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HEAT TRANSFER IN THE POWER-LAW FLUID FLOW IN A POROUS MEDIUM

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Abstract. *This work presents an analytical solution for fully developed forced convection of power-law fluid in a channel partially filled with porous medium. The lower part of the channel is occupied by a clear fluid while the upper part is occupied by a fluid-saturated porous medium. A uniform heat flux is imposed at the lower part of the channel. The energy and the momentum macroscopic equations are utilized for obtained the analytical solution for temperature distributions, as well as for the Nusselt number.*

Keywords: *analytical solution, heat transfer, porous medium, power-law fluid, shear stress jump.*

1. INTRODUCTION

In petroleum production there are many parameters that must be analyzed before deciding if an oil well is economically viable and whether the oil exploitation from a natural reservoir is possible or not. One of the main parameters usually considered by financial analysts and the technical staff to enable oil production is the estimation of the reservoir Productivity Index (PI). In this sense, mathematical tools can be crucial to take a decision, since it is usually not possible to reproduce the problem in experimental simulation because of the high complexity of the problem in question.

In this context, the mathematical tool plays a significant role in several industries. It has been the focus of many private and public agencies for development and researches around the world.

Ochoa-Tapia and Whitaker (1995a-*b*) proposed an analytical expression to take into account the variation of the shear stress jump at the interface between clear fluid and porous medium.

Kuznetsov (1996-99) used in his works the boundary condition proposed by Ochoa-Tapia and Whitaker (1995a-*b*) to obtain an analytical solution for a Newtonian fluid flow in a channel partially filled with porous material.

Inoue and Nakayama (1998) investigated the viscous and inertia effects in pressure drop in non-Newtonian fluid flow across a porous medium. The porous medium was simulated by periodic spatially array of cubes. The numerical results were used to obtain a macroscopic relationship between pressure gradient and mass flow rate.

Pearson and Tardy (2002) presented an overview on the continuum transport models in porous media, and the length scale needed to transport the phenomena from the pore scale to Darcy continuum scale, using variables average. The authors examined the influence of non-Newtonian rheology to the parameters of transport mono and multi-phase, that is, the Darcy viscosity, the dispersion length and the relative permeability.

Papatzacos and Skjæveland (2006) studied the diffuse-interface model for the two-phase flow of a one-component fluid in a porous medium with the following characteristics: (i) a unified treatment of two phases as manifestations of one fluid with a van der Waals type equation of state, (ii) the inclusion of wetting, and (iii) the absence of relative permeabilities. The authors show that relative permeabilities depend on the spatial derivatives of the saturation.

Chandesris and Jamet (2006) investigated the velocity boundary condition that must be imposed at an interface between a porous medium and a free fluid. They concluded that the continuity of the velocity is recovered and a jump in the stress built using the viscosity appears. These results also indicated an explicit dependence of the stress jump coefficient to the internal structure of the transition zone and its sensitivity to this microstructure is recovered. de Lemos and Silva (2006) studied the turbulent fluid flow in channel partly filled with porous material. The results indicated that depending on the value of the stress jump parameter, substantially dissimilar fields for the turbulence energy are obtained. Negative values for the stress jump parameter gave results closer to experimental data for the turbulent kinetic energy at the interface. de Lemos and Silva (2006) analyzed the turbulent flow fluid in the channel composed by porous region and a clear fluid region. They conclude that the penetration extent of turbulence was Darcy number and porosity-dependent.

Al-Amiri, et al., (2008) investigated the wall heat conduction effect on the natural-convection heat transfer within a two-dimensional cavity, filled with a fluid-saturated porous medium. The authors concluded that the temperature of the interface is sensitive the dimensionless groups: Darcy, Rayleigh numbers and the porosity, area ratio, etc. Kumar, et al., (2009) analyzed the fully developed combined free and forced convective flow in a fluid saturated porous medium channel bounded by two vertical parallel plates, where the fluid flow was modeled using Brinkman equation model. Kumar, et al., (2009) obtained analytical solutions for the governing ordinary differential equations by perturbation series method and found that the presence of porous matrix in one of the region reduces the velocity and temperature.

Valdés-Parada, et al., (2009) studied the momentum transfer between a homogeneous fluid and a porous medium in a system analogous to the one used by Beavers and Joseph (1967) , using volume averaging techniques. The authors proposed a closed generalized momentum transport equation that is valid everywhere and is expressed in terms of position-dependent effective transport coefficients.

Nield and Kuznetsov (2009) modeled analytically the fluid flow in a three-layer channel composed by a transition layer sandwiched between a porous medium and a fluid clear of solid material. Kuznetsov and Nield (2010) presented an analytic investigation of forced convection in parallel-plate channel partly occupied by a bidisperse porous medium and partly by a fluid clear of solid material. They authors found a singular behavior of the Nusselt number for the case of asymmetric heating. Saito and de Lemos (2010) proposed a model for turbulent flow and heat transfer in a highly porous medium applied to a porous channel bounded by parallel plates. Saito and de Lemos (2010) showed that for laminar and turbulent flows the thermal dispersion mechanism leads to larger local temperature differences.

Aguilar-Madera, et al., (2011) solved effective-medium equations for modeling momentum and heat transfer in a parallel-plate channel partially filled with a porous insert. The authors found that the thermal performance is improved by either increasing the size of the porous insert or by favoring mixing inside the channel.

Singh, et al., (2011) studied the transient as well as non-Darcian effects on laminar natural convection flow in a vertical channel partially filled with porous medium. The authors obtained, using perturbation technique, approximate solutions for velocity field with Darcy number, Grashof number, kinematic viscosity ratio, distance of interface and variations in temperature distribution with thermal conductivity ratio.

Silva and de Lemos (2011) investigated the turbulent flow in channel with a centered porous material. This work showed that the increasing the size of the porous material pushes the flow outwards, increasing the levels of turbulent kinetic energy at the macroscopic interface.

Nimvari, et al., (2012) studied the turbulent flow and heat transfer through a partially porous channel. The authors found that the turbulent kinetic energy is significant in both the clear fluid region and the porous region. Furthermore, they show that the peak of turbulent kinetic energy occurs around the porous/fluid interface and penetration depth of turbulent kinetic energy in the porous layer is independent of *Da* number.

The work of Cekmer, et al., (2012) studies the fully developed heat and fluid flow in a parallel plate channel partially filled with porous layer. The authors show results that for a partially porous filled channel, the value of overall performance is highly influenced from Darcy number, but it is not affected from thermal conductivity ratio (k_r) when k_r>2.

However, so far, there seems to be in the literature no mathematical tool able to reproduce adequately the power-law fluid flow in a channel with porous material. Therefore, this work aims to present analytical solution, along with an appropriate boundary condition for variation of the exchange of momentum in the interface for the steady state fully developed power-law fluid flow, with constant properties, permeating a channel partially filled with homogeneous and isotropic porous material saturated by an incompressible and monophasic fluid.

2. PROBLEM FORMULATION

2.1 Geometry

Figure (1) describes a schematic diagram of a fully developed power-law fluid flow in steady state in a channel partially filled with porous material. The fluid flow with constant properties flows from left to right, permeating the porous structure (porous medium) and the clear region (clear fluid). A uniform heat flux is imposed at the lower plate. The boundary conditions used are: y=0, no-slipping condition; y=H, symmetry condition.

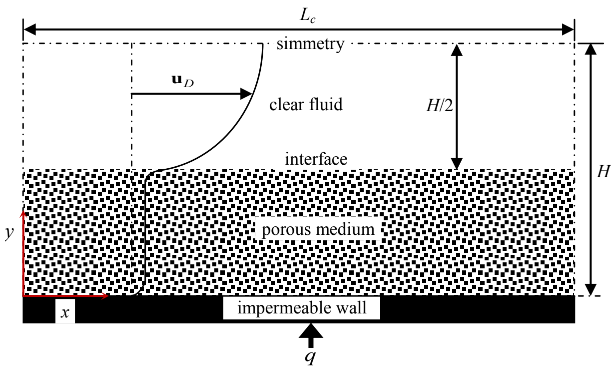

Figure 1: Scheme of a fluid flow in a channel partially filled with porous material.

2.2 Governing Equations

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The macroscopic equations governing the power-law fluid flow in channel with a rigid, homogeneous, isotropic porous medium saturated by an incompressible fluid, without taking into account the Forchheimer drag term, has the following form:

$$\frac{du_D}{dx}=0 \qquad \text{for} \qquad 0\le y\le H \tag{1}$$

$$-\frac{d\left(\phi\langle p\rangle^i\right)}{dx}+\frac{d}{dy}\left(\eta_{PM}\frac{du_D}{dy}\right)-\frac{m\phi}{K}\left(\frac{|u_D|}{\phi\sqrt{K}}\right)^{n-1}u_D=0 \qquad \text{for} \qquad 0\le y\le \frac{H}{2} \tag{2}$$

$$\rho c_f u_D\frac{\partial\langle T\rangle^i}{\partial x}=k_{eff}\frac{\partial^2\langle T\rangle^i}{\partial y^2} \qquad \text{for} \qquad 0\le y\le \frac{H}{2} \tag{3}$$

$$-\frac{d\langle p\rangle^i}{dx}+\frac{d}{dy}\left(\eta\frac{du_D}{dy}\right)=0 \qquad \text{for} \qquad \frac{H}{2}\le y\le H \tag{4}$$

$$\rho c_f u_D\frac{\partial\langle T\rangle^i}{\partial x}=k_f\frac{\partial^2\langle T\rangle^i}{\partial y^2} \qquad \text{for} \qquad \frac{H}{2}\le y\le H \tag{5}$$

where

$$\eta_{PM}=m\left|\frac{1}{\phi}\frac{du_D}{dy}\right|^{n-1} \text{ and } \eta=m\left|\frac{du_D}{dy}\right|^{n-1} \tag{6}$$

are, respectively, the apparent viscosity in a porous medium, η_{PM}, and the apparent viscosity in a clear fluid, η. The variable K is the permeability of the porous media, ϕ is the porosity of the porous media, $\langle p\rangle^i$ is the intrinsic average pressure, u_D is the Darcy velocity, n is the flow behavior index, m is the consistency index of the fluid, c_f is the specific heat of the fluid, k_{eff} is the effective thermal conductivity of the porous medium, k_f is the thermal conductivity of the fluid, $\langle T\rangle^i$ is the intrinsic average temperature.

2.3 Boundary Conditions

In $y=0$, the no-slipping condition and the uniform heat flux have been applied, i.e.,

$$u_D=0 \tag{7}$$

$$q=-k_{eff}\frac{\partial T}{\partial y} \tag{8}$$

In $y=\frac{H}{2}$, condition of continuity of temperature and of velocity (see [0], [0]), shear stress jump condition, proposed by Silva and de Lemos (2013), and the local thermal equilibrium are assumed:

$$u_D\big|_{CF}=u_D\big|_{PM}=u_{Di} \tag{9}$$

$$\langle T\rangle^i\Big|_{PM}=\langle T\rangle^i\Big|_{CF}=\langle T\rangle^i_i \tag{10}$$

$$\frac{\eta_{PM}}{\phi}\frac{du_D}{dy}\bigg|_{PM}-\eta\frac{du_D}{dy}\bigg|_{CF}=\eta_i^*\frac{\beta}{\sqrt{K}}u_{D_i} \tag{11}$$

$$k_{eff}\left.\frac{\partial\langle T\rangle^i}{\partial y}\right|_{PM}=k\left.\frac{\partial\langle T\rangle^i}{\partial y}\right|_{CF} \tag{12}$$

where

$$\eta_i^*=m\left(\frac{\left|u_{D_i}\right|}{\phi\sqrt{K}}\right)^{n-1} \tag{13}$$

where η_i^* represents the apparent viscosity of the fluid at the interface, u_{D_i} is the Darcy velocity component parallel to the interface and β an adjustable coefficient which accounts for stress jump at the interface.

The shear stress jump condition is applied at interface between porous medium and clear fluid to have a matching of diffusion fluxes across the interface.

Note that by making the flow behavior index, n, equal to one and the fluid consistency index, m, equal to the dynamic viscosity, μ, in the Eqs. (6) and (13), the apparent viscosities presented in Eq. (11) resumes its original form, first proposed by Ochoa-Tapia and Whitaker (1995a-b).

In $y=H$, the symmetry condition has been applied:

$$\left.\frac{du_D}{dy}\right|_{y=H}=\left.\frac{\partial\langle T\rangle^i}{\partial y}\right|_{y=H}=0 \tag{14}$$

2.4 Dimensionless Variables

$$Da=\frac{K}{H^2},\quad u=u_D\left(\frac{m}{GH^{(n+1)}}\right)^{1/n},\quad Y=\frac{y}{H},\quad X=\frac{x}{L_c},\quad G=-\frac{d\langle p\rangle^i}{dx},\quad \theta=\frac{\langle T\rangle^i-T_w}{\overline{T}-T_w},\quad R=\frac{k_f}{k_{eff}} \tag{15}$$

where T_w is the temperature at the wall and q is the uniform heat flux imposed at the lower plate.

2.5 Dimensionless Governing Equations

For a fully developed region of the channel with a uniform wall heat flux, the first law of thermodynamics gives:

$$\frac{\partial\langle T\rangle^i}{\partial x}=\frac{\partial\overline{T}}{\partial x}=\frac{q}{\rho c_f H\overline{u}}=\text{constant} \tag{16}$$

where the mean flow velocity is defined as:

$$\overline{u}=\frac{1}{H}\int_0^H u_D\,dy \tag{17}$$

The Nusselt number for this problem can then be defined as:

$$Nu=\frac{qH}{k_{eff}\left[T_w-\overline{T}\right]} \tag{18}$$

where the mean flow temperature is defined as:

$$\overline{T}=\frac{1}{\overline{u}H}\int_0^H u_D\langle T\rangle^i\,dy \tag{19}$$

Substituting the apparent viscosities (Eq. 6) and the dimensionless variables (Eq. (15)) in the Eqs. (1) to (5), we obtain:

$$\frac{du}{dX}=0 \qquad \text{for} \qquad 0\le Y\le 1 \tag{20}$$

$$1+\frac{1}{\phi^n}\frac{d}{dY}\left(\frac{du}{dY}\right)^n-\frac{1}{\phi^{(n-1)}Da^{(n+1)/2}}u^n=0 \qquad \text{for} \qquad 0\le Y\le \frac{1}{2} \tag{21}$$

$$-Nu\left(\frac{u_D}{\bar{u}}\right)=\frac{d^2\theta}{dY^2} \qquad \text{for} \qquad 0\le Y\le \frac{1}{2} \tag{22}$$

$$1+\frac{d}{dY}\left(\frac{du}{dY}\right)^n=0 \qquad \text{for} \qquad \frac{1}{2}\le Y\le 1 \tag{23}$$

$$-Nu\left(\frac{u_D}{\bar{u}}\right)=R\frac{d^2\theta}{dY^2} \qquad \text{for} \qquad \frac{1}{2}\le Y\le 1 \tag{24}$$

where R is the thermal conductivity ratio and Da is the Darcy number.

2.6 Dimensionless Boundary Conditions

Applying the apparent viscosities (Eqs. (6), (13)) and the dimensionless variables (Eq. (15)) in the boundary conditions – Eqs. (9) – (14), we obtain:

At wall, $Y=0$:

$$u=\theta=0 \tag{25}$$

At interface, $Y=\frac{1}{2}$:

$$u\big|_{CF}=u\big|_{PM}=u_i \tag{26}$$

$$\theta\big|_{PM}=\theta\big|_{CF}=\theta_i \tag{27}$$

$$\left(\frac{1}{\phi}\frac{du}{dY}\bigg|_{PM}\right)^n-\left(\frac{du}{dY}\bigg|_{CF}\right)^n=\frac{\beta}{\phi^{(n-1)}Da^{n/2}}u_i^n \tag{28}$$

$$\frac{\partial\theta}{\partial Y}\bigg|_{PM}=R\frac{\partial\theta}{\partial Y}\bigg|_{CF} \tag{29}$$

At symmetry, $Y=1$:

$$\frac{du}{dY}\bigg|_{Y=1}=\frac{d\theta}{dY}\bigg|_{Y=1}=0 \tag{30}$$

2.7 Velocity Distribution in the Channel

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The dimensionless velocity distribution in the channel is obtained through the solutions of the dimensionless momentum equations, Eqs. (21) and (23), and utilizing the boundary conditions given by Eqs. (25), (26) and (30) (more details can be found in Silva and de Lemos (2013)):

$$u = -\left(\frac{B\left(e^{-A/2} - 1\right) + u_i}{\left(e^{-A/2} - e^{A/2}\right)}\right) e^{AY} + \left(\frac{B\left(e^{A/2} - 1\right) + u_i}{\left(e^{-A/2} - e^{A/2}\right)}\right) e^{-AY} + B \qquad \text{for} \qquad 0 \le Y \le \frac{1}{2} \tag{31}$$

$$u = \frac{n}{n+1}\left[\left(\frac{1}{2}\right)^{n+1/n} - \left(-Y+1\right)^{n+1/n}\right] + u_i \qquad \text{for} \qquad \frac{1}{2} \le Y \le 1 \tag{32}$$

where

$$A = \left(\frac{\phi}{n}\right)^{\frac{1}{n+1}} \frac{1}{Da^{1/2}}, \qquad B = \frac{Da^{(n+1)/2n}}{\phi^{\frac{(1-n)}{n}}}, \qquad c_1 = -\left(\frac{B\left(e^{-A/2} - 1\right) + u_i}{\left(e^{-A/2} - e^{A/2}\right)}\right), \qquad c_2 = \frac{B\left(e^{A/2} - 1\right) + u_i}{\left(e^{-A/2} - e^{A/2}\right)}, \qquad c_3 = 1 \qquad \text{and}$$

$$c_4 = u_i + \frac{n}{n+1}\left(\frac{1}{2}\right)^{n+1/n} \tag{33}$$

Applying the boundary condition (Eq. (28)), it is obtained:

$$\left(-\frac{A}{\phi}\left(\frac{B\left(2 - e^{A/2} - e^{-A/2}\right) + u_i\left(e^{A/2} + e^{-A/2}\right)}{\left(e^{-A/2} - e^{A/2}\right)}\right)\right)^n - \left[\frac{1}{2} + \frac{\beta}{\phi^{(n-1)}}\left(\frac{u_i}{Da^{1/2}}\right)^n\right] = 0 \tag{34}$$

The u_i, is the velocity at interface between porous medium and clear fluid, obtained from the transcendental Eq. (34) by Newton's method.

2.8 Temperature Distribution in the Channel

The dimensionless temperature in the porous medium ($0 \le Y \le \frac{1}{2}$) is obtained by the integrating the dimensionless energy equation, Eq. (22), and utilizing the boundary conditions given by Eqs. (25), (27) and (30):

$$\frac{\theta_{PM}}{Nu} = -\frac{1}{A^2 \bar{u}}\left[c_1\left(e^{AY} - 1\right) + c_2\left(e^{-AY} - 1\right) + B\frac{A^2 Y^2}{2} - 2\left[c_1\left(e^{0.5A} - 1\right) + c_2\left(e^{-0.5A} - 1\right) + BA^2\frac{1}{8}\right]Y\right] + 2\frac{\theta_i}{Nu}Y \tag{35}$$

where $\bar{u}\left(= \bar{u}_{PM} + \bar{u}_{CF}\right)$ is the mean flow velocity in the channel, given by:

$$\bar{u} = \frac{1}{A}\left[c_1\left(e^{\frac{A}{2}} - 1\right) - c_2\left(e^{-\frac{A}{2}} - 1\right)\right] + \frac{B}{2} + \frac{n}{n+1}\left(\frac{1}{2}\right)^{n+1/n}\left(\frac{1}{2} - \frac{n}{4n+2}\right) + \frac{u_i}{2} \tag{36}$$

The dimensionless temperature in the clear fluid ($\frac{1}{2} \le Y \le 1$) is obtained by the integrating the dimensionless energy equation, Eq. (24), and utilizing the boundary conditions given by Eqs. (27) and (30):

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$$\frac{\theta_{CF}}{Nu} = -\frac{1}{\bar{u}R}\left[\frac{n}{n+1}\left(\frac{1}{2}\right)^{n+1/n} + u_i\right]\frac{Y^2}{2} + \frac{1}{\bar{u}R}\frac{n^3}{(n+1)(2n+1)(3n+1)}(-Y+1)^{3n+1/n} + \frac{1}{\bar{u}R}\left[\left(\frac{1}{2}\right)^{n+1/n}\frac{n}{n+1} + u_i\right]Y + \frac{\theta_i}{Nu}$$
$$-\frac{1}{\bar{u}R}\frac{n}{n+1}\left(\frac{1}{2}\right)^{3n+1/n}\left[\frac{n^2}{(2n+1)(3n+1)} + \frac{3}{2}\right] - \frac{3}{8}\frac{1}{\bar{u}R}u_i \tag{37}$$

Where the θ_i is the temperature at interface between porous medium and clear fluid, obtained by using the boundary conditions given by Eq. (29):

$$\frac{\theta_i}{Nu} = -\frac{1}{2\bar{u}}\left\{\frac{n}{n+1}\left(\frac{1}{2}\right)^{n+1/n}\left[\frac{n}{4n+2} - \frac{1}{2}\right] - u_i\frac{1}{2} - \frac{1}{A^2}\left[c_1\left[e^{0.5A}(A-2)+2\right] - c_2\left[e^{-0.5A}(A+2)-2\right] + \frac{BA^2}{4}\right]\right\} \tag{38}$$

The Nusselt number can found from the following compatibility condition (Bejan, 2004):

$$\bar{u} = \int_0^1 \theta u dY \tag{39}$$

2.9 Nusselt Number

The Nusselt number can be found by substituting the velocity and temperature distributions into compatibility condition given by Eq (39). This results in the following equation for the Nusselt number:

$$Nu = \frac{\bar{u}}{\psi_1 + \psi_2} \tag{40}$$

The parameters ψ_1 e ψ_2 are defined as:

$$\psi_1 = \int_0^{0.5}\frac{\theta_{PM}}{Nu}udY \quad \text{and} \quad \psi_2 = \int_{0.5}^{1}\frac{\theta_{CF}}{Nu}udY \tag{41}$$

The value of ψ_1 can then be found as:

$$\psi_1 = -\frac{c_1}{2A^3\bar{u}}\left[c_1e^{A} + Ac_2 + 2Be^{0.5A} - 2c_1e^{0.5A} + 2c_2e^{-0.5A} - AB - 2B + c_1 - 2c_2\right]$$
$$-\frac{c_2}{2A^3\bar{u}}\left[-c_2e^{-A} + Ac_1 - 2Be^{-0.5A} - 2c_1e^{0.5A} + 2c_2e^{-0.5A} - AB + 2B + 2c_1 - c_2\right]$$
$$-\frac{B}{2\bar{u}}\left\{c_1\left[\frac{0.25e^{0.5A}}{A} - \frac{e^{0.5A}}{A^2} + \frac{2e^{0.5A}}{A^3}\right] - c_2\left[\frac{0.25e^{-0.5A}}{A} + \frac{e^{-0.5A}}{A^2} + \frac{2e^{-0.5A}}{A^3}\right] + \frac{B}{24} - \frac{2}{A^3}(c_1 - c_2)\right\}$$
$$+\left[2\frac{c_1\left(e^{0.5A}-1\right)}{A^2\bar{u}} + 2\frac{c_2\left(e^{-0.5A}-1\right)}{A^2\bar{u}} + \frac{B}{4\bar{u}} + 2\frac{\theta_i}{Nu}\right]\left[c_1\left(\frac{Ae^{0.5A} - 2e^{0.5A} + 2}{2A^2}\right) - c_2\left(\frac{Ae^{-0.5A} + 2e^{-0.5A} - 2}{2A^2}\right) + B\frac{1}{8}\right] \tag{42}$$

The value of ψ_2 can then be found as:

$$
\begin{aligned}
\psi_2 = & \frac{11}{48\bar{u}R}\left[\frac{n}{n+1}\left(\frac{1}{2}\right)^{n+1/n}+u_i\right]^2 \\
& +\frac{1}{\bar{u}R}\frac{n^2}{(n+1)(2n+1)}\left(\frac{1}{2}\right)^{4n+1/n}\left[\frac{n}{n+1}\left(\frac{1}{2}\right)^{n+1/n}+u_i\right]\left[\frac{1}{2}+\frac{n}{3n+1}\left(\frac{5n+1}{4n+1}\right)\right] \\
& +\frac{1}{\bar{u}R}\frac{n^4}{(n+1)(2n+1)(3n+1)(4n+1)}\left[\frac{n}{n+1}\left(\frac{1}{2}\right)^{n+1/n}+u_i\right]\left(\frac{1}{2}\right)^{4n+1/n}+ \\
& -\frac{1}{\bar{u}R}\frac{n^5}{(n+1)^2(2n+1)(3n+1)(5n+2)}\left(\frac{1}{2}\right)^{5n+2/n}-\frac{1}{\bar{u}R}\frac{n^2}{(n+1)(2n+1)}\left(\frac{1}{2}\right)^{3n+1/n}\left[\left(\frac{1}{2}\right)^{n+1/n}\frac{n}{n+1}+u_i\right]\left[\frac{4n+1}{3n+1}\right]+ \\
& \left\{\frac{\theta_i}{Nu}-\frac{1}{\bar{u}R}\frac{n}{n+1}\left(\frac{1}{2}\right)^{3n+1/n}\left[\frac{n^2}{(2n+1)(3n+1)}+\frac{3}{2}\right]-\frac{3}{8}\frac{1}{\bar{u}R}u_i\right\}\left[\frac{n}{n+1}\left(\frac{1}{2}\right)^{2n+1/n}+\frac{u_i}{2}-\frac{n^2}{(n+1)(2n+1)}\left(\frac{1}{2}\right)^{2n+1/n}\right]
\end{aligned}
\qquad (43)
$$

3. RESULTS AND DISCUSSION

Table 1 shows the influence of Darcy number, *Da*, and the thermal conductivity ratio, *R*, in the fluid velocity at the interface, u_i, in the mean flow velocity, $\bar{u}$, in the flow temperature at the interface, θ_i, in the mean flow temperature, $\bar{\theta}$, and the Nusselt number for stress jump coefficient, β=0, flow behavior index *n*=0.6 and porosity ϕ=0.6. Note that increasing the Darcy number implies increased velocities, u_i and $\bar{u}$ and temperatures θ_i and $\bar{\theta}$ which leads to an increase in the Nusselt number. Furthermore, it is possible to verify that the increase in thermal condutivity ratio causes also an increase in the Nusselt number.

Table 1: Influence of Darcy number, *Da*, and thermal conductivity ratio, *R*, in the fluid flow parameters.

β=0, *n*=0.6, ϕ=0.6						
Da	*R*	u_i	$\bar{u}$	θ_i	$\bar{\theta}$	*Nu*
10^{-5}	0.5	5.98×10^{-4}	2.17×10^{-2}	0.519	0.590	1.038
	1			0.684	0.646	1.367
	1.5			0.764	0.673	1.528
10^{-4}	0.5	1.90×10^{-3}	2.24×10^{-2}	0.522	0.595	1.044
	1			0.686	0.648	1.372
	1.5			0.766	0.674	1.532
10^{-3}	0.5	6.12×10^{-3}	2.49×10^{-2}	0.533	0.603	1.068
	1			0.696	0.655	1.394
	1.5			0.775	0.679	1.552
10^{-2}	0.5	2.19×10^{-2}	3.55×10^{-2}	0.591	0.651	1.212
	1			0.750	0.695	1.537
	1.5			0.823	0.715	1.687

In Fig. (2) is presented the influence of the Darcy, *Da* $(=K/H^2)$, in the behavior pseudoplastic fluid flow, for a shear stress jump coefficient, β=0, porosity, ϕ=0.6 and thermal ratio, *R*=1. It can be seen that an increase in the Darcy number causes an increase in mass flow rate in the permeable layer, which indicates an increase in the permeability value of the porous structure, which propagates throughout the channel and leads to increased mass flow rate through the clear fluid.

Figure (3) displays the distributions of the dimensionless temperature, θ, for pseudoplastic fluid, *n*=0.6, with shear stress jump coefficient, β=0, porosity, ϕ=0.6 and thermal ratio, *R*=1. This figure shows a decrease of the temperature with a decrease in the Darcy number.

Figure (4) shows the influence of the Darcy number and of flow behavior index, *n*, on Nusselt number for β=0, porosity, ϕ=0.6 and thermal ratio, *R*=1. It can be seen that until $Da=1\times10^{-3}$, the higher the flow behavior index, *n*, the smaller the value of the Nusselt number. However, the growth rate of the Nusselt number is higher for *n*=1.4, which leads to an inversion in the behavior of the curve from $Da=1\times10^{-3}$, causing Nusselt number increases with *n*. It should be noted that we use a Nusselt number based on the half distance between the plates, which is *H*, while a Nusselt number used in Kuznetsov (1998) (which we denote by Nu_K) is based on 2*H*. Therefore, $Nu=Nu_K/2$. Moreover, the figure shows a good agreement between the analytical solution here presented and the analytical solution proposed by Kuznetsov (1998) for a Newtonian fluid.

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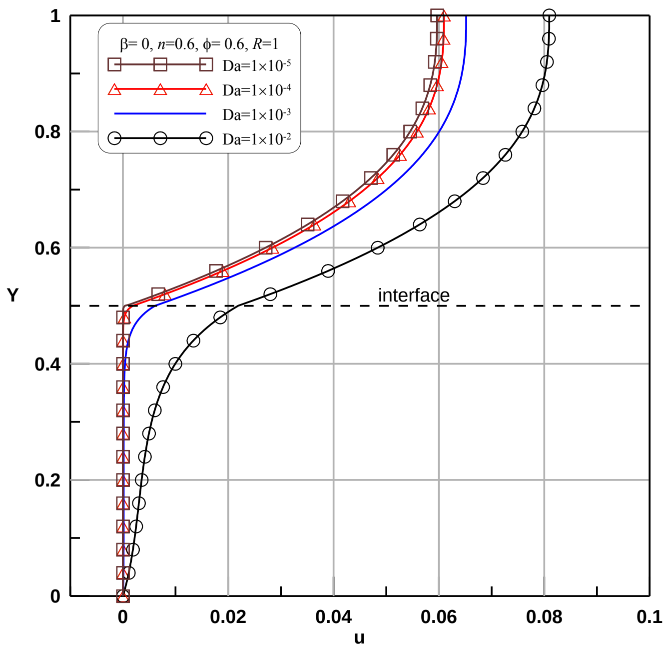

Figure 2 : Effect of Darcy on velocity distribution

Figure (5) displays the effect of Darcy and of porosity, ϕ, on Nusselt number for β=0, porosity, n=0.6 and thermal ratio, R=1. It can be seen that the influence the porosity on Nusselt number is only important for $Da>1\times10^{-3}$. For $Da>1\times10^{-3}$, the lower the porosity the greater the value of the Nusselt number.

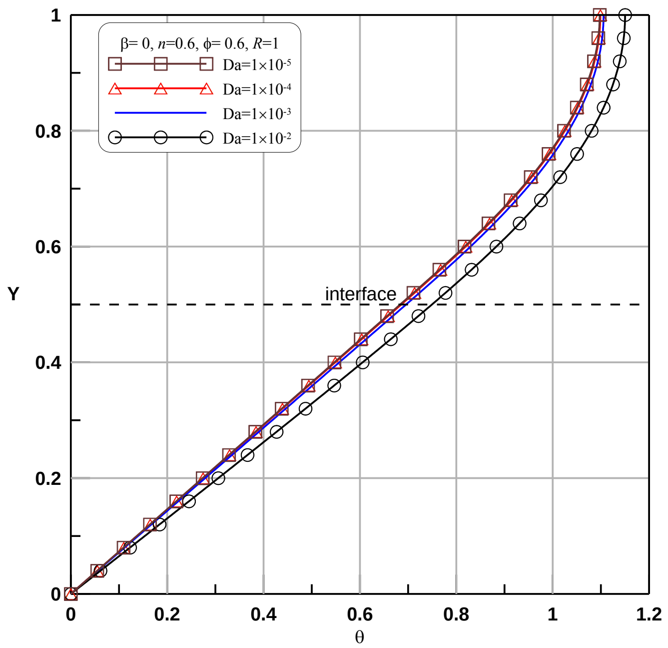

Figure 3: Effect of Darcy on temperature distribution

Figure (6) displays the dependence of the Nusselt number on the Darcy number and on the thermal conductivity ratio, $R = k_f / k_{eff}$, for β=0, porosity, ϕ=0.6 and thermal ratio, n=0.6. The case R>1 corresponds to the situation when thermal conductivity of the fluid is larger than thermal conductivity of the porous material and R<1 the inverse. As it can be seen, the Nussel number remains practically constant until $Da=1\times10^{-3}$, after this value Nusselt number increases with thermal conductivity ratio. Furthermore, as expected, the increase R value causes an in the Nusselt number.

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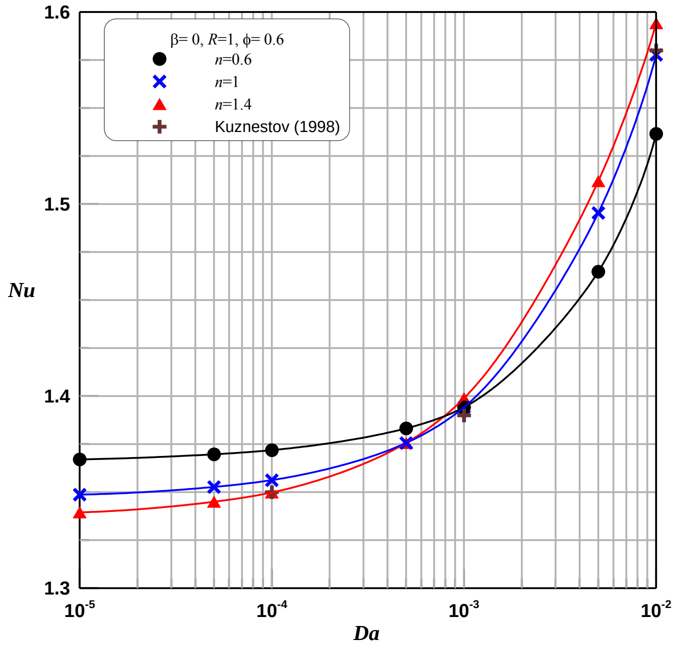

Figure 4: Effect of Darcy and of flow behavior index, *n*, on Nusselt number

Figure (7) depicts the dependence of the Nusselt number on the Darcy number and on the shear stress jump coefficient β for the boundary condition indicating jump in the shear stress at interface between porous medium and clear fluid. Figure (7) is computed for n=0.6, porosity, ϕ=0.6 and thermal ratio, R=1. It can be seen that the coefficient β have appreciable influence on Nusselt number. On the other hand, the increase of the Darcy number implies in the increase of the Nusselt number.

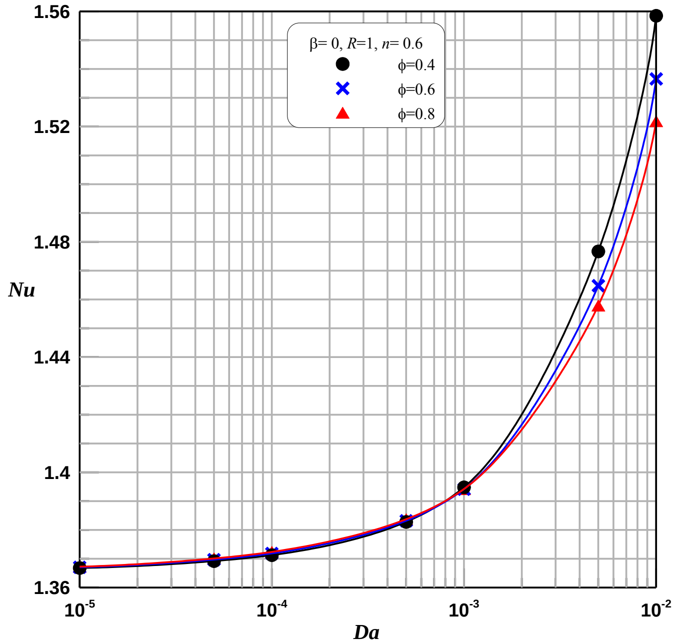

Figure 5: Effect of Darcy and of porosity, ϕ, on Nusselt number

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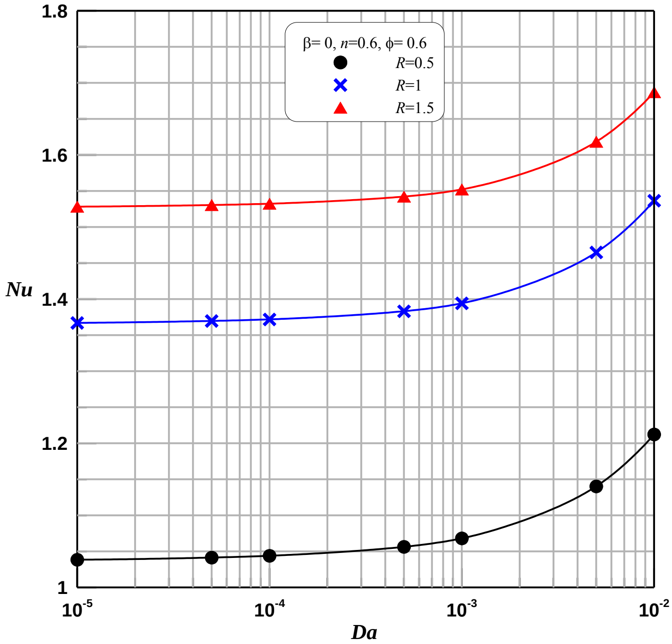

Figure 6: Effect of Darcy and of thermal ratio, R, on Nusselt number

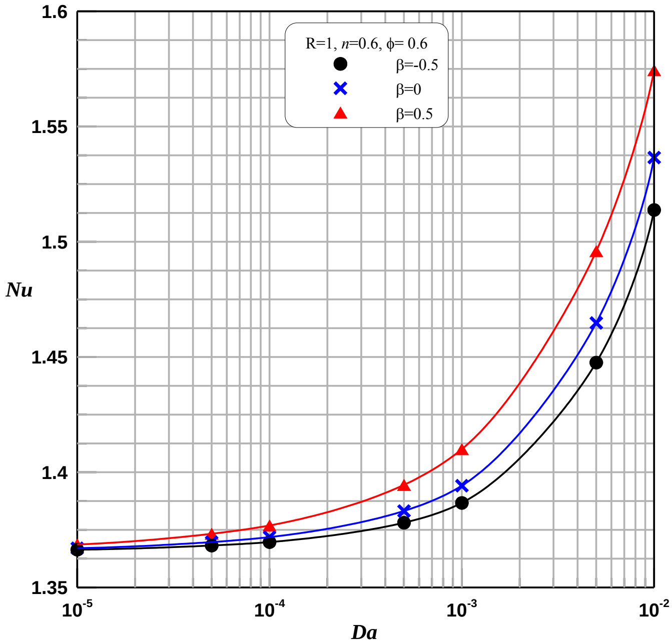

Figure 7: Effect of Darcy and of stress jump coefficient, β, on Nusselt number

4. CONCLUSIONS

This work presents an analytical solution for fully developed forced convection of power-law fluid in a channel partially filled with porous medium. Comparison of this work with the previously published results of Kuznetsov (1998) for the case of a Newtonian fluid has shown a good agreement. It was observed that the Nusselt number strongly depends of the number Darcy, *Da*, and of the thermal conductivity ratio, *R*. The new analytical solution obtained in this paper makes it possible to extensively investigate possibilities of enhanced heat transfer by changing values of pertinent parameters. It is valuable for gaining a deeper insight in understanding the transport processes at the interface between the porous medium and the clear fluid.

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