ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil


# INFLUENCE OF A PURELY NONLINEAR ABSORBER ON THE VIBRATIONS AND STABILITY OF A TALL BUILDING

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***Abstract.*** *The importance of vibration control in civil engineering is increasing, especially in the design of tall buildings that are vulnerable to the actions of dynamic loads, for example, wind and earthquake. Usually hybrid or passive control is used. One of the most used passive vibration control devices is the Tuned Mass Damper (TMD). The TMD, in its simplest form, consists of a linear mass-spring-damper system (secondary system) that is installed in a convenient position of the main structure (primary system) and is tuned to a particular excitation frequency, dissipating in this way most of the energy of the primary system. Recently it has been advocated that, if properly designed, purely nonlinear dampers can be more efficient in dissipating the energy of the primary systems. The aim of this paper is to investigate the behavior of a two degree of freedom system composed of a linear spring-mass-damper system connected to a mass through a purely nonlinear spring with cubic nonlinearity in the main resonance region. A detailed parametric analysis is conducted using time histories, Poincaré maps, bifurcation diagrams and basins of attraction to verify the behavior of this dynamical system.*

***Keywords: Passive control, tuned mass damper, purely nonlinear damper, cubic nonlinearity, energy sink, bifurcation analysis, dynamic integrity.***

## 1. INTRODUCTION

The nonlinear energy sink (NES) has been defined as an essentially nonlinear single-degree-of-freedom (SDOF) structural element with relatively small mass and weak dissipation, attached to a primary structure via essentially nonlinear coupling (Gendelman, 2001; Vakakis and Gendelman, 2001; Manevitch *et al*., 2007). If the primary structure is excited by a shock whose energy is above a certain critical threshold, the NES can act as broadband passive and adaptive controller by absorbing vibration energy from the primary structure in an almost irreversible manner. This process is referred to as passive targeted energy transfer (TET) (Quinn *et al*., 2008; Sapsis *et al*., 2009). TET normally occurs via transient resonance captures, made possible by the essential (nonlinearizable) stiffness nonlinearity of the NES which prevents a preferential resonance frequency.

One of the pioneering works in this area is due to Gendelman (2001), who was the first to identify the transference of energy from the main linear system to the nonlinear system, that is, the NES. He studied the dynamics of a two freedom degrees system composed of a linear oscillator weakly coupled to a strongly nonlinear main system coupled by linear stiffness and linear damping. He indicated that this study could help in the design of nonlinear tuned mass dampers (TMDs), the main advantage being that a nonlinear TMD have not a preferred oscillation frequency. This nonlinear TMD interacts in a resonant way with main system modes in an arbitrary range of frequencies. Gendelman also showed that while the input energy is transmitted to the linear oscillator, one nonlinear normal mode (NNM) can be excited if the transmitted energy is greater than a critic value. As a result, TET happens and a significant portion of transmitted energy is dissipated by the nonlinear TMD.

Recently (Vakakis *et al*,. 2008) summarized in two volumes the main contributions of the preceding years on this subject and presented a detailed literature review on targeted energy transfer and nonlinear energy sink.

The present work investigates through a detailed parametric analysis the effect of a purely nonlinear system with cubic nonlinearity on the behavior of a linear structural system, in particular the energy transfer of the main system to the nonlinear system. In this work the nonlinear part has a mass of comparable magnitude as the linear one. The aim is to verify if under these conditions energy transfer occurs under different dynamic conditions, namely initial displacement, impulse and harmonic excitation.

ISSN 2176-5480



This work consists of five sections. The first, this one, presents a brief introduction to the subject and the motivation of this paper. The second section presents the mathematical model and the governing equations of motion. The third one consists of the free vibration analysis. The next, presents the problem formulation considering external forcing. The fifth section presents the numerical results and, finally, the last one contains the conclusions of this paper.

## 2. FORMULATION

The simplest Nonlinear Energy Sink (the NES) is composed of a linear mass-spring-damper system connected to the nonlinear secondary system, as shown in Fig. 1, where $f(t)$ is the external force, $M$, $K$ and $C$ are, respectively, the mass, linear stiffness and damping of the main system, while $m$, $k$ and $c$ are the NES mass, linear stiffness and damping and $\beta$ is nonlinear stiffness coefficient.

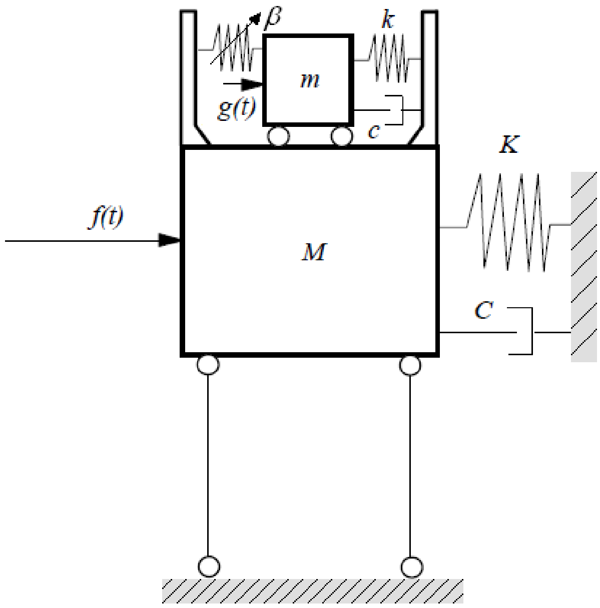


Figure 1 – Building (main system) with the attached NES.

The dynamics of the system is described by the two coupled equations of motion:

$$\begin{gathered}(M+m)\cdot \ddot{x}_1(t)+m\cdot \ddot{x}_2(t)+C\cdot \dot{x}_1(t)+K\cdot x_1(t)=f(t)+c\cdot \dot{x}_2(t)+k\cdot x_2(t)\\ m\cdot \ddot{x}_1(t)+m\cdot \ddot{x}_2(t)+c\cdot \dot{x}_2(t)+k\cdot x_2(t)+x_2^{\,3}(t)\beta=g(t)\end{gathered} \tag{1}$$

where $x_1(t)$ is the absolute displacement of the mass $M$, while $x_2(t)$ is the relative displacement of the mass $m$ with respect to the mass $M$. This allows the analysis of purely nonlinear damper and classical linear and nonlinear damper.

## 3. FREE VIBRATION

This section presents a parametric study of the linear NES optimum damping ($c_o$). For this the equations of motion (1) are integrated using the Runge-Kutta algorithm and, for each $c$ value, the main system's first amplitude peak is evaluated, as well as the time that the main system takes to reduce to 1% of the initial displacement.

### 3.1 Determination of the NES optimal damper ($c_o$)

Two initial conditions applied to the main system are tested in the study of the NES optimum damper ($c_o$). First an initial displacement and then one pulse is considered. The $m/M$ ratio considered was equal to 0.5, the damper ($C$) and the linear stiffness ($K$) of main system were considered equal to 0.02 Ns/m and 6.283 N/m, respectively. The nonlinear stiffness coefficient ($\beta$) is equal to 100 N/m$^3$.

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### 3.1.1 Initial Displacement

Fig. 3 shows the NES damper ($c$) effect considering an initial displacement equal to 0.12m applied to the main system. In this figure, it is evaluated for each $c$ value the time required for the displacement of the main system to be less than 1% of its initial displacement.

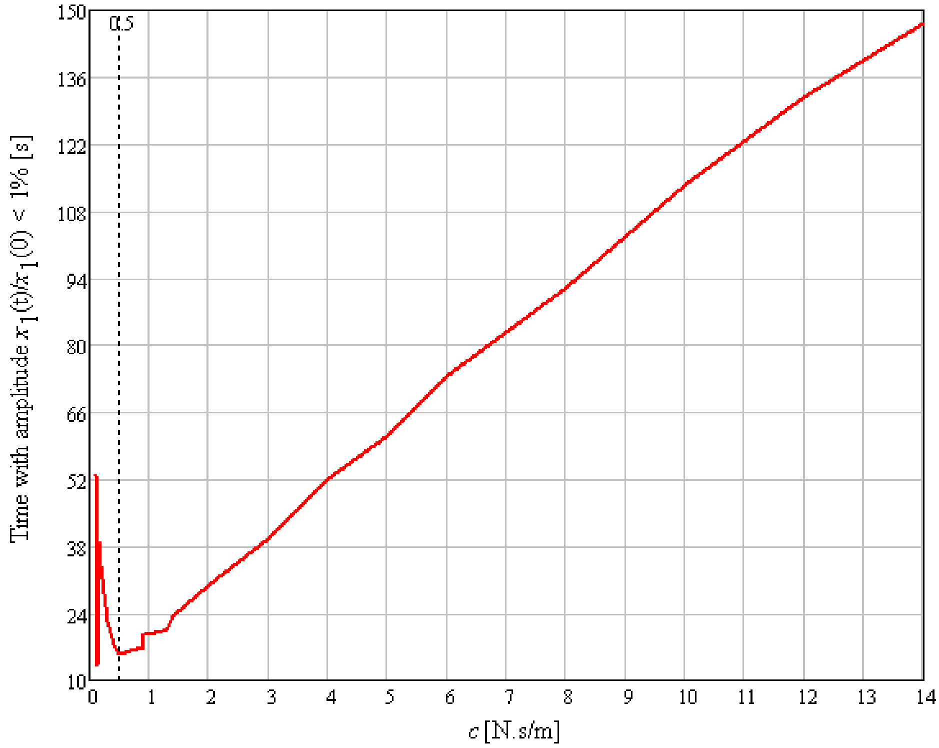
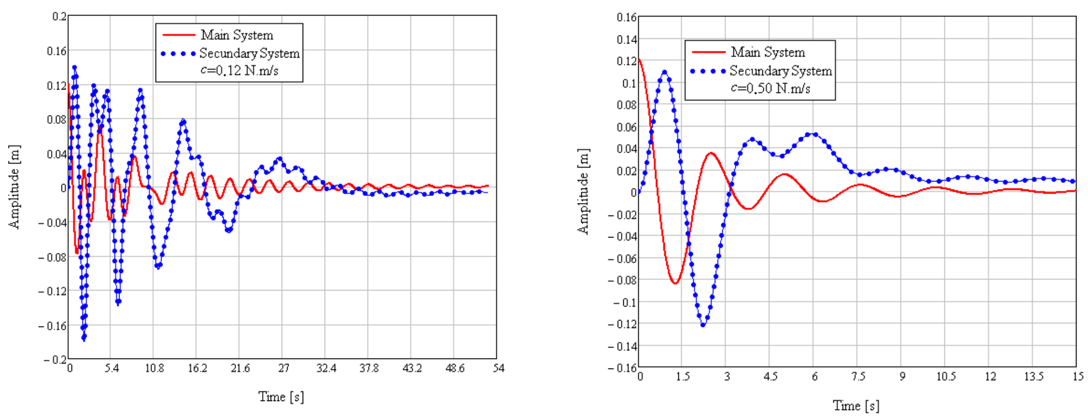



Figure 3 – Time required for the displacement of the main system to be less than 1% of its initial displacement as a function of the damping coefficient $c$.

Analyzing Fig. 3, it is observed that for $c$ = 0.5Ns/m the NES leads to the fastest reduction of the main system displacements and that a small level of damping should be used in practice.

Fig. 4 shows the time history of main system and the NES displacements for four selected values of $c$, where we can observe that for large values of $c$ practically no energy transfer occurs from the linear to the nonlinear system.



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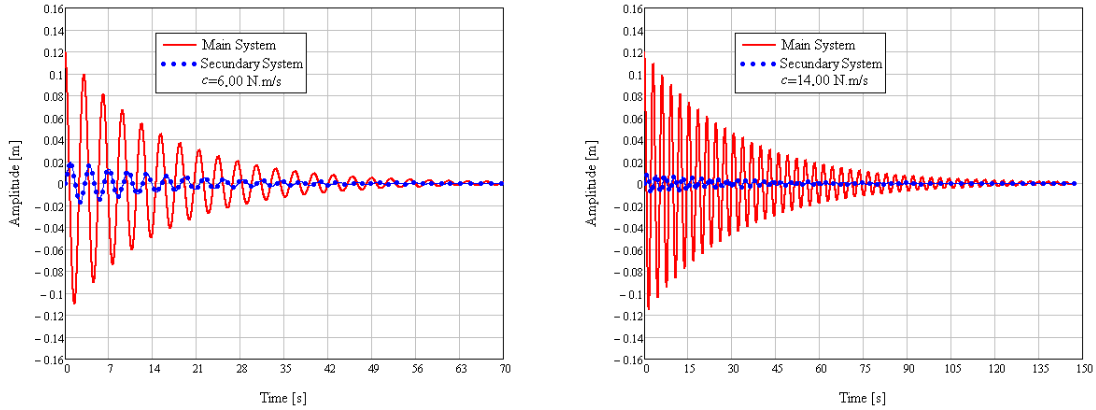
(a) (b)




(c) (d)

Figure 4 – Time history of main system and the NES displacements for (a) $c$ = 0.12 Ns/m, (b) $c$ = 0.50 Ns/m, (c) $c$ = 6.00 Ns/m and (d) $c$ = 14.00 Ns/m.

### 3.1.2 Initial Pulse

Fig. 5 shows the effect of the NES damper on the first peak amplitude of the main system, considering an initial pulse equal to 0.20m /s applied to the main system.

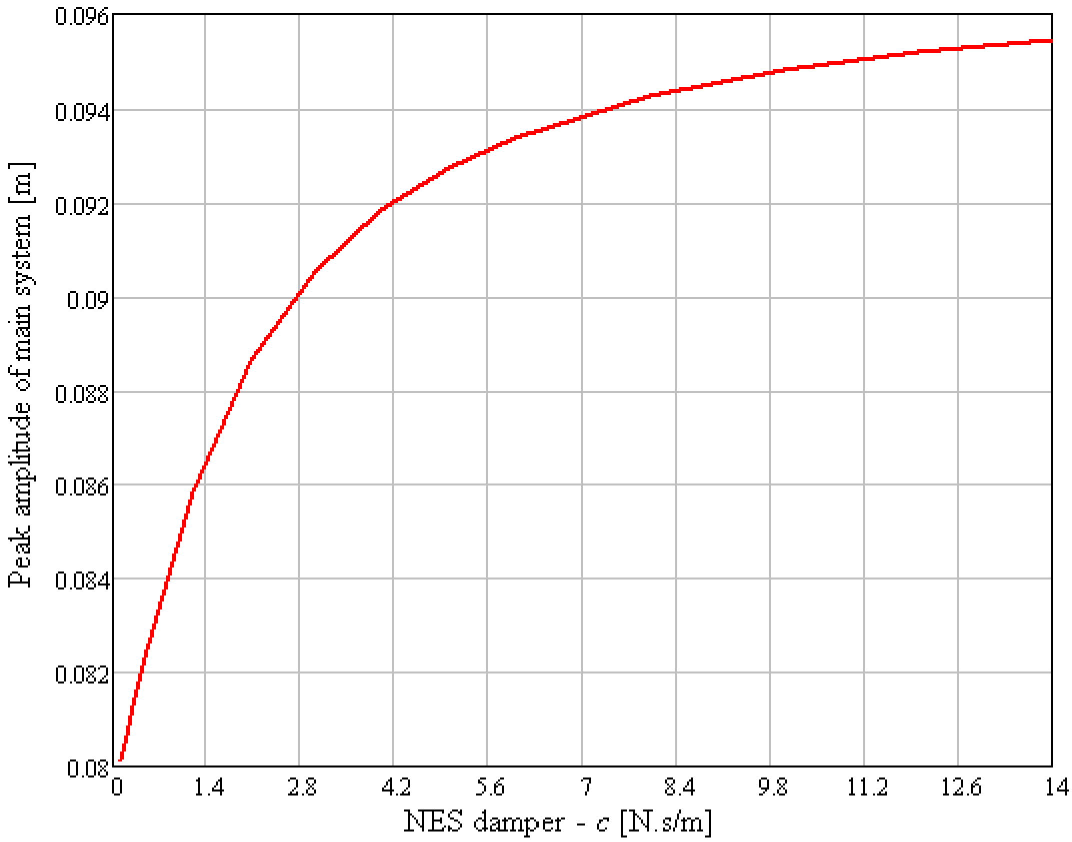



Figure 5 – First peak displacement amplitude of the main system as a function of the NES damping $c$.

Analyzing Fig. 5, it is observed that the first peak displacement amplitude increases asymptotically from a value of approximately 0.080m to a value of around 0.096m.

Fig. 6 shows the time history of main system and the NES displacements for four values of $c$.

ISSN 2176-5480



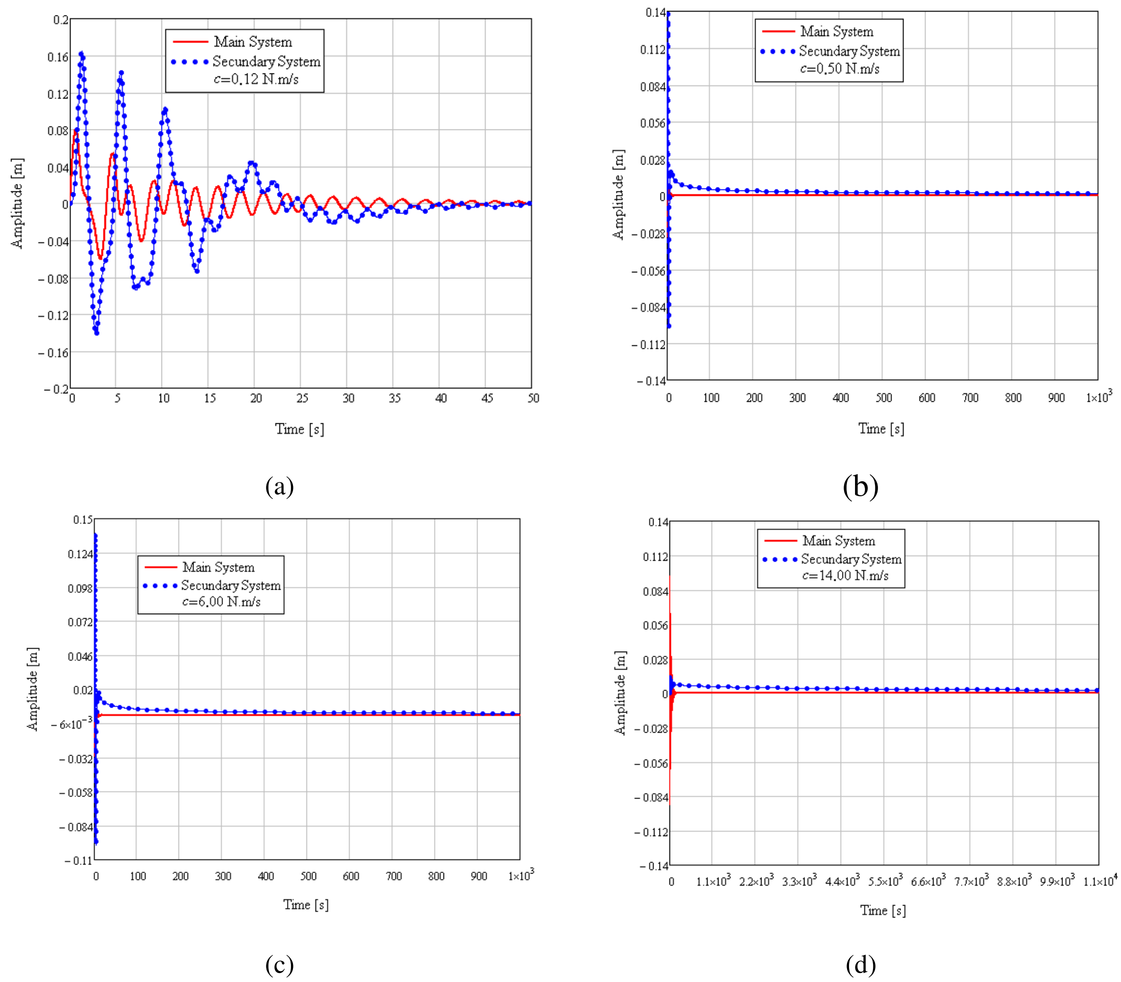



Figure 6 – Time history of main system and the of the NES for (a) $c$ = 0.12 Ns/m, (b) $c$ = 0.50 Ns/m, (c) $c$ = 6.00 Ns/m and (d) $c$ = 14.00 Ns/m.

## 4. HARMONIC LOADING VIBRATION

This section is divided into two subsections. The first presents a parametric study of the NES optimum damper ($c_o$) for a given value of $\beta$. Using this value of $c_o$, the second subsection analyzes the optimal value of the nonlinearity coefficient $\beta_o$ of the NES. The optimal values of $\beta_o$ and $c_o$ is obtained based on the evaluation of the main system and the NES displacements, where the Runge-Kutta algorithm is used again for the time integration of Eq. (1). The root mean square (rms) of the response is used as a means of evaluating the effectiveness of the NES parameters. The initial conditions are equal zero. The force has a magnitude of $A$ = 2.5 N and an excitation frequency $\Omega$ = 2.04 rad/s.

### 4.1 Determination of the NES optimal damper ($c_o$)

In this study $m/M$ ratio considered was equal to 0.5, the damper ($C$) and the linear stiffness ($K$) of main system are equal to 0.02 Ns/m and 6.283 N/m, respectively. The nonlinear stiffness coefficient $\beta$=100 N/m$^3$. Fig.7 depicts the NES damper effect in the first main system displacement amplitude peak.

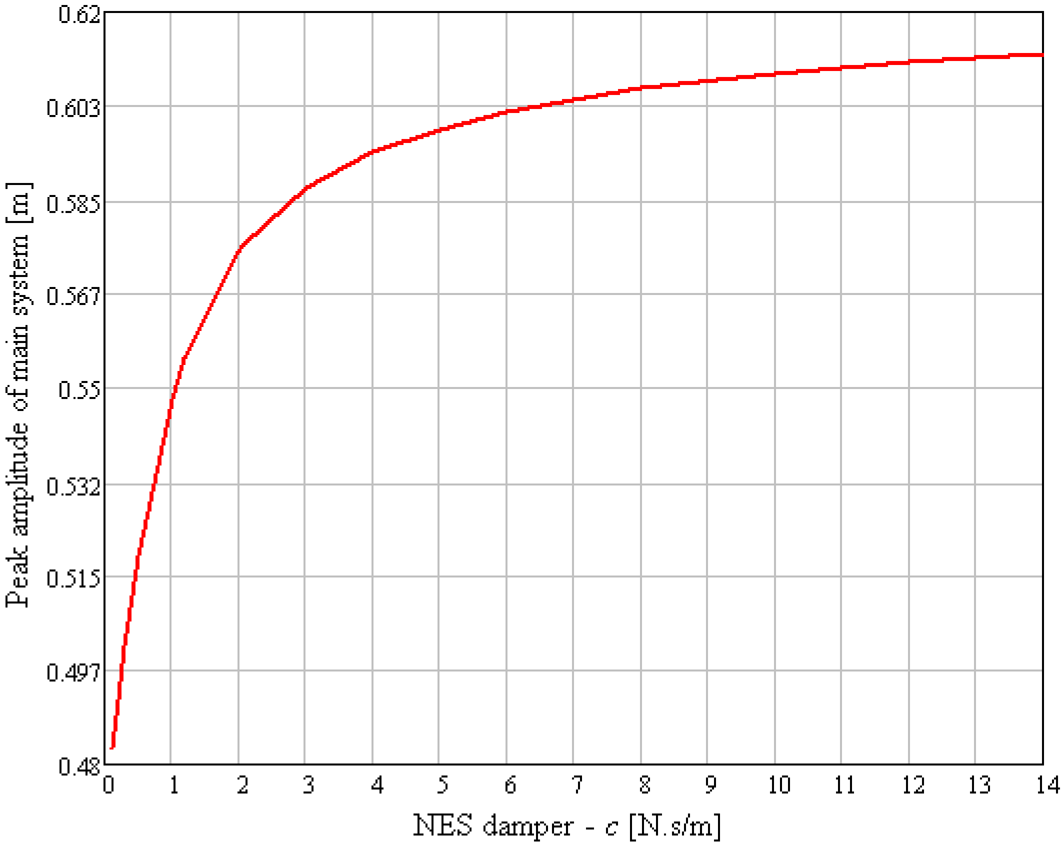



Figure 7 – The first main system displacement amplitude peak according to an increase of the NES damper ($c$) for loading vibration.

Fig.8 shows the evaluation of the value of efficiency of the NES damper from the RMS measure.

ISSN 2176-5480



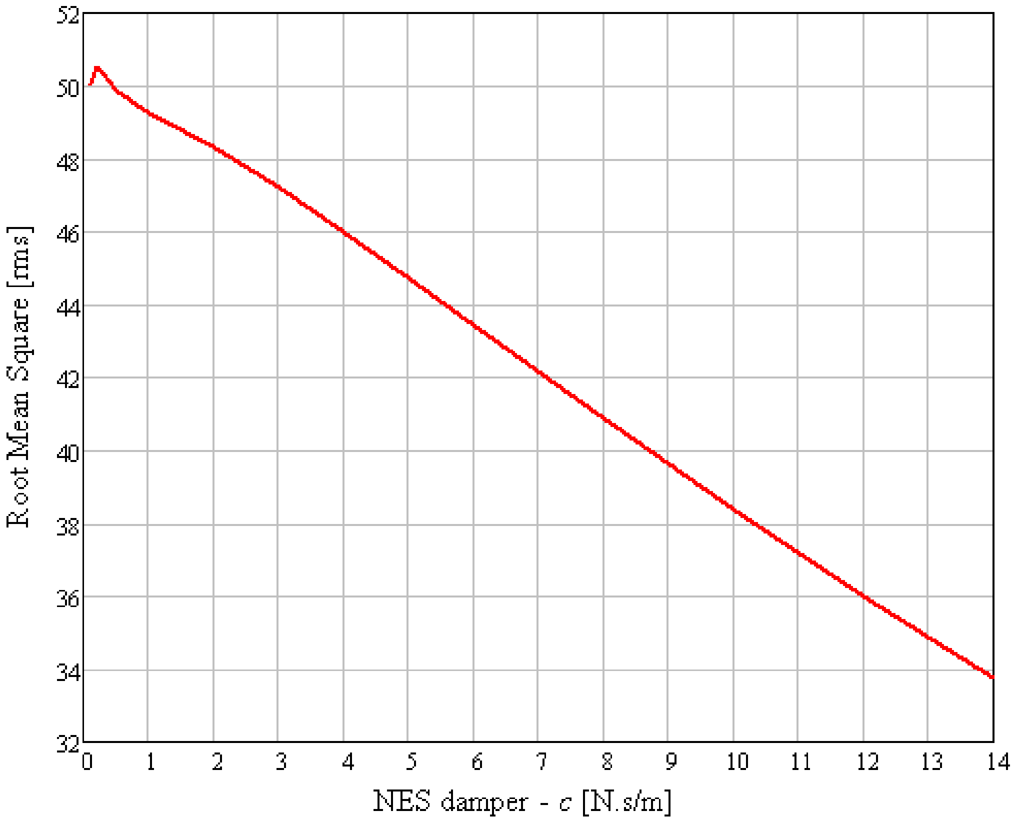



Figure 8 – RMS measure of efficiency of the NES damper value (*c*) under loading vibration.

According Fig. 8, the NES optimal damper ($c_o$) would be around 0.20 Ns/m.

## 4.2 Determination of the NES optimal nonlinear stiffness coefficient ($\beta_o$)

The *m*/*M* ratio, and *C* and *K* values were the same used in determining *c* optimal value ($c_o$). The $c_o$ = 0.20 Ns/m aforementioned is kept constant. Sixteen coefficients of nonlinear stiffness (β) were tested in a range, equally spaced, of 50 N/m$^3$ to 800 N/m$^3$.

Fig. 9 depicts the NES nonlinear stiffness coefficient ($\beta$) effect in the first peak of amplitude of displacement main system.

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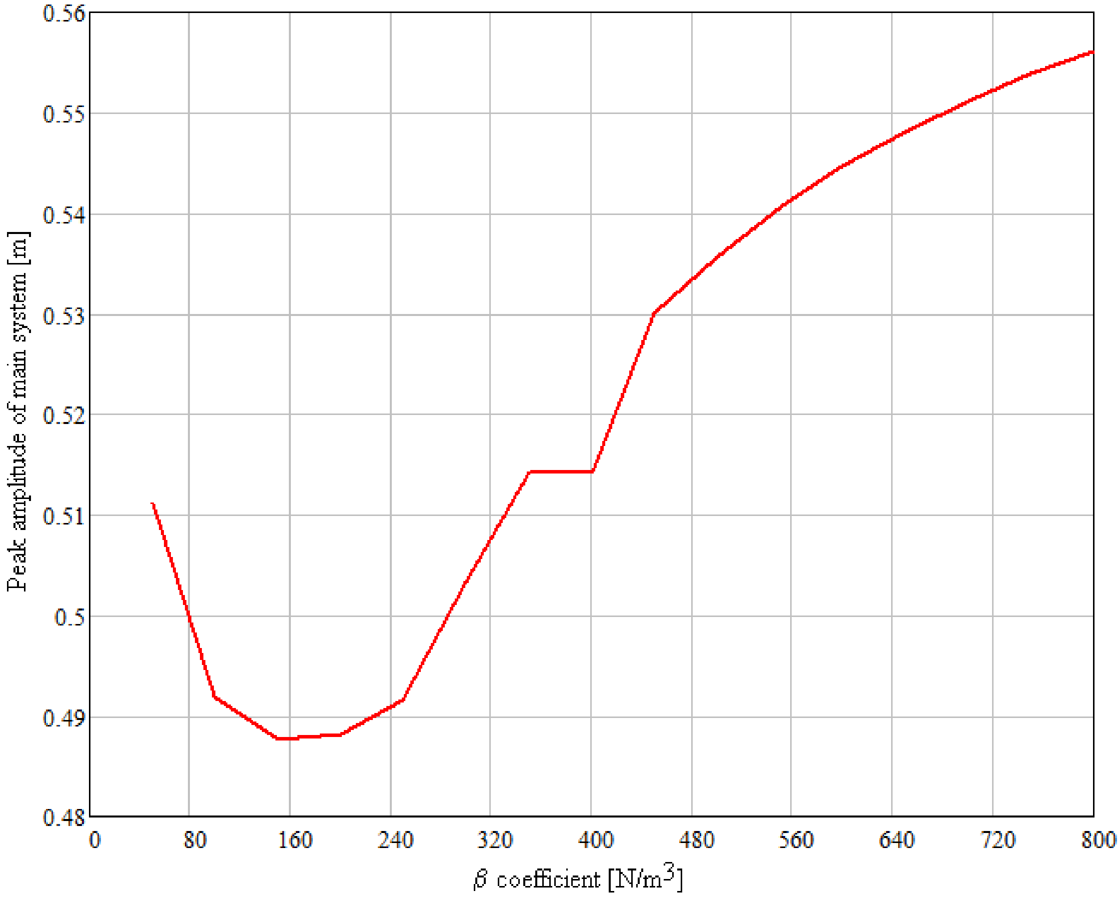


Figure 9 – First peak displacement amplitude of the main system according to the increase of the NES nonlinear stiffness coefficient ($\beta$) for loading vibration.

Fig. 10 shows the evaluation of the value of efficiency of $\beta$ from the RMS measure.

ISSN 2176-5480



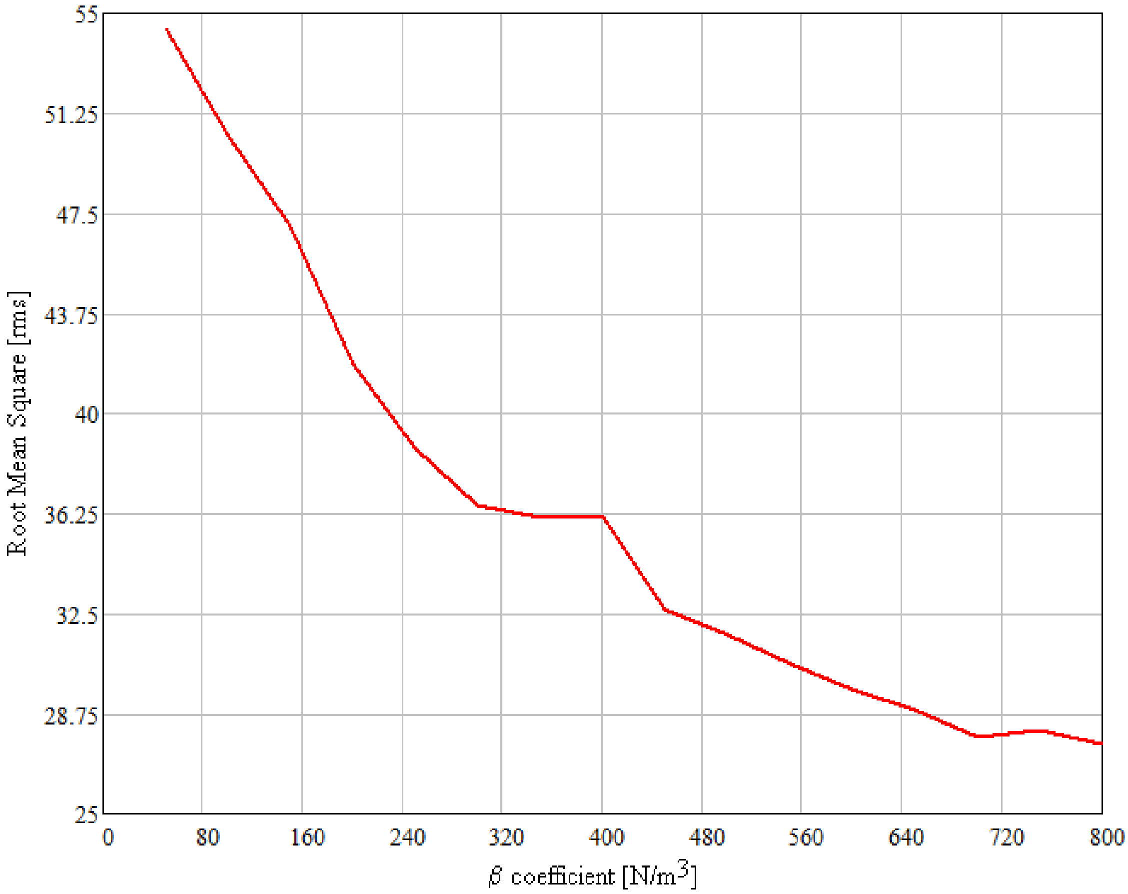


Figure 10 – RMS of the solution as a function of the nonlinear stiffness coefficient ($\beta$) under forced vibration.

## 5. STABILITY OF THE SOLUTIONS

This section shows the responses of the main system with nonlinear system. The force brute algorithm was used with $m/M$=0.5, damping of the main system $C$=0.02Ns/m, linear stiffness main system term $K$=6283N/m and the nonlinear stiffness term ($\beta$) being 100 N/m3. The frequency of the excitation is 1.0 rad/s and the load is a sine whose amplitude varies from 2.0 up to 7.0 N.

The Fig. 11 shows the diagram bifurcation to the response of the main linear system while Fig. 12 shows the bifurcation diagram to the nonlinear system as a function of the forcing magnitude. As the magnitude of the force increases, the system undergoes several bifurcations leading to periodic solutions of various orders, quasi-periodic and chaotic solutions. Also coexisting solutions are also observed. For example, for $A$=3N four coexisting attractors are observed. Figure 13 shows Poincaré Maps and the Phase Plans for these four solutions. This shows that the system has a highly non-linear behavior, in spite of the linearity of the main system.

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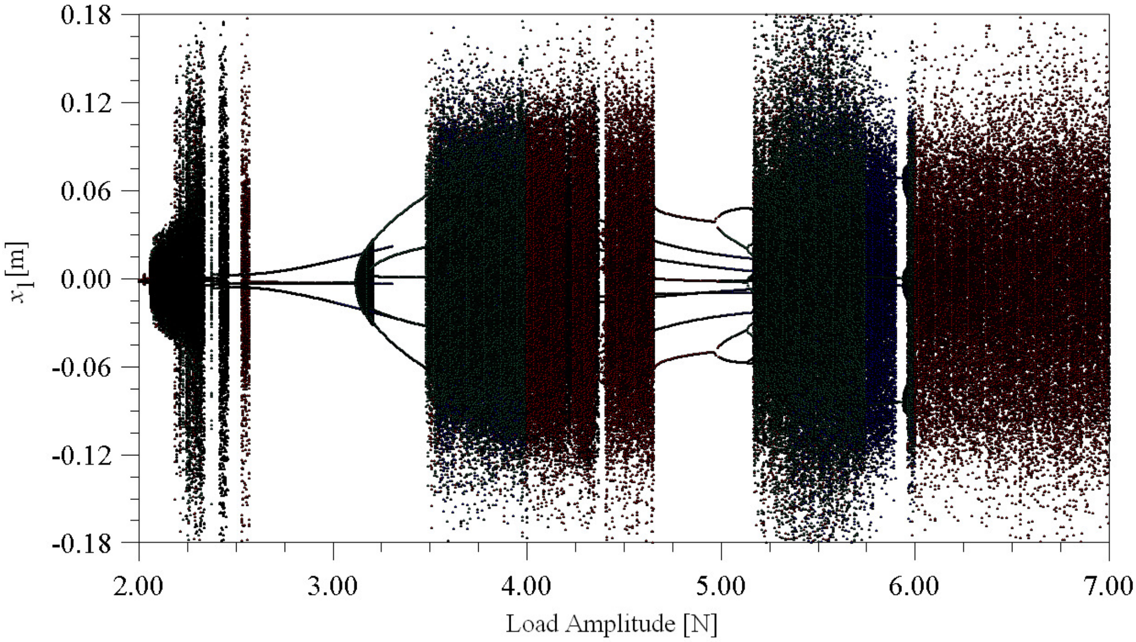



Figure 11 – Bifurcation diagram of the main system displacement to the sine load with rad/s of excitation and the load amplitude varies from 2.0 up to 7.0 N.

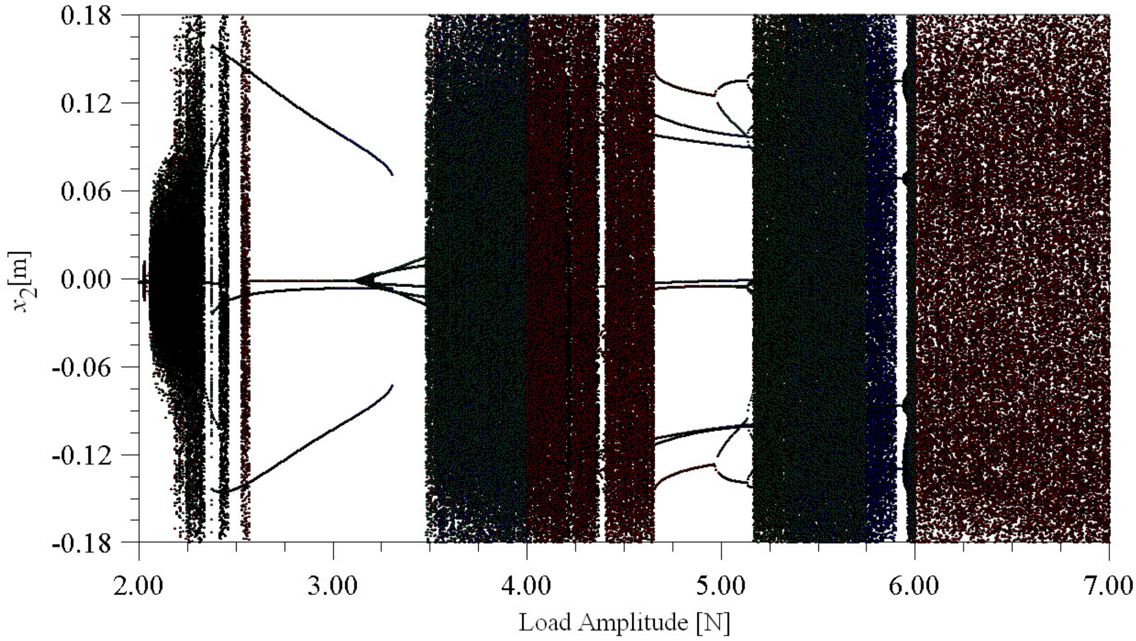
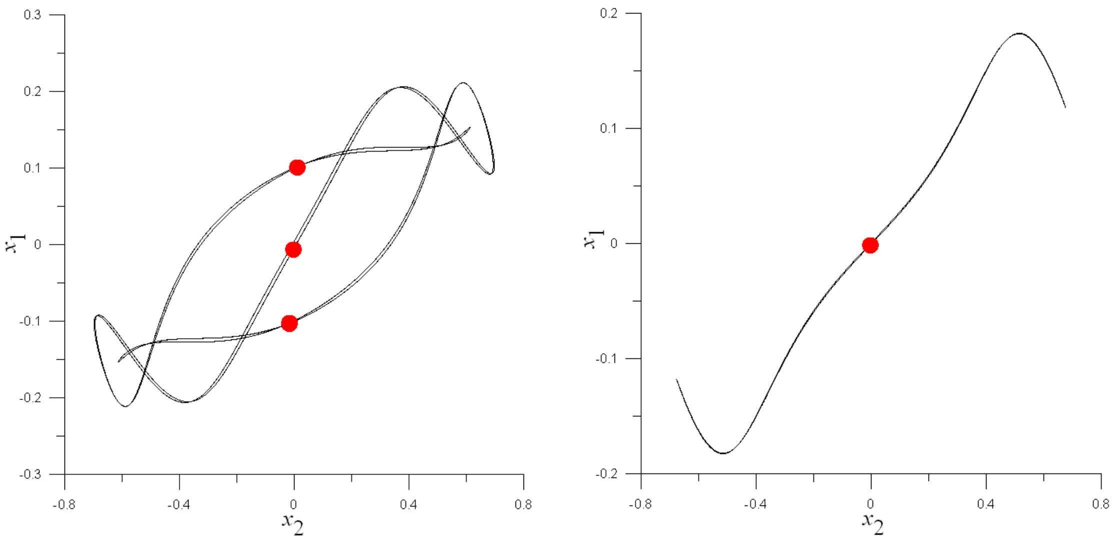



Figure 12 – Bifurcation diagram of the displacement to the main system under a sine load with rad/s of excitation and the load amplitude varies from 2.0 up to 7.0 N.

ISSN 2176-5480






(a) (b)

Figure 13 – Poincaré Maps and Phase Plans of the coexisting solutions for $A$=3.0N to: (a) first solution (b) the only solution having period one.

## 6. CONCLUSIONS

The results show that, for a two degree of freedom model with comparable masses, under specific circumstances energy transfer may occur from the linear to the nonlinear part of the two degree of freedom system. But the results also show that the response of the system becomes highly nonlinear with several bifurcations, leading to periodic solutions of various orders as well as quasi-periodic and chaotic motions.

This is part of a work in progress and future research will focus on the behavior of a linear system with a purely nonlinear system acting as a vibration absorber and a tuned mass damper with linear and cubic stiffness.

## 7. ACKNOWLEDGEMENTS

The authors, specially the first who has post doctoral fellow, would like to thank the CAPES Foundation of the Ministry of Education of Brazil.

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