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# DESIGN OF A THERMAL CONTROL SYSTEM FOR EXTENDED SURFACES

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***Abstract.*** *This study presents the design of a control system for a test bench used in investigations of a fin cooled by free convection with ambient air. This work applies the modern control theory with the state space formulation. The test bench is composed of a block made of aluminum, wherein a cylindrical brass fin and an electrical resistor are fixed. The electrical resistor and the aluminum block are modeled as lumped systems. The purpose of the control system is to maintain a constant temperature at the base of the fin, that is, the point where it emanates from the aluminum block, by manipulating the heat rate generated by the electrical resistance. The control system is designed by using an optimum Linear-quadratic regulator and a Luenberger state observer. The control system is implemented in Matlab© and simulated with the Computer-Aided Control System Design (CACSD) Simulink©. The control system is compatible with the National Instruments™ modular data acquisition system, which was also used in the experimental implementation of the present study.*

***Keywords: fin, control, heat transfer, transient equation***

## 1. INTRODUCTION

Initial value problems with Dirichlet boundary conditions are an important class of heat transfer problems. This type of boundary condition, with a constant temperature, can be approximately imposed on the body surface through heat transfer with materials undergoing phase-change, which generally involve very large heat transfer coefficients. Alternatively, a system can be implemented to control the surface temperature by acting on the heat flux imposed on the surface.

Thermal control systems have industrial applications in numerous fields, e.g. the food industry, chemical processes, power plants, automotive sectors, electronics, etc. Early in the last century, bi-metallic actuating switches were applied in thermal control systems (Taylor *et al*., 1948). Due to the development of electronics, the use of thermistors (Blackburn, 1987) surpassed those mechanical control devices in terms of both economic and accuracy criteria. Currently, state of the art technology involves optimum regulators and optimum observers (Kalman, 1960a; Kalman, 1960b) in thermal control dynamic systems (Karvounis *et al*., 2008).

This study presents the modeling, simulation and control of a thermal system. The thermal system presented is comprised of a test bench composed of an aluminum block with a cylindrical brass fin, in which an electrical resistor is inserted. The thermal system is modeled using lumped systems for the electrical resistance and the aluminum block, and by employing the transient fin heat transfer equation. Values for the fin heat transfer coefficient are obtained in experiments carried out on the test bench. The transient fin equation is numerically solved using the forward-time, centered-space (FTCS) finite difference explicit scheme, while the solution stability condition was calculated by the von Neumann stability analyses (Charney *et al.*, 1950). Convergence, in the sense of Lax-Richtmyer (Lax and Richtmyer, 1956), inferred by theoretical analysis is supported by the experiments.

The model state variables are converted to dimensionless values, so that the surrounding environment temperature assumes a zero value and the reference temperature at the fin base assumes a unitary value. The dimensionless model is used to determine the control equations in state space. The Modern Control theory is characterized by expressing the problem in a single vector matrix differential equation. This approach has known advantages over the Classical Theory in Laplace state (Ogata, 2003), as it reduces the multiple input/multiple output problems to a single equation.

The control is designed in this work with an optimum linear-quadratic regulator and a Luenberger state observer (Luenberger, 1979). The regulator controls the electrical resistance, while the state observer gathers information from two thermocouples in the test bench, in order to estimate the state variables. The numerical simulations were carried out within the Matlab© platform and the Computer-Aided Control System Design Simulink©. The data acquisition module used in the test bench was procured from National Instruments™.

Results of numerical simulation are compared with the controlled experimental bench response. Two reference conditions were tested: a constant reference temperature, and a three-step reference. A good convergence between the results reveals an appropriate modeling of the physical problem and of the control system

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## 2. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The test bench of this study has an aluminum block with two through-holes, wherein an electrica l resistor and a fin are fixed, as shown in Fig. 1. Due to the large thermal conductivity of aluminum and the low heat transfer on the external surface of the aluminum block, it is formulated in terms of a lumped system. Equation (1) presents the formulation of the aluminum block.

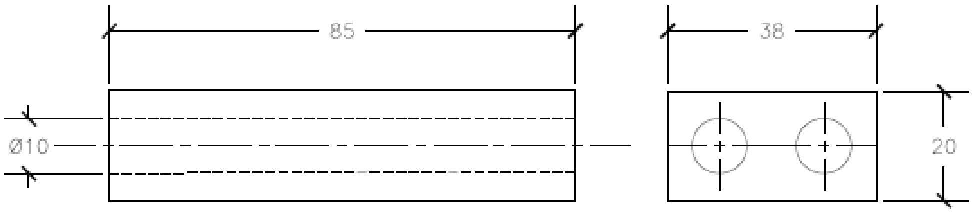



Figure 1. The aluminum block dimensioned in millimeters

$$V_{block}\, c_{p\,Al}\, \rho_{Al} \frac{dT_1}{dt} = \dot{g}(t) V_g - h_c A_c [T_1(t) - T_2(t)] - \dot{Q}_{loss}(t) \quad (1)$$

where $V_{block}$ and $V_g$ are the volumes of the block and of the resistor, respectively; $A_c$ is the contact area between the block and the fin; $c_{pal}$ and $\rho_{al}$ are the specific heat and density of the aluminum, respectively; and $h_c$ is the thermal contact conductance coefficient between the block and the fin, taken as a constant value of $10^3$ W/m$^2$K (Ayers, 2003), regardless of its possible dependency on the interface heat flux. The variable $\dot{g}$ is the heat generation per unit volume inside the electrical resistor, and $Q_{loss}$ is the heat loss from the aluminum block to the ambient. In equation (1), $T_1$ is the temperature of the aluminum block, while $T_2$ is the temperature of the fin inside the aluminum block, which is also assumed to be uniform for the sake of simplicity.

The fin was discretized by finite differences, so that $T_2$ also gives the temperature of the boundary node that is located at the fin base. Hence, the other $n$ nodes along the fin surface are numbered as illustrated by figure 2. Figure 3 is a photograph of the test bench. The thermophysical properties used in the formulation are presented in Table 1.

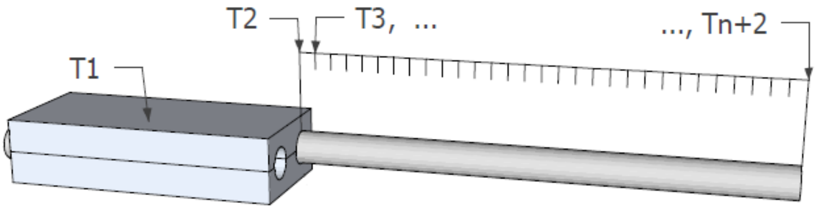
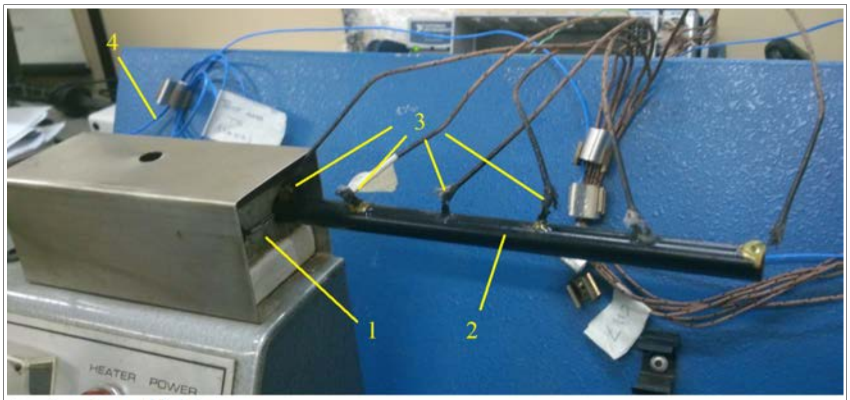



Figure 2. The test bench one-dimensional mesh model with state variables.




1 - aluminum block
2 - brass fin
3 - thermocouples type J (fin state sensors)
4 - thermocouple type K (environment sensor)

Figure 3. The test bench

ISSN 2176-5480



Table 1. Nominal Material Properties at 300K ( Touloukian and Ho, 1972).

| Material Properties | Aluminum Alloy 6061 | Brass 360 0,7 Cu; 0,3 Zn |
|---|---|---|
| Thermal Conductivity, k (W/m.K )[(1)] | 154,9 | 110 |
| Density, ρ ($kg/m^3$)[(1)] | 2700 | 8530 |
| Specific Heat, cp (J/kg.K)[(1)] | 962,9 | 380 |

[(1)] reported uncertainties in these values are within 3,5%

The heat loss, $Q_{loss}$, is calculated using the McAdams correlation (McAdams, 1954), which yields in a non-linear equation that relates the heat loss to the state *T1* and the surrounding environment temperature $T_\infty$, in the form of equation (2a), where $A_{block}$ is the surface area of the block that exchanges heat with the surrounding environment. Equation (2a) is then linearized in terms of the reference control set point temperature $T_{ref}$, as shown in Eq. (2b).

$$\dot{Q}_{loss} = 1{,}36 \ A_{block}(T_1 - T_\infty)^{5/4} \quad (2a)$$

$$\dot{Q}_{loss} = 1{,}36 \ A_{block}(T_1 - T_\infty)(T_{ref} - T_\infty)^{1/4} \quad (2b)$$

The portion of the fin located inside the aluminum block was all considered in a uniform temperature $T_2(t)$. The lumped formulation for that region of the fin is given Eq. (3):

$$V_b \, c \, \rho \, \frac{dT_2(t)}{dt} = h_c A_c [T_1(t) - T_2(t)] - q_b(t) A \quad (3)$$

where $q_b$ is the heat conducted to the brass fin at its base, $V_b$ is the volume of the fin inside the aluminum block, $A$ is the cross-section area of the fin, while $c$ and $\rho$ are the specific heat and density of the brass, respectively.

The region of the fin that exchanges heat with the surrounding environment by free convection is modeled in terms of the classical transient fin equation, where partial lumping is used in the fin cross section, as given by Eq. (4). The boundary condition at the fin base is the heat flux $q_b(t)$ that is transferred from the fin region inside the aluminum block, as in Eq. (4). At the fin tip, convective heat transfer is assumed with the same heat transfer coefficient of the lateral fin surface, $h$, as in Eq. (5). The mathematical formulation for the fin is then given by

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{hP}{kA}[T_\infty - T(x,t)] \qquad (for: \ 0 < x < L \ , \ t > 0) \quad (4)$$

$$-k\frac{\partial T}{\partial x} = q_b(t) \qquad (for: \ x = 0) \quad (5)$$

$$k\frac{\partial T}{\partial x} + h\,T = h\,T_\infty \qquad (for: \ x = L) \quad (6)$$

where $P$ and $A$ are the perimeter and area of the fin cross section respectively. The length $L$ of the fin portion outside the aluminum block is about 150 millimeters, and the thermal diffusivity is $\alpha = \rho c_p/k$. The density ρ, the specific heat $c_p$, and the thermal conductivity $k$ of brass can be found in Tab 1. The heat transfer coefficient $h$ is 1296 $W/m^2K$. Its uncertainty is within 9,5%, obtained experimentally for this test bench at steady state conditions, with a constant reference temperature of 155 ºC at the fin base, and an environment temperature of 25 ºC.

The mathematical modeling for the problem is provided by two ordinary differential equations for the block and the fin inside the block, and one partial differential equation for the fin outside the block. These equations are subjected to initial conditions that involve equilibrium with the surrounding medium. That is, at time $t = 0$, the block and the fin are at the uniform temperature $T_\infty$.

By using the second order space-center finite-difference explicit scheme of Eq. (7), the partial differential equation (4) reduces to a system of $n$ ordinary differential equations for each of the finite-difference nodes. These equations were discretized explicitly in time.

$$\frac{\partial^2 T(x_0)}{\partial x^2} \approx \frac{T(x_0+\Delta x) - 2T(x_0) + T(x_0-\Delta x)}{\Delta x^2} \quad (7)$$

For the boundary nodes, a central difference with a fictitious node was used for the computation of the temperature gradients at the surfaces, so that only the temperatures appear as state variables for the control problem state in Eq. (8).

$$\frac{\partial T(x_0)}{\partial x} \approx \frac{T(x_0+\Delta x) - T(x_0-\Delta x)}{2\Delta x} \quad (8)$$

Von Neumann's Stability Analysis (Charney *et al.*, 1950) is carried out, so that the explicit finite difference system of equations is stable if the condition in Eq. (9) is satisfied:

$$\Delta t \leqslant \frac{2}{\alpha\left(\frac{hP}{KA}+\frac{3}{\Delta x^2}\right)} \quad (9)$$

## 3. MODERN CONTROL DESIGN

While conventional control theory is based on the input-output relationship, modern control theory is based on a convenient description of system equations in a single first-order vector-matrix differential equation (Ogata, 2003). Equations (10) and (11) stands for the system dynamics and the observable output dynamics, respectively. The system of equations below is the classical representation of a linear control system in space-state.

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{B}\,u \quad (10)$$

$$y = \mathbf{C}\,\mathbf{x} + D\,u \quad (11)$$

where, for the present case, **x** is the state vector composed of the dimensionless temperatures at the block, fin base and fin nodes; u is the input signal of the electrical resistance in Watts, which is the control; and y is the output signal from a thermocouple located at the fin base. The matrix vectors **A**, **B** and **C,** and the scalar D represent the system's and instruments' dynamics. The matrices **A** and **B** result from the discretization of the formulation of the physical problem, as described above.

For convenience in the analysis, the formulation of the physical problem was written in dimensionless form, with the state variables given in Eq. (12):

$$\mathbf{x}_i = \theta_i = \frac{T_i - T_\infty}{T_{ref} - T_\infty} \quad (12)$$

The only observable output in the test bench is considered to be the temperature at the fin base. Therefore, by neglecting the sensor dynamics, the line vector **C** is [0 1 0 0 … 0] and the scalar D equals to zero. A proportional control is assumed, with the gain **k** for the input signal u as a function of the estimated state $\hat{\mathbf{x}}$, so that,

$$u(t) = -\mathbf{k}\,\hat{\mathbf{x}}(t) \quad (13)$$

The error of this estimation is given by

$$\mathbf{e}(t) = \mathbf{x}(t) - \hat{\mathbf{x}}(t) \quad (14)$$

By rewriting Eq. (10) in terms of the estimated state $\hat{\mathbf{x}}$, we obtain:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\,\hat{\mathbf{x}} + \mathbf{B}\,u \quad (15)$$

Subtracting Eq. (15) from Eq.(10) results in:

$$\dot{\mathbf{e}} = \mathbf{A}\,\mathbf{x} - \mathbf{A}\,\hat{\mathbf{x}} = \mathbf{A}\,\mathbf{e} \quad (16)$$

Equation (16) shows that the error variation is not dependent on the control signal. Hence, if **A** is unstable, the error grows unlimited. In order to mitigate this possible instability a correction term **L** is introduced, as applied in Eq. (17) and (18). The term L is known as the Luenberger state observer (Luenberger, 1979).

$$\dot{\mathbf{x}} = \mathbf{A}\,\hat{\mathbf{x}} + \mathbf{B}\,u + \mathbf{L}(y - \hat{y}) \quad (17)$$

$$\hat{y} = \mathbf{C}\,\hat{\mathbf{x}} \quad (18)$$

ISSN 2176-5480



By substituting Eq. (17) into Eq.(16a) we obtain

$$\dot{\mathbf{e}} \;=\; \mathbf{Ax} - \mathbf{A}\,\hat{\mathbf{x}} - \mathbf{L}\left(\mathbf{Cx} - \mathbf{C}\,\hat{\mathbf{x}}\right) \;=\; \left(\mathbf{A} - \mathbf{LC}\right)\mathbf{e} \tag{19}$$

And, by using equations (13) and (14), equation (10) becomes:

$$\dot{\mathbf{x}} \;=\; \mathbf{A}\,\mathbf{x} - \mathbf{BK}\,\hat{\mathbf{x}} \;=\; \left(\mathbf{A} - \mathbf{BK}\right)\mathbf{x} \;+\; \mathbf{BK}\;\mathbf{e} \tag{20}$$

According to the Principle of Separation of Estimation and Control (Joseph and Tou, 1961; Potter, 1964), the closed-loop control stated by Eqs. (19) and (20) will be stable if the regulator dynamic (**A** – **BK**) is stable; and, the estimator dynamics (**A** – **LC**) is also stable. Figure (4) illustrates this principle in a control system.

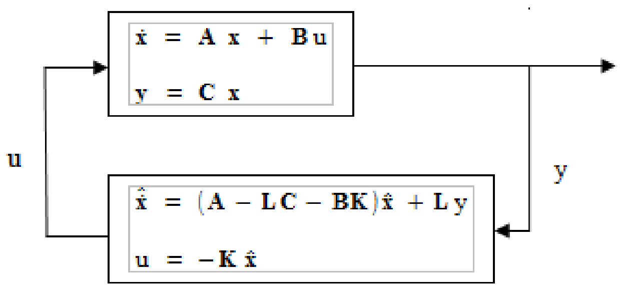



Figure 4. Close-loop control system with proportional gain and Luenberger observer.

When designing the control system, the gain **L** and **K** must be calculated. To determine the optimal control gain as in a quadratic optimal control problem (Kalman, 1960a), the objective function in Eq. (21) is minimized:

$$J \;=\; \int_0^{\infty} \left(\mathbf{x}^*\mathbf{Q}\;\mathbf{x} \;+\; \mathbf{u}^*\mathbf{R}\;\mathbf{u}\right)\,dt \tag{21}$$

where the weight matrices **Q** and **R** represent the relative importance of the error and the energy expenses in the control. The notation $^*$ on the state variables and control denotes the matrix conjugate transpose.

The order of magnitude for the values of those weights can be found with Bryson's rule (Bryson and Ho,1975), Eq. (22). Although Bryson's rule generally renders good results, its application is often just the starting point of a trial-and-error iterative design procedure aimed at obtaining desirable properties for the closed-loop system (Hespanha, 2007). Therefore, a small iterative correction was carried out in this study after using Bryson's rule, in order to satisfy tighter control specifications of the test bench application.

$$\begin{aligned} \mathbf{Q}_{i\,i} &= \frac{1}{maximum\; acceptable\; value\; of\;\; x_i^2} \\ \mathbf{R}_{j\,j} &= \frac{1}{maximum\; acceptable\; value\; of\;\; u_j^2} \end{aligned} \tag{22}$$

The steps to solve the optimization problem stated in Eq. (21) leads to an algebraic Riccati equation – ARE – that is shown in Eq. (23). The ARE solution **P** will be used in Eq. (24) to find the optimum proportional gain **K**. With this procedure, a stable regulator dynamic (**A** – **BK**) is obtained, in the sense of Lyaponov stability (Kalman, 1960a; Lyaponov, 1892).

$$\mathbf{A}^*\,\mathbf{P} + \mathbf{P}\;\mathbf{A} - \mathbf{P}\;\mathbf{B}\;\mathbf{R}^{-1}\;\mathbf{B}^*\;\mathbf{P} + \mathbf{Q} \;=\; 0 \tag{23}$$

$$\mathbf{K} \;=\; -\mathbf{R}^{-1}\;\mathbf{B}^*\;\mathbf{P} \tag{24}$$

The quadratic optimal control problem has strong similarities with the estimator gain determination problem. Indeed, the poles that make the dynamic of (**A** – **BK**) stable will also make (**A** – **LC**) stable, and this property is called duality. The resultant eigenvalue dual problem is stated in Eq. (25).

$$\left|s\,\mathbf{I} - \mathbf{A} + \mathbf{BK}\right|\,\left|s\,\mathbf{I} - \mathbf{A} + \mathbf{LC}\right| \;=\; 0 \tag{25}$$

Aiming at the same poles of the regulator dynamic, finding **L** is straightforward. Equations (26) and (27) represent a

ISSN 2176-5480



space-state equation of a dual problem as stated in Eq. (25).

$$\dot{\mathbf{z}} = \mathbf{A}^* \mathbf{z} + \mathbf{C}^* \mathrm{v} \tag{26}$$

$$\mathrm{n} = \mathbf{B}^* \mathbf{z} \tag{27}$$

where v is obtained in terms of the proportional dual gain, as in Eq. (28):

$$\mathrm{v} = -\mathbf{k_d}\, \mathbf{z} \tag{28}$$

The dual gain $\mathbf{k_d}$ is calculated so that the eigenvalue problem ($\mathbf{A}^*$ - $\mathbf{C}^*\mathbf{k_d}$) has the same poles of the quadratic optimal control problem given by Eq. (20). Furthermore, if the problem ($\mathbf{A}^*$ - $\mathbf{C}^*\mathbf{k_d}$) has the same eigenvalues of ($\mathbf{A} - \mathbf{k_d}^*\mathbf{C}$), $\mathbf{L}$ is given by Eq. (29).

$$\mathbf{L} = \mathbf{k_d}^* \tag{29}$$

The desired control set-point value must then be established. Equations (30) and (31) show the system equilibrium state. To maintain a constant temperature, the generation term of the electrical resistance has to compensate for the heat losses through convection. In an equilibrium state ($\partial \mathbf{x}/\partial t = 0$), a non-zero reference output signal y is related to a reference state **x,** and demands a certain reference input signal u.

$$\dot{\mathbf{x}} = [0] = \mathbf{A}\,\bar{\mathbf{x}} + \mathbf{B}\,\bar{\mathrm{u}} \tag{30}$$

$$\mathrm{y} = \bar{\mathrm{y}} = \mathbf{C}\,\bar{\mathbf{x}} + \mathrm{D}\,\bar{\mathrm{u}} \tag{31}$$

The solution of Eq. (30) and (31) are in the form of Eq. (32) and (33), respectively.

$$\bar{\mathbf{x}} = \mathbf{F}\,\bar{\mathrm{y}} \tag{32}$$

$$\bar{\mathrm{u}} = \mathrm{N}\,\bar{\mathrm{y}} \tag{33}$$

Equation (13) states an arbitrary proportional gain that leads to a convergence towards a zero signal, because the reference is zero. It is desirable to achieve a non-zero set point state, therefore, a more suitable non-zero set-point proportional gain can be expressed by Eq. (34) (Hespanha, 2007).

$$\mathrm{u} - \bar{\mathrm{u}} = -\mathbf{K}\left(\hat{\mathbf{x}} - \bar{\mathbf{x}}\right) \tag{34}$$

From Eq.(32), (33) and (34) the non-zero set-point regulator equation is given by Eq. (35).

$$\mathrm{u} = -\mathbf{k}\,\hat{\mathbf{x}} + \left(\mathbf{K}\,\mathbf{F} + \mathrm{N}\right)\bar{\mathrm{y}} \tag{35}$$

Figure (5) shows the schematic block diagram of the closed-loop control system with a non-zero set-point.

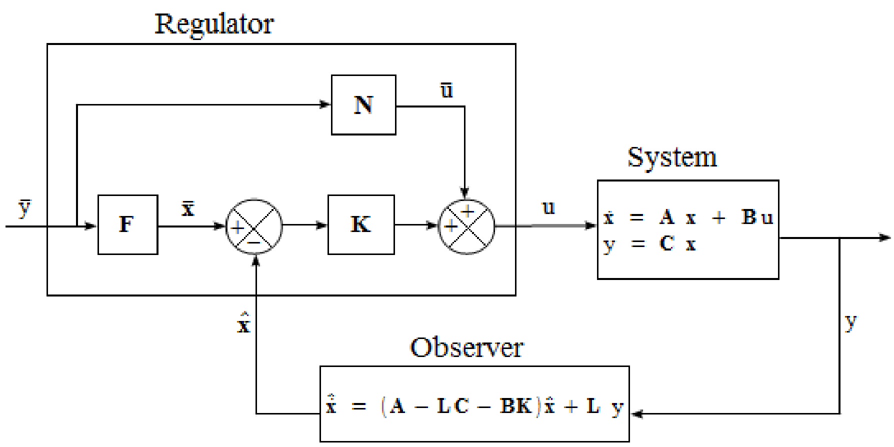



Figure 5. The close-loop control system

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## 4. RESULTS AND DISCUSSION

The control system described above is applied to regulate the temperature at the fin base. The designed control is tested against the numerical model and in the real test bench for various energy cost weights **R**, while the error cost **Q** is kept at a set constant value, calculated using Bryson's rule, for a maximum acceptable error of 1 ºC.

For the numerical tests, the set-point involved three-step input reference signals, with dimensionless values of 0.35, 0.5 and 1.0, the reference 1.0 being related to the reference temperature of 100 ºC. Figure (6) shows the results obtained for the dimensionless base temperature, in numerical tests of the control system for **R** = 0.1, 0.2, 0.5, 1 and $1 \times 10^{3}$. In this figure, the black line represents the set-point, and the color lines represent the responses of the controlled system. All the numerical tests are carried out considering the environment temperature of 24 ºC.

Figure (6) shows that setting a low energy weight results in a large overshoot, and requires significant time for convergence. Setting up the energy weight from 0.1 to 0.2 incurs in a slightly increased energy cost, but still yields an overshoot which disappears for **R** = 0.5. Tests demonstrate that the system is very sensitive for **R** in the range from 0.1 to 0.5. In fact, the response of the controlled system is practically unaffected for larger values of the energy weight R, as illustrated by figure below. A good compromise between error and energy expenditure is then achieved with **R** = 0.5, due to its low overshoot and fast convergence.

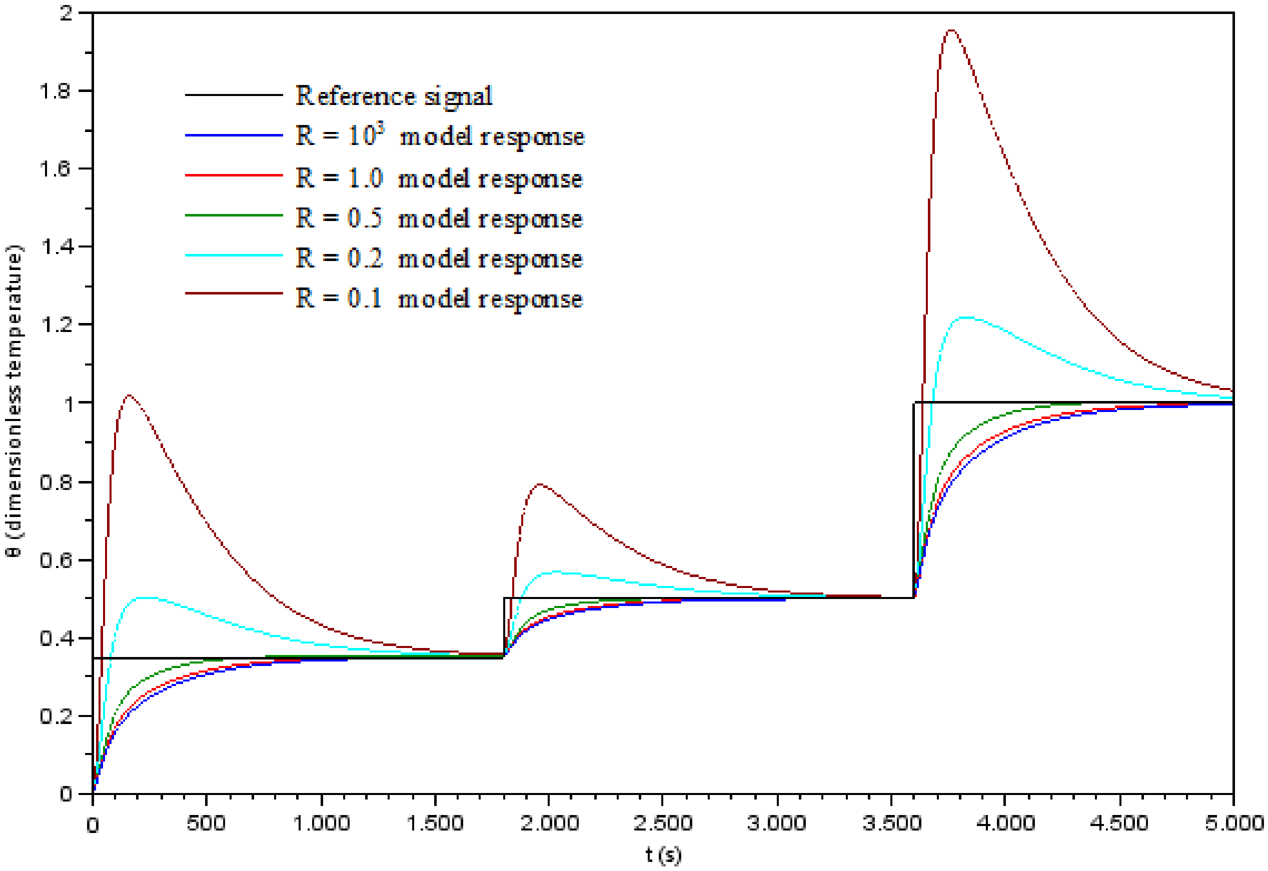


Figure 6. Model Response vs. Reference three-step input signal

Figure (7) shows the control acting upon the real test bench. Three constant reference temperatures are set for the experiment: 35 ºC, 50 ºC and 100 ºC. The error cost is kept constant and given by Bryson's rule and the energy cost is set R=1.0. This figure shows that the control system designed in the present work is capable of accurately responding to the set-point changes, in a stable and fast manner. Oscillations observed in the actual base temperatures can be due to perturbations in the environment temperature, which is considered as constant and equal to 23 ºC in the model.

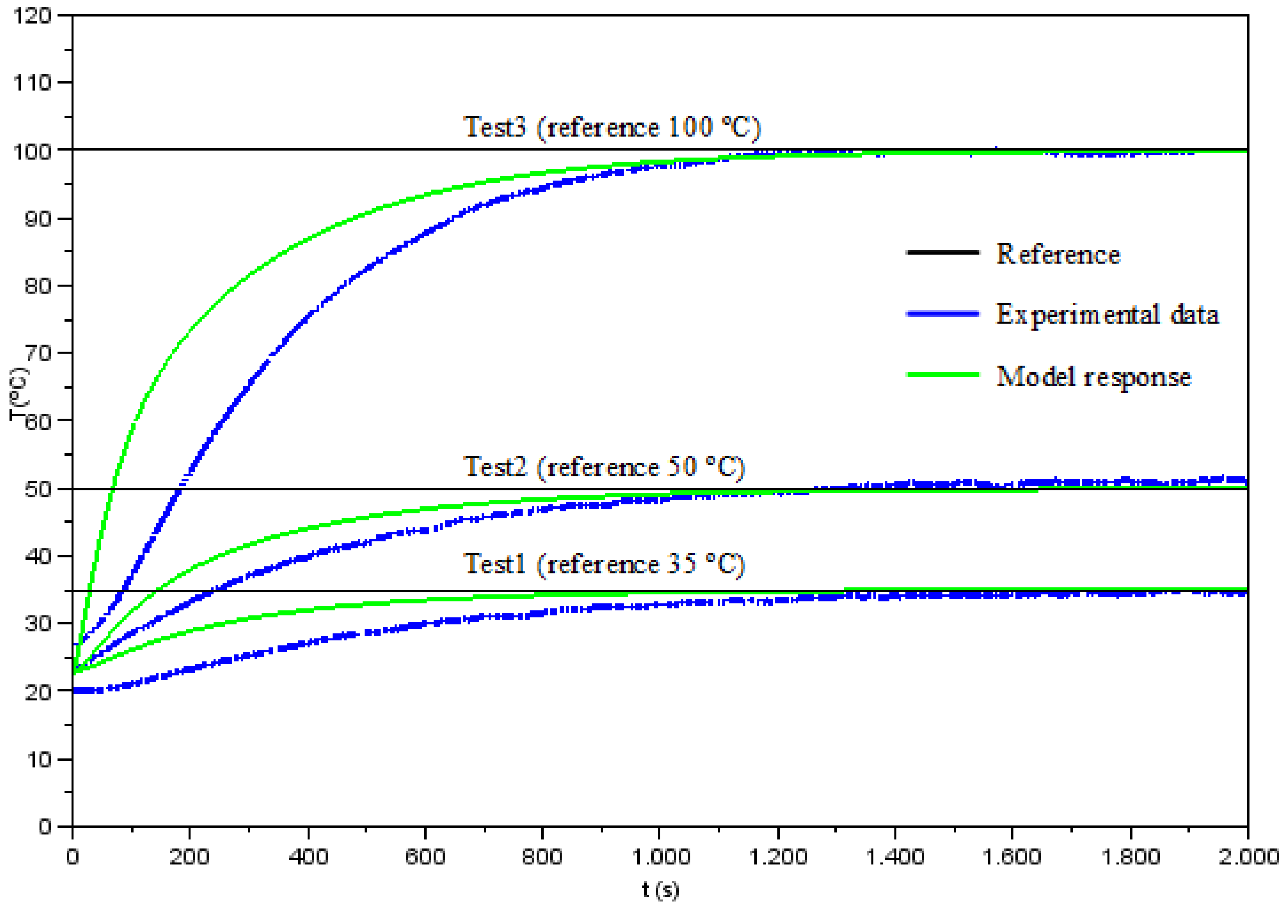


Figure 7. Model response vs. Experiment data for three reference inputs at **R**=1.0

## 5. CONCLUSION

This work deals with the control of the base temperature of a cylindrical fin. The formulation of the physical problem is presented and consists of two ordinary and one partial differential equations. The partial differential equation is solved by explicit finite-differences, and its stability criteria is verified by von Neumann analysis. An optimal control approach is applied in this work, with a formulation in terms of state variables.

The control system is implemented in Matlab, which is compatible with the input/output system manufactured by National Instruments, also used in the experiment. The control system is applied to both simulated and actual experimental data. For both cases, the utilized approach resulted in stable behavior, with overshot and response time that could be appropriately tuned by the weight matrices of the objective function of the optimal control problem.

## 6. ACKNOWLEDGMENTS

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