

## ACOUSTIC CHARACTERIZATION OF A LIQUID-PROPELLANT ROCKET COMBUSTION CHAMBER BY NUMERICAL AND ANALYTICAL MODELS

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**Abstract.** *The purpose of this work is to characterize the acoustic behavior of a liquid-propellant rocket combustion chamber cavity. Simple analytical models are used to calculate the acoustic resonance frequencies and these results are compared to natural frequencies obtained by the finite element method. The combustion chamber is characterized on four different conditions: room temperature, operational conditions and two intermediate conditions. The parameters of sound speed and gas density within the chamber have been acquired by applying a chemical equilibrium software for the combustion of the propellants (natural gas and air in equivalence ratio of 0.15). The results showed good agreement among numerical and analytical results and the largest discrepancy (less than five percent) was observed in the fifth tangential mode for all cases. The results suggest that the adopted procedures are able to model the real cavity geometry but experimental work in room and operational conditions can validate the numerical model in a more completely way.*

**Keywords:** *combustion chamber, combustion instability, acoustic resonance, liquid-propellant rocket engine.*

### 1. INTRODUCTION

Combustion instabilities are a severe problem in the development of a rocket engine, since they can seriously impair the engine operation (Culick, 2002). There are basically three types of combustion instabilities in liquid-propellant rocket engines: low, medium and high frequency. Low frequency instabilities are caused by pressure interactions between propellant feed system, if not the entire vehicle, and combustion chamber. Medium frequency instabilities are linked with mechanical vibrations of propulsion structure, injector manifold, flow eddies, propellant ratio fluctuations, and propellant feed system resonances. And the high frequency instabilities are the most potentially dangerous and not well-understood ones. It occurs due to coupling of the combustion process forces (pressure waves) and chamber acoustical resonance properties (Sutton and Biblarz, 2001).

By analyzing the energy spectrum of the sound pressure levels measured during the operation of a combustion chamber one can check that it is an inherently noisy environment. According to Bunrley and Culick (1997) combustion instability is characterized by the presence of sound pressure peaks with well-defined magnitudes summed to the background noise across the frequency range. These peaks are correlated with the resonance frequencies of the chamber cavities, where the sound pressure on each position of acoustic fluid space represent oscillation attributed to the acoustic modes of these cavities. This coupling of the acoustic natural frequency and the burning oscillation in the combustion chamber can cause efficiency loss or even explosion of the engine.

The combustion chamber of a liquid-propellant rocket engine has longitudinal and transversal (tangential and radial) acoustic modes, and combined modes too. These three basic types of acoustic modes are shown in Fig. 1.

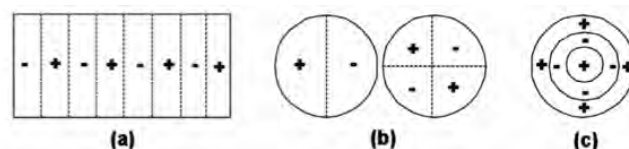


Figure 1. Longitudinal (a), tangential (b) and radial (c) modes of cylindrical cavities.

The acoustic behavior of a combustion chamber is generally determined by performing measurements on cold tests (without combustion). The experimental modal analysis is a well-applied technique in structure dynamics. However, due to the development of commercial acoustic sources, experimental acoustic modal analysis can be an appropriate choice of extracting the acoustical frequency response functions. In addition, the mathematical approach of the modal parameters extraction of structures can be applied to acoustic systems by assuming linear behavior. The acoustic dynamics in combustion environment can be obtained by shifting the resonant frequencies obtained by a cold test by a scaling factor defined by the ratio of sound speed at cold test temperature and real operation temperature, since investigations have shown that the combustion does not affect the chamber acoustics except by the increasing in the gas temperature (Laudien et al., 1994).

This paper describes two mathematical models used to determine the acoustic behavior of a cylindrical combustion chamber: a finite element model and an analytical model based on the solution of Helmholtz equation on a cylinder showed in Laudien et al. (1994). These methods are applied in four different situations: cold or room conditions, hot - considering the combustion environment, and at other two intermediate temperatures.

## 2. NUMERICAL ACOUSTIC MODAL ANALYSIS

The most commonly used finite element implementation for acoustic problems uses pressure as nodal variable (Desmet and Vandepitte, 2001). The fluid domain  $V$  is discretized into a number of finite elements. Each element possess some nodes that are defined at some element particular locations, usually at the element corner positions. Provided that the finite element discretization ('mesh') of the fluid domain  $V$  contains a total of  $n$  nodes, the acoustic pressure field  $p$  in the domain  $V$  is approximated as described in Eq. (1).

$$p(\vec{r}) \approx \sum_{i=1}^n N_i(\vec{r}) \cdot p_i = [N_i] \cdot \{p_i\} \quad (1)$$

Each function  $N_i$  in the  $(1 \times n)$  matrix  $[N_i]$  is a nodal shape function, associated with node  $i$  ( $i = 1..n$ ). After some developments, a finite element model in the unknown nodal pressure values  $p_i$  can be obtained,

$$([K_a] + j\omega[C_a] - \omega^2[M_a]) \cdot \{p_i\} = \{F_a\} \quad (2)$$

The matrices  $[K_a]$  and  $[M_a]$  are respectively, the acoustic 'stiffness' matrix and the acoustic 'mass' matrix. The acoustic damping matrix  $[C_a]$  can result, for instance, from the normal impedance boundary condition. The excitation vector  $\{F_a\}$  results from the external acoustic source distribution  $q$  and the normal velocity boundary condition.

The calculation of natural frequencies and mode shapes of an interior acoustic system is obtained by solving the following eigenvalue problem (Desmet and Vandepitte, 2001),

$$[K_a]\{\Phi_m\} = \omega_m^2 \cdot [M_a]\{\Phi_m\} \quad (m = 1..n_a) \quad (3)$$

where: each  $(n_a \times 1)$  eigenvector  $\{\Phi_m\}$  represents a mode shape and where the associated eigenvalue corresponds to the square of the natural frequency  $\omega_m$  of that mode. Note that, due to the discretization of the acoustic system, which has an infinite number of degrees of freedom and, hence, an infinite number of modes, into a system with  $n_a$  degrees of freedom, only  $n_a$  mode shapes are obtained.

## 3. ANALYTICAL CALCULATIONS

Equation 4 describes the considered mathematical model for the acoustic natural frequencies calculation of inner cavity. The inner acoustic environment is treated as an acoustically closed system, even though the nozzle is in contact with the external fluid or external acoustic environment. Such approximation has shown good theoretical versus experimental agreement (Laudien et al., 1994).

$$f_{lmn} = \frac{c}{2\pi} \sqrt{\frac{\lambda_{mn}^2}{R_c^2} + \frac{l^2 \pi^2}{L_c^2}} \quad (4)$$

where  $l, m, n = 0, 1, 2, \dots$  (longitudinal, tangential and radial mode number directions),  $c$  is the sound speed,  $\lambda_{mn}$  the transverse eigenvalue listed in Tab. 1 for the first few modes,  $R_c$  the chamber radius,  $L_c$  the chamber length or effective acoustic length (distance between injectors faceplate and nozzle throat, less approximately one-half of the converging nozzle length).

Table 1. Transversal eigenvalues.

$n$	$m$					
	0	1	2	3	4	5
0	0	1.8412	3.0542	4.2012	5.3176	6.4156
1	3.8317	5.3314	6.7061	8.0152	9.2824	10.5199
2	7.0156	8.5363	9.9695	11.3459	12.6819	13.9872
3	10.1730	11.7060	13.1704	14.5859	15.9641	17.3128

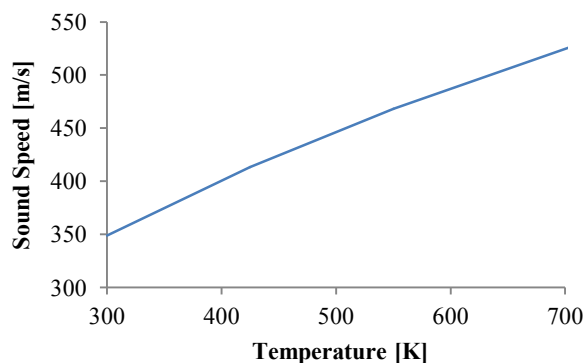
Note that in Tab. 1 the values are associated with natural frequencies, which in turn are function of the chamber geometry and the three orthogonal directions  $l$ ,  $m$  and  $n$ . Due to limitations of the Tab. 1 is only possible to calculate the natural frequencies relating to the first five longitudinal and tangential modes, first three radial modes, and the combined modes in the same range of frequencies.

#### 4. METHODOLOGIES

To obtain modes of all kinds, the mode number directions  $l$ ,  $m$  and  $n$ , are shifted. Then Eq. (4) is applied to the following conditions:

- . At room condition - the same conditions in which experimental acoustic modal analysis is made for physical characterization of a combustion chamber,
- . In burning condition, where the combustion environment is considered,
- . And at two intermediated temperatures to the previous conditions in order to analyze transition “states” from the combustion start to the chamber fully operational condition.

The studied cylindrical combustion chamber works at atmospheric pressure and has 0.15m of diameter and 1.2m of effective acoustic length. The sound speed values are obtained using chemical equilibrium software. In hot conditions, the temperature of combustion gases reaches 700K for the propellants natural gas and air to equivalent ratio 0.15. The low equivalent ratio is adopted with aiming to achieve incomplete burning conditions favorable to occurrence of combustion instabilities, which will be studied experimentally in next steps of this work. Beyond the burning temperature are considered the temperatures of 300, 425 and 550K. Figure 2 contains the curve of sound speed versus temperature built with parameters from chemical equilibrium simulations.

Figure 2. Sound speed curve for mixed methane and air ( $\phi = 0.15$ ).

The manufacturing procedures of the original combustion chamber are described in Corá (2010) and its drawing is shown in Fig. 3. The acoustic cavity of this combustion chamber is modeled using the finite element method, which provide the natural frequencies and mode shapes of cavity geometry. The speaker is installed to generate an acoustic stimulus (noise) that coupled to the shock waves from the combustion processes would increase the possibility of occurrence of combustion instabilities. But the analyses presented in this work do not consider the influences of speaker cavity, because it cannot be included in the analytical calculations, which would prevent comparisons between results from the two models.

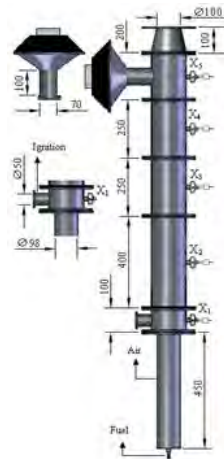


Figure 3. Combustion chamber drawing.

## 5. RESULTS

From theoretical calculations acoustic modes are easily obtained, for example, by making  $l, m, n$  equal to 1, 0, 0 one can calculate the natural frequency related to the first longitudinal mode, and equal to 0, 2, 0, the frequency from the second tangential mode.

Using the finite element method the acoustic volume was modeled using 85,044 linear solid tetrahedral elements in ANSYS software, which yielded 16,134 nodes, as shown in Fig. 4.

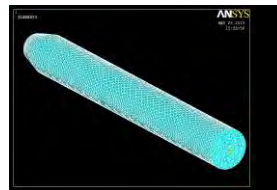


Figure 4. Acoustic cavity mesh.

From the two presented models the natural frequencies of the first five longitudinal and tangential modes are obtained and the results are listed in Tab. 2, considering the parameters of sound velocity ( $c$ ) and gas density ( $d$ ) for each condition. The latter is necessary only for the numerical model.

Table 2. Natural frequencies of acoustic modes in combustion chamber.

		<i>Room conditions</i> ( $c = 348.7\text{m/s}$ and $d = 1.1639\text{kg/m}^3$ )			$T = 425\text{K}$ ( $c = 413.4\text{m/s}$ and $d = 0.8216\text{kg/m}^3$ )			$T = 550\text{K}$ ( $c = 468.3\text{m/s}$ and $d = 0.6349\text{kg/m}^3$ )			<i>Burning conditions</i> ( $c = 528.2\text{m/s}$ and $d = 0.4923\text{kg/m}^3$ )		
		NUMERICAL FREQUENCY [HZ]	ANALYTICAL FREQUENCY [HZ]	DEVIATION [%]	NUMERICAL FREQUENCY [HZ]	ANALYTICAL FREQUENCY [HZ]	DEVIATION [%]	NUMERICAL FREQUENCY [HZ]	ANALYTICAL FREQUENCY [HZ]	DEVIATION [%]	NUMERICAL FREQUENCY [HZ]	ANALYTICAL FREQUENCY [HZ]	DEVIATION [%]
Longitudinal modes	1 <sup>st</sup>	137.2	139.5	1.65	162.7	165.4	1.63	184.3	187.3	1.60	207.8	211.3	1.66
	2 <sup>ed</sup>	274.3	279.0	1.68	325.2	330.7	1.66	368.4	374.6	1.66	415.6	422.6	1.66
	3 <sup>rd</sup>	411.1	418.4	1.74	487.4	496.1	1.75	552.1	562.0	1.76	622.7	633.8	1.75
	4 <sup>th</sup>	547.5	557.9	1.86	649.0	661.4	1.87	735.2	749.3	1.88	829.3	845.1	1.87
	5 <sup>th</sup>	817.9	836.9	2.27	809.8	826.8	2.06	917.4	936.6	2.05	1034.7	1056.4	2.05
Tangential modes	1 <sup>st</sup>	1371.2	1362.4	-0.65	1625.6	1615.2	-0.64	1841.5	1829.7	-0.64	2077.1	2063.8	-0.64
	2 <sup>ed</sup>	2286.7	2260.0	-1.18	2711.0	2679.3	-1.18	3071.0	3035.2	-1.18	3463.8	3423.4	-1.18
	3 <sup>rd</sup>	3175.5	3108.7	-2.15	3764.7	3685.6	-2.15	4264.6	4175.0	-2.15	4810.1	4709.0	-2.15
	4 <sup>th</sup>	4054.9	3934.8	-3.05	4807.3	4664.9	-3.05	5445.7	5284.4	-3.05	6142.3	5960.4	-3.05
	5 <sup>th</sup>	4951.9	4747.3	-4.31	5870.8	5628.2	-4.31	6650.4	6375.6	-4.31	7501.1	7191.1	-4.31

The visualization of the acoustic mode shapes in numerical analysis is necessary to identify the kinds of acoustic modes. Figure 5 shows the first five longitudinal and tangential numerical modes in burning conditions. Note the similarity with Fig. 1.

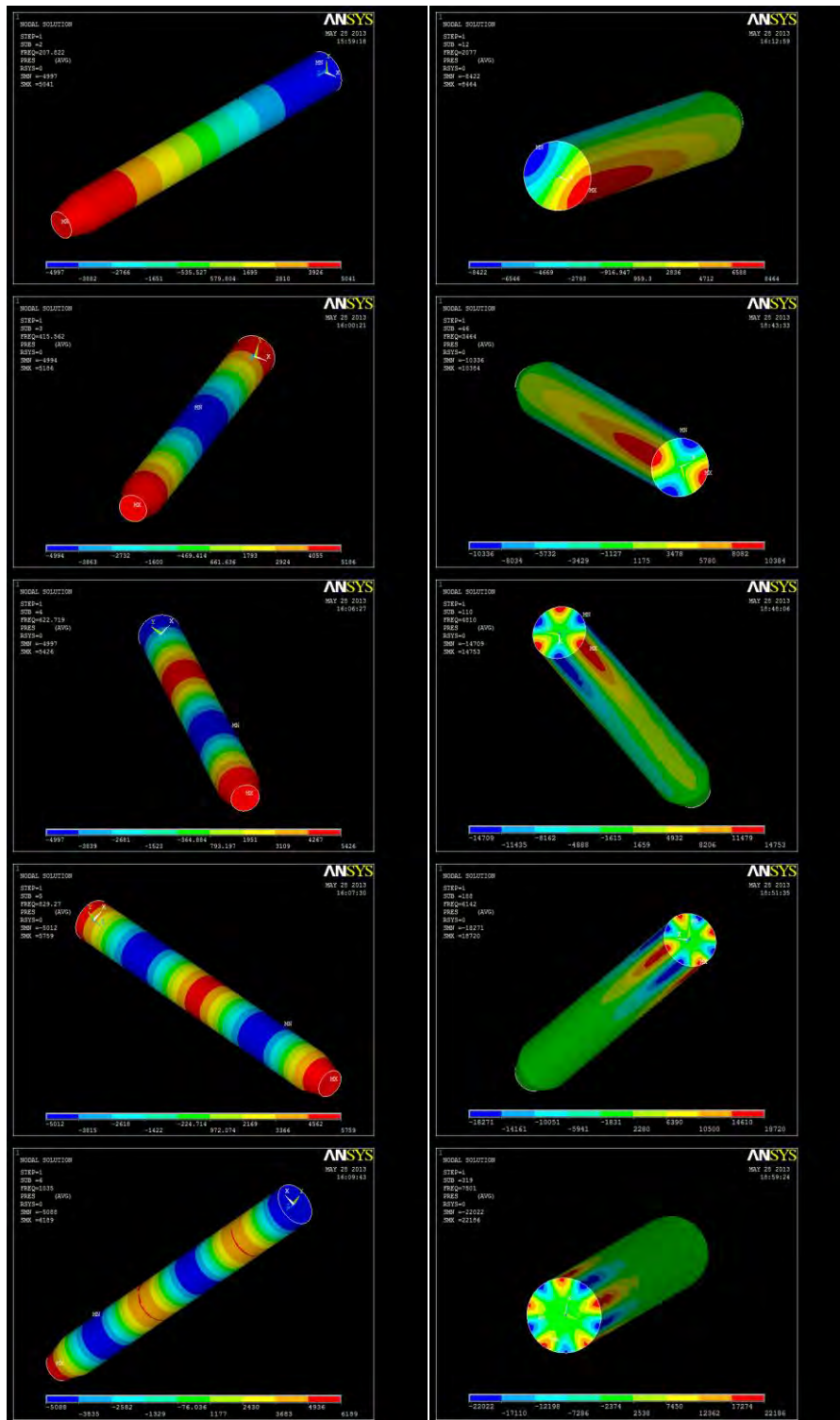


Figure 5. Longitudinal (left) and tangential (right) modes.

It is not usual the plotting of three-dimensional modes from the analytical method. You can do it by Eq. (5), but it is painful defining the color map. According Blevins (1979):

$$\phi_{lmn} = J_m \left( \lambda_{mn} \frac{r}{R_c} \right) \cos \frac{l\pi x}{L_c} \cos \frac{m}{\theta} \tag{5}$$

where  $\phi_{lmn}$  is the mode shape of velocity potential,  $J_m$  is the Bessel function,  $r$  is the radial coordinate,  $R_c$  is the combustion chamber radius,  $x$  is the axial coordinate and  $\theta$  is the phase angle showed in Fig. 6.

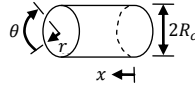


Figure 6. Closed right cylindrical volume.

The two-dimensional modes from analytical model are determined by Eq. (6) described in Beranek and VÉR (1992), where  $\psi$  is the amplitude,  $z$  is the displacement along the chamber length and  $j = 0,1,2 \dots$  (the acoustic mode).

$$\psi_j = \cos \frac{j\pi z}{L_c} \quad (6)$$

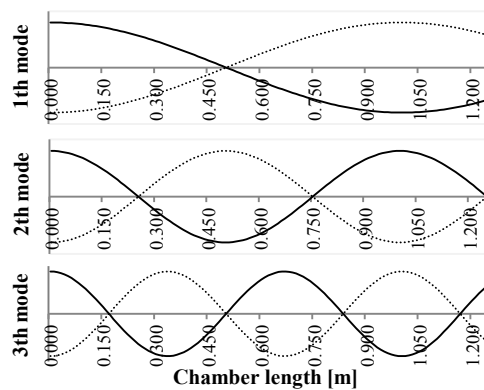


Figure 6. First three longitudinal modes.

## 6. CONCLUSION

The two models used to characterize the inner cavity of the combustion chamber showed quite compatible results. The largest deviation occurred on the fifth tangential mode, where the frequencies varied approximately four percent, which occurred in all conditions. The best agreement was identified in the first tangential mode, being the deviation of the frequencies less than 0.7 percent in all conditions. The results are quite reasonable for the range of frequencies analyzed. The results suggest that the procedures adopted are able to model the real cavity geometry but experimental work in room and operational conditions can validate the numerical model in a more completely way.

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