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DYNAMIC ANALYSIS OF A SERPENTINE BELT DRIVE WITH AUTOMATIC TENSIONER

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Abstract. *Abstract Serpentine belt drives are a recurrent application in automotive industry. A proper understanding of its dynamic behavior is of major importance for designing and to ensure adequate operation in different load cases. Aiming a better comprehension of the subject, it was implemented a mathematical model to describe its behavior. Torsional vibration degrees of freedom are used to describe the motion of pulleys and spring loaded tensioner arm, as well, while the belt spans are modeled as axially moving strings with transverse displacement. Infinitesimal deformations were regarded on belt spans to induce nonlinear effects due to elasticity and tension fluctuations. Elastic creep on belt due to friction was not considered, though. Theoretical modal analysis was performed to identify natural frequencies based on the system equations and properties. An experimental setup was used to validate some of the parameters calculated through the mathematical model and to verify steady state nonlinear effects which occur on the system. Good correlation between theoretical model and experimental setup was achieved, making it possible to verify variations on belt tension during dynamic analysis and its response to torque fluctuations applied on accessory pulleys.*

Keywords: *belt drive, vibration, serpentine belt, tensioner, dynamics*

1. INTRODUCTION

Belt drives are a widely used mechanism for power transmission, being capable of transmitting power between non concentric shafts, with advantages such as shock and vibration absorption with low cost of production and maintenance (Shigley *et al.*, 2008). A prominent application is observed on automotive industry, where it is being used on both driving camshaft and front end accessory drive.

Our attention relies here on front end accessory drive, or FEAD. Belt is used to transmit mechanical power generated by crankshaft motion to vehicle accessories placed on engine's front end, like alternator, steering pump, water pump, air conditioner compressor and others. To ensure proper belt tensioning and its operation, a belt tensioner is often used, particularly in cases where loads due to accessories or tension variation on belt are considerable.

Automotive engines powered by combustion cycles typically deliver variable power, in such way that torque and speed fluctuations are observed at the output. Torque fluctuations are the main excitation mechanisms, producing variation on operational tension and rotational vibration on driving pulleys (Hawker, 1991).

Several authors studied the dynamic behavior of belt drives with automatic tensioner. Ulsoy *et al.* (1985) describe moments acting on tensioner arm due to belt tension and parametric excitation produced by Mathieu instabilities. Beikmann (1992) performs a very complete theoretical and experimental analysis on belt drive vibration with coupling effects introduced by tensioner, evaluating belt tension variation as function of system operational speed (Beikmann *et al.*, 1997). A theoretical method is developed to determine natural frequencies and modeshapes (Beikmann *et al.*, 1996a), where a prototypical system is studied and modal properties are also evaluated experimentally by impact tests.

Nonlinear effects produced by belt infinitesimal elongation were investigated numerically through system's time response, determined by numerical integration of differential equations derived with modal superposition (Beikmann *et al.*, 1996b).

In this study, Beikmann's nonlinear method to determine time response is used to evaluate a real system where inertial properties of pulleys and tensioner are previously unknown. To deal with the leak of information, Experimental Modal Analysis is performed on system, in order to determine parameters such as natural frequencies, damping factors and modeshapes. With this data, nonlinear model is feed, together with belt and tensioner elastic properties, that are known. And then, one can derive system's time response, to conditions that would demand greater time and expensive equipment to test in laboratory.

Experimental Modal Analysis was performed through system excitation by random tangential force on pulley, applied by electromagnetic shaker, while responses were measured with accelerometers on discrete components and proximity probe on belt spans. Least Squares Complex Exponential Method (LSCE) is used for modal parameters extraction.

Once modal properties are known, theoretical time response can be determined. Numerically, system model is then excited by torque fluctuation with different amplitudes and frequencies in order to determine whether nonlinear effects occur and what are the conditions for this.

2. THEORETICAL MODEL

Aiming the study of a belt drive mechanism, it is considered a simplified mathematical model, in which all necessary components that takes part on a regular tensioner-loaded automotive front-end are used. For this, it is considered a system similar to the presented in Fig. 1, with three pulleys. Pulley 1 represents crankshaft, pulley 2 belongs to tensioner and pulley 4 is the accessory pulley. Tensioner moves rotationally and is loaded by a torsional spring while dry friction effects, usual in this device, is neglected.

This discrete components have vibrational degree of freedom $\theta_i(t)$, besides constant speed c/r_i given by belt motion, except for tensioner.

The belt has constant section A and elastic modulus E, and it is considered that the belt presents longitudinal and transverse displacements. Damping due to viscoelasticity and bending stiffness are neglected and is supposed the belt has constant translating speed c. Degrees of freedom $w_i(x,t)$ and $u_i(x,t)$ that represent transverse and longitudinal displacement of belt spans are considered continuous, depending on position x on the span, and time. Following the as-

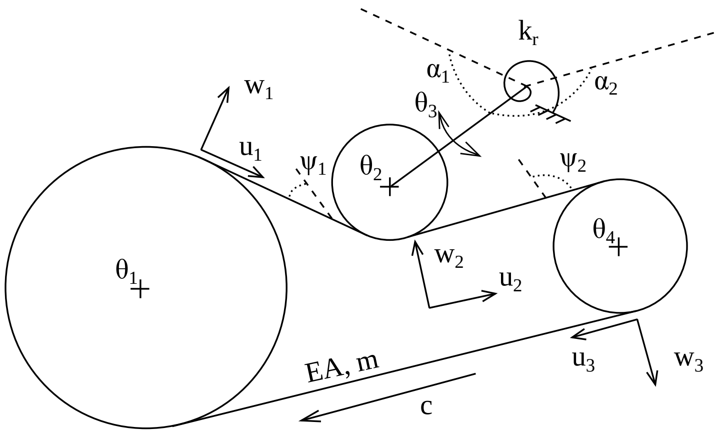

Figure 1. Geometrical model of the system.

sumptions made by Beikmann *et al.* (1996b), the governing equations for the problem, taking into account the elimination of equilibrium terms and assuming the infinitesimal belt stretching behaves in a quasi-static way, become:

$$mw_{i,tt} + 2mcw_{i,xt} - P_{ti}w_{i,xx} = P_{di}w_{i,xx} \qquad i = 1,2,3 \tag{1}$$

$$m_1\chi_{1,tt} = k_1\left(\chi_3\cos\psi_1 + \chi_2 - \chi_1\right) - k_3\left(\chi_1 - \chi_4\right) + F_{d1} + P_{d1NL} - P_{d3NL} \tag{2}$$

$$m_2\chi_{2,tt} = k_2\left(\chi_3\cos\psi_2 + \chi_2 - \chi_1\right) - k_1\left(\chi_3\cos\psi_1 + \chi_2 - \chi_1\right) + F_{d2} + P_{d2NL} - P_{d1NL} \tag{3}$$

$$\begin{aligned} m_3\chi_{3,tt} = & \left[-P_{t1}w_{1,x}(l_1,t) + mcw_{1,t}(l_1,t)\right]\sin\psi_1 + \left[P_{t2}w_{2,x}(0,t) - mcw_{2,t}(0,t)\right]\sin\psi_2 + \\ & - k_1(\chi_3\cos\psi_1 + \chi_2 - \chi_1)\cos\psi_1 - k_2(\chi_3\cos\psi_1 + \chi_2 - \chi_1)\cos\psi_2 + \\ & + F_{d3} - k_4\chi_3 - P_{d1NL}\cos\psi_1 + P_{d2NL}\cos\psi_2 \end{aligned} \tag{4}$$

$$m_4\chi_{4,tt} = k_3\left(\chi_1 - \chi_4\right) - k_2\left(\chi_3\cos\psi_1 + \chi_2 - \chi_1\right) + F_{d4} + P_{d3NL} - P_{d2NL} \tag{5}$$

where $k_4 = k_s + k_{gr}$, and

$$k_s = \frac{k_r}{r_3^2} \qquad k_{gr} = \frac{1}{r_3}\left(P_{t1}\sin\psi_1 - P_{t2}\sin\psi_2\right)$$

Subscripts i,j represent partial derivative of coordinate i with respect to variable j. For example $w_{1,xx}$ is the second derivative of span 1 transverse displacement with respect to coordinate x. Considering the substitutions $F_{di} = M_{di}/r_i$, $m_i = J_i/r_{i,}^2$, $\chi_i = r_i\theta_i$ e $k_i = EA/l_i$, where M_{di} is the dynamic torque applied externally on pulleys and tensioner, J_i are discrete elements moment of inertia, k_i are the equivalent linear stiffness of belt spans. Belt operational belt tension $P_{oi} = P_{ti} + mc^2$ are defined, where P_{ti} are tensions due to belt traction and mc^2 is the centrifugal tension.

It is important to note that equations for belt spans longitudinal displacement are not presented, once it's found that its motion is uncoupled of the remaining components (Beikmann, 1992).

We have also dynamic tension that arise from belt infinitesimal elongation, which has linear and nonlinear terms.

$$P_{d1} = k_1\left(\chi_3\cos\psi_1 + \chi_2 - \chi_1\right) + P_{d1NL} \tag{6}$$

$$P_{d2} = k_2\left(\chi_3\cos\psi_2 + \chi_2 - \chi_1\right) + P_{d2NL} \tag{7}$$

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$$P_{d3} = k_3 (\chi_1 - \chi_4) + P_{d3NL} \tag{8}$$

$$P_{diNL} = \frac{k_i}{2} \int_0^{l_i} w_{i,x}^2 dx \tag{9}$$

Equations 1-5 can be written on matrix form

$$[M] \left\{ \ddot{W} \right\} + [G] \left\{ \dot{W} \right\} + [K] \{W\} = \{Q\} \tag{10}$$

where excitation vector $\{Q\}$ comprehends external excitation forces and nonlinear terms.

$$\{Q\} = \left\{ \begin{array}{c} P_{d1} w_{i,xx} \\ P_{d2} w_{2,xx} \\ P_{d3} w_{3,xx} \\ F_{d1} + P_{d1NL} - P_{d3NL} \\ F_{d2} + P_{d2NL} - P_{d1NL} \\ +F_{d3} - P_{d1NL} \cos\psi_1 + P_{d2NL} \cos\psi_2 \\ F_{d4} + P_{d3NL} - P_{d2NL} \end{array} \right\} \tag{11}$$

This problem can be solved by means of state space transformation, the system is written as

$$[A] \left\{ \dot{U} \right\} + [B] \{U\} = \{X\} \tag{12}$$

$$[A] = \left[\begin{array}{cc} M & 0 \\ 0 & K \end{array} \right] \quad \text{e} \quad [B] = \left[\begin{array}{cc} G & K \\ -K & 0 \end{array} \right] \tag{13}$$

with $\{U\} = \left\{ \begin{array}{cc} \dot{W} & W \end{array} \right\}^T$ and $\{X\} = \left\{ \begin{array}{cc} Q & 0 \end{array} \right\}^T$.

Performing a Theoretical Modal Analysis on the homogeneous part of Eq. 12 produces eigenvalues $\pm i\omega_r$ and eigenvectors $\overline{U}_r$ associated to the eigenvalues with real and imaginary parts, $\overline{Y}_r$ and $\overline{Z}_r$ (Meirovitch, 1974).

According with Beikmann *et al.* (1996a) and Zhang and Zu (1999), orthogonality properties given by Eq. 14 are valid to this system, due to symmetry properties of generalized mass and stiffness matrices $[A]$ and $[B]$. Inner products are used instead of matrix products because belt span terms in eigenvectors have spatial dependence with coordinate x and inner product of eigenvectors with matrices $[A]$ and $[B]$ is able to describe orthogonality properties properly (Moon and Wickert, 1997).

$$\begin{array}{lll} \left\langle \overline{Y}_r, A\overline{Y}_s \right\rangle = \delta_{rs} & \left\langle \overline{Z}_r, A\overline{Z}_s \right\rangle = \delta_{rs} & \left\langle \overline{Z}_r, A\overline{Y}_s \right\rangle = 0 \\ \left\langle \overline{Y}_r, B\overline{Y}_s \right\rangle = 0 & \left\langle \overline{Z}_r, B\overline{Z}_s \right\rangle = 0 & \left\langle \overline{Z}_r, B\overline{Y}_s \right\rangle = \omega_s \delta_{rs} \end{array} \tag{14}$$

By the principle of modal superposition, the response for the system of equations can be achieved through a finite expansion of eigenvectors.

$$\{U(t)\} \approx \sum_{r=1}^{n} \left\{ \xi_r(t) \overline{Y}_r + \eta_r(t) \overline{Z}_r \right\} = [P] \{V(t)\} \tag{15}$$

where $[P] = \left[\begin{array}{ccccc} \overline{Y}_1 & \overline{Z}_1 & \overline{Y}_2 & \overline{Z}_2 & \ldots \end{array} \right]$ is the modal matrix with the modeshapes considered on the expansion and vector V contains the generalized modal coordinates for real (ξ_r) and imaginary (η_r) parts of eigenvector.

Substituting Eq. 15 on Eq. 12, and completing the inner products with $[P]$, we have

$$\langle P, AP \rangle \left\{ \dot{V}(t) \right\} + \langle P, BP \rangle \{V(t)\} = \langle P, X(t) \rangle \tag{16}$$

$$[I] \left\{ \dot{V}(t) \right\} + [H] \{V(t)\} = \{R(t)\} \tag{17}$$

Matrix $[I]$ is the identity and $[H]$ is a block-diagonal matrix with the natural frequencies of the system. Each couple of equations from Eq. 17 describe time behavior of a mode r. To introduce damping to the system, we can consider a equivalent viscous damping factor, ζ_r, for each modeshape. Equations become then,

$$\dot{\xi}_r(t) - \omega_r \eta_r(t) = \left\langle \overline{Y}_r, X(t) \right\rangle - 2\zeta_r \omega_r \xi_r(t) \tag{18}$$

$$\dot{\eta}_r(t) + \omega_r \xi_r(t) = \left\langle \overline{Z}_r, X(t) \right\rangle \tag{19}$$

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With this set of equations, it is possible to investigate nonlinear effects produced by dynamic tensions upon the system. For this, one can make use of modeshapes, natural frequencies and damping factors obtained by means of a Experimental Modal Analysis to feed Eqs. 18-19, and then determine time response with influence of nonlinear terms, without the need to instrument a mechanism under uncontrolled conditions to measure this effects.

Excitation considered is produced by inner product between eigenvectors obtained through Modal Analysis and excitation vector described by motion equations of the system and depends only of physical properties of the belt, tensioner spring and system geometry. Previous knowledge on inertia properties of the discrete components, such as pulleys and tensioner arm, is not really required.

3. EXPERIMENTAL TEST

For tests it was used a belt drive assembly built on a inertial test bench, and consists of a real engine front-end with two pulleys, one for the crankshaft and another for alternator, and a spring loaded tensioner with idler pulley on its ending.

It was performed a Experimental Modal Analysis on the system. Excitation was provided by an electromagnetic shaker attached to the mechanism by thin stinger bolted with a load cell on pulley 1. Responses were acquired through piezoelectric accelerometers fixed tangentially to the pulleys and tensioner arm, and for belt spans it was used a proximity probe close to the span on non nodal points at lower frequencies.

Figure 2 shows test setup and details for stinger and proximity sensor assembly. Random excitation was applied to the system and Frequency Response Functions were obtained using H_1 estimator based on cross-correlation spectral densities. Acceleration signals acquired by accelerometers were filtered and integrated to displacement signals and FRFs representing system's receptance as shown on Fig. 3. Data acquisition was performed using 10kHz as sample rate and a total of 1×10^6 points were recorded.

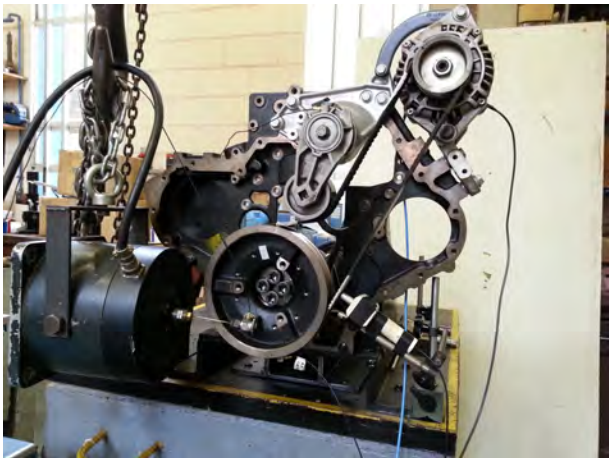

Figure 2. Experimental Setup.

Modal parameter estimation were applied by means of Least Squares Complex Exponential Method (LSCE) which is a SIMO technique based on system's Impulse Response Function. This method considers that some modal properties, such as natural frequencies and damping factors, are global, being the same at any degree of freedom, while the modal participation of each mode is estimated individually for each degree of freedom, based on values of poles obtained (Maia and Silva, 1997).

Due to system's high damping, produced by viscoelastic properties of belt and dry friction effects on tensioner, extracted modes have high damping factors, and it is verified a large number of computational modes. In some modes, coupling of rotational motion of pulleys with transverse modes of spans is increased, as a consequence of belt bending stiffness. Natural frequencies and damping factor estimated by a 37 order model are shown on Tab. 1. It should be noted

Table 1. Natural Frequencies and Damping Factors estimated.

Mode	Frequency [Hz]	Damping Factor [%]	Description
1	58.95	0.66	First rotational mode
2	71.83	0.20	First transverse mode of span 3
3	109.58	4.53	Second rotational mode
4	121.56	0.30	First transverse mode of span 2
5	124.79	2.60	Rotational + bending mode
6	143.96	0.23	Second transverse mode span 3

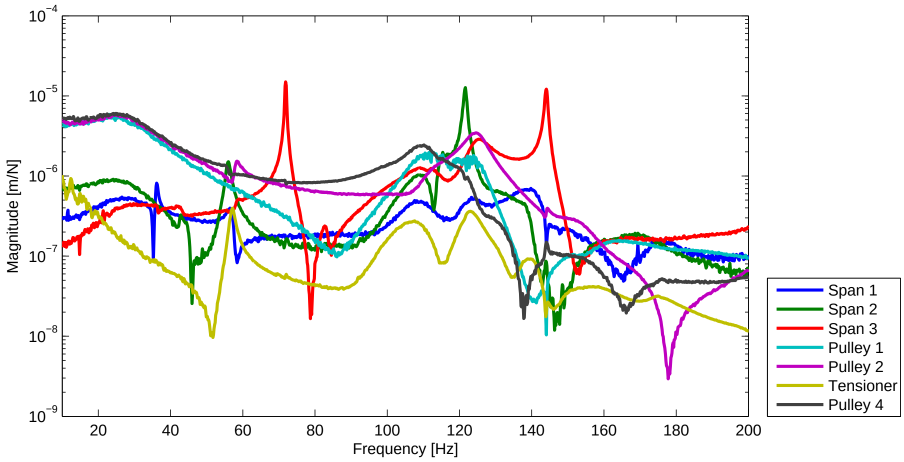

Figure 3. Frequency Response Functions Estimated for the system.

though, that modal amplitudes for belt spans are obtained locally. So to correct this, one can consider a function that describes each belt span modeshape. Sack (1954), proposed a solution for the homogeneous equation of a string transverse vibration with axial motion, and it was applied by Beikmann *et al.* (1996a) to describe belt span behavior.

Applying the same solution for the case in study, we have

$$\overline{v}_{ir} = \overline{a}_{3r} \sin\left(\frac{\omega_r x}{c_i'}\right) \exp\left(i\frac{\omega_r x}{c_{ai}'}\right) \tag{20}$$

$$c_1' = \frac{c_i^2 - c^2}{c_i} \qquad c_{ai}' = \frac{c_i^2 - c^2}{c} \tag{21}$$

where $c_1 = \sqrt{P_{o1}/m}$ and c is the transport speed of the system.

To determine modal amplitude for span 1 at a mode r, $\overline{a}_{1r}$, when the measurement was performed at a point $x = x_{k1}$, one can simply divide the modal amplitude, A_{1r}, obtained in the parameter estimation, by the function that describes the modeshape.

$$\overline{a}_{1r} = A_{1r}\left[\sin\left(\frac{\omega_r x_{k1}}{c_1'}\right) \exp\left(i\frac{\omega_r (x_{k1} - l_1)}{c_a'}\right)\right]^{-1} \tag{22}$$

Modeshapes for first 4 natural frequencies are presented on Fig. 4. One can see that first mode's motion is ruled by rotations of tensioner and pulley 2, mode 2 is governed by belt span 3 transverse vibration, mode 3 presents coupling between span 2 transverse displacement and tensioner rotation and mode 4 is purely transverse mode of span 2.

4. NUMERICAL SIMULATION

Once modeshapes and poles were obtained, we can define modal matrix. Eigenvectors, written on state space become

$$\{\overline{Y}_r\} = \begin{Bmatrix} -\omega_r \overline{v}_{1r}^I \\ -\omega_r \overline{v}_{2r}^I \\ -\omega_r \overline{v}_{3r}^I \\ 0 \\ 0 \\ 0 \\ 0 \\ \overline{v}_{1r}^R \\ \overline{v}_{2r}^R \\ \overline{v}_{3r}^R \\ \overline{a}_{4r} \\ \overline{a}_{5r} \\ \overline{a}_{6r} \\ \overline{a}_{7r} \end{Bmatrix} \qquad \{\overline{Z}_r\} = \begin{Bmatrix} \omega_r \overline{v}_{1r}^R \\ \omega_r \overline{v}_{2r}^R \\ \omega_r \overline{v}_{3r}^R \\ \omega_r \overline{a}_{4r} \\ \omega_r \overline{a}_{5r} \\ \omega_r \overline{a}_{6r} \\ \omega_r \overline{a}_{7r} \\ \overline{v}_{1r}^I \\ \overline{v}_{2r}^I \\ \overline{v}_{3r}^I \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{23}$$

where $\overline{v}_{ir}^R$ and $\overline{v}_{ir}^I$ are eigenfunctions that describe real and imaginary parts of spatial behavior of belt spans.

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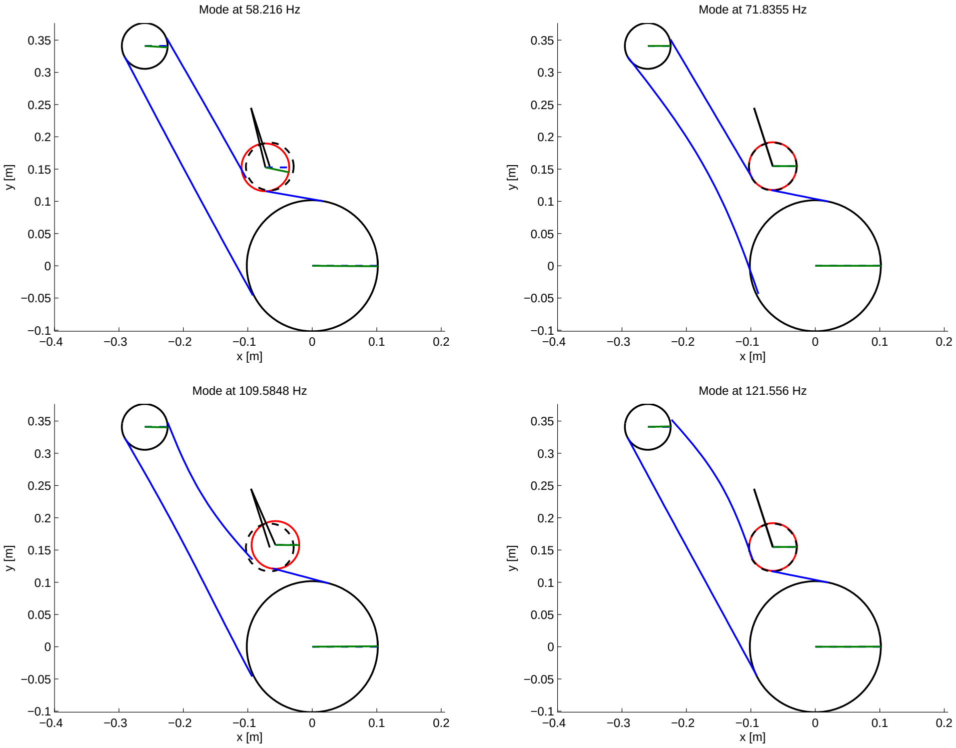

Figure 4. Modeshapes obtained through modal estimation.

Belt spans displacement w_i which occurs on nonlinear dynamic tensions can be expressed in terms of modal superposition, becoming

$$w_i(x,t) \approx \sum_{r=1}^{n} \left(\xi_r \overline{v}_{ir}^{R} + \eta_r \overline{v}_{ir}^{I} \right) \tag{24}$$

Finally, using a 3 mode expansion, it is possible to describe system's time response to initial conditions and external excitations in frequency range that is adequate do describe the system in real operating conditions. Modal matrix is

$$[P] = \left[\begin{array}{cccccc} \overline{Y}_1 & \overline{Z}_1 & \overline{Y}_2 & \overline{Z}_2 & \overline{Y}_3 & \overline{Z}_3 \end{array} \right] \tag{25}$$

Simulations were performed on Matlab version 7.1 R14. As mentioned before, some elastic properties of the system and its geometry must be known in order to apply the modal superposition with nonlinear effects. In test case, belt modulus is $EA = 155000\text{N}$ and linear density, $m = 0.1082\text{kg/m}$.

For a first test case, it is considered that system is excited by a torque fluctuation applied by pulley 1 with 10Nm magnitude and frequency of 30Hz. Figure 5 shows time behavior of generalized coordinates related to real part of eigenvectors and Fig. 6 presents physical response of discrete elements to excitation conditions proposed. As one can see from Figs. 5 and 6, time responses of the system have usual periodic behavior. In contrast, when system is excited by high levels of torque fluctuation, nonlinearities are excited and time responses present different harmonics and its amplitude is modulated. Figure 7 shows time response for a system excited in pulley 1 by a 250Nm torque fluctuation at 30Hz.

In a second test case, with high excitation amplitude, it is verified that mode 3 is more excited than in a situation with low amplitude, even being out of its resonance frequency range. Mode 3 presents a slight energy exchange with mode 2, while mode 1 is barely excited, due to its high damping factor.

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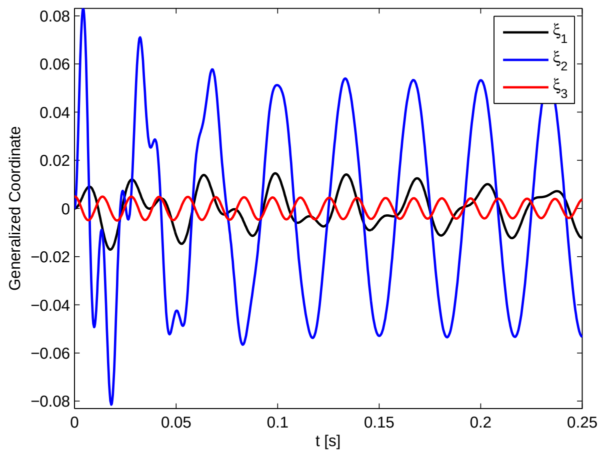

Figure 5. Response to 30Hz excitation with low amplitude (10Nm).

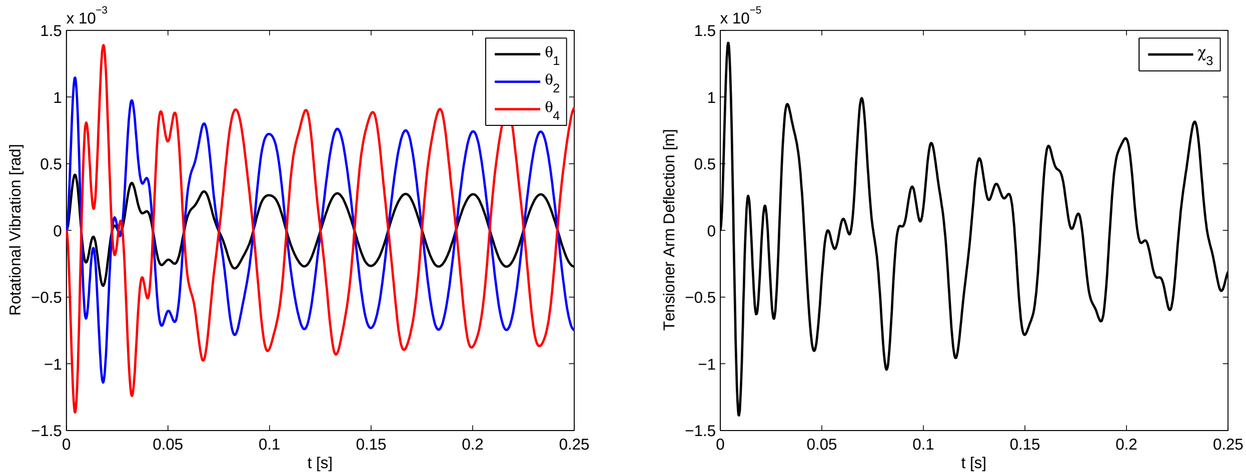

Figure 6. Physical response of system to 30Hz excitation with low amplitude (10Nm).

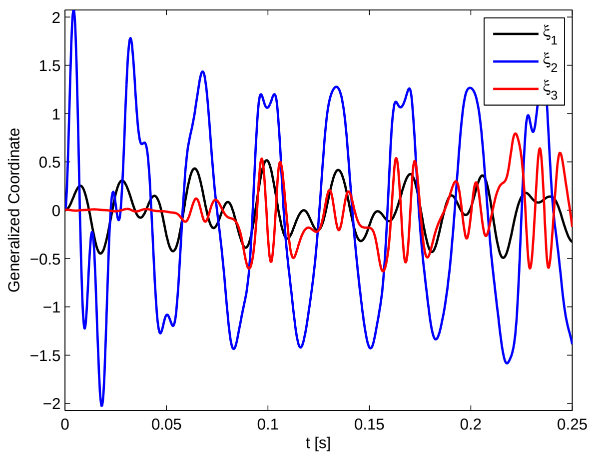

Figure 7. Response to 30Hz excitation with high excitation (250Nm).

Another test case was evaluated considering excitation on third resonance. First using 10Nm amplitude torque fluctuation and then using 50Nm amplitude torque fluctuation. It is observed, from Fig. 8, that for lower amplitude of excitation, time response is just as expected, presenting typical resonance pattern for second mode, but for higher excitation third mode is also excited, producing nonlinear effects changing second mode's response.

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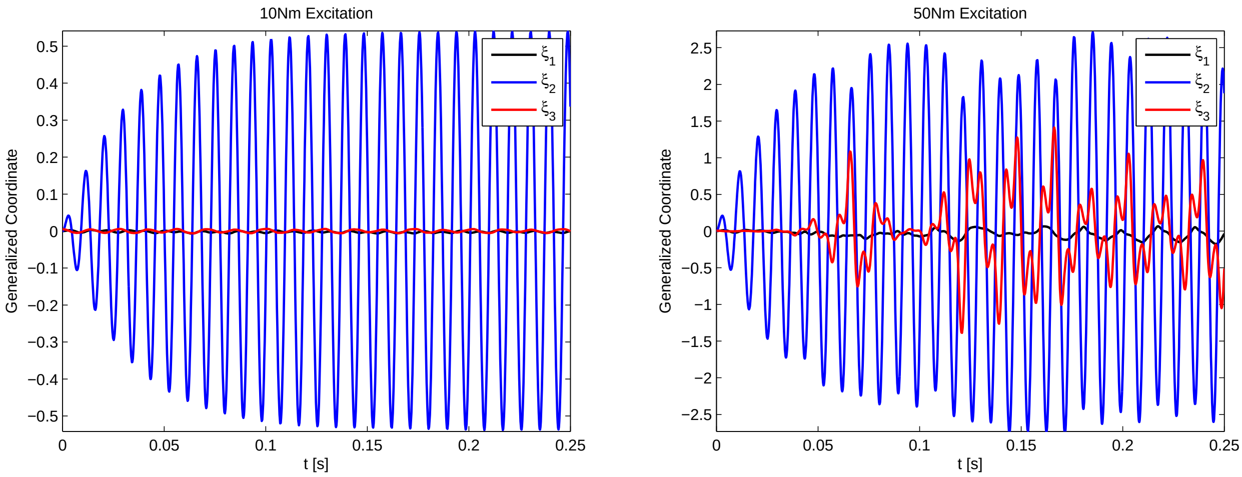

Figure 8. Response to 109.58Hz excitation with 10Nm and 50Nm amplitudes.

Dynamic tension for excitation at 109.58Hz (third resonance) and 10Nm amplitude is presented in Fig. 9. As one can see, even to low amplitudes tension variation is great. In this case, tension fluctuation is only due to linear terms, while nonlinear terms are not excited, in contrast to the situation observed on Fig. 10, where dynamic tension is increased by nonlinear terms.

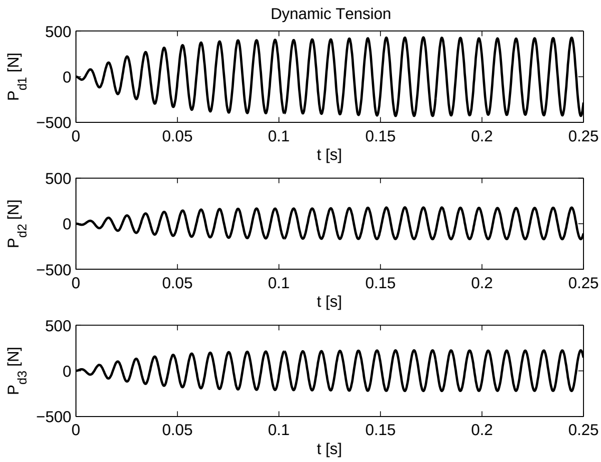

Figure 9. Dynamic tension on belt spans due to excitation of 10Nm at 109.58Hz.

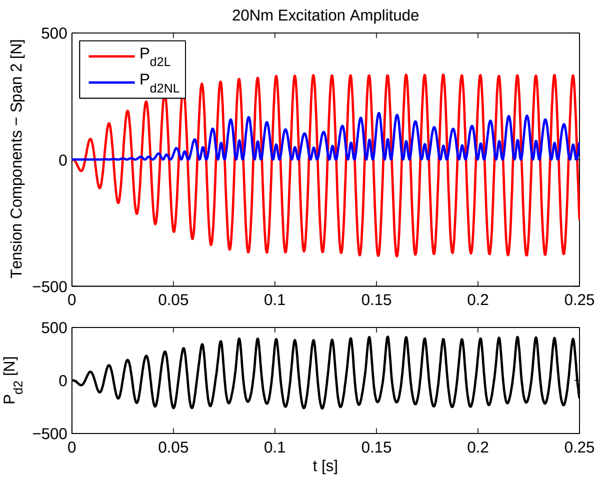

Figure 10. Dynamic tension on belt spans due to excitation of 20Nm at 109.58Hz for span 2 with linear (P_{d2L}) and nonlinear (P_{d2NL}) components.

5. CONCLUSIONS

As proposed, belt drive system dynamic behavior was evaluated by experimental and theoretical studies. In a first step equations needed to model mechanism functioning and nonlinear effects, as derived by Beikmann *et al.* (1996b), under vibration conditions are presented. Modal superposition is used to eliminate equations' spatial dependence and make them adequate to numerical integration procedures.

To obtain necessary data to use the model, a Experimental Modal Analysis was conducted, where natural frequencies, damping factors and modeshapes of a real system were determined. Great amount of damping in system is verified mainly due to dry friction on tensioner arm and belt viscoelasticity. In some modeshapes, coupling between free span and tensioner subsystem is observed, pointing out to bending stiffness effects, in contrast to theoretical model where this aspect is neglected.

Once a set of data containing estimated modal parameters was defined, theoretical model was fed and time responses determined for different exciting conditions. It was observed that the system presents major rotational modes of vibration in frequency range of operation, with transverse modes of belt spans occurring in higher frequencies than ones affected by engine normal operation.

Exciting system model through torque fluctuations produce linear harmonic responses to small excitations and when excitations have high amplitude, nonlinear effects take place. Modes that were not excited before, present elevated responses. Nonlinear effects are intensified by responses of modeshapes or combination of modeshapes where occur greater belt span displacements, due to physical characteristics of nonlinear tension.

Method showed good results, being capable of adjusting mathematical model to experimental data, making it possible to perform a preliminar evaluation of time response of the system and nonlinear effects identification produced by torque fluctuation excitation. This kind of condition could only be experimentally evaluated instrumenting a real engine on a dynamometer, while cost and time to perform a single modal analysis is really inferior.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Beikmann, R.S., Perkins, N.C. and Ulsoy, A.G., 1996a. "Free vibration of serpentine belt drive systems". *Journal of Vibration and Acoustics*, Vol. 118, pp. 406–413.

Beikmann, R.S., Perkins, N.C. and Ulsoy, A.G., 1996b. "Nonlinear coupled vibration response of serpentine belt drive sistems". *Journal of Vibration and Acoustics*, Vol. 118, pp. 567–574.

Beikmann, R.S., 1992. *Static and Dynamic Behavior of Serpentine Belt Drive Systems: Theory and Experiment*. Ph.D. thesis, The University of Michigan.

Beikmann, R., Perkins, N. and Ulsoy, A., 1997. "Design and analysis of automotive serpentine belt drive systems for steady state performance". *Journal of Mechanical Design*, Vol. 119, p. 162.

Hawker, L.E., 1991. *A Vibration Analysis of Automotive Serpentine Accessory Drive Systems*. Ph.D. thesis, University of Windsor.

Maia, N. and Silva, J., 1997. *Theoretical and Experimental Modal Analysis*. Engineering dynamics series]. Research Studies Press Limited. ISBN 9780471970675.

Meirovitch, L., 1974. *Computational Methods in Structural Dynamics*. Monographs and textbooks on mechanics of solids and fluids. Mechanics, Dynamical systems. Springer. ISBN 9789028605800.

Moon, J. and Wickert, J.A., 1997. "Non-linear vibration of power transmission belts". *Journal of Sound and Vibration*, Vol. 200, pp. 419–431.

Sack, R.A., 1954. "Transverse oscillations in travelling strings". *British Journal of Applied Physics*, Vol. 5, No. 6, p. 224.

Shigley, J., Nisbett, J. and Budynas, R., 2008. *Shigley's mechanical engineering design*. McGraw-Hill series in mechanical engineering. McGraw-Hill. ISBN 9780073121932.

Ulsoy, A.G., Whitesell, J.E. and Hooven, M.D., 1985. "Design of belt-tensioner systems for dynamic stability". *Journal of Vibration, Acoustics, Stress and Reliability in Design*, Vol. 107, pp. 282–290.

Zhang, L. and Zu, J., 1999. "Modal analysis of serpentine belt drive systems". *Journal of Sound and Vibration*, Vol. 222, No. 2, pp. 259 – 279. ISSN 0022-460X. doi:10.1006/jsvi.1998.2078.

8. RESPONSIBILITY NOTICE

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