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CALCULATION OF THE FLOW OF A MAGNETIC FLUID IN CAPILLARIES TUBES WITH VARYING MAGNETIZATION BY VORTICITY

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Abstract. *This work presents analytical and numerical results for unidirectional flow in capillaries of a symmetric magnetic fluid free of Brownian motion and fluid inertia effects. We consider a condition that deviates from the standard equation of equilibrium magnetization due to vorticity of the flow on the orientation of the magnetic dipoles in the fluid. The solutions are developed with the fluid undergoing each a pressure gradient and an applied magnetic field gradient. The governing equations are made non-dimensionalized and a physical parameter denoted magnetic Reynolds number is identified as measuring the relative importance between inertia and magnetic forces. The resulting equation is solved numerically and by a regular perturbation method. Explicit expressions for the dimensionless velocity profile, effective viscosity and the friction factor of the flow as function of the dimensionless magnetic and hydrodynamic physical parameters are presented. Numerical and analytical results are compared and a good agreement is verified for the range where the analytical solutions can be used. The studies are applied to explore the emerging propose of drag reducing fluids by applying a gradient of magnetic field favorable to the flow direction.*

Keywords: *Magnetic Fluids, Magnetization, Capillary Flow, Drag Reduction, Effective viscosity*

1. INTRODUCTION

Magnetic fluids, or ferrofluids, are colloidal suspensions of small magnetic particles dispersed in a carrier liquid, water or oil in most cases . These solid particles, typically ferrite and magnetite, with diameters varying from 3 to 15 *nm*, can be treated as permanent nano-magnets in random motion due to the Brownian forces, which prevents particles from settling under gravity (Rinaldi *et al.*, 2005). The distribution of electric charges or a surfactant layer around each particle ensures the magnetic fluid stabilization, avoiding irreversible aggregates formation. A typical magnetic fluid has about 10^{23} particles per cubic meter, is opaque to the visible light and the particle volumetric fraction are of 1% order (Cunha, 2012).

After the experimental works of Rosensweig *et al.* (1969) and McTague (1969), a lot of studies was developed on viscous properties of ferrofluids. The main attraction of these suspensions comes from their ability to achieve a wide range of viscosity in a fraction of second (Bossis *et al.*, 2002). This enable a flow control by an external magnetic field, providing an efficient way to control force or torque transmission. The study of these interactions between a fluid and a magnetic field is the objective of the branch of physics called *ferrohydrodynamics* (Rosensweig, 1997).

Presently, the magnetic fluids are used in several technical and medical applications (Berkovsky and Bashtovoy, 1996). Ferrofluids are largely used as dynamical sealing in hard disk of computers and as dampers in loudspeakers (Scherer and Neto, 2005). In magnetic separation, accelerating the process of separation oil from water (Cunha and Sobral, 2004). Magnetic fluids can also accelerate the convective cooling and make it possibly in low gravity environments (Gontijo and Cunha, 2012). In medicine, biocompatible magnetic fields can do the transport of drugs into the human body and, by controlling an external applied magnetic field, carry these drugs to the exact required place (Lacava *et al.*, 1999).

In this work is developed a study of the influence of an applied magnetic field in the laminar flow of a magnetic fluid in a capillary tube. The fluid studied here is superparamagnetic, i.e., there are no internal torques due to the magnetic field. It is demonstrated how the drag varies with the application of the magnetic field, which can be interpreted as a variation in the effective viscosity. In previous works, Cunha and Sobral (2004) and Cunha and Sobral (2005) developed an analytical solution for this problem. Here we focuses in a numerical solution and a unitary volume rate is always guaranteed.

2. GOVERNING EQUATIONS

The solution of the problem of a magnetic fluid flow in a capillary is based on the couple of the hydrodynamics and magnetic equations.

Hydrodynamic equations includes the continuity and the balance of linear momentum equations. The former is given by:

$$\nabla \cdot \mathbf{u} = 0\,, \tag{1}$$

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where $\mathbf{u}$ represents the velocity field. Continuity equation was simplified considering all compressible effects negligible. Balance of linear momentum is expressed by Cauchy's equation (Batchelor, 1967):

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=\nabla\cdot\boldsymbol{\Sigma}+\rho\mathbf{g} \tag{2}$$

where ρ denotes the fluid density and $\mathbf{g}$ is the gravity acceleration. The stress tensor $\boldsymbol{\Sigma}$ results from the sum of the tensor due to hydrodynamic forces Σ_h and the tensor due to magnetic forces $\boldsymbol{\Sigma}_m$,

$$\boldsymbol{\Sigma}=\boldsymbol{\Sigma}_h+\boldsymbol{\Sigma}_m \tag{3}$$

The stress tensor due to hydrodynamic effects is given by the constitutive equation for newtonian fluids (Chandrasekharaiah and Debnath, 1994):

$$\boldsymbol{\Sigma}_h=-p_h\mathbf{I}+2\eta\mathbf{D} \tag{4}$$

where p_h represents the hydrostatic pressure, $\mathbf{I}$ is the unit second order tensor and $\mathbf{D}$ represents the rate of strain tensor of the flow. The magnetic stress tensor is given by (Rosensweig, 1997):

$$\boldsymbol{\Sigma}_m=-p_m\mathbf{I}+\mathbf{BH} \tag{5}$$

where $p_m=\frac{1}{2}\mu_0H^2$ denotes the magnetic pressure, $\mathbf{H}$ represents the magnetic intensity vector and $\mathbf{B}$ is the magnetic induction vector. The substitution of Eq. (4) and Eq. (5) in Eq. (3) give the expression for the complete stress tensor. Now, the stress tensor is expressed as the sum of an isotropic tensor and a deviatoric tensor given by:

$$\boldsymbol{\Sigma}=-P\mathbf{I}+\boldsymbol{\Sigma}^d \tag{6}$$

where

$$P=p_h+p_m-\frac{1}{3}\mathbf{B}\cdot\mathbf{H} \tag{7}$$

and

$$\boldsymbol{\Sigma}^d=2\eta\mathbf{D}+\mathbf{BH}-\frac{1}{3}\mathbf{B}\cdot\mathbf{H}\ . \tag{8}$$

Here P is the flow's mechanical pressure and defined by $P=-tr(\Sigma/3)$. Finally, substitution of Eq. (6) in Eq. (2) results:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=-\nabla p+\eta\nabla^2\mathbf{u}+\mu_0\mathbf{M}\cdot\nabla\mathbf{H} \tag{9}$$

In equation Eq. (9), $p=P-\rho\mathbf{g}\cdot\mathbf{x}$ represents the modified pressure, including the hydrostatic effects. Equation (9) differs from the classical Navier-Stokes equation due to the presence of the last therm, directly proportional to the local fluid magnetization and to the applied magnetic field gradient.

The symmetry of the deviatoric tensor $\boldsymbol{\Sigma}^d$ presented in Eq. (8) is guaranteed if there is no internal magnetic torque, i.e., $\mathbf{M}\times\mathbf{H}=\mathbf{0}$. This condition implies no hysteresis nor magnetic memory. Magnetic fluids with such properties are called superparamagnetic fluids and are the focus of this work. In this conditions $\mathbf{M}=\chi(H)\mathbf{H}$, and, consequently $\mathbf{B}=\mu\mathbf{H}$. This means that the magnetization vector $\mathbf{M}$ is always aligned with the applied magnetic field $\mathbf{H}$. So, using this approximation:

$$\mathbf{BH}=\mu\mathbf{HH} \tag{10}$$

With $\mathbf{M}$ and $\mathbf{H}$ collinear, the last term in Eq. (9) can be replaced by $\mu M\nabla H$, where M and H are the absolute values of magnetization and applied magnetic field vectors, respectively. Under these considerations, the modified magnetic fluid momentum equation is given by:

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\nabla\mathbf{u}\right)=-\nabla p+\eta\nabla^2\mathbf{u}+\mu_0M\nabla H\ . \tag{11}$$

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2.1 Magnetization evolution equation

To solve Eq. (11) an evolution equation for $\mathbf{M}$ is needed. In this work is used the equation proposed by Shliomis and Morozov (1994) and slightly modified by Cunha and Sobral (2004) for a symmetric fluid with small magnetization:

$$\frac{\partial \mathbf{M}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{M} = \mathbf{\Omega} \times \mathbf{M} - \frac{1}{\tau_s}\left(\mathbf{M} - \mathbf{M^0}\right) \tag{12}$$

where $\Omega = \frac{1}{2}\nabla \times \mathbf{u}$ represents the fluid angular velocity, $\mathbf{M^0}$ is the equilibrium magnetization and τ_s is the characteristic time associated with the particle's relaxation.

Equation (12) couples in continuity scale the magnetization $\mathbf{M}$ and the hydrodynamic velocity field $\mathbf{u}$. The equilibrium magnetization module M is obtained from the classical Langevin model (Rosensweig, 1997) :

$$M^0 = L(\alpha)\phi M_d = L(\alpha) M_s \tag{13}$$

with

$$L(\alpha) = coth(\alpha) - 1/\alpha \qquad and \qquad \alpha = \frac{mH}{kT} \ . \tag{14}$$

In Eq. (13) M_d represents the particle magnetization, i.e., from the solid material from which it is done and M_s is the saturation magnetization (the maximum magnetization that the magnetic fluid can reach). The magnetic particles volume fraction is represented by ϕ and L denotes the Langevin function. The parameter α can be interpreted as a dimensionless applied magnetic field. This last is a ratio between magnetic forces and brownian forces. For $\alpha \ll 1$ there is a complete domain of the brownian forces and the magnetization is small. For $\alpha \gg 1$, the magnetic particles response to an applied field is fast and results in an appreciable magnetization.

2.2 Dimensionless equations

The governing equations are made dimensionless in order to make an appropriate study of the dependency between the parameters and the magnetic fluid response. Using as characteristic scales the average velocity U, the capillary tube diameter a and the intensity of the applied magnetic field H_0 results in the following dimensionless variables:

$$\tilde{\mathbf{u}} = \frac{\mathbf{u}}{U} \ , \qquad \tilde{\mathbf{x}} = \frac{\mathbf{x}}{a} \ , \qquad \tilde{t} = \frac{t}{a/U} \ , \qquad \tilde{p} = \frac{p}{8\rho U^2} \ , \qquad \tilde{H} = \frac{H}{H_0} \ , \qquad \tilde{M} = \frac{M}{H_0} \quad and \quad \tilde{\nabla} = a\nabla \ , \tag{15}$$

where tilde indicates that the variable is dimensionless. The pressure p is made dimensionless by $8\rho U^2$ to result in an normalized volume rate, as is show in the next section. The substitution of the dimensionless variables from Eq. (15) in Eq. (11) and Eq. (12) results (the tilde is not used here to avoid a dense notation):

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -8\nabla p + \frac{1}{Re}\nabla^2 \mathbf{u} + \frac{1}{Re_m} M \nabla H \tag{16}$$

and

$$\frac{\partial \mathbf{M}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{M} = \mathbf{\Omega} \times \mathbf{M} - \frac{1}{\hat{\omega}}\left(\mathbf{M} - \mathbf{M^0}\right) \ . \tag{17}$$

The dimensionless parameter are the classical Reynolds number Re, a magnetic parameter called Magnetic Reynolds number Re_m and a dimensionless magnetic relaxation time $\hat{\omega}$, each one defined as:

$$Re = \frac{\rho U a}{\eta} \ , \qquad Re_m = \frac{\rho}{U}\left(\frac{U}{H_0}\right)^2 \qquad and \qquad \hat{\omega} = \frac{U\tau_s}{a} \ . \tag{18}$$

The Reynolds is the ratio between inertial and viscous forces. The Magnetic Reynolds number is a relation between inertial and magnetic forces. Finally, the dimensionless magnetic relaxation time indicates the tendency of the magnetization to vary from its initial direction and module in comparison to the time scale of the flow a/U.

3. ANALYTICAL SOLUTIONS

The problem solved is the laminar flow of a diluted magnetic fluid in a capillary tube with unitary radius and aspect ratio $\ell/a = 100$ (ℓ is the tube length). The magnetic field, stationary and with linear decay, is applied in the axial direction. This flow is uniaxial and axisymmetric if the condition $Re(a/\ell)^2 \ll 1$ is guaranteed, which is the case in all situations assessed in this work. Equation (16) reduces to:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) + \frac{Re}{Re_m} M \frac{dH}{dz} = -8ReG \tag{19}$$

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where G is the constant pressure gradient and dH/dz is the constant magnetic field gradient.

Two analytical solutions developed in this work are presented in this section: the first, with constant magnetization, is obtained by directly integration of the resulting differential equation and for the other is proposed a regular perturbation method.

3.1 Constant magnetization

In the first solution the magnetic particles relaxation time τ_s is much smaller than the characteristic flow time a/U, resulting in $\hat{\omega} \ll 1$. In this situation the magnetic particles are always aligned with the magnetic field, which means that the flow does not alter the magnetization. Therefore Eq. (17) reduces to:

$$\mathbf{M} = \mathbf{M^0} \tag{20}$$

In this case the magnetization M in Eq. (19) is constant and one can obtain the velocity profile by directly integration, resulting in:

$$u(r) = \left(2ReG + \frac{Re}{Re_m} M \frac{dH}{dz}\right)\left(1 - r^2\right) \ . \tag{21}$$

For Newtonian fluids the volume rate, in dimensional form, is given by Poiseuille's law (Pao, 1967):

$$Q = -\frac{\pi a^4 \Delta P}{8\eta\ell} \tag{22}$$

The dimensionless form is given by:

$$Q = 1 = ReG \tag{23}$$

where the volume rate is made dimensionless by $\pi U a^2$, becoming a unitary value. The product ReG equals 1 just for Newtonian fluids, i.e., in the absence of a magnetic field. When a field is applied the product deviates from unity, but the dimensionless volume rate always equals 1.

Integrating the velocity field given by Eq. (21) results:

$$Q = 2\int_0^1 u(r) r dr = ReG + \frac{1}{8}\frac{Re}{Re_m} M \frac{dH}{dz} \ . \tag{24}$$

But $Q = 1$, therefore an expression to the dimensionless gradient pressure is given by:

$$G = \frac{1}{Re}\left(1 - \frac{1}{8}\frac{Re}{Re_m} M \frac{dH}{dz}\right) \tag{25}$$

The result in Eq. (25) shows that G is now given by the Newtonian contribution $1/Re$ added by a non-newtonian contribution. The substitution of G in the Eq. (21) results

$$u(r) = 2(1 - r^2) \tag{26}$$

to all applied magnetic field, i.e., the velocity profile is parabolic when there is no couple between magnetization and hydrodynamic. Taking again Poiseiulle's law, the effective viscosity is given by (dimensional form):

$$\eta_{ef} = -\frac{\pi a^4}{8Q}\frac{\Delta P}{l} \tag{27}$$

Making Eq. (27) dimensionless results:

$$\frac{\eta_{ef}}{\eta_0} = ReG \ . \tag{28}$$

With no applied magnetic field, $ReG = 1$, i.e., Newtonian behavior. In Eq. (28) η_0 is the effective viscosity with no applied field. For example, if a magnetic fluid with 1% of magnetic particle volume fraction is studied, η_0 is the effective viscosity with no applied field. Therefore ReG takes into account only magnetic effects, eliminating hydrodynamics effects due to the presence of the particles.

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3.2 Varying magnetization

To develop a solution with with the magnetization changed by the flow, the linkage between the motion equation and the magnetization evolution equation is required. With permanent flow and with

$$|\mathbf{u} \cdot \nabla \mathbf{M}| \ll |\mathbf{\Omega} \times \mathbf{M}| \ , \tag{29}$$

Eq. (17) reduces to:

$$\hat{\omega}\mathbf{\Omega} \times \mathbf{M} = \mathbf{M} - \mathbf{M^0} \ . \tag{30}$$

Therefore the magnetization deviates from its equilibrium value due to the flow vorticity.

In cylindrical coordinates, Eq. (30) can be written as:

$$M_z \frac{du}{dr} = -\frac{2}{\hat{\omega}} \left(M_r - M_r^0 \right) \tag{31}$$

and

$$M_r = \frac{2}{\hat{\omega}} \left(M_z - M_z^0 \right) \tag{32}$$

where M_r and M_z represent the radial and axial magnetization, respectively, and M_r^0 and M_z^0 represent the equilibrium magnetization in each direction. Presuming an initial magnetization aligned with the applied magnetic field ($M_r^0 = 0$), the system given by Eq. (31) and Eq. (32) results in:

$$M_r = -\frac{1}{2} \hat{\omega} M_z \frac{du}{dr} \tag{33}$$

and

$$M_z = M_z^0 \left[\frac{1}{4} \hat{\omega}^2 \left(\frac{du}{dr} \right)^2 + 1 \right]^{-1} \ . \tag{34}$$

The magnetization absolute value is then:

$$M = \sqrt{M_z^2 + M_r^2} = M_z^0 \left[\frac{1}{4} \hat{\omega}^2 \left(\frac{du}{dr} \right)^2 + 1 \right]^{-\frac{1}{2}} \ . \tag{35}$$

After an expansion and getting just the leader order term results:

$$M \approx \left[1 - \frac{1}{8} \hat{\omega}^2 \left(\frac{du}{dr} \right)^2 \right] \ . \tag{36}$$

The substitution of Eq. (36) in Eq. (19) results in magnetic fluid flow equation in a capillary tube with coupling vorticity-magnetization:

$$\frac{d^2u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{1}{8} \hat{\omega}^2 \beta \left(\frac{du}{dr} \right)^2 = -\gamma \tag{37}$$

where

$$\beta = \frac{Re}{Re_m} M_z^0 \frac{dH}{dz} \qquad and \qquad \gamma = 8ReG + \frac{Re}{Re_m} M_z^0 \frac{dH}{dz} \ . \tag{38}$$

The nonlinearity from Eq. (37) comes from the term with $(du/dr)^2$. This makes impossible a simple integration of the equation like in the case of constant magnetization. Assuming that $\epsilon = \hat{\omega}^2 \beta / 8$ is small, Eq. (37) becomes slightly nonlinear and can be solved by a regular perturbation method (Logan, 2006). Expanding $u(r)$ and taking only terms until ϵ^2 order, results:

$$u(r) = u_0(r) + \epsilon u_1(r) + \epsilon^2 u_2(r) \ . \tag{39}$$

And the substitution of $u(r)$ in Eq. (37):

$$\left[\frac{d^2u_0}{dr^2} + \frac{1}{r} \frac{du_0}{dr} + \gamma \right] + \epsilon \left[\frac{d^2u_1}{dr^2} + \frac{1}{r} \frac{du_1}{dr} - \left(\frac{du_0}{dr} \right)^2 \right] + \epsilon^2 \left[\frac{d^2u_2}{dr^2} + \frac{1}{r} \frac{du_2}{dr} - 2 \frac{du_0}{dr} \frac{du_1}{dr} \right] = 0 \tag{40}$$

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Therefore the system to be solved is given by:

$$\frac{d^2u_0}{dr^2}+\frac{1}{r}\frac{du_0}{dr}=-\gamma \tag{41}$$

$$\frac{d^2u_1}{dr^2}+\frac{1}{r}\frac{du_1}{dr}=\left(\frac{du_0}{dr}\right)^2 \tag{42}$$

$$\frac{d^2u_2}{dr^2}+\frac{1}{r}\frac{du_2}{dr}=2\frac{du_0}{dr}\frac{du_1}{dr} \tag{43}$$

The boundary conditions are given by the non-slip condition in the wall and the symmetry in the center of the tube:

$$u_i(1)=0 \qquad\qquad i=0,1,2 \tag{44}$$

$$\left.\frac{du_i}{dr}\right|_{r=0}=0 \qquad\qquad i=0,1,2\ . \tag{45}$$

The solutions are:

$$u_0=-\frac{\gamma}{4}r^2+\frac{\gamma}{4}\ , \qquad u_1=\frac{\gamma^2}{64}r^4-\frac{\gamma^2}{64} \qquad \mathit{and} \qquad u_2=-\frac{\gamma^3}{576}r^6+\frac{\gamma^3}{576}\ . \tag{46}$$

Therefore, the resulting velocity profile can be written as:

$$u(r)=\frac{\gamma}{4}\left(1-r^2\right)-\frac{\gamma^2}{64}\epsilon\left(1-r^4\right)+\frac{\gamma^3}{576}\epsilon^2\left(1-r^6\right)\ . \tag{47}$$

The expression obtained in Eq. (47) represents a parabolic velocity profile only when there is no magnetic applied field, i.e., the Newtonian behavior is observed when the applied magnetic field is null. The dimensionless volume rate in this case is given by:

$$Q=2\int_0^1 u(r)rdr=\frac{\gamma}{8}-\epsilon\frac{\gamma^2}{96}+\epsilon^2\frac{\gamma^3}{768}\ . \tag{48}$$

But, by definition $Q=1$, resulting:

$$ReG\left[1+\frac{1}{8}\frac{M_z^0}{Re_mG}\frac{dH}{dz}-\epsilon\frac{\gamma^2}{96ReG}+\epsilon^2\frac{\gamma^3}{768ReG}\right]=1\ . \tag{49}$$

With no applied magnetic field Eq. (49) shows that $ReG=1$. This condition is not obtained when a field is applied, revealing the influence of the magnetic field in the flow.

Figure 1 shows velocity profiles of magnetic fluids for different values of Re_m for unfavorable (1a) and favorable (1b) applied magnetic fields. The initial magnetization is $M_z^0=0.13$ and $\hat{\omega}^2/8=0.1$. The magnetic applied field gradient dH/dz is 0.01 when favorable and -0.01 when unfavorable. In Fig. 2 is plotted the dimensionless effective viscosity ReG as a function of Re_m^{-1} for different values of pressure gradient. This figure shows a comparison between the analytical solutions developed in this section. The first solution with no coupling between magnetization and vorticity shows good results only for small values of Re_m^{-1}. This solution overestimate the effective viscosity because the magnetization is not decreased by the flow.

4. NUMERICAL SOLUTION

From the fact that analytical solutions, even the one obtained by perturbation methods, are restricted to specific situations where $\epsilon \ll 1.0$, a numerical treatment of the problem was developed to extend the validity of the solution. To solve equation Eq. (37) a fourth order Runge-Kutta method is used.

An important parameter to define is the step h used in the radial direction, which determines the number of points where the velocity is calculated in the tube radius a. The step h must be small to generate a precisely result, but not too small, increasing the computational cost. As a first approximation is proposed $h/a=0.01$, which guarantees that $h/a \ll 1$. It is possible, by scale analysis, find a minimum value to h. Considering the equilibrium between viscous (F_η) and magnetic (F_m) forces:

$$F_\eta \sim F_m \qquad\qquad \Leftrightarrow \qquad\qquad \eta hU \sim \mu_0 M_d H_0 h^2 \tag{50}$$

resulting:

$$\frac{h}{a}=min\left(10^{-2},10^{-2}\left(\frac{H_0}{M_d}\right)\frac{Re_m}{Re}\right)\ . \tag{51}$$

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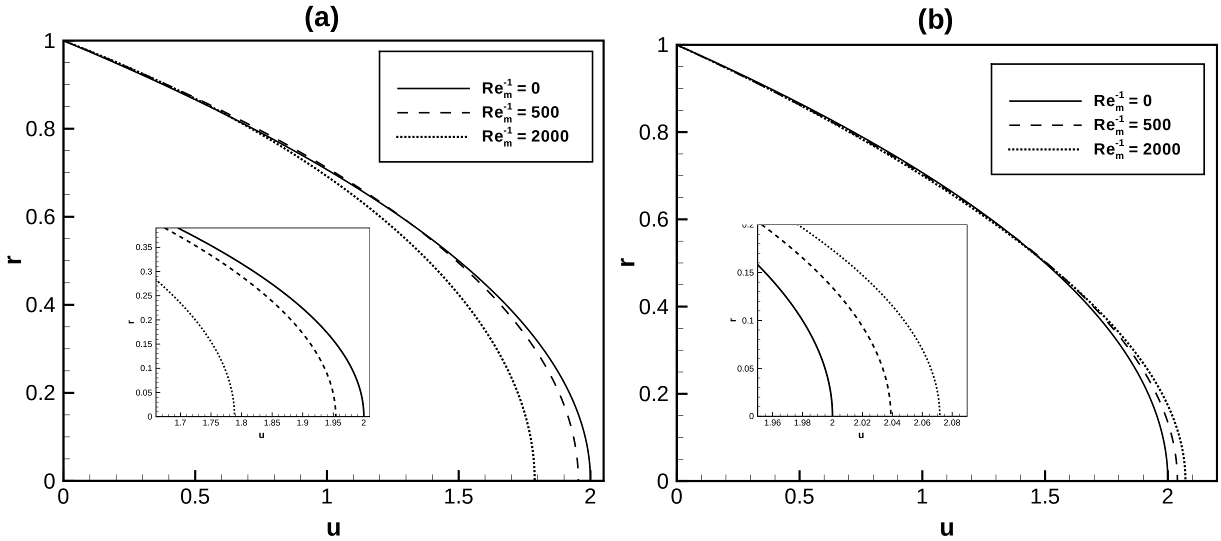

Figure 1. Velocity profile of a magnetic fluid flow in a capillary tube with applied magnetic field for $G = 0.5$. In (a) the field is unfavorable ($dH/dz = -0.01$) and in (b) is favorable ($dH/dz = 0.01$).

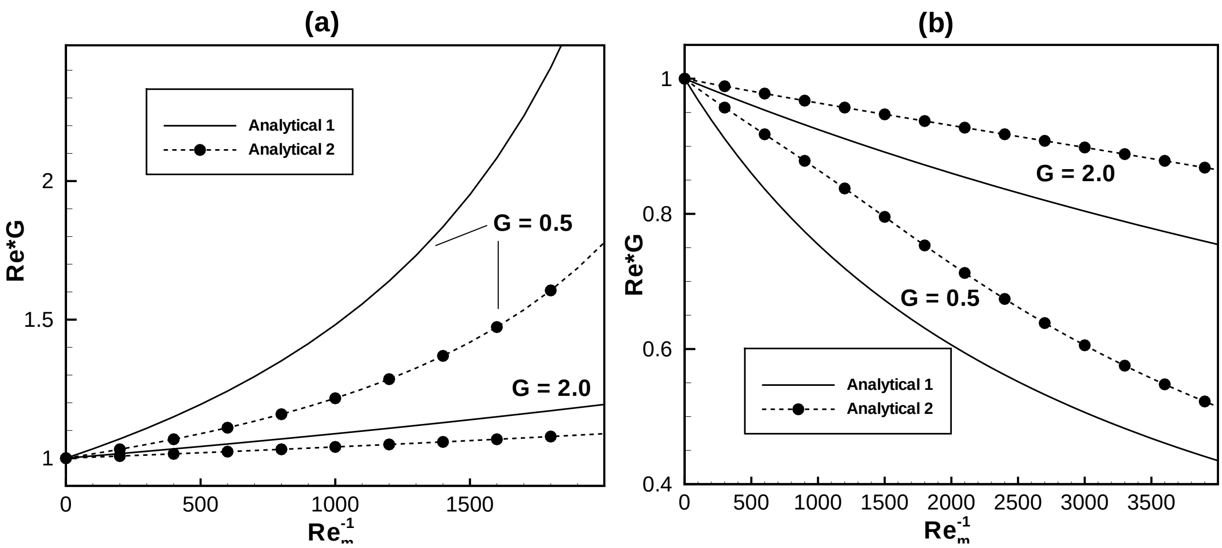

Figure 2. Comparison of dimensionless effective viscosity ReG obtained from analytical solutions: analytical 1 refers to Eq. (25) and analytical 2 to Eq. (49). In (a) the applied field is unfavorable and in (b) it is favorable.

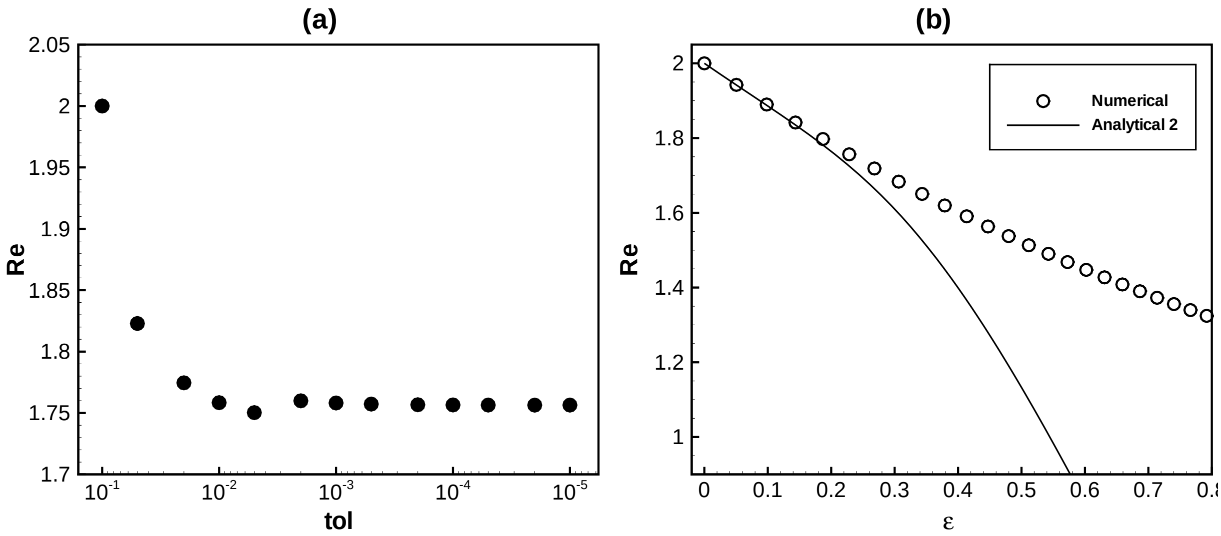

Figure 3. (a) The influence of the parameter tol in the result of the numerical solution. In this solution, $G = 0,5$, $Re_m^{-1} = 1000$ and $dH/dz = 0.01$. (b) Comparison between the analytical solution in Eq. (49) given by perturbation method and the numerical solution, for $G = 0,5$ and $dH/dz = 0.01$.

The boundary conditions of the problem impossibilities a direct solution with a Runge-Kutta method because the conditions are given in different points at the initial time. With G as an input parameter, a value of $Re = 1/G$ is assumed as first approximation. With this value is determined a velocity profile. If the volume rate obtained is unitary, which is necessary by definition in Eq. (23), then this is the correct value of Re in this condition. If not, a new value of Re is calculated by a Newton-Raphson procedure. This routine is resumed in the following algorithm:

Numerical Algorithm

1 Input Re_m, G, dH/dz, ϕ, M_d and α
2 Re $\leftarrow$ 1/G
3 u(r=0) $\leftarrow$ 2
4 if $|u(r=a)| < tol \rightarrow$ go to 8
5 $u_+(r=0) \leftarrow u(r=0) + \Delta u$
6 $u(r=0) \leftarrow u(r=0) - \frac{(\Delta u)u(r=a)}{u_+(r=a)-u(r=a)}$
7 Back to 4
8 $\Theta(Re) \leftarrow Q(Re) - 1$
9 if $\Theta(Re) < tol \rightarrow$ END
10 $\Theta(Re + \Delta Re) = Q(Re + \Delta Re) - 1$
11 Re $\leftarrow$ Re $- \frac{\Delta Re\Theta(Re)}{\Theta(Re+\Delta Re)-\Theta(Re)}$
12 Back to 8

where $Q(Re)$ is the volume rate found for the assumed Re. The parameter tol is the tolerance and defines when the iteration to determine the velocity profile and the final Re stops. Figure 3a shows a study of the Reynolds number convergence with varying tol for $G = 0.5$, $Re_m^{-1} = 1000$ and $dH/dz = 0.01$. As it is seem from Fig. 3a, with a tolerance of 10^{-3} the results have already converged. In the present work we have used a $tol = 10^{-4}$ in order to this convergence to be always rigorously guaranteed.

In Fig. 3b is made a comparison between the numerical solution described in this section and the analytical solution obtained by a regular perturbation method, developed in the previous section. For small ϵ the solutions show a good agreement. As ϵ increases there is a divergence between the solutions because the solution given by the asymptotic expansion is valid only in the limit of small ϵ. Therefore, as ϵ approximates 1 this solution is not anymore confident, as the Fig. 3b has shown.

5. RESULTS AND DISCUSSIONS

In this section are presented the results obtained by the numerical solution. The magnetic particles fraction is $\phi = 0.03$ and $\omega^2 = 0.8$. With $\alpha = 10$ and a dimensionless particle magnetization $M_d/H_0 = 5$, the resulting initial magnetization

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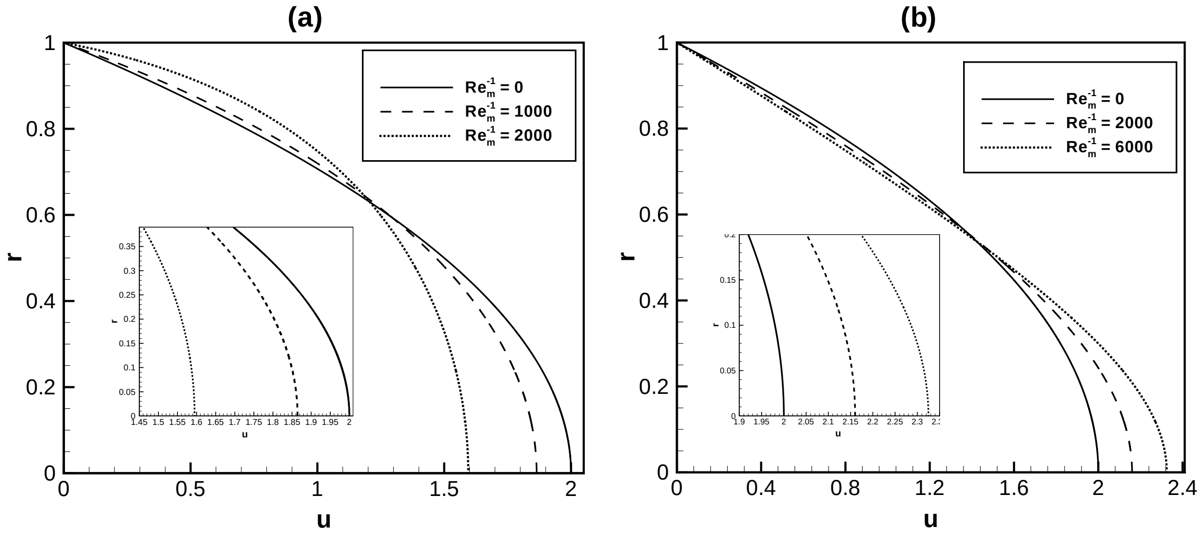

Figure 4. Velocity profiles for different Re_m^{-1} with $G = 0,4$. (a) Unfavorable applied magnetic field: $dH/dz = -0.01$. (b) Favorable applied magnetic field: $dH/dz = 0.01$.

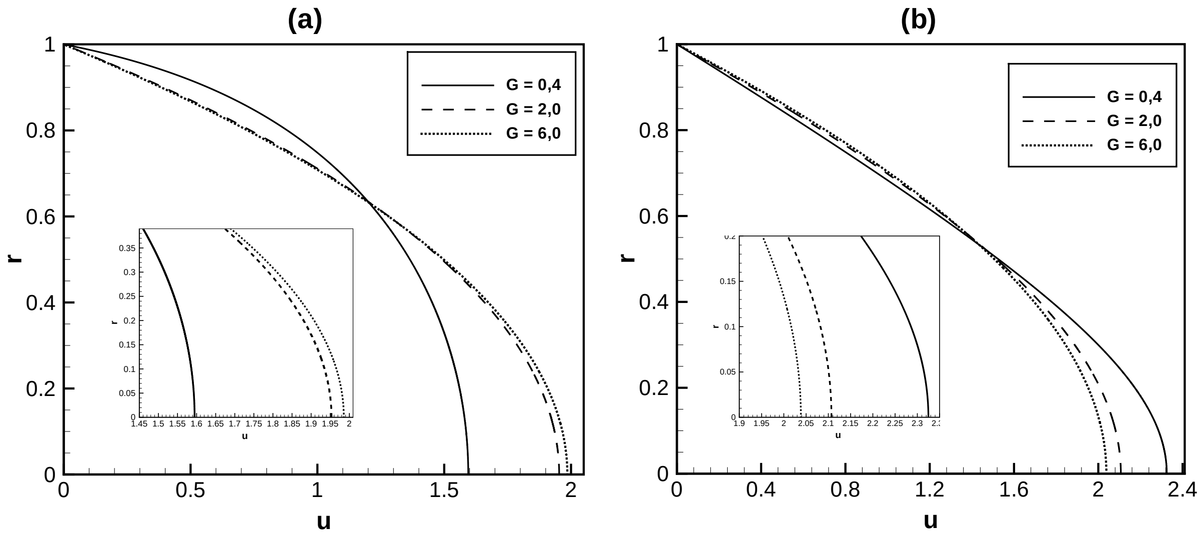

Figure 5. Velocity profiles for different G. (a) Unfavorable applied magnetic field: $dH/dz = -0.01$, with $Re_m^{-1} = 2000$. (b) Favorable applied magnetic field: $dH/dz = 0.01$, with $Re_m^{-1} = 6000$.

calculated by Eq. (13) is $M_z^0 = 0.13$. For favorable applied magnetic fields, the gradient is $dH/dz = 0.01$ and for unfavorable $dH/dz = -0.01$.

Figure 4 shows the velocity profile for favorable magnetic field (a) and unfavorable (b) with dimensionless pressure gradient $G = 0,4$ for different values of the magnetic Reynolds number. When the magnetic field gradient is negative there is a flatness in the profile and the shear rate in the wall is bigger than in the Newtonian case. With $Re_m^{-1} = 2000$ the reduction in the maximum velocity is about 20%. In Fig. 4b , with favorable magnetic field, there is an increse in the maximum velocity reached by the flow. With $Re_m^{-1} = 6000$ this increase is about 16%.

In Fig. 5 the velocity profiles are compared keeping a constant Re_m^{-1} and varying G to determine the influence of the pressure gradient on the flow. In Fig. 5a the magnetic field is favorable ($dH/dz = 0.01$) and $Re_m^{-1} = 2000$. As can be noted, the less the value of G the less the modification in the velocity profile and, therefore, the fluid response. This happens because a small G results in a large Re, which means a small viscous force. So the magnetic force has a major influence and the fluid response to an external magnetic field increases. With $G = 6$, the behavior is closely Newtonian, indicating small influence of the magnetic field. For Fig. 5b the same analysis can be done. With $G = 0.4$ the velocity profile is great different from a parabolic form, and with $G = 6$ this difference is minimum, even with a large inverse magnetic Reynolds number (6000, in this case).

The maximum velocity for each case is plotted in Fig. 6. As was discussed in the previous paragraph, for favorable magnetic fields the maximum velocity is increased from that in the Newtonian flow, as one can see in Fig. 6b. In the other

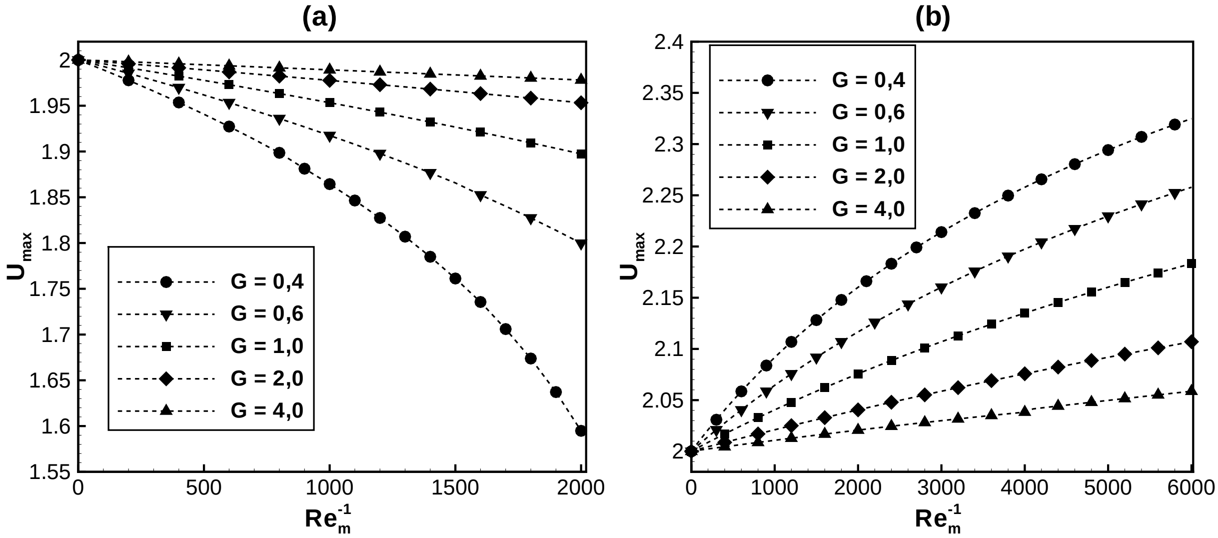

Figure 6. Maximum velocity for each profile in terms of the magnetic Reynolds number Re_m^{-1} with unfavorable magnetic field in (a) and favorable in (b).

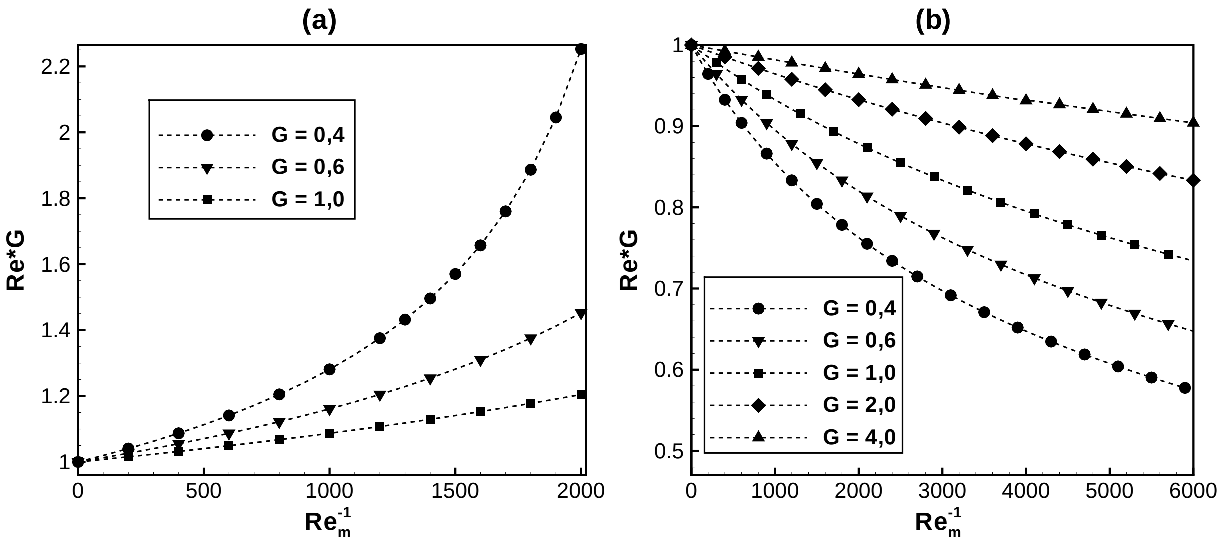

Figure 7. Dimensionless effective viscosity ReG in terms of the magnetic Reynolds number Re_m^{-1} with unfavorable magnetic field in (a) and favorable in (b).

side, Fig. 6a shows the decreasing caused in the maximum velocity reached in the flow by the applied magnetic field.

As defined in Eq. (28), the dimensionless product ReG can be interpreted as an effective dimensionless viscosity. This value is unitary for Newtonian flows. Figure 7 show this product as a function of the magnetic Reynolds number for different values of G. In (a) the applied magnetic field is unfavorable and one can see that this product increases with Re_m^{-1}. As showed, the maximum is reached in 2.2, that means that the viscosity doubles. In (b), applying a favorable magnetic field results in a decrease of the effective viscosity. For the most dramatic case, this reduction is more than 40%.

Finally, in Fig. 8 the drag factor is showed, with the dimensionless pressure gradient G as a function of the Reynolds number Re. For small values of Re there is a convergence of the values found. In Fig. 8a, with the unfavorable condition, the points with $Re_m^{-1} = 1000$ and $Re_m^{-1} = 2000$ are upper than the points with $Re_m^{-1} = 0$, which represents the Newtonian case. This means that for constant Reynolds numbers Re the gradient pressure G necessary to keep the flow increases with Re_m^{-1}. This can be interpreted as an increase in the effective viscosity. In Fig. 8b , with a favorable applied magnetic field, the opposite can be concluded. For $Re = 1.4$ one have $G = 0.7$ in a Newtonian flow and $G = 0.4$ when $Re_m^{-1} = 6000$. This means that less work is needed in these flow conditions when there is a favorable applied magnetic field. This can be interpreted as an decrease in the effective viscosity.

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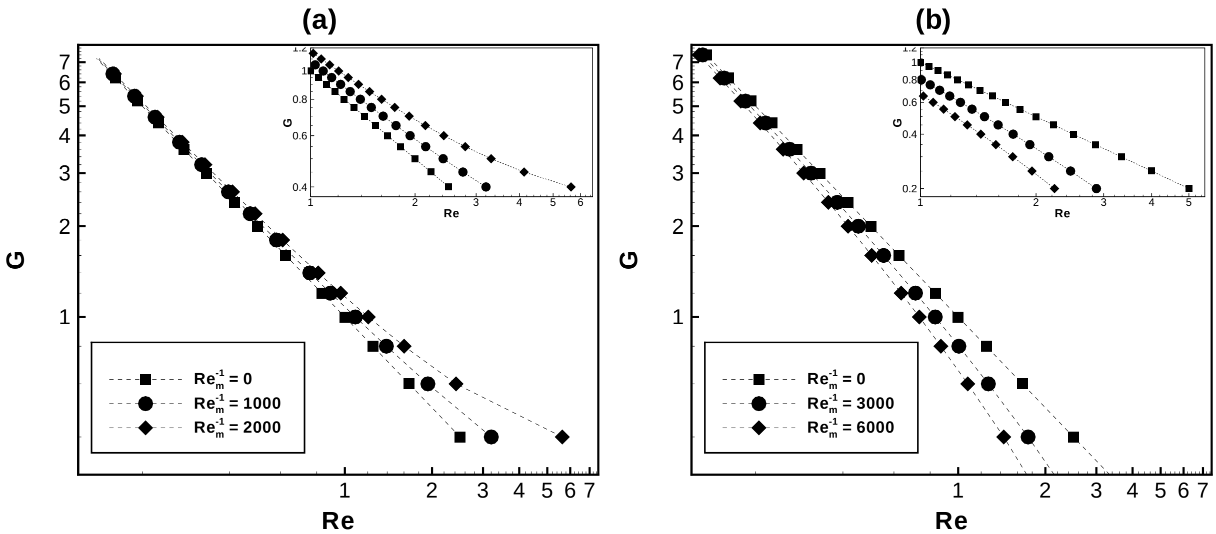

Figure 8. Drag factor for different Re_m^{-1}. (a) Unfavorable magnetic field. (b) Favorable magnetic field.

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