eq,p}$) and yaw axis ($J_{eq,y}$) in Eq. 13 and Eq. 14 are a composition of the inertial moments shown in the Eqs. 15, 16, 17, 18 e 19.

$$J_{eq,p} = J_{m,p} + J_{body,p} + J_p + J_y \tag{13}$$

$$J_{eq,y} = J_{m,y} + J_{body,y} + J_p + J_y + J_{shaft} \tag{14}$$

ISSN 2176-5480



These partial moments of inertia are due to the movement of the axis and the mass concentration. The $J_{m,p}$ and $J_{m,y}$ are internal parameters of the motor, the moment of inertia of the rotor.

$$J_{body,p} = \frac{m_{body,p} L_{body}^2}{12} \tag{15}$$

$$J_{body,y} = \frac{m_{body,y} L_{body}^2}{12} \tag{16}$$

$$J_{shaft} = \frac{m_{shaft}(R_1^2 + R_2^2)}{2} \tag{17}$$

$$J_p = (m_{m,p} + m_{suporte,p}) r_p^2 \tag{18}$$

$$J_y = (m_{m,y} + m_{suporte,y}) r_y^2 \tag{19}$$

### 4.1.3 Non Linear Movement Equations

In the movement analysis, the concept of analytic mechanical will be used. This approach is taken because the lagrangian formalism, that is scalar, is simplest than newtonian, that is vectorial. Also, it allows to describe low speed systems in the same way.

To use this formalism, the generalized coordinates must be determined, represented by $\{q_i\}$, that describes the system behavior. The lagrangian function $L$, according to Cederwall (2009), can depend of the generalized coordinates, generalized velocity, $\dot{q}_i = \frac{\partial q_i}{\partial t}$, and of the time $t$, i.e., $L(q_i, \dot{q}_i, t)$.

To this system, four generalized coordinates, $[q1 \;\; q2 \;\; q3 \;\; q4]^T$, equivalent to $[\theta \;\; \psi \;\; \dot{\theta} \;\; \dot{\psi}]^T$, are defined. These variables describe the behavior of the system given by the angular position and displacement of pitch and yaw axis.

Cederwall (2009) presents the lagrange variable as the difference between the kinetic energy and potential energy $L = T - V$ and define the Euler Lagrange equation,

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}_1} - \frac{\partial L}{\partial q_1} = Q_1 \tag{20}$$

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}_2} - \frac{\partial L}{\partial q_2} = Q_2 \tag{21}$$

The $Q_1$ and $Q_2$ terms are generalized forces existing due to the non conservativity feature of the system. These equations are correspondent to the linked torque, that is dependent of the voltage in the motors. Also, it has a term of viscosity rotation in each axis that is contrary to the torque.

$$Q_1 = K_{pp} V_{m,p} + K_{py} V_{m,y} - B_p \dot{\theta} \tag{22}$$

$$Q_2 = K_{yp} V_{m,p} + K_{yy} V_{m,y} - B_y \dot{\psi} \tag{23}$$

The values of $K_{pp}$, $K_{py}$, $K_{yp}$ e $K_{yy}$ are determined from the torque and thrust constants and the electric resistance of the motor, as

$$K_{pp} = K_{f,p} r_p \tag{24}$$

$$K_{yy} = K_{f,y} r_y \tag{25}$$

$$K_{py} = \frac{K_{t,y}}{R_{m,y}} \tag{26}$$

$$K_{yp} = \frac{K_{t,p}}{R_{m,p}} \tag{27}$$

Using Eqs. 20 and 21 and the kinetic and potential energy given in Eqs. 7 and 6, the results are two differential equations that describe the system,

$$(J_{eq,p} + m_{heli} l_{cm}^2)\ddot{\theta} = K_{pp} V_{m,p} + K_{py} V_{m,y} - B_p \dot{\theta} - m_{heli} g l_{cm} cos\theta - m_{heli} l_{cm}^2 sin\theta cos\theta \dot{\psi}^2 \tag{28}$$

$$(J_{eq,y} + m_{heli} l_{cm}^2 cos\theta^2)\ddot{\theta} = K_{yy} V_{m,y} + K_{yp} V_{m,p} - B_y \dot{\psi} + 2 m_{heli} l_{cm}^2 sin\theta cos\theta \dot{\psi} \dot{\theta} \tag{29}$$

To analyze Eqs. 28 and 29, Quanser Inc. (2006) makes a linearization of these differential equations around a point of stability, where $\theta_0 = 0^o$, $\psi_0 = 0^o$, $\dot{\theta}_0 = 0^o$, $\dot{\psi}_0 = 0^o$. In this way, Eqs. 30 and 31 are obtained, as following

$$(J_{eq,p} + m_{heli} l_{cm}^2)\ddot{\theta} = K_{pp}V_{m,p} + K_{py}V_{m,y} - B_p\dot{\theta} - m_{heli} g l_{cm} \quad (30)$$

$$(J_{eq,y} + m_{heli} l_{cm}^2)\ddot{\psi} = K_{yy}V_{m,y} + K_{yp}V_{m,p} - B_y\dot{\psi} + 2m_{heli} l_{cm}^2 \theta\dot{\psi}\dot{\theta} \quad (31)$$

Using the states $x = [\,\theta \quad \psi \quad \dot{\theta} \quad \dot{\psi}\,]^T$, Quanser Inc. (2006) gives the linear state-space model as in Eq. 32 and 33, where $u = [\,V_{m,p} \quad V_{m,y}\,]^T$.

$$\dot{x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{B_p}{J_{eq,p}+m_{heli}l_{cm}^2} & 0 \\ 0 & 0 & 0 & -\frac{B_y}{J_{eq,p}+m_{heli}l_{cm}^2} \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{K_{pp}}{J_{eq,p}+m_{heli}l_{cm}^2} & \frac{K_{py}}{J_{eq,p}+m_{heli}l_{cm}^2} \\ \frac{K_{yp}}{J_{eq,p}+m_{heli}l_{cm}^2} & \frac{K_{yy}}{J_{eq,p}+m_{heli}l_{cm}^2} \end{bmatrix} u \quad (32)$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x \quad (33)$$

### 4.2 PARAMETRIC MODELLING

From the previously mentioned steps presented by Aguirre (2007), the process of black box modeling has begun to be studied, as its ways to be realized considering the MIMO system available.

As the system has non linear characteristics, especially in the pitch axis, it was decided to make the tests for modeling by means of applying several unit steps in the workstation with its axis in a balanced point, in order to obtain a linear model that would properly work in that limited track. The steps are applied in each motor individually, and the response is singly analyzed in each of the DOFs.

A data acquisition system using embedded electronics and computational resources was developed in order to execute the tests. The workstation motors are connected to a H-bridge, which control pins have voltage signals coming from an Arduino UNO board, responsible for enforcing the input signals and acquiring the values that come from the positioning sensors by means of an A/D converter. The computational software National Instruments® LabVIEW was used as a GUI platform for supervising the data acquisition and as the master of the network, enslaving the Arduino. Figures 4, 5 and 6 show images of the block diagram and the front panel of the virtual instrument developed for this application.

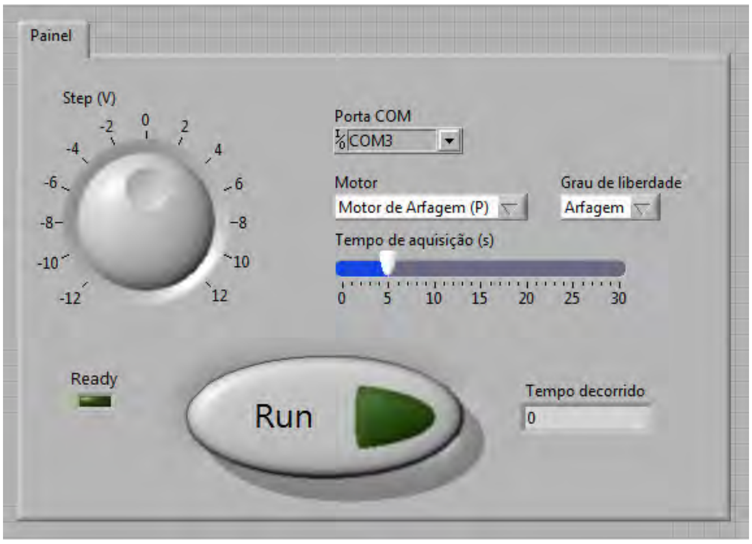


Figure 4. Front panel of the acquisition system

In this software, the user is able to select the step amplitude, from -12 V to 12 V, the motor that will be tested, the sensor that will be read and the test duration. In the end, the software provides to the user two spreadsheets, with information of the values read in the sensor and the time that these values have been read. The average sample time is about 10 ms.

## 5. RESULTS

In this section, the phenomenological modeling achieved results are presented.

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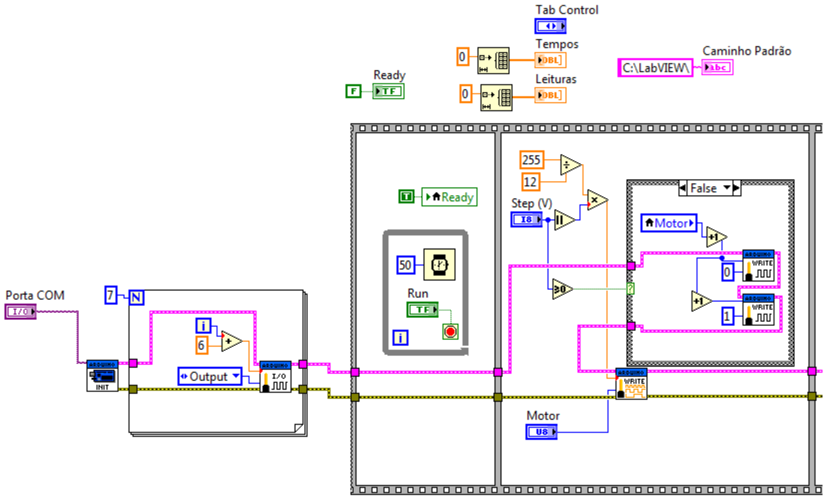


Figure 5. Block diagram of the acquisition system - Part 1

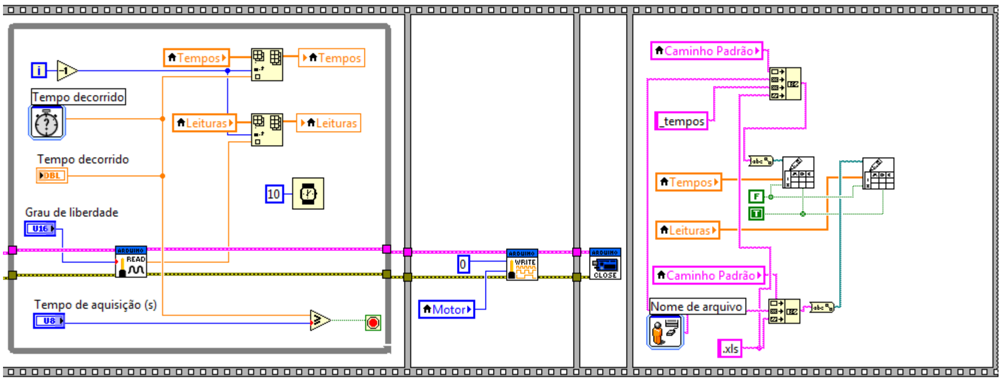


Figure 6. Block diagram of the acquisition system - Part 2

## 5.1 PHENOMENOLOGICAL MODELING

With the phenomenological modeling done, a practical stage began in order to to estimate the necessary parameters and finally achieve the resultant model.

The modeling takes in consideration almost the totality of the involved parameters presented in Section 4. However, some of them were disconsidered. The disconsidered values include the inertial moments of the rotor of the motors $J_{m,p}$ and $J_{m,y}$, that have a minor influence in the inertial moment equivalent in the pitch and yaw axis. In a first moment, the viscous damping, $B_p$ and $B_y$, were disconsidered to simplify the model. Later, at the validation process this choice will be analyzed. To obtain the intended parameters, tests were made in order to estimate direct measures (as mass, length and others) and tests to estimate indirect parameters, as angular displacement according to the current in the motors to discover the torque constants, friction coefficients and others.

In this way, in Tab. 1 the measures directly found are presented and, in an upcoming subsection, the procedures used to find some indirectly parameters through mathamatic manipulation are explained.

From the values found in Tab. 1, the inertial moments $J_{eq,p}$ and $J_{eq,y}$ can now be calculated, given by the Eq. 13 and Eq. 14, and the $l_{cm}$ given in Eq. 4. The obtained values are presented in Tab. 2.

### 5.1.1 Tests to Identify Parameters of the DC Motor

According to Gomes and Fenili (2009), it is known that a DC motor can be modeled through its electrical and mechanical characteristics, as shown in Eq. 34 and Eq. 35.

$$L_m \dot{i_a} + R_a i_a + K_b \dot{\theta} = U \qquad (34)$$

$$I_m \ddot{\theta} + C_m \dot{\theta} + K_t i_a = 0 \qquad (35)$$

ISSN 2176-5480



Table 1. Measured values

| Simbol | Description | Value | Unit |
|---|---|---|---|
| $m_{m,p}$ | Mass of pitch motor | $57.57 \times 10^{-3}$ | $kg$ |
| $m_{e,p}$ | Mass of pitch motor prop | $65.93 \times 10^{-3}$ | $kg$ |
| $m_{m,y}$ | Mass of yaw motor | $50.60 \times 10^{-3}$ | $kg$ |
| $m_{e,y}$ | Mass of yaw motor prop | $33.41 \times 10^{-3}$ | $kg$ |
| $r_p$ | Distance of the lever arm due to the pitch motor | -0.255 | $m$ |
| $r_y$ | Distance of the lever arm due to the yaw motor | 0.387 | $m$ |
| $L_{body}$ | Total length of the pitch axis body | 0.801 | $m$ |
| $m_{heli}$ | Total moving mass of the helicopter | $384.98 \times 10^{-3}$ | $kg$ |
| $m_{body,p}$ | Mass moving over the pitch axis | $274.10 \times 10^{-3}$ | $kg$ |
| $m_{body,y}$ | Mass moving over the yaw axis | $384.98 \times 10^{-3}$ | $kg$ |
| $m_{shaft}$ | Mass of the shaft rotating over the yaw axis. | $63.74 \times 10^{-3}$ | $kg$ |

Table 2. Equivalent inertial moments of the axis

| Simbol | Description | Value | Unit |
|---|---|---|---|
| $J_{eq,p}$ | Equivalent inertial moment of the pitch axis | $35.267 \times 10^{-3}$ | $kg.m^2$ |
| $J_{eq,y}$ | Equivalent inertial moment of the yaw axis | $41.218 \times 10^{-3}$ | $kg.m^2$ |
| $l_{cm}$ | Center of mass | $4.91 \times 10^{-3}$ | $m$ |

where $\dot{\theta}$ is the angular displacement of the motor axis, $i_a$ is the electrical current in the motor, $R_a$ is the armor resistance, $K_b$ is the counter-electromotive constant force, $U$ is the applied voltage, $I_m$ is the motor inertia, $C_m$ is the internal friction coefficient of the motor and $K_t$ is the torque constant of the motor.

Gomes and Fenili (2009) makes an analysis which was assumed in this paper. The temporal variation of the current $\dot{i_a}$ and the angular displacement $\ddot{\theta}$ are disconsidered, in way to obtain a simplified equation given by:

$$R_a i_a + K_b\dot{\theta} = U \quad (36)$$

$$C_m\dot{\theta} + K_t i_a = 0 \quad (37)$$

Through the obtained simplified models, it is possible to realize tests to discover the values of $R_a$ and $K_b$ by way of voltage, current and angular displacement read in the motors.

To make this test, an optocoupler system was built so that, through the frequency sampled by the propeller passing in a fix point, the analysis of angular displacement in function of the voltage and current values applied could be done. In way to find the best adjusted curve, the two values with the bigger standard deviation of $d\theta/dt$ was disconsidered. Using the least square method, the values of $R_a$ and $K_b$ for each motor have been obtained.

Further the consideration that the $\dot{i_a}$ and $\ddot{\theta}$ are discarded, Gomes and Fenili (2009) consider yet that $K_t$ and $K_b$ are sufficiently close to equal this terms. The same procedure is assumed in this paper, and, in this way, to determinate the values of $K_{t,y}$ and $K_{t,p}$.

Table 3. Values of the torque constants and electrical resistence of the motors

| Simbol | Description | Value | Unit |
|---|---|---|---|
| $K_{t,y}$ | Current-torque constant of yaw motor | 0.0156 | $N.m/A$ |
| $K_{t,p}$ | Current-torque constant of pitch motor | 0.0187 | $N.m/A$ |
| $R_{m,y}$ | Electrical resitence of yaw motor | 5.3228 | $\Omega$ |
| $R_{m,p}$ | Electrical resitence of pitch motor | 10.7942 | $\Omega$ |

For realizing the calculus of the generalized forces in the pitch and yaw axis, it is necessary to obtain the thrust force constant of the motors, given in $N/V$. For this purpose was made a test where was applied different values of voltage to the pitch and yaw motors hanged in a dynamometer. Through the data treatment, it was possible to build an adjusted curve to the obtained values and, then, to find the coefficient shown in Tab. 4.

By means of the values experimentally obtained, it is possible to determine the values of $K_{pp}$, $K_{yy}$, $K_{py}$ e $K_{yp}$ given by the Eqs. 24, 25, 26 and 27.

Table 4. Calculated values to the thrust force constants of the motors

| Simbol | Description | Value | Unit |
|---|---|---|---|
| $K_{f,p}$ | Thrust force constant of pitch motor/propeller | 0.0620 | $N/V$ |
| $K_{f,y}$ | Thrust force constant of yaw motor/propeller | 0.0957 | $N/V$ |

Table 5. Calculated values to the torque constants

| Simbol | Description | Value | Unit |
|---|---|---|---|
| $K_{pp}$ | Thrust torque constant acting on pitch axis from pitch motor/propeller | 1.581 | $N.m/V$ |
| $K_{yy}$ | Thrust torque constant acting on yaw axis from yaw motor/propeller. | 3.704 | $N.m/V$ |
| $K_{py}$ | Thrust torque constant acting on pitch axis from yaw motor/propeller. | $2.930 \times 10^{-3}$ | $N.m/V$ |
| $K_{yp}$ | Thrust torque constant acting on yaw axis from pitch motor/propeller. | $1.732 \times 10^{-3}$ | $N.m/V$ |

### 5.1.2 Resulting Model

The resulting model is given by using the found values in Eq. 30 and Eq. 31, and considering $g = 9,80665 m/s^2$,

$$35.28 \times 10^{-3}\ddot{\theta} = 1.581 V_{m,p} + 2.930 \times 10^{-3} V_{m,y} - 18.54 \times 10^{-3} \quad (38)$$

$$41.23 \times 10^{-3}\ddot{\psi} = 3.704 V_{m,y} + 1.732 \times 10^{-3} V_{m,p} + 18.56 \times 10^{-6}\theta\dot{\psi}\dot{\theta} \quad (39)$$

Using the linear state-space model given in Eq. 32 and 33, the response to the step is shown in Fig.7, where the columns represent the results to the step voltage $V_{m,p}$ and $V_{m,y}$, respectively. These responses show the behavior of $\theta$, $\psi$, $\dot{\theta}$ and $\dot{\psi}$, respectively, according to the input voltages in the motors.

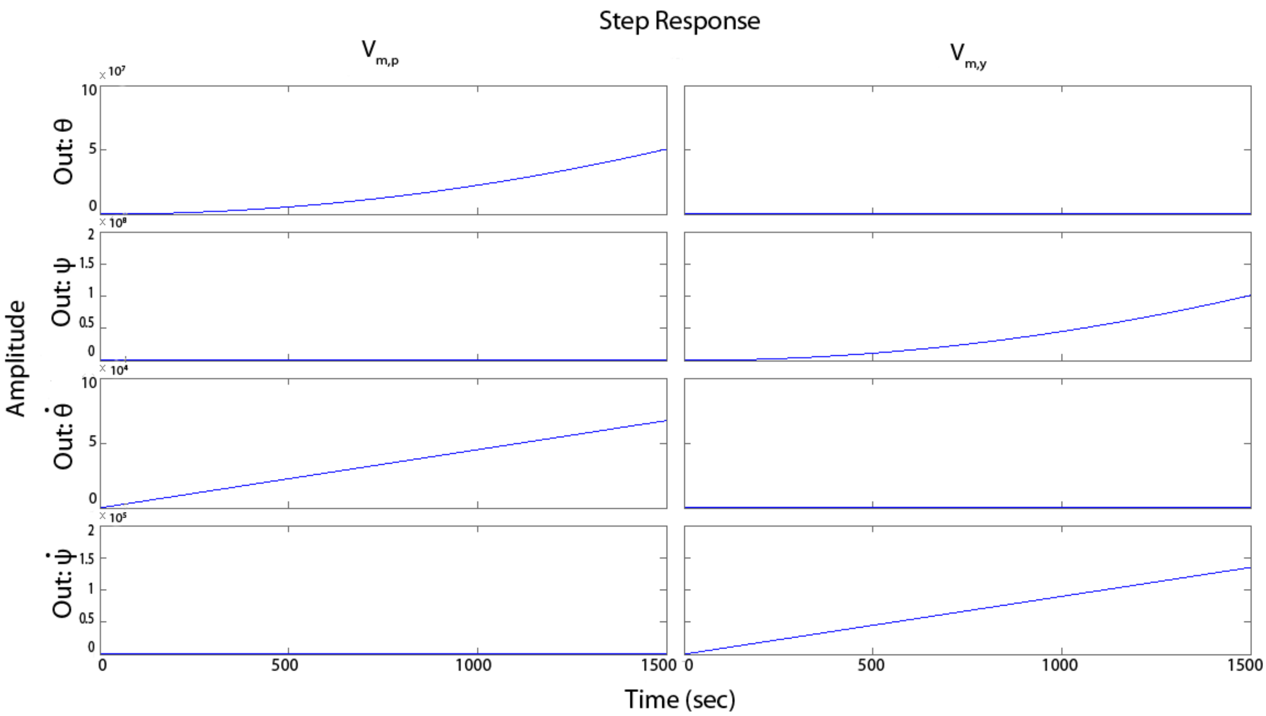


Figure 7. The response of the system to the step

## 6. CONCLUDING REMARKS

The main goals proposed in the beginning of this paper have been acquired. The several tests that have been made in the workstation gave an embracing idea of the main dynamics that must be handled in order to control it.

In the phenomenological modeling, a reasonable model was obtained. Though using some approximations to simplify the analysis, the linearization shows a fair representation of the system, in which the major influence of each motors

ISSN 2176-5480



voltage in each angular displacement is evident. The future perspectives for this reasoning embrace the comparison of this model with the real system in way to make adjustments and validate the technique.

The parametrical modeling has not achieved a model, but preliminary tests done with the data acquisition system have shown the main non linearities found in this workstation's behavior, mostly not seen in the white box model. All the efforts spent in these tests, which were not shown in this work because of the lack of conclusive and undisputed results, were worth to give an idea of the way that the workstation behaves in manifold situations. Some of these behaviors include non linearities in the step response, a large saturation in steady state analysis, a dead-zone region when the input is too low, delays in the response of the actuators, variance on time, among other shallow conclusions. Even so, all this achieved knowledge will be extremely useful in future works with this system, aiming to finally achieve a functional parametric model for applying control techniques.

The current works are concentrated in improving and validating the phenomenological model, studing the effects of the disregarded parameters. Also, the structure of the workstation has been improved to have an easier access to signals and apply control techniques.

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## 8. RESPONSIBILITY NOTICE

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