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PUMPING POWER MINIMIZATION IN REGULAR CIRCULAR BANKS OF TUBES

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Abstract. *A semi-empirical model aimed at identifying an optimal tube spacing, $s/(2b)$, in a bank tube heat exchanger is presented in this article. The pumping power is computed from estimates of the mass flow rate and pressure drop experienced by the fluid flowing over the tubes while accounting for space constraints, by making use of geometrical characteristics, mass conservation and simple semi-empirical expressions for the pressure drop and friction factor. Pumping power reductions of up to one order of magnitude are obtained when the tube spacing is varied in the range $0.25 < s/(2b) < 3$, highlighting the importance of searching for the optimal spacing if minimizing pumping power is of concern.*

Keywords: *heat exchangers, pumping power, bank tube heat exchanger*

1. INTRODUCTION

Bank tube heat exchangers are widely used in the heating ventilation, air conditioning and refrigeration (HVAC-R) industries as well as in power generation systems. A large share of the total energy consumption worldwide is associated with HVAC-R systems. This fact, has in part motivated investigations on the heat transfer enhancement and pumping power minimization in bank tube heat exchangers.

Bejan *et al.* (1995) conducted an study of the optimal spacing between cylinders in cross-flow forced convection in which the optimal cylinder-to-cylinder spacing was found by maximizing the overall thermal conductance. Recent studies have shown numerically and experimentally that the finned elliptical tube arrangements have better heat transfer performance than the circular finned arrangement (Mainardes *et al.* (2007), Matos *et al.* (2004))Mainardes *et al.* (2012) conducted an experimental study of the pumping power minimization for staggered finned circular and elliptical-tube heat exchangers.

Event though there is sufficient experimental and computational evidence that the pumping power required to flow the fluid over the tube arrangement can be minimized by properly selecting the tube-to-tube spacing. There is a need for simple analytical tools to extend available scale analysis studies (Mainardes *et al.* (2012)) in order to produce more refined, yet quick, estimates of near optimal arrangements.

The objective of this study is to present a simple methodology to estimate the optimal tube spacing in a bank tube heat exchanger.

2. THEORY

A typical bank tube heat exchanger with a regular (non-staggered) configuration is depicted in Figure 1. The heat exchanger occupies a volume, $H \times W \times L$, which is frequently constrained due to space limitations. An important engineering problem consists of minimizing the pumping power under fixed total volume. The pumping power is defined as follows:

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$$\dot{W} = \frac{\dot{m}\Delta p}{\rho} \tag{1}$$

where $\dot{m}$ is the total mass flow rate (kgs^{-1}) entering the heat exchanger, Δp the pressure drop and ρ the density.

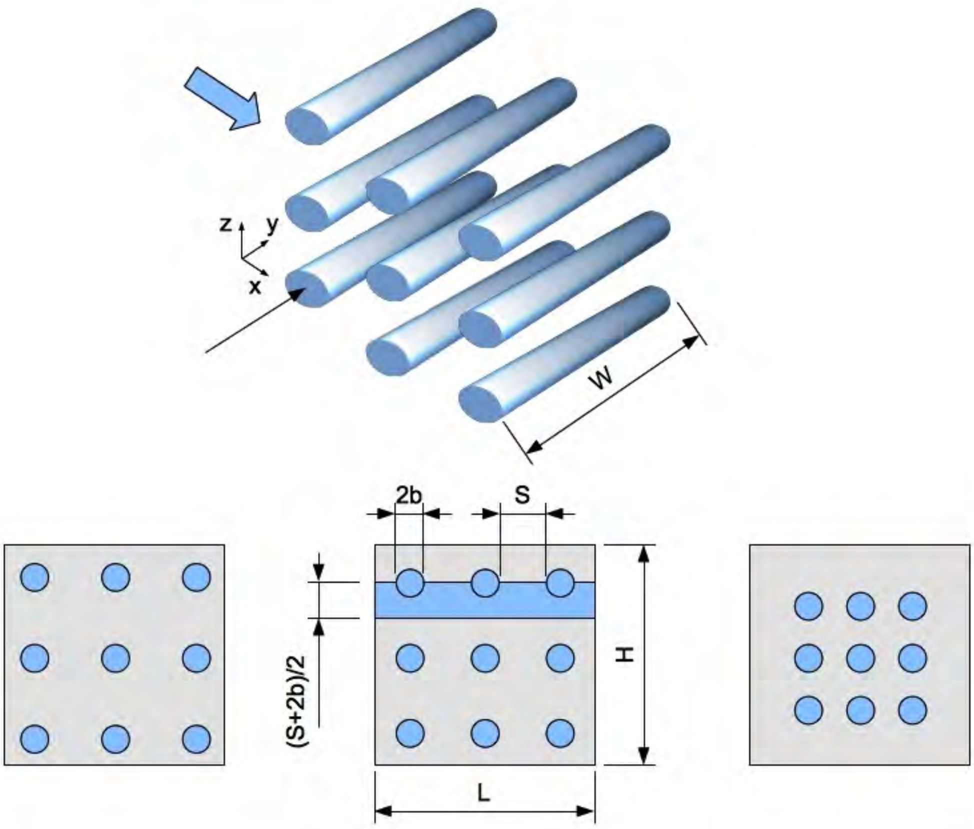

Figure 1. Top: Tube bank heat exchanger with regular (non-staggered). Bottom: three configurations with different ratios $S/(2b)$

On the bank side (outside the tubes), an elemental channel is defined as the sum of all unit cells in direction z (Fig. 1). Therefore, the total mass flow rate entering the heat exchanger on the bank side is given by the product between the heat exchanger resulting number of elemental channels, N_{ec}, by the mass flow rate through one elemental channel, $\dot{m}_{ec}$, as follows:

$$\dot{m} = N_{ec}\dot{m}_{ec} = N_{ec}\rho u_\infty \left[(S+2b)/2\right] W = N_{ec}\left(\frac{S}{2b}+1\right)\rho u_\infty bW \tag{2}$$

u_∞ is the free stream velocity entering the heat exchanger. The dimensionless pressure drop is given by,

$$\Delta\tilde{p} = \Delta p / \left(\rho u_\infty^2\right) \tag{3}$$

One possible expression to compute the pressure drop is Zukauskas (1987),

$$\Delta p = N_r \chi \left(\frac{\rho u_{max}^2}{2}\right) f_k \tag{4}$$

where N_r is the number of rows, $\chi = 1$ for configurations in which the spacing between columns is equal to the spacing between rows,and f_k represents a friction coefficient.

The maximum velocity can be related to u_∞ by,

$$u_{max} = \frac{S+2b}{S} u_\infty \tag{5}$$

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Making the dimensionless pressure drop on the bank side,

$$\Delta\tilde{p} = N_r \frac{f_k}{2} \left(\frac{\frac{S}{2b} + 1}{\frac{S}{2b}} \right)^2 \tag{6}$$

Incorporating Zakuskas expression for the pressure drop, the pumping power in the bank side, in a regular tube arrangement equal number of rows and columns, is given by,

$$\dot{W} = N_t \left(\frac{S}{2b} + 1 \right) \left(\frac{\frac{S}{2b} + 1}{\frac{S}{2b}} \right)^2 \rho u_\infty^3 f_k W b \tag{7}$$

Thus a dimensionless pumping power for the bank side can be written as follows:

$$\tilde{W} = \frac{\dot{W}}{N_t \rho u_\infty^3 b W} = \left(1 + \frac{S}{2b} \right) \Delta\tilde{p}_k = \left(\frac{S}{2b} + 1 \right) \left(\frac{\frac{S}{2b} + 1}{\frac{S}{2b}} \right)^2 f_k \tag{8}$$

3. Results and Discussion

For a constant friction factor, f_k, it is possible to find the spacing that minimizes the pumping power analytically by taking the derivative of the pumping power (Eq. 8) with repespect to $s/(2b)$ and solving $\partial\tilde{W}/\partial(s/(2b)) = 0$. By doing this, we obtain an optimal spacing, $(s/(2b))_{opt} = 2$.

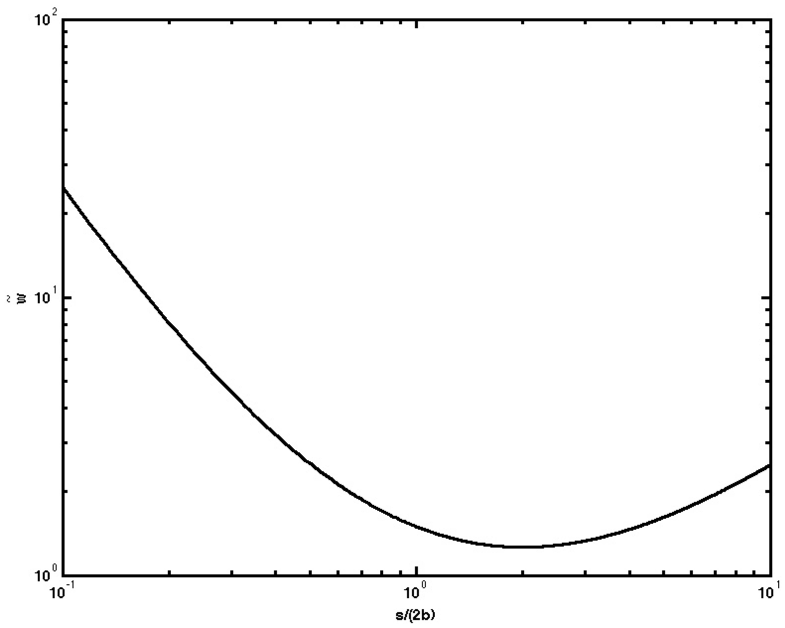

Figure 2. Pumping power as a function of tube spacing $s/(2b)$ - constant f_k

Variations in the friction factor, f_k, associated with the Reynolds number and the tube spacing in the range $0.25 < s/(2b) < 1.5$ can obtained from Zukauskas (1987), where the friction factor is reported graphically as a function of the spacing, $s/2b$, and the Reynolds number based on the tube diameter and the maximum velocity u_{max}. Since the free stream velocity u_∞ is being used in the present study, we have reploted Zakuskas plot for the friction factor using the Reynolds number based on tube diameter and and the free stream velocity, u_∞. The result is shown in Figure (3).

Our group has conducted experiments with related heat exchangers (finned staggered tube arrangements) using Reynolds numbers of 2650, 5300, 7950 and 10600 (Mainardes *et al.* (2012)). Notice that the heat exchangers under study here are not finned and are aligned instead of staggered. In spite of the difference, we select the same Reynolds numbers values for the analysis here. Markers in Figure 4 correspond to readings from Figure 3 and the trend dashed lines, correspond to a curve fit for the values in the domain $0.25 < s/(2b) < 1.5$, covered by Fig. 3.

f_k can be properly correlated in the range $0.25 < s/(2b) < 1.5$ and $2650 < Re_{2b,u_\infty} < 10600$ by an expression of the form,

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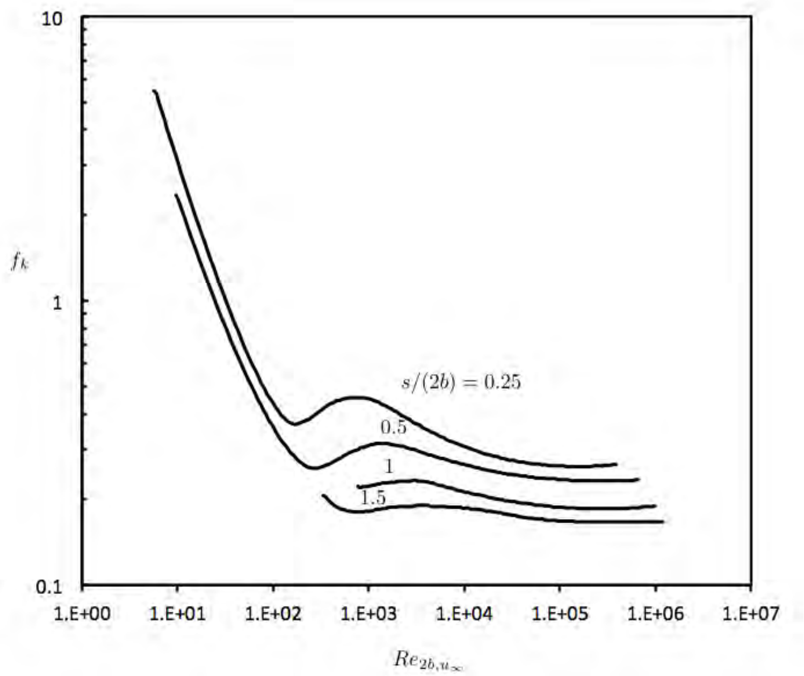

Figure 3. Friction factor for tube bank heat exchanger with regular (non-staggered) configuration for different ratios $S/(2b)$

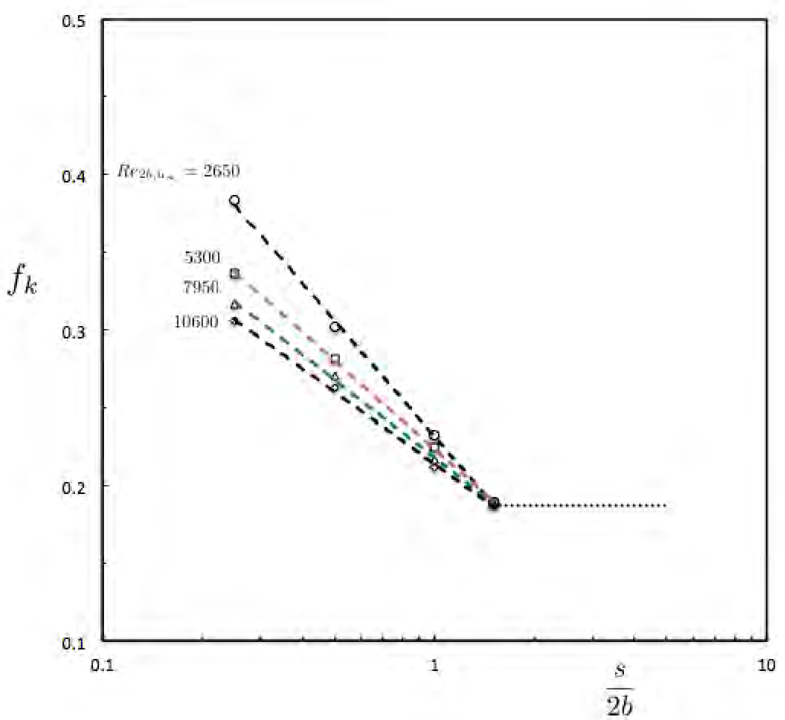

Figure 4. Effect of Reynolds and tube spacing on the friction factor used in Eq. 4

$$f_k = -B_1 \ln\left(\frac{s}{2b}\right) + B_2 \tag{9}$$

where the constants B_1 and B_2 are reported in Table 1.

In order to explore the domain of $s/(2b)$ larger than 1.5, we extrapolate the value of f_k taking it as a constant (dotted line in Fig. 4). The constant value for f_k for $s/(2b)$ larger than 1.5, could be reasoned as follows: when the tubes are far

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Table 1. Interpolation constants for Eq. 9.

Re_{2b,u_∞}	B_1	B_2
2650	-0.108	0.2315
5300	-0.082	0.2235
7950	-0.072	0.2179
10600	-0.067	0.2138

apart, the effects of one tube in the flow, do not extend to infinity for a non-zero Reynolds number (Panton (2006)). If the tubes are far enough apart, they will behave individually as a single tube. For flow over a single cylinder, the pumping power required to overcome the drag can be computed as the product of the drag force, F_D and the free stream velocity, and the drag force can be estimated from the drag coefficient, C_D,

$$\dot{W} = F_D u_\infty = C_D \frac{1}{2} \rho u_\infty^2 (2b) W \tag{10}$$

Then, for N_t non-interacting cylinders we would have,

$$\tilde{W} = \frac{\dot{W}}{N_t \rho u_\infty^3 bW} = C_D \tag{11}$$

We can use Eq. 11 to estimate a value for f_k, to be used in Eq. 8:

$$\tilde{W} = \frac{\dot{W}}{N_t \rho u_\infty^3 bW} = C_D = \left(\frac{S}{2b} + 1\right) \left(\frac{\frac{S}{2b} + 1}{\frac{S}{2b}}\right)^2 f_k \tag{12}$$

In the range $1 < s/(2b) < 10$, the group $\left(\frac{S}{2b} + 1\right) \left(\frac{\frac{S}{2b}+1}{\frac{S}{2b}}\right)^2$ is of order 10, in fact, $8 < \left(\frac{S}{2b} + 1\right) \left(\frac{\frac{S}{2b}+1}{\frac{S}{2b}}\right)^2 < 13.18$, then we can argue that,

$$f_k \sim \frac{C_D}{10} \quad for 1 < s/(2b) < 10 \tag{13}$$

The drag coefficient for flow over a cylinder is illustrated in Figure 5, which shows that in the range $10^3 < Re < 10^4$ the drag coefficient $C_D \sim 1$, which leads to a constant $f_k \sim 1/10$. This result agrees well with the extrapolation suggested by the dashed lines in Figure 4.

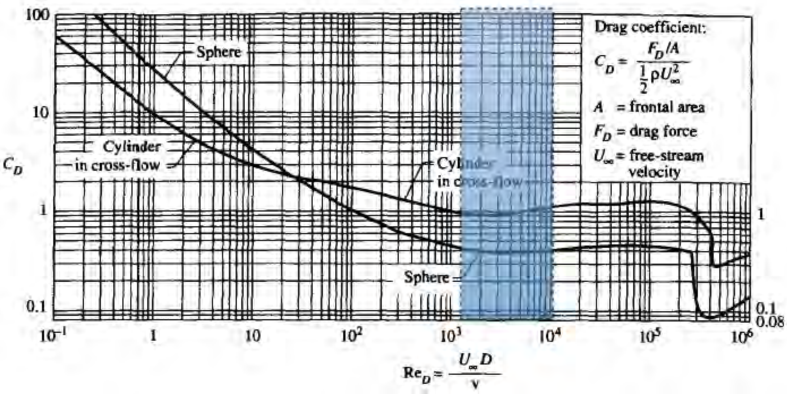

Figure 5. Drag coefficient for flow over a sphere and a single cylinder in cross flow. Drawn after Bejan (2004)
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With the Incorporation of the dependence on Reynolds and tube spacing into f_k in Eq. 8 it is convenient to re-scale the dimensionless pumping power, as now the free stream velocity could potentially vary. Defining a reference free steam velocity, u_∞^*, such that $Re^* = Re_{2b,u_\infty} \left(\frac{u_\infty^*}{u_\infty}\right)$, then we can track the dimensionless power,

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$$\tilde{W}^* = \tilde{W}\frac{Re}{Re^*} \tag{14}$$

Figure 6 illustrates the minimization of pumping power for cases in which the variation of f_k with both tube spacing and Reynolds number are accounted for. It can be observed that the optimal value still lies above $s/(2b) = 1.5$, where f_k is constant. For this reason, in the range considered here, the incorporation of Eq. 9 does not shift the optimal value of $s/(2b)$.

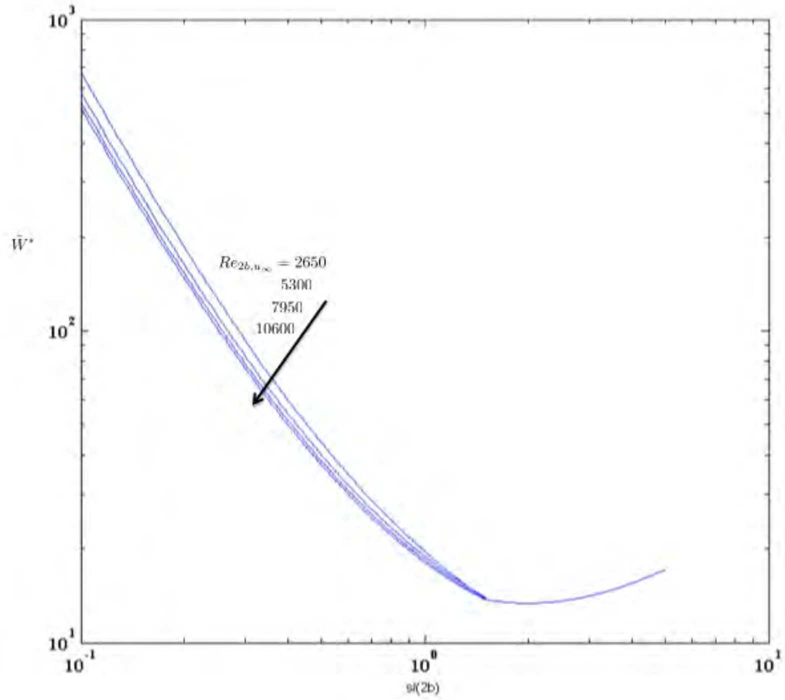

Figure 6. Minimization of pumping power, $\tilde{W}^*$. We have set $Re^* = 10^3$.

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4. Conclusions

A simple analysis to study the optimal tube spacing in heat exchangers consisting of a bank of circular tubes has been presented. The objective is to minimize pumping power under a total volume constraint. The results complement exiting experimental and computational studies in the sense that they provided a simple semi-empirical approach to quickly estimate optimal tube spacing. Appropriate dimensionless groups were identified to report the results to allow for generalization of the results. Specifically, key conclusions of this article are:

- An optimal tube spacing $(s/(2b))_{opt}$ = 2 was found to minimize the pumping power required for the fluid flowing outside the bank of tubes to flow.

- The optima obtained are sharp, highlighting their importance in actual engineering designs. For the case of constant f_k (Fig. 2), variations in the dimensionless pumping power of an order of magnitude are observed in the range $0.1 < s/(2b) < 10$

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