

## THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS OF THE LOAD DISTRIBUTION BEHAVIOR ON BOLTS IN DIFFERENT JOINTS CONFIGURATIONS

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**Abstract.** The load distribution behavior along the bolt body presents a great contribution to define it principally because the bending arm which determines the applied moment. Thus, this study presents different bolted joints configurations, usually applied in high responsibility structures in such way to analyze the load distribution behavior all along the bolt body. The joints present different plates materials and thicknesses as like aluminum and steel alloys, considering shims or not and standard bolts with a variety of diameter. The analyses regard the analytical method which presents the load distribution type, triangular, trapezoidal or rectangular and finite elements method (FEM) analysis considering three-dimensional models regarding contact between the joined parts and consequently nonlinear behavior. Therefore, after the analysis of the results from the mentioned methods, stated the shims and the plates thicknesses have a significant influence in the differences between the methods. Even though the differences exist, the reliability of the analytical method is confirmed once all bending arms are bigger than those resulted by the finite element method. Hence, the joints dimensioned by the analytical method are conservative, present satisfactory margins of safety and, consequently, low probability of bolt failure due bending.

**Keywords:** Bolted joints, load distribution, bolt, finite elements.

### 1. INTRODUCTION

Practically all mechanical system applies bolted joints in order to put together independent components, fitting up structural gaps to maintain the geometrical envelope in accordance with design and the alignment between the joined parts. Normally, the bolts withstand tension and shear loads which the last one generates bending moment in the bolt body due the contact effect with the hole surface. The load distribution behavior along the bolt body presents a great contribution to define it principally because the bending arm which determines the applied moment.

Some bibliographies like Bruhn (1973) and Shigley (1984), consider in the bolt bending dimensioning, an approximate bending arm, judging the load distribution between the hole and the bolt body as uniform. Figure 1 shows a joint under double shear.

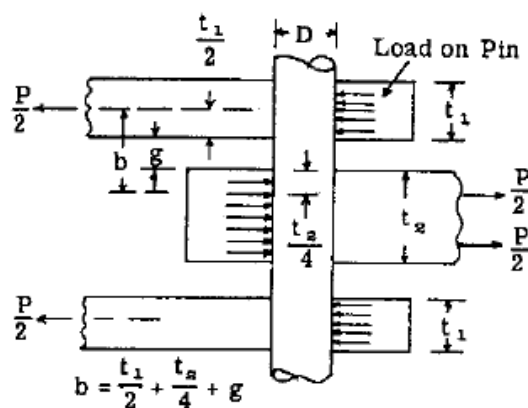


Figure 1. Rectangular load distribution in accordance with Bruhn (1973).

However, Niu (2005) mentions once the bolt bends, the stress distribution tends to present a pick instead of present a uniform distribution as cited previously. This variation along the contact between bolt and hole results in a triangular or trapezoidal load distribution which decrease the bending arm and consequently the stress. Figure 2 shows a non uniform load distribution in a bolt joint.

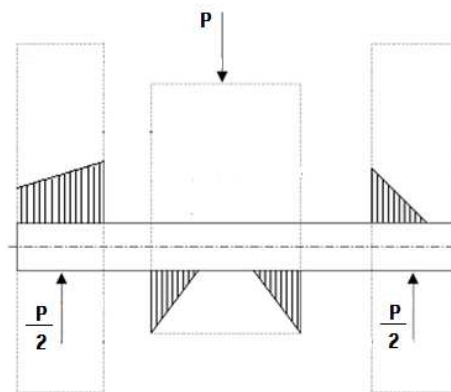


Figure 2. Trapezoidal and triangular load distribution.

The main purpose of this paper is to demonstrate the load distribution behavior in different bolted joints configurations using a conventional analytical method and the finite element method (FEM). The last one regards three-dimensional models with non linear analysis allowing contact and friction among the involved bodies.

Thus, the results correlation between the performed analyses will show some variations regarding the cited methods as the kind of load distribution, the effective length of the bolt body which withstands the applied loads, the joints behavior and the particularity of each studied method.

## 2. ANALYTICAL METHOD

In accordance with Bruhn (1973), the arm is defined by the Eq. (1) for single shear and Eq. (2) for double shear:

$$b = \frac{t_1}{2} + \frac{t_2}{2} + g \quad (1)$$

$$b = \frac{t_1}{2} + \frac{t_2}{4} + g \quad (2)$$

Where:  $b$  bending arm;  
 $t_1$  part 1 thickness;  
 $t_2$  part 2 thickness;  
 $g$  gap between the parts or shim thickness.

The load distribution behavior depends on the thickness of the joined parts, the bolt diameter and the bearing allowables of the involved materials. For this study, the configuration is a single shear bolted joint which is demonstrated in the Fig. 3 with the involved parameters in the method.

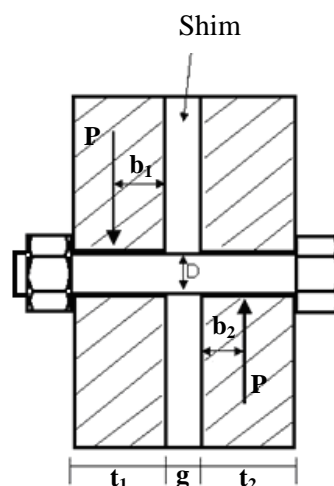


Figure 3. Bolted joint configuration.

Where:  $b_1$  distance from the applied load to plate 1 edge;  
 $b_2$  distance from the applied load to plate 2 edge;  
 $t_1$  part 1 thickness;  
 $t_2$  part 2 thickness;  
 $g$  shim thickness;  
 $P$  applied force;  
 $D$  bolt diameter.

First, it is considered a triangular load distribution and the maximum stress is the minimum bearing stress allowable among the joined parts and the bolt. See Fig. 4.

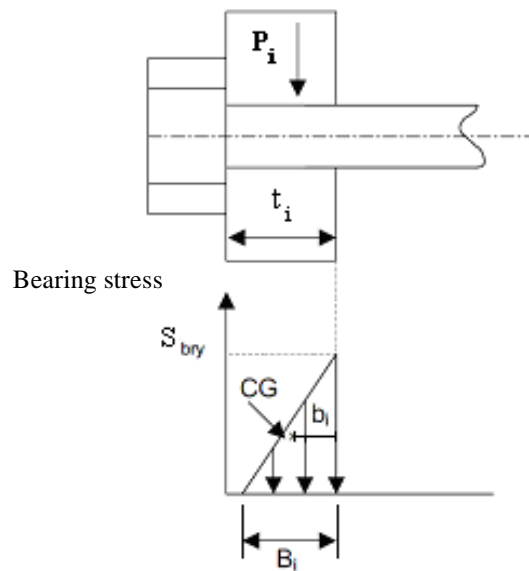


Figure 4. Triangular stress distribution.

From Fig. 4, if  $D$  is the bolt diameter, the reaction on the bolt can be written as follows:

$$P = \frac{S_{bry} \times B_i}{2} \times D \quad (3)$$

Where  $S_{bry}$  is the minimum bearing stress allowable among the joined parts and the bolt and  $B_i$  is the length of the load distribution. And hence,

$$B_i = \frac{2 \times P}{S_{bry} \times D} \quad (4)$$

The value of  $b_i$  depends on the length obtained for  $B_i$ :

If  $B_i \leq t_i$ , it is assumed there is a triangular load distribution and  $b_i$  will be:

$$b_i = \frac{B_i}{3} \quad (5)$$

If  $B_i > t_i$ , the load distribution may be trapezoidal or rectangular.

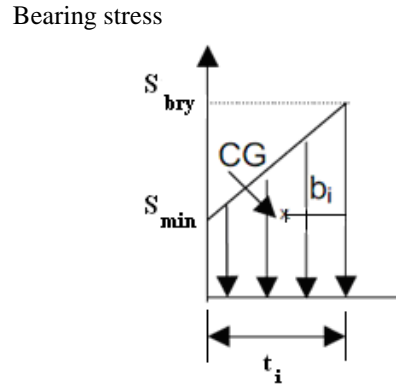


Figure 5. Trapezoidal stress distribution.

From Fig. 5, the reaction on the bolt can be written as follows:

$$P = \frac{S_{bry} + S_{min}}{2} \times t_i \times D \quad (6)$$

Thus,

$$S_{min} = \frac{2 \times P_i}{t_i \times D} - S_{bry} \quad (7)$$

If  $S_{min} \geq S_{bry}$ , plastic deformation (or failure in ultimate condition) occurs and the load distribution becomes uniform presenting a rectangular behavior. Consequently,  $b_i$  will be:

$$b_i = \frac{t_i}{2} \quad (8)$$

However, if  $S_{min} < S_{bry}$ , the load distribution will present a trapezoidal behavior and  $b_i$  will be:

$$b_i = \frac{t_i}{3} \times \left( \frac{S_{bry} + 2 \times S_{min}}{S_{bry} + S_{min}} \right) \quad (9)$$

Therefore, this study regards three different configurations of single shear bolted joints. Table 1 presents them below.

Table 1. Configurations of the bolted joints.

| Configuration | Bolt      | D<br>(mm) | Material – Plate 1 | t <sub>1</sub><br>(mm) | Material – Plate 2 | t <sub>2</sub><br>(mm) | g<br>(mm) | P (N) |
|---------------|-----------|-----------|--------------------|------------------------|--------------------|------------------------|-----------|-------|
| 1             | NASM21134 | 11.09     | Al7475-T7351       | 12.0                   | Al7475-T7351       | 9.0                    | 2.0       | 40000 |
| 2             | NASM21134 | 6.35      | Al7475-T7351       | 4.0                    | Al7475-T7351       | 3.0                    | 2.5       | 13333 |
| 3             | NASM21134 | 12.68     | AISI4130N          | 11.0                   | Al7050-T7451       | 21.0                   | 0         | 62867 |

Moreover, a FEM analysis was done respecting the three configurations above, in accordance with section 3.

### 3. FEM ANALYSIS

First of all, using the *software* MSC Patran® 2010, it is defined an analysis code based on MSC Marc® structural type which allows, during the modeling, the contact between the involved bodies and to define the coefficient of friction due these interfaces. The defined materials are in accordance with the Tab. 1 and the constitutive model is defined as linear elastic, once the applied loads are presented in this region. Regarding the contact simulation, since the main numerical purpose is to detect the movement between the bodies, five deformable bodies are determined in order to

evaluate the joint behavior: bolt, nut, plate 1, plate 2 and shim. Moreover, the coefficients of friction applied in the model are described in Tab. 2.

Table 2. Coefficient of friction between the bodies (OBERG et al, 2004).

| Body    | Coefficient of friction |      |             |         |             |
|---------|-------------------------|------|-------------|---------|-------------|
|         | Bolt                    | Nut  | Plate 1     | Plate 2 | Shim        |
| Bolt    | -                       | 0.8  | 0.45 (0.8*) | 0.45    | 0.8         |
| Nut     | 0.8                     | -    | -           | 0.45    | -           |
| Plate1  | 0.45 (0.8*)             | -    | -           | 0.8*    | 0.45 (0.8*) |
| Plate 2 | 0.45                    | 0.45 | 0.8*        | -       | 0.45        |
| Shim    | 0.8                     | -    | 0.45 (0.8*) | 0.45    | -           |

\* : Applicable for configuration 3.

The type of contact defined between the plates and shim with the bolt and nut is the large-sliding, which allows separation, relative slide but there is no penetration in the finite elements meshes. The same occurs between the plates and shim. The interface between the bolt and nut respects the small-sliding formulation to induce no relative movement to the bolt after the contact considering the pre torque is effective. Furthermore, in order to decrease the computational work time, the contact boundary conditions of the parts which physically do not touch are disabled.

In these models, the solid elements CTETRA4 are used even to represent the bolt and nut as to the plates and shim. The mesh is refined in the bolt body and around the holes of the plates and shim because these regions will supply important data due the contact between the parts and the possibility to define the load distribution behavior along the bolt body. This type of element was chosen once the model is not isoparametric and the computational work time can be enhanced, maintaining the reliability of the results. In general, the three-dimensional models present approximately 20000 elements and 5000 nodes. In the Fig. 6 is showed the model of configuration 1.

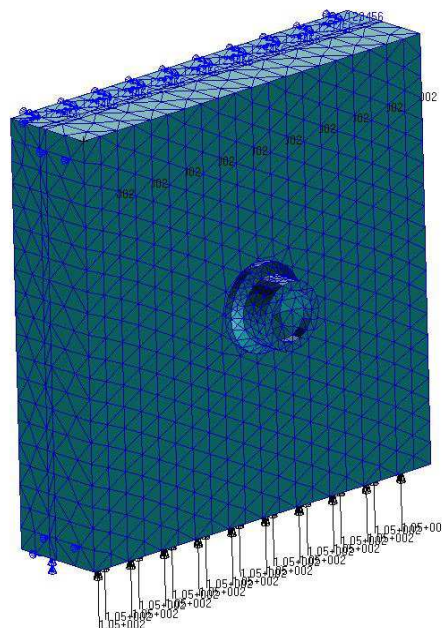


Figure 6. Three-dimensional model of joint configuration 1.

The boundary conditions consider not only the contact but also the loading and constraints.

The distributed load is applied uniformly in the two opposite faces of one plate in such way to avoid secondary and non uniform responses in this study.

Similarly, all degrees of freedom of the adjacent plate faces are restrained to allow only the others components displacements due the applied loads and the existent contacts. Moreover, the degree of freedom  $u_y$  (translational in y axis) is restrained in the faces perpendicular to the applied loads of the plate and the shim to induce a behavior without distortion in the direction of this axis. In the Fig. 7, the boundary conditions in the models are illustrated.

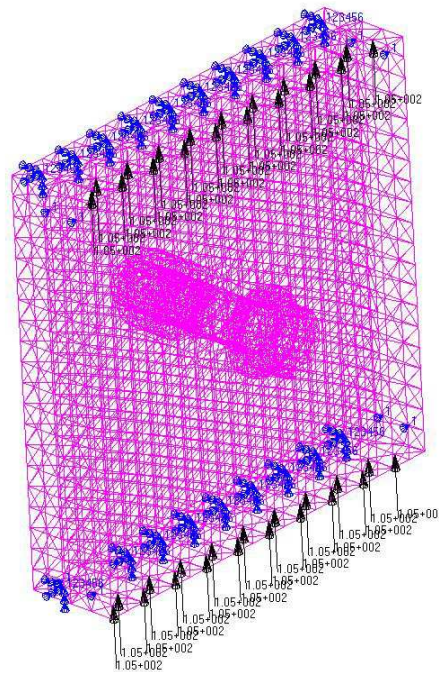


Figure 7. Three-dimensional model boundary conditions.

Thus, the simulation of these bolted joints offers as response the contact forces in the nodes and the displacement of the involved parts, which allow the evaluation of the load distribution behavior along the bolt body.

#### 4. RESULTS AND DISCUSSION

The three bolted joints configurations were analyzed by the analytical method (Section 2) and FEM analysis (Section 3). In order to obtain the results from the analytical formulas, a Microsoft Excel spreadsheet was developed. Table 3 shows the results and the type of the load distribution for each configuration.

Table 3. Analytical method results.

| Configuration | $S_{bry1}$<br>(MPa)<br>(1) | $S_{bry2}$<br>(MPa)<br>(1) | $S_{brybolt}$<br>(MPa)<br>(1) | $B_1$<br>(mm) | $B_2$<br>(mm) | $S_{min1}$<br>(MPa) | $S_{min2}$<br>(MPa) | $b_1$<br>(mm) | $b_2$<br>(mm) | Load<br>distribution<br>1 | Load<br>distribution<br>2 |
|---------------|----------------------------|----------------------------|-------------------------------|---------------|---------------|---------------------|---------------------|---------------|---------------|---------------------------|---------------------------|
| 1             | 668.30                     | 668.30                     | 2383.94                       | 10.79         | 10.79         | N/A                 | 105.52              | 3.60          | 3.50          | Triangular                | Trapezoidal               |
| 2             | 668.30                     | 668.30                     | 2383.94                       | 6.28          | 6.28          | 353.87              | 703.83              | 1.82          | 1.50          | Trapezoidal               | Rectangular               |
| 3             | 1956.76                    | 695.90                     | 2383.94                       | 5.07          | 14.25         | 13.70               | N/A                 | 1.69          | 4.75          | Trapezoidal               | Triangular                |

(1): MMPDS, 2008.

In accordance with Tab. 3, the joint configurations 1 and 3 present a triangular or trapezoidal load distribution. Assessing the  $B$  values, it verifies the contact region presents a lower length than the plate thickness where triangular load distribution happens. For the trapezoidal load distribution region, all the plate thickness is in contact with the bolt body, confirmed by the  $B$  values greater than the plate thickness. Moreover, the  $S_{min}$  values are lower than the  $S_{bry}$  ones, disregarding a rectangular load distribution. On the other hand, the load distribution in the configuration 2 presents a rectangular load distribution on plate 2. This behavior is defined by the contact all along the plate thickness and the beginning of plastic deformation on the plate once  $S_{min2}$  surpass  $S_{bry2}$ .

The FEM analysis with three-dimensional models uses a methodology which, each load increment is analyzed the contact between the bodies nodes and, when this effect happens, the contact force is generated. Therefore, the nodes interaction in the model allows the evaluation of the load distribution along the bolt body. This methodology is applied for all joints configurations.

The contact forces in the same coordinate of the bolt longitudinal axis are summed in such way to analyze the load distribution along the bolt body. In the Fig. 8 is showed the contact regions in the bolt for joint configuration 1.

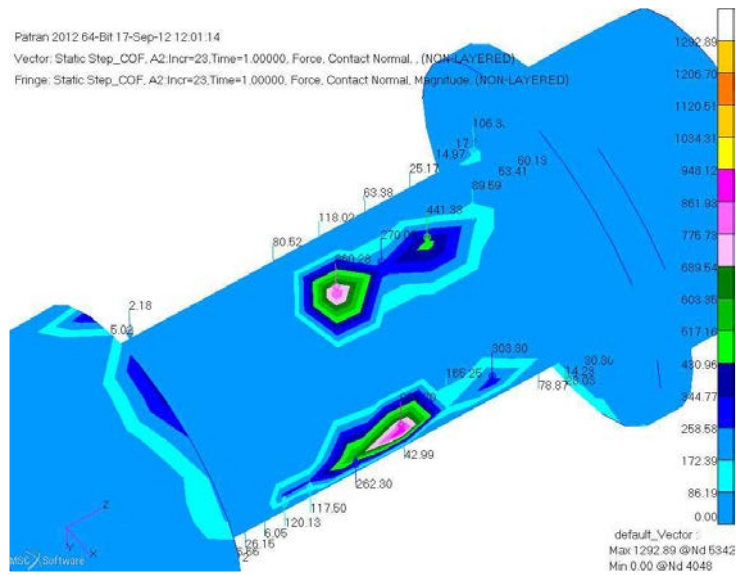


Figure 8. Contact regions for configuration 1.

In the Tab. 4 is showed the contact forces of the configuration 1 between plate 1 and bolt.

Table 4. Contact forces between plate 1 and bolt in configuration 1.

| Bolt longitudinal axis position [mm] | Contact force [N] |
|--------------------------------------|-------------------|
| -1.24461                             | 510.18            |
| -2.34623                             | 683.85            |
| -3.86653                             | 1147.69           |
| -6.48898                             | 8080.06           |
| -9.11142                             | 5533.52           |
| -11.7339                             | 18794.97          |

Taking in account the results in Tab. 4, the load distribution between the plate 1 and the bolt body can be defined as trapezoidal. Consequently, the  $B_1$  value is 12 mm and  $b_1$  value is 4.06 mm. In the Tab. 5 is showed the contact forces of the configuration 1 between plate 2 and bolt.

Table 5. Contact forces between plate 2 and bolt in configuration 1.

| Bolt longitudinal axis position [mm] | Contact force [N] |
|--------------------------------------|-------------------|
| -14.36                               | -16951.25         |
| -16.98                               | -7610.85          |
| -18.44                               | -6441.36          |
| -19.60                               | -2267.90          |
| -21.05                               | -2633.27          |
| -22.22                               | -2176.44          |

Analogously, for the plate 2 and the bolt, the results define a trapezoidal load distribution with  $B_2$  value equal 9 mm and 3.28 mm for the  $b_2$ .

In the Fig. 9 is showed the contact regions in the bolt for joint configuration 2.



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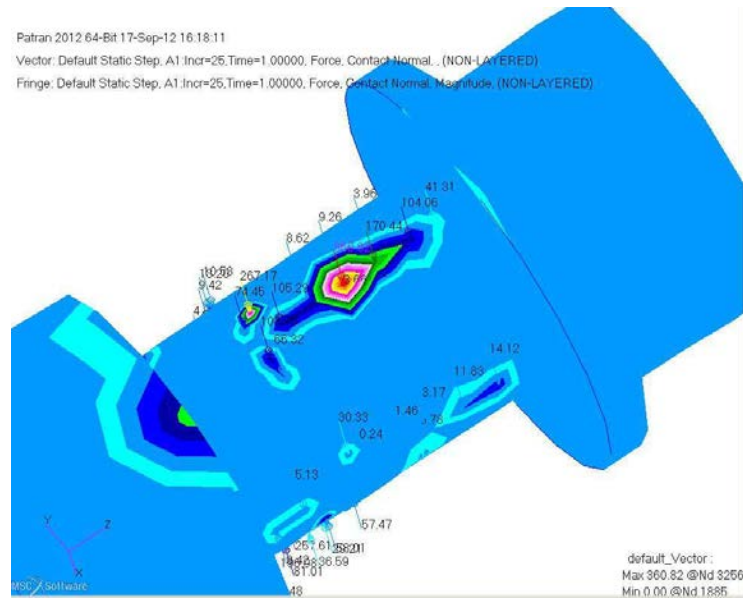


Figure 9. Contact regions for configuration 2.

In the Tab. 6, the contact forces are showed for the configuration 2 between the plate 1 and bolt.

Table 6. Contact forces between plate 1 and bolt in configuration 2.

| Bolt longitudinal axis position [mm] | Contact force [N] |
|--------------------------------------|-------------------|
| -0.71                                | 180.59            |
| -1.47                                | 1221.46           |
| -2.59                                | 3393.44           |
| -3.61                                | 4554.06           |

The results in the Tab. 6 presents a triangular load distribution behavior between the plate 1 and bolt for joint configuration 2 with values of 3.42 mm for  $B_1$  and 1.14 mm for  $b_1$ . In the Tab. 7, the contact forces are showed for the configuration 2 between the plate 2 and bolt.

Table 7. Contact forces between plate 2 and bolt in configuration 2.

| Bolt longitudinal axis position [mm] | Contact force [N] |
|--------------------------------------|-------------------|
| -6.68                                | -1061.75          |
| -7.28                                | -2612.26          |
| -7.37                                | -3155.01          |
| -7.97                                | -4602.58          |
| -8.51                                | -1835.08          |

Analyzing the contact forces in Tab. 7, the load distribution along the bolt body due the plate 2 is trapezoidal with  $B_2$  equal to 3 mm e  $b_2$  equal to 1.07 mm.

In the Fig. 10 is showed the contact regions in the bolt for joint configuration 3.



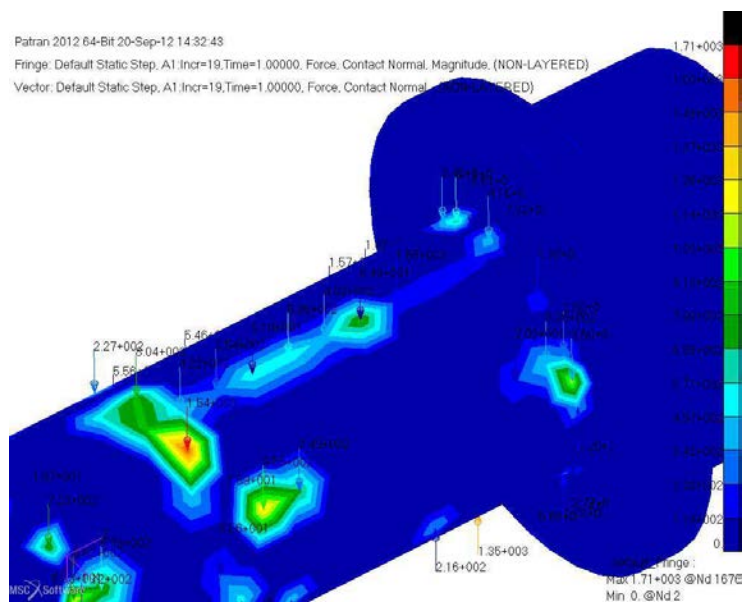


Figure 10. Contact regions for configuration 3

In the Tab. 8, the contact forces are showed for the configuration 3 between the plate 1 and bolt.

Table 8. Contact forces between plate 1 and bolt in configuration 3.

| Bolt longitudinal axis position [mm] | Contact force [N] |
|--------------------------------------|-------------------|
| -21.15                               | -28303.33         |
| -22.21                               | -10531.10         |
| -23.27                               | -16350.53         |
| -24.45                               | -1086.27          |
| -25.56                               | 2752.29           |
| -27.13                               | -14821.79         |
| -28.58                               | 13661.45          |

It can be observed in the position -24.76 mm the contact force is zero, i.e., there is a triangular load distribution behavior between the plate 1 and the bolt in configuration 3 with values of 3.76 mm for  $B_l$  e 1.25 mm for  $b_l$ . In the Tab. 9, the contact forces are showed for the configuration 3 between the plate 2 and bolt.

Table 9. Contact forces between plate 2 and bolt in configuration 3.

| Bolt longitudinal axis position [mm] | Contact force [N] |
|--------------------------------------|-------------------|
| -1.40                                | 938.64            |
| -2.38                                | 133.35            |
| -2.99                                | -153.29           |
| -4.13                                | -45.56            |
| -5.10                                | -1517.23          |
| -6.13                                | -202.59           |
| -7.12                                | 355.58            |
| -9.13                                | -150.66           |
| -10.14                               | 16.49             |
| -11.13                               | 401.95            |
| -12.14                               | -45.14            |
| -13.13                               | 534.67            |
| -14.14                               | 492.08            |
| -15.13                               | -113.51           |
| -16.14                               | 235.37            |

Continuation of Table 9. Contact forces between plate 2 and bolt in configuration 3.

| Bolt longitudinal axis position [mm] | Contact force [N] |
|--------------------------------------|-------------------|
| -17.13                               | -14.53            |
| -18.14                               | 1011.46           |
| -19.13                               | 47.44             |
| -20.15                               | 3473.83           |

The contact forces presented a considerable dispersion along the bolt longitudinal axis. Although, it can be determined a triangular load distribution with  $B_2$  equal to 14.08 mm e  $b_2$  equal to 4.69 mm.

Evaluating the obtained results by the analytical method and the finite element method, the Tab. 10, Tab. 11 and Tab. 12 were created.

Table 10. Methods comparison in configuration 1.

| Joint configuration 1 |                         |            |            |                        |                                  |
|-----------------------|-------------------------|------------|------------|------------------------|----------------------------------|
|                       | Method                  | $b_i$ (mm) | $B_i$ (mm) | Load distribution type | Bending arm difference $b_i$ (%) |
| Plate 1               | Analytical              | 3.6        | 10.79      | Triangular             | -                                |
|                       | FEM - three-dimensional | 4.06       | 12         | Trapezoidal            | 12.78                            |
| Plate2                | Analytical              | 3.5        | 10.79      | Trapezoidal            | -                                |
|                       | FEM - three-dimensional | 3.28       | 9          | Trapezoidal            | -6.29                            |

Table 11. Methods comparison in configuration 2.

| Joint configuration 2 |                         |            |            |                        |                                  |
|-----------------------|-------------------------|------------|------------|------------------------|----------------------------------|
|                       | Method                  | $b_i$ (mm) | $B_i$ (mm) | Load distribution type | Bending arm difference $b_i$ (%) |
| Plate 1               | Analytical              | 1.82       | 6.28       | Trapezoidal            | -                                |
|                       | FEM - three-dimensional | 1.14       | 3.42       | Triangular             | -37.36                           |
| Plate2                | Analytical              | 1.5        | 6.28       | Rectangular            | -                                |
|                       | FEM - three-dimensional | 1.07       | 3          | Trapezoidal            | -28.67                           |

Table 12. Methods comparison in configuration 3.

| Joint configuration 3 |                         |            |            |                        |                                  |
|-----------------------|-------------------------|------------|------------|------------------------|----------------------------------|
|                       | Method                  | $b_i$ (mm) | $B_i$ (mm) | Load distribution type | Bending arm difference $b_i$ (%) |
| Plate 1               | Analytical              | 1.69       | 5.07       | Triangular             | -                                |
|                       | FEM - three-dimensional | 1.25       | 3.76       | Triangular             | -26.04                           |
| Plate2                | Analytical              | 4.75       | 14.25      | Triangular             | -                                |
|                       | FEM - three-dimensional | 4.69       | 14.08      | Triangular             | -1.26                            |

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The analytical method was defined in this study as the reference methodology to the finite element method, because it is based in consolidated theory and in some cases empirically validated.

Considering the results, the types of load distribution in the bolt configurations and the found differences, it is possible to analyze the FEM analysis presents satisfactory results comparing with the analytical method once it takes in account non linear effects like contact and allows interface between the plates and bolt with a representative geometry to the real parts.

Another important factor which must be discussed is the shim influence in a bolted joint analysis. As it can be evaluated, the great variations occurred for the joint configuration 2 where the shim thickness is very significant, representing 62.5% of the plate 1 thickness and 83.3% of the plate 2 thickness.

Additionally, the bending arm differences show the analytical method presents values greater than the finite element method.

## 5. CONCLUSIONS

In this paper, three different bolted joint configurations were analyzed by the analytical and finite element method. It can be stated all analyses were performed with success and reliable results were achieved.

After the results comparison, the three-dimensional modeling using the finite element method presented responses with low deviation referencing the analytical analysis, once non linear consideration is took in account representing in a trustworthy way the real joints.

The shim in a bolted joint and the plate thickness contribute with the differences between the methods. The joint without shim and with the greatest thicknesses plates presented only 2% of difference between them, since the analytical one does not regard the shim contribution. Furthermore, the analytical method considers only the geometry and the mechanical properties of the used materials in joints to determine the load distribution type along the bolt body.

Even though the differences between the methods exist, the choice and use in one of them will depend the cost to process the data and the liability required by the project, where the FEM allows an enhanced structure, reducing weight and acceptable and coherent results. On the other hand, the analytical method leads a more conservative results and, consequently, lower margin of safety once all bending arm are greater than the values achieved by the finite element method.

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