ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

LATIN HYPERCUBE DESIGN OF EXPERIMENTS AND SUPPORT VECTOR REGRESSION METAMODEL APPLIED TO BUCKLING LOAD PREDICTION OF COMPOSITE PLATES

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Abstract. *The structural design of laminate composites commonly involves an optimization process in order to get the optimal stacking sequence. However, optimization processes have high computational costs. An alternative is to use a low cost approximation (a metamodel) to substitute the complex model during the optimization process. Metamodels are usually built by fitting the response of a high fidelity model evaluated in some sampling points of the design space and can be especially useful in optimization problems of composite structures. This work uses the latin hypercube design of experiments to choose the sampling points and the support vector regression (SVR) to develop a metamodel for buckling loads of composite plates under compression. Support vector regression is one extension of support vector machine for function estimation. It is based on the framework of statistical learning theory. The metamodel development was divided into three stages. In the first stage the samples were selected applying latin hypercube design in database construction. In the second stage the finite element method was used to obtain the buckling load. Finally, the support vector regression was used to obtain the metamodel. The performance of the metamodel was evaluated and the results are presented for buckling load responses*

Keywords: *Laminated composite, metamodels, latin hypercube, support vector regression*

1. INTRODUCTION

Laminated composites are a class of materials that have anisotropic properties. They require optimal stacking sequence when applying in engineering structures. In order to obtain an effective design of composite materials this work focuses on buckling stability. In this way a metamodel or surrogate model was developed using design of experiments and support vector regression techniques to predict the buckling load of composite plates. Since the optimization process has high computational cost, it is necessary to seek helpful resources for perform the buckling analysis of composite laminates. In this context the approximation of solutions for metamodels has become attractive. A metamodel is a procedure which creates an approximation or interpolation of a given phenomenon, that is, it is the response of its behavior. The response may be a physical experiment or a simulation of a number of points in the field. These methods seek an answer known for certain numbers of points based on statistical techniques and the metamodel is developed as surrogate of the expensive simulation process. Support vector regression (SVR) is one extension of support vector machine (SVM) for function estimation. It is based on the framework of statistical learning theory or machine learning. Here, SVR was developed associated with latin hypercube to estimate or map the buckling load of composite laminated. The metamodel development was divided into three stages. In the first stage the samples were generated varying the angle in increments of 5 degrees to simulate by analytical processes of the buckling load. In the second stage the finite element method was used to obtain a buckling load or critical load with the same samples. The relationship between design variables and the predict response were models in finite elements analysis or in analytical equations. In the third stage the experimental samples were determined applying latin hypercube design and SVR was used to obtain a metamodel for approximating the buckling load. The proposed technique for predicting the buckling load was demonstrated by numerical examples. The supervised learning applied with support vector regression was computed for training and validate samplings. In both analyses the correlation factor, a statistical measurement of data, are much close to the output vector modeling.

ISSN 2176-5480

2. LAMINATED COMPOSITES BY METAMODELS

As described by Mendonça (2005), the composite material is a set of two or more different materials combined in a macroscopic scale, to function as a unit, aiming to obtain a set of properties that none of the individual components presents. Laminated composite materials consist of layers of at least two different materials that are bonded together (Jones, 1999). More specifically, a laminate is a stack of lamina with different orientations of the principal material directions. Lamina is a single layer or ply of unidirectional composite material (Staab, 1999). The Figure 1 shows a laminated composite stacking sequence and the discrete angles or orientations fibers of each lamina. The numbers of plies may also tailor with many laminas. As these materials can be easily tailored with specific sequence that required their applications, many different kinds of applications are increasing in diversity and quantities.

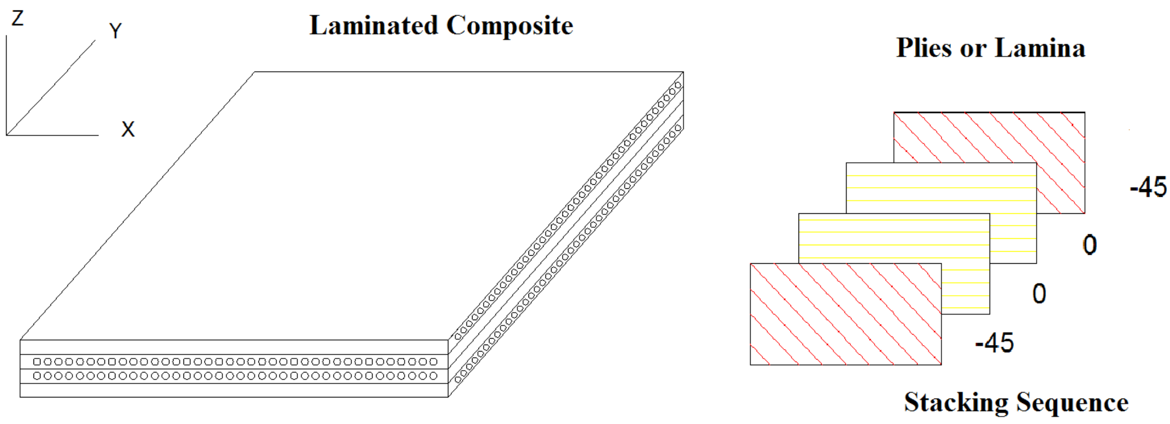

Figure 1. Laminated composite plate

The potential advantages of composite materials are their ultrahigh strength and stiffness fibers that can be tailored to efficiently meet design requirements. These advantages are used in many industries applications as automobilist, spatial and aeronautic.

Laminated composite structures with buckling behavior are studied to obtain a safe and reliable design. The accurate knowledge of critical buckling loads is essential for lightweight structural design as exposed Leissa (1983). Classical buckling studies are also presented by Leissa (1993) and Arnold and Mayers (1984). Sundaresan *et al.* (1996) investigated the buckling and post-buckling of thick laminated rectangular plates with first-order shear deformation theory associated with non-linear strain-displacement relationships and the finite element analysis. Shukla *et al.* (2005) also studied the critical buckling load of laminated composite rectangular plates under in-plane uniaxial and biaxial loadings based on the first-order shear deformation theory and von-Karman-type nonlinearity. The influences of boundary conditions on the buckling load for rectangular plates are analyzed by Baba (2007). Recently, Srinivasa *et al.*, (2012) presented the buckling studies on laminated composite skew plates using finite element. In order to obtain the best stacking sequence of laminated many optimization techniques are developed to solve these combinatorial optimization. Le Riche and Haftka (1993) optimized the stacking sequence of a composite laminate for buckling loads maximization by genetic algorithm. Erdal and Sonmez (2005) developed the method to find globally optimum designs of composite laminates using simulated annealing in a stacking sequence of layers. The ant colony optimization metaheuristic is applied in optimization of laminated stacking sequence for maximum buckling load by Aymerich and Serra (2008). Furthermore, the optimization techniques or metaheuristics algorithm have improved the methods to solve hard problem or combinatorial optimization problems the computational time spend a lot of time yet. In order to minimize these problems the design of experiments (DOE) and metamodels are applied for stacking sequence optimization. Todoroki and Ishikawa (2004) studied the design of experiments for stacking sequence optimization with genetic algorithm. Reddy *et al.* (2011) presented the design of experiments and artificial neural networks for stacking sequence optimizations of laminated composite plates. Pan *et al*. (2010) worked with metamodel and support vector regression for lightweight design of B-pillars. Reddy *et al.* (2012) also used artificial neural networks to predict natural frequencies of laminated composite plates. Nik *et al.* (2012) reported the surrogate optimization algorithm to examine the simultaneous stiffness and buckling load of laminated plate with curvilinear fiber paths. Buckling analysis solved through the ratio of polynomial response surface is presented by Alibrandi *et al.* (2010). As informed above there are few works applying the support vector in the analysis of laminated composite plates. However, this work aims to study computer simulations applying the design of experiments and surrogate models. Latin hypercube, one of design of experiments, and support vector regression, one of metamodels, are proposed to analyze the buckling load by stacking sequence for composite laminate plate.

ISSN 2176-5480

3. METAMODELING TECHNIQUES AND DESIGN OF EXPERIMENTS (DOE)

A model generated from experimental tests or approximated computational functions are called metamodel or surrogate model. Metamodeling is defined by Wang and Shan (2007) as a process of constructing a metamodel. Initially, surrogate models have been applied in expensive analysis and simulation processes in order to reduce computational time. The approximate model is considered also a surrogate of simulation model. Recently, metamodeling techniques have been applied in design and optimization. Applications in the design, development, or improvements design or processes are using this response surface methodology. The response surface means a collection of statistical and mathematical techniques for development, improving and optimizing processes (Meyers and Montgomery, 2002). The response surface can be also the physical experiment or computational simulation for a phenomenon based in a certain numbers of points (Simpson *et al.*, 2008). In the review of Wang and Shan (2007) there are research in sampling, metamodels, model fitting, model validation, and design space exploration and optimization methods. A model approximation or sampling methods are based on the theory of design of experiment (DOE) which focuses on planning experiments across the design space. The design space is considered for maximum exploration of engineering design problem and design of experiments is the process of selecting the better data point. DOE is a method for determining the relationship between factors affecting a process and the output of that process as explained Reddy *et al.* (2012). The response approach is based on sequential experimentation explored in region of interest generating an approximate polynomial function. In case that the true response surface is complex due to the wide range of variable and extending in entire region of design space, the design are called space-filling as stated Meyers and Montgomery (2002). In the review of metamodeling presented by Simpson *et al.* (2008), this technique reduces computational demand and provides a rapid design space exploration with visualization. In this way fast response is developed for design of experiment and performance analyses in graphical approach. Meyers and Montgomery (2002) and Wang and Shan (2007) also presented many kinds of experimental design as central composite, box-behnken, latin hypercube, montecarlo and others sampling methods. In relations to metamodel there are kriging, radial basis functions, artificial neural network, and decision tree and support vector machine and so on. The analysis of this work is based on computer simulation of latin hypercube and support vector machine. A short definition about both is presented.

3.1 Latin hypercube design

Metamodel is based on samples and the mathematical formulation for this surrogate model is considered in a sampling plan. Its means a spatial arrangement of the observations (experimental or computational). Engineering design can become very expensive if the evaluation of the problem worked with every possible combination. In statistics this type of analysis is full factorial. To minimize this kind of problem the experiments can take some minimum number of samples that the dimensionality of the sampling can be reduced. One of experiments that used this concept is called latin hypercube design. Latin hypercube sampling is generating points by stratification of sampling plan on all of its dimensions (Forrester *et al.*, 2008). Cheng and Druzdzel (2000) also reported that the purpose of latin hypercube is to ensure that each value (or a range of values) of a variable is represented in the sample, no matter which value might turn out to be more important. Those points are generated in projections onto the variable axes in a uniform distribution. The techniques consist in building a latin square, an uniform projection plans. As exposed in Forrester *et al.* (2008), if n designs are required, an n x n square is built by filling every column and every line with the permutation of $\{1, 2, \ldots, n\}$. However, every number must only selected once in every column and line. An example of latin square with $n = 3$ is presented in Table 1.

Table 1. Latin square.

2	1	3
3	2	1
1	3	2

A multidimensional latin hypercube are building in a similar way. Dividing the design space into equal sized hypercubes and placing points one in each (Forrester *et al.*, 2008). Considering hypercube samples of size n for m variables through randomly selecting m permutations of the sequence of integers 1, 2,…,n and assigning them to each of the m colums of the table or latin square resulting in latin hypercube design (Cheng and Druzdzel, 2000). This technique is applied normalizing the plan (n and m variables) into the [0, 1] or the variables are uniformly distributed in [0, 1].

An example of latin hypercube is showed in Figure 2. The design matrix based on two variables (angle in degree from 0° to 90°), is constructing randomly generating the samples for approximation model. Eight-point latin hypercube sampling plan are shown in Figure 2, along with its two-dimensional projections. All eight points are visible where each row and column of hypercube contains only one point.

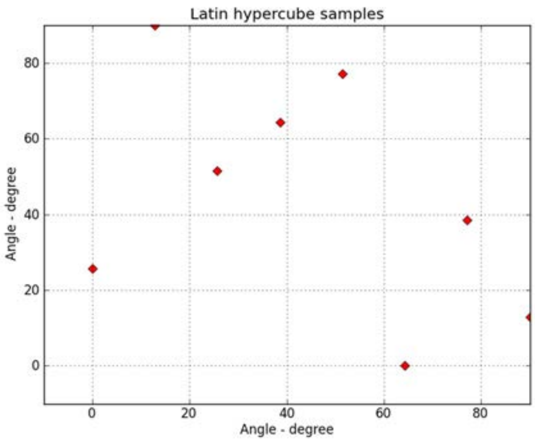

Figure 2. Example of a two-dimensional latin hypercube design of experiments

3.2 Support vector machine and support vector regression

The support vector machine (SVM) is a learning algorithm inspired by statistical learning theory with machine learning characteristics recognized as artificial intelligence. An overview of statistical learning theory was stated by Vapinik (1999) and the capacity of learning machines was reported by Vapinik (1993). SVM have their origins in nonlinear algorithm developed by Vapnik and Lerner *apud* in Smola and Schölkopf (2004). The background of SVM and its theory are found in Vapinik (1999) and Vapinik and Vashist (2009). As SVM is based in a learning method, it is used training procedures (Sánchez, 2003). This method is employed for solving classification or regression problems, mainly due to its possibility to model nonlinear data relationships in high dimensional feature space applying a kernel function or kernel methods (Üstün *et al.*, 2012). The advantage of kernel trick is due to their capacity to transform the original input space into a high dimensional feature space, a nonlinear relationship, modeling as a linear problem. A tutorial about the SVM can be finding in Smola and Schölkopf (2004), additional explanation can be find in (Sànchez A., 2003, Pan *et al.*, 2010, Üstün *et al.*, 2012, Suttorp and Igel, 2006, Che, 2013). As reported Guo and Zhang (2007), in SVM the decision function of support vector classification or support vector regression are determined by support vector. In classification the support vector machine generates the hyperplane, i.e., a function that classifies some set of samples, for example pattern recognition. In a regression case the support vector determines the approximation function.

This subsection introduces the approach of SVR. It is one of the variations of SVM. Basically, SVR technique search the multivariate regression function $f(x)$ based on data set or specifically the training set X to predict the desired output (Vapinik, 1993, Vapinik, 1999, Smola and Schölkopf, 2004, Üstün *et al.*, 2012, Guo and Zhang, 2007). The regression function for training SVM can be formulated as

$$f(x) = \sum_{i=1}^{n}(\alpha_i - \alpha_i^*)\, K(x_i, x) + b \quad (1)$$

where $K(x_i, x)$ is a kernel function, n is the number of training data, b is the parameter of the model and α_i, α_i^* are the Lagrange multipliers. The training data set considered as

$$X = \{(x_i, y_i),\ i = 1, \dots, n\} \quad (2)$$

where x_i is the ith input vector for ith training sample and y_i is the target value or output vector for ith training sample. When the output of SVR regression $f(x)$ is quite similar of required output vector y_i the results achieved a good approximation metamodel. The kernel methods use the kernels or kernel functions as (Üstün *et al.*, 2012, Sánchez, 2003)

$$K(x_i, x_j) = \langle \phi(x_i).\phi(x_j) \rangle \quad (3)$$

It is an inner product and ϕ is in general a nonlinear mapping from input space onto characteristic space. As reported Üstün *et al.* (2012) this kernel transformation to work with non-linear relationships in the data in an easier way. More specifically, for nonlinear regression problem, the kernel function can do the extension the linear regression of support machine or the linear regression on the higher dimensional space will be similar to the nonlinear regression on the input patterns (Che, 2013). The $\alpha_i,\ \alpha_i^*$ are found applying the optimization process, minimizing the objective function

$$\frac{1}{2}\sum_{i,j}(\alpha_i-\alpha_j^*)(\alpha_j-\alpha_j^*)K(x_i,x_j)+e\sum_{i=1}^{n}(\alpha_i+\alpha_i^*)-y_i\sum_{i=1}^{n}(\alpha_i-\alpha_i^*) \quad (4)$$

subject to

$$\sum_{i=1}^{n}(\alpha_i-\alpha_i^*)=0\,;\ 0\le\alpha_i\,;\ \alpha_i^*\le C,\ i=1,\dots,n \quad (5)$$

where e and C are the function optimization parameters. The accuracy of a SVR models is influenced by some parameters. In this case the constant C and e. It means that better combinations of those parameters representing the smaller error in a surrogate model. Regularization constant C determines the trade-off between the training error and model approximation. Parameter e is a precision in a feasible convex optimization problem, i.e., in some cases the "soft margin" is accepted as loss function or the amount of deviations tolerated in functions searching by vector machines (Smola and Schölkopf, 2004). Solving the transformed regression problem, for example by quadratic programming and considering that only non-zero α contribute to the final regression model (Üstün *et al.*, 2012). The vectors obtained in this way are named support vectors. The regression function of SVR based on support vector may be formulated as (Guo and Zhang, 2007)

$$f(x)=\sum_{x_i\in SV}(\alpha_i-\alpha_i^*)K(x_i,x)+b \quad (6)$$

where SV is the support vector set.

There are many kernel functions based on variance-covariance, polynomial, radial basis function (RBF). Those kinds of equations can be applied in nonlinear problem to tackle the regression problem. Some kernel functions examples are summarized in Table 2.

Table 2. Kernel functions examples (Sánchez, 2003)

Kernel functions	General use in
$\tanh(\vec{x}.\vec{y}-\theta)$	Multilayer perceptron
$exp(-\lVert\vec{x}-\vec{y}\rVert^2)$	Gaussian RBF network
$(1+\vec{x}.\vec{y})^d$	Polynomial of degree d
$\lVert\vec{x}-\vec{y}\rVert^{2n+1}$	Thin plate
$\lVert\vec{x}-\vec{y}\rVert^{2n}ln(\lVert\vec{x}-\vec{y}\rVert)$	Splines
$\dfrac{sin(d+1)(x-y)}{sin(x-y)/2}$	Trigonometric polynomial of degree d

4. LATIN HIPERCUBE AND SUPPORT VECTOR REGRESSION APPLIED TO LAMINATED COMPOSITES BEHAVIOR

This study is focused on buckling of laminated composites. In the first analysis the laminated was considered with four plies for compute the data sampling and the buckling load value. The discrete angles or orientations fiber of each lamina varies from 0 until 180 degrees, varying in increments of 5 degrees. The results are presented in subsection 5.1 and 5.2. The training set with input vector (orientations angle) and target value (buckling value) to predict the desired output are computed in two manners. First of all in analytical Matlab® code for laminate composite buckling load (Jones, 1999) and second with finite element method simulation in Abaqus®. The regression function by SVR was obtained training these data as support vector machine. Support vector regression was coded in Python script and the tests presented used three types of kernel functions: linear, polynomial (up 2) and radial based function (Gaussian function). In the second analysis the design of experiments adopted was latin hypercube as showed in Figure 2 for generate the sampling data set. It was developed in Matlab® and Python script. SVR applied in this case are showed in subsection 5.2.

5. NUMERICAL RESULTS

In this subsection numerical results are presented for analytical buckling load and finite element method. Both methods generated the sampling data set to estimate the regression function by SVR. To investigated the analysis the laminated are made of graphite/epoxy layers of thickness $h=0{,}127$ mm, length $a=508$ mm and width $b=127$ mm (Aymerich and Serra, 2008). The material engineering elastic modules are $E_1=$ 127.59 GPa, $E_2=$ 13.03 GPa, $G_{12}=$

6.41 GPa and $\nu_{12} = 0.3$. The critical buckling load of a simply supported laminated plate was studied with the biaxial load $N_x = 175$ N/m and $N_y/N_x = 0.125$. Analytical buckling load factor is calculated over possible values of half-waves p and q by the following equation

$$\lambda = \pi^2 \frac{D_{11}\left(\frac{p}{q}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{p}{q}\right)^2\left(\frac{q}{p}\right)^2 + D_{22}\left(\frac{q}{p}\right)^4}{\left(\frac{p}{q}\right)^2 N_x + \left(\frac{q}{p}\right)^2 N_y} \tag{7}$$

where coefficients D_{16} and D_{26} are neglected and D_{ij} are the coefficients of the flexural stiffness matrix. A combination of p and q applied in Eq. 7 reaches the buckling load. The critical buckling load of the plate is given by the minimum of $\lambda(p,q)$ defined in Jones (1999) and Aymerich and Serra (2008).

5.1 Analytical and finite element models for buckling load and SVR

In this subsection the laminated composite are made by 4 plies. The discrete angles or orientations fiber of each lamina varies from 0 to 180 degree with intervals of 5 degrees. Those samples are simulated in analytical buckling load, developed in Matlab® code, to generate the training support vector machine. Adopting the finite element method (FEM) the sampling vector also was simulated in Abaqus®, computing this data set by finite element method.

The results are influenced by kernel transformations of the input data space with dimension (*N* x *M*) into a feature space with dimension (*N* x *N*), where *N* is the number of samples and *M* the numbers of variables (buckling load). The input matrix is transformed into a square matrix as reported Üstün *et al.* (2012). The regression fitting are computed with support vector regression based on the sampling above. SVR are in Python code and the kernel function was tested with linear, polynomial and Gaussian function. Figure 3 shows the graphics of regression results and the correlation factor for analytical data sampling. The better regression fitting was achieved by Gaussian kernel function with the parameters values as $C = 1\text{x}10^7$ and $e = 0.0007$. The regression with polynomial kernel up to degree two was tested but the regression fitting was not better and demanded a lot of computation time.

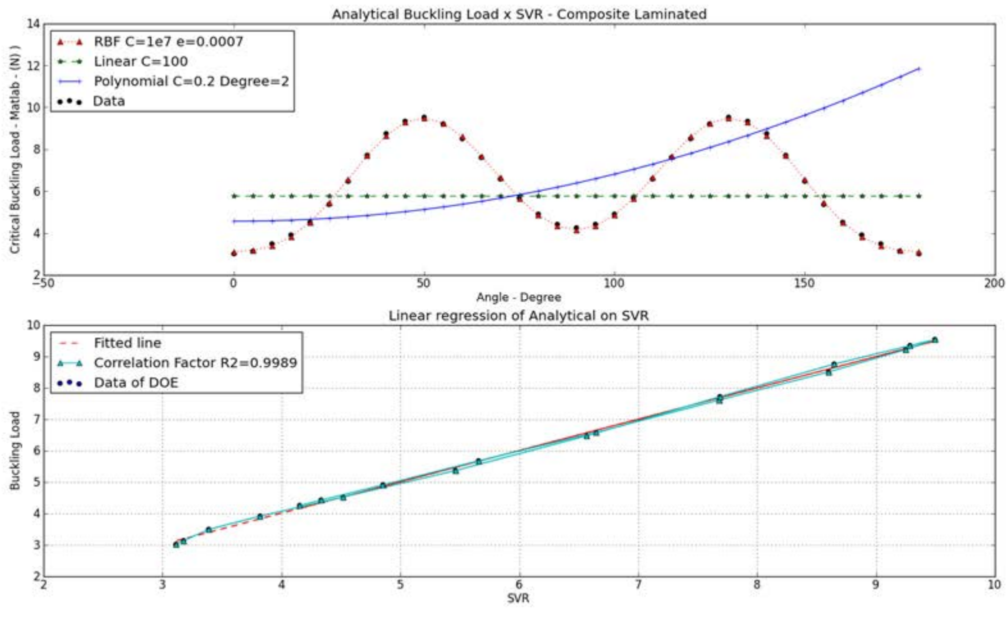

Figure 3. Regression fitting with SVR and correlation factor for analytical buckling load prediction

The correlation factor is calculated as (Meyers and Montgomery, 2002, Reddy *et al.*, 2011)

$$R2 = 1 - \left[\frac{\sum_j (t_j - o_j)^2}{\sum_j (o_j)^2}\right] \tag{8}$$

where t_j are the targets or experimental values and o_j are the outputs or predicted values from SVR. This regression coefficient estimates the correlation between SVR predicted values and targets values. The correlation factor calculated for the sampling set was R2=0.9989. It means that the quality of results, based in statistics theory, are good and the regression fitting with SVR by RBF kernel function is also good.

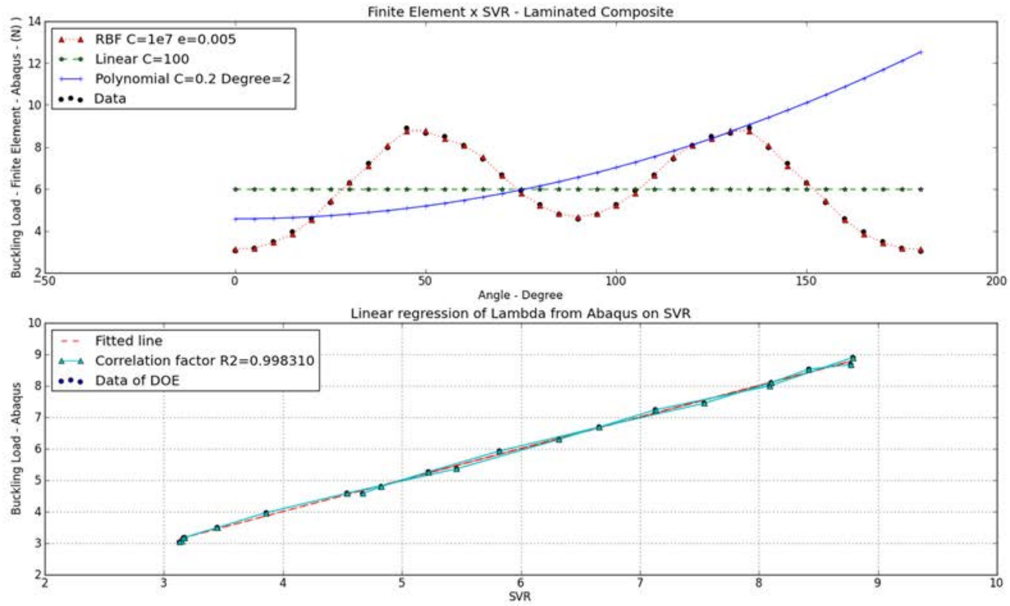

Figure 4. Regression fitting with SVR and correlation factor for finite element buckling load prediction

The regression fitting for finite element sampling by SVR is also computed with linear, polynomial and Gaussian function. RBF function presented better regression fitting, however linear and polynomial with degree 2 was not appropriated in this case. Parameters $C = 1x10^7$ and $e = 0.005$ are computed in regression analysis. The Figure 4 shows the regression results and the correlation factor of the finite element data sampling. This regression coefficient estimates the correlation between SVR predicted values with SVR approximation model and the finite element values.

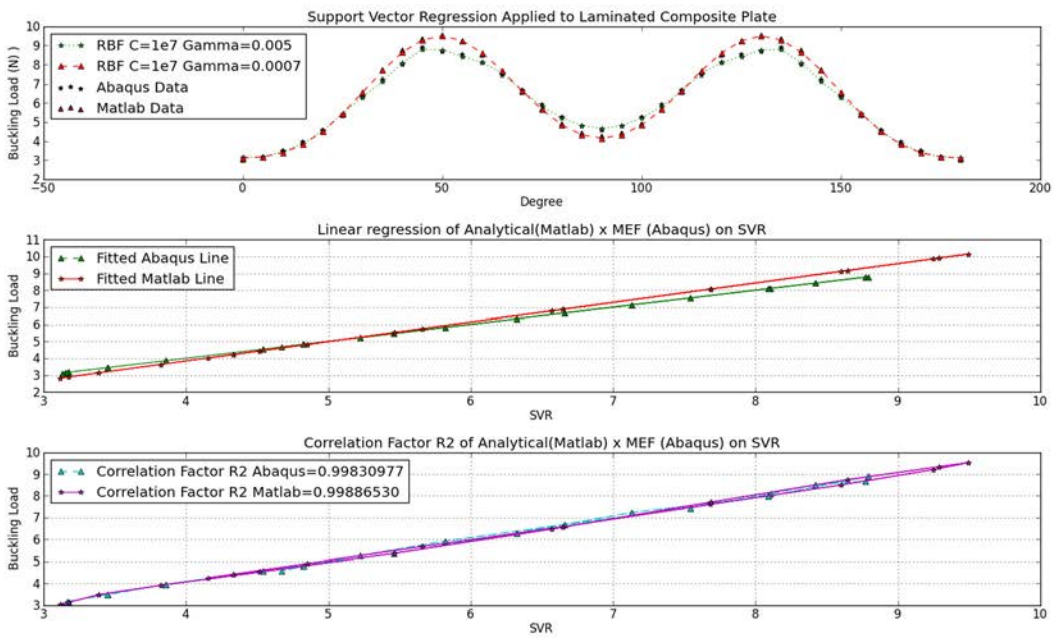

Figure 5. SVR regression comparative for analytical and finite element method

The correlation factor obtained for sampling with finite element method is also good. The value of it is R2= 0,998310 and the quality of this estimated regression fitting is very similar of the factor computed with analytical method. A better kernel function in this case is also RBF function. SVR achieved a better performance with radial basis function for both analytical and MEF buckling data sampling regression as observed in Figure 3 and Figure 4.

An overview of complete results of sampling training is presented in Figure 5. Comparative results for analytical and MEF adopting the SVR with radial basis function are showed. Fitting line regression related to Abaqus® and analytical data simulated in Matlab® are also presented. The correlation factors R2 for both analyses are plotted. The values presented for both methods are quite closed.

5.2 Sampling with latin hypercube for analytical buckling load and SVR

In this subsection the laminated composite are made with 48 plies, symmetric and balanced, with the characteristics presented in section 5. The stacking sequence of laminated are selected from discrete angles or orientations fiber of each lamina (0 degree, ±45 degree or 90 degree). As exposed in subsection 3.1, latin hypercube was adopted to obtain randomly the samples (stacking sequence) for approximation model or the design of experiments matrix. 60 staking sequence samples were generated with latin hypercube. The sample points are $5N_{dv}$, where N_{dv} is the total number of design variables. These quantities are based on the studies of Pan *et al*. (2010) and Yang and Gu (2004). As the laminated is symmetric and balanced with 48 plies, $N_{dv} = 12$. Table 3 presents the matrix with latin hypercube training support vectors.

Table 3. Matrix with latin hypercube training support vectors by analytical buckling load.

Laminated Composite	Stacking sequence	Buckling load
1	$[\pm 45\ 90_2\ \pm 45\ 0_4\ 90_2\ 0_4\ \pm 45\ 0_4\ 90_2]_S$	10956.77
2	$[90_2\ 0_2\ 90_2\ \pm 45_3\ 0_2\ \pm 45\ 90_4\ 0_4]_S$	12407.53
3	$[90_2\ \pm 45\ 0_4\ 90_2\ \pm 45\ 90_2\ 0_4\ \pm 45_3]_S$	12079.96
4	$[90_4\ 0_2\ 90_2\ \pm 45\ 0_2\ \pm 45\ 90_4\ 0_2\ 90_2\ 0_2]_S$	11251.83
5	$[0_2\ 90_2\ \pm 45_2\ 90_4\ 0_2\ 90_2\ 0_2\ 90_2\ 0_2\ \pm 45]_S$	10932.84
6	$[\pm 45_2\ 0_2\ 90_4\ 0_4\ \pm 45\ 90_2\ 0_4\ \pm 45]_S$	13026.76
7	$[\pm 45_2\ 0_2\ 90_2\ 0_2\ \pm 45_2\ 90_2\ 0_4\ \pm 45_2]_S$	12985.38
8	$[90_2\ 0_4\ \pm 45_3\ 90_2\ \pm 45\ 0_2\ 90_2\ \pm 45_2]_S$	12294.21
9	$[\pm 45\ 0_2\ \pm 45_2\ 0_2\ \pm 45\ 0_2\ 90_2\ 0_2\ \pm 45\ 0_2\ 90_2]_S$	12717.56
10	$[0_2\ \pm 45_2\ 0_4\ 90_2\ \pm 45\ 90_2\ 0_2\ \pm 45\ 90_4]_S$	11027.56
11	$[90_2\ 0_2\ 90_2\ \pm 45_2\ 0_2\ 90_2\ 0_4\ 90_2\ \pm 45\ 90_2]_S$	11715.58
12	$[0_4\ 90_2\ \pm 45\ 90_2\ 0_2\ 90_2\ 0_2\ \pm 45_2\ 0_2\ \pm 45]_S$	10768.61
13	$[0_2\ \pm 45_2\ 0_4\ 90_2\ 0_2\ \pm 45_2\ 90_2\ \pm 45\ 0_2]_S$	11370.74
14	$[\pm 45_2\ 0_2\ \pm 45_4\ 90_4\ \pm 45_3]_S$	14612.57
15	$[0_2\ \pm 45_2\ 90_4\ \pm 45\ 0_4\ \pm 45_3\ 90_2]_S$	10059.12
16	$[0_4\ \pm 45_2\ 90_4\ \pm 45_2\ 0_2\ \pm 45\ 0_4]_S$	11775.31
17	$[90_2\ \pm 45\ 90_2\ 0_2\ 90_2\ \pm 45\ 90_2\ 0_4\ \pm 45\ 90_2\ \pm 45]_S$	12054.19
18	$[90_2\ \pm 45_2\ 0_4\ \pm 45_3\ 90_4\ \pm 45_2]_S$	13566.91
19	$[\pm 45\ 90_4\ \pm 45\ 90_2\ 0_2\ \pm 45\ 90_2\ 0_2\ \pm 45_2\ 0_2]_S$	13258.42
20	$[\pm 45\ 90_4\ \pm 45\ 90_2\ 0_2\ \pm 45\ 90_2\ 0_2\ \pm 45_2\ 0_2]_S$	12848.37
21	$[\pm 45\ 0_2\ 90_2\ 0_2\ \pm 45\ 90_4\ 0_4\ \pm 45\ 0_2\ \pm 45]_S$	12236.34
22	$[\pm 45\ 90_2\ \pm 45\ 90_2\ 0_4\ \pm 45\ 0_2\ \pm 45_2\ 0_2\ \pm 45]_S$	13259.35
23	$[90_2\ \pm 45\ 0_2\ \pm 45\ 0_4\ 90_4\ 0_4\ \pm 45\ 0_2]_S$	12335.68
24	$[90_2\ 0_2\ 90_4\ \pm 45\ 0_2\ 90_2\ \pm 45\ 90_2\ 0_2\ \pm 45\ 90_2]_S$	11167.17
25	$[0_2\ 90_2\ \pm 45_2\ 90_2\ \pm 45\ 90_2\ 0_2\ 90_4\ 0_4]_S$	10932.84
26	$[0_2\ 90_2\ \pm 45\ 0_2\ \pm 45\ 90_2\ 0_2\ 90_2\ \pm 45\ 90_2\ 0_4]_S$	11874.33
27	$[\pm 45\ 90_2\ 0_4\ 90_2\ \pm 45_2\ 90_4\ 0_2\ 90_4]_S$	12558.43
28	$[\pm 45_2\ 0_2\ \pm 45\ 90_4\ \pm 45_2\ 90_4\ \pm 45_2]_S$	14361.92
29	$[90_2\ \pm 45_2\ 90_2\ \pm 45\ 90_2\ \pm 45\ 90_2\ \pm 45\ 90_4\ \pm 45]_S$	13316.09
30	$[0_2\ 90_2\ \pm 45\ 90_2\ 0_2\ 90_4\ 0_2\ \pm 45\ 90_2\ \pm 45_2]_S$	10046.05
31	$[0_2\ 90_4\ 0_2\ \pm 45\ 0_4\ \pm 45\ 90_2\ 0_2\ 90_4]_S$	11045.04
32	$[\pm 45\ 90_2\ 0_4\ 90_2\ \pm 45_2\ 0_2\ \pm 45_3\ 0_2]_S$	12771.83
33	$[90_4\ 0_2\ \pm 45\ 90_2\ 0_2\ 90_4\ 0_4\ \pm 45_2]_S$	11123.00
34	$[90_2\ \pm 45\ 90_4\ \pm 45_3\ 90_4\ \pm 45\ 90_4]_S$	12415.30

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35	$[90_4\ 0_2\ \pm45\ 0_2\ \pm45_3\ 90_4\ 0_2\ 90_2]_S$	12120.44
36	$[0_2\ 90_4\ 0_2\ \pm45\ 90_2\ 0_2\ 90_2\ 0_2\ 90_2\ \pm45\ 90_2]_S$	10525.93
37	$[0_4\ \pm45_4\ 0_2\ \pm45\ 0_4\ 90_2\ \pm45]_S$	10976.93
38	$[90_4\ 0_4\ \pm45\ 90_2\ 0_2\ \pm45_3\ 0_2\ 90_2]_S$	11347.52
39	$[0_2\ \pm45_2\ 90_2\ \pm45\ 90_2\ 0_2\ \pm45\ 90_2\ 0_2\ \pm45_2]_S$	10059.61
40	$[\pm45_3\ 0_2\ \pm45_2\ 90_2\ 45_5]_S$	14904.86
41	$[\pm45\ 0_2\ 90_4\ \pm45\ 90_2\ \pm45_2\ 0_2\ \pm45\ 90_2\ 0_2]_S$	13007.46
42	$[\pm45\ 90_2\ 0_4\ 90_4\ 0_2\ 90_2\ \pm45_2\ 90_4\ 0_2]_S$	10824.31
43	$[\pm45_2\ 90_2\ 0_2\ 90_4\ \pm45\ 90_2\ \pm45_3\ 0_2]_S$	13541.15
44	$[0_2\ 90_2\ 0_2\ 90_2\ \pm45_3\ 0_2\ 90_2\ 0_2\ \pm45_2]_S$	11597.33
45	$[0_2\ \pm45_2\ 90_2\ 0_2\ \pm45_2\ 90_2\ \pm45\ 90_2\ 0_2\ 90_2]_S$	9811.85
46	$[\pm45_2\ 90_2\ \pm45_2\ 0_4\ \pm45_3\ 0_4\ \pm45]_S$	14730.56
47	$[0_2\ 90_4\ \pm45\ 90_2\ 0_2\ 90_4\ 0_2\ \pm45_3]_S$	9493.31
48	$[90_2\ \pm45\ 0_2\ 90_2\ 0_2\ \pm45\ 90_2\ \pm45\ 0_4\ 90_2\ 0_2]_S$	12205.09
49	$[\pm45_3\ 90_4\ \pm45\ 90_2\ \pm45\ 0_2\ \pm45_2\ 0_2]_S$	14759.42
50	$[90_2\ \pm45\ 0_4\ 90_2\ \pm45\ 90_2\ 0_2\ \pm45\ 0_2\ \pm45\ 90_2]_S$	12142.53
51	$[\pm45\ 90_4\ \pm45_3\ 0_4\ \pm45\ 90_2\ \pm45_2]_S$	13809.83
52	$[0_4\ \pm45_3\ 90_2\ \pm45\ 90_4\ 0_4\ 90_2]_S$	11296.05
53	$[90_2\ \pm45\ 90_2\ 0_2\ \pm45\ 90_4\ 0_2\ \pm45\ 90_2\ 0_4]_S$	12271.35
54	$[0_2\ \pm45_2\ 0_2\ \pm45_2\ 90_2\ 0_2\ 90_2\ 0_2\ \pm45\ 0_2]_S$	12340.19
55	$[\pm45\ 90_2\ \pm45\ 90_4\ \pm45\ 90_2\ 0_2\ 90_2\ 0_2\ 90_2\ 0_2]_S$	13041.42
56	$[0_2\ 90_4\ \pm45_2\ 90_4\ 0_2\ \pm45\ 90_2\ 0_4]_S$	10932.84
57	$[90_2\ 0_4\ \pm45_3\ 0_2\ \pm45\ 0_2\ \pm45_2\ 90_2]_S$	11910.83
58	$[0_4\ \pm45\ 90_2\ \pm45\ 0_4\ 90_4\ \pm45\ 90_2\ \pm45]_S$	10835.70
59	$[0_2\ \pm45\ 90_2\ 0_2\ \pm45\ 90_2\ 0_4\ \pm45\ 90_2\ 0_2\ 90_2]_S$	11665.05
60	$[\pm45\ 0_2\ \pm45_5\ 90_2\ \pm45\ 90_2\ \pm45\ 90_2]_S$	14310.21

Figure 6 presents the latin hypercube sampling and SVR for buckling load laminated composite. Correlation factor R2 for those regression simulations is also presented. The statistical learning methods with R2 = 0.999987 for metamodel approximation with C = 1x10^{10} and e = 0.1525 resulted that SVR is good method for response approximated in this case.

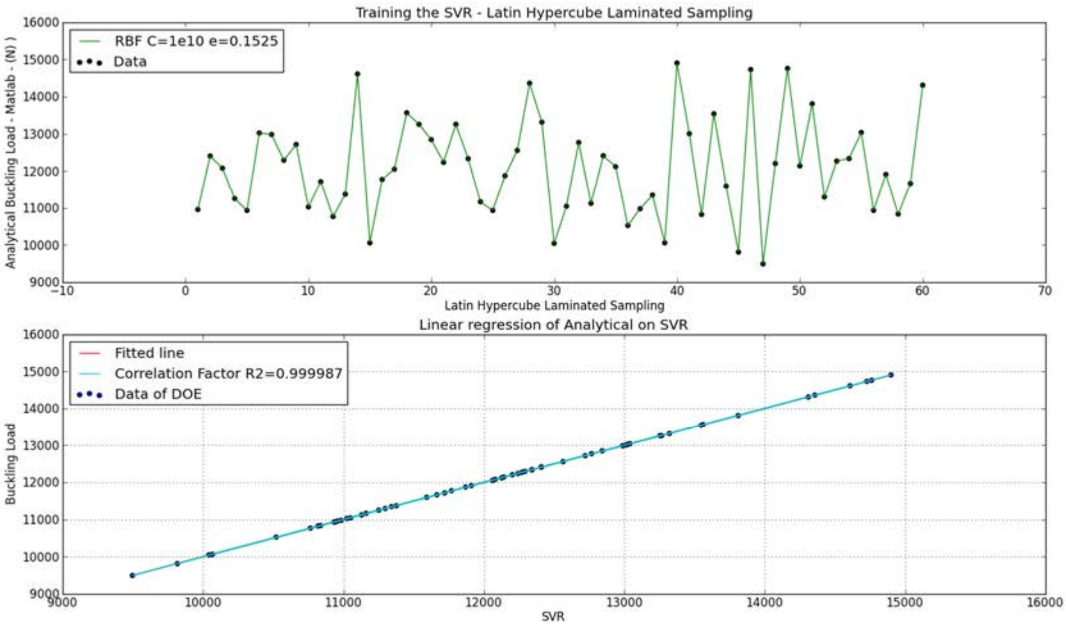

Figure 6. Latin hypercube and SVR for training buckling critical load sampling

ISSN 2176-5480

In order to validate the metamodel 15 new samples from latin hypercube were generated and tested in the same processes above. The parameters C = 1x10^{10} and e = 0.1525 was simulated in a support vector regression. The Table 4 presented those values. Figure 7 shows the approximation modeling for those buckling data set. The output vectors are much closed according with correlation factor R2=0.99999994 and the support vector regression is a good supervising learning method for approximating the buckling critical load of laminated composite plates.

Table 4. Matrix with latin hypercube training support vectors by analytical buckling load.

Laminated Composite	Stacking sequence	Buckling load
1	$[\,90_2\ \pm45_3\ 0_2\ 90_2 \pm 45\ \ 0_2 \pm45_2\ 90_2\ \pm45\,]_S$	13784.07
2	$[\pm45\ 90_4\ \ 0_4\ \ 90_2\ \pm 45_2\ \ 90_4\ \pm 45\ 90_2]_S$	12341.28
3	$[\pm45_2\ 0_4\ 90_2\ 0_2\ \pm45_2\ 0_4\ \ 90_2\ \pm 45\ ]_S$	12423.90
4	$[90_2 \pm 45\ 90_4\ \ 0_4 \pm 45\ 0_2\ \ 90_2\ \pm 45_2\ 0_2\ ]_S$	11943.76
5	$[0_2 \pm 45\ 90_4\ 0_4\ 90_2 \pm45\ \ 0_2 \pm45\ 0_2 \pm 45\ ]_S$	11710.18
6	$[\pm45_2\ \ 90_2\ \ 0_2\ 90_2\ \pm 45\ \ 0_4 \pm45_3\ 0_2]_S$	13673.65
7	$[90_2\ \pm 45\ \ 90_4\ \ \pm 45\ 0_2\ \pm 45\ \ 90_4\ 0_2\ \ \pm 45_2]_S$	12203.52
8	$[90_2\ 0_4\ \pm 45\ \ 90_2 \pm45_2\ \ 0_4\ 90_2\ 0_4\ ]_S$	11858.33
9	$[0_4\ \pm 45_3\ 0_2\ \pm 45\ \ 0_2\ \pm 45_4]_S$	10670.87
10	$[0_2\ 90_2 \pm 45\ \ 90_2\ \ 0_2\ \ 90_4\ 0_2\ \ \pm45_2\ \ 0_4\ ]_S$	11498.42
11	$[\pm45_5\ \ 0_4 \pm45\ \ 90_2\ 0_2 \pm45\ 90_2\ ]_S$	14846.85
12	$[\pm45\ 0_4\ 90_2\ 0_2 \pm 45\ \ 0_2\ 90_4\ 0_2\ 90_4\ ]_S$	11178.99
13	$[\pm45\ \ 90_2\ 0_2\ 90_2 \pm 45_2\ 0_2\ \ 90_2\ \ 0_2\ \pm 45_2\ 90_2\ ]_S$	12915.45
14	$[90_2\ \pm 45\ 0_4\ \pm 45\ \ 90_4\ \pm 45_2\ 90_2\ \pm 45_2]_S$	12525.31
15	$[90_2\ \pm 45_2\ 90_2\ 0_2\ 90_2\ \pm 45\ \ 0_2\ \ 90_4\ 0_4\ ]_S$	12845.52

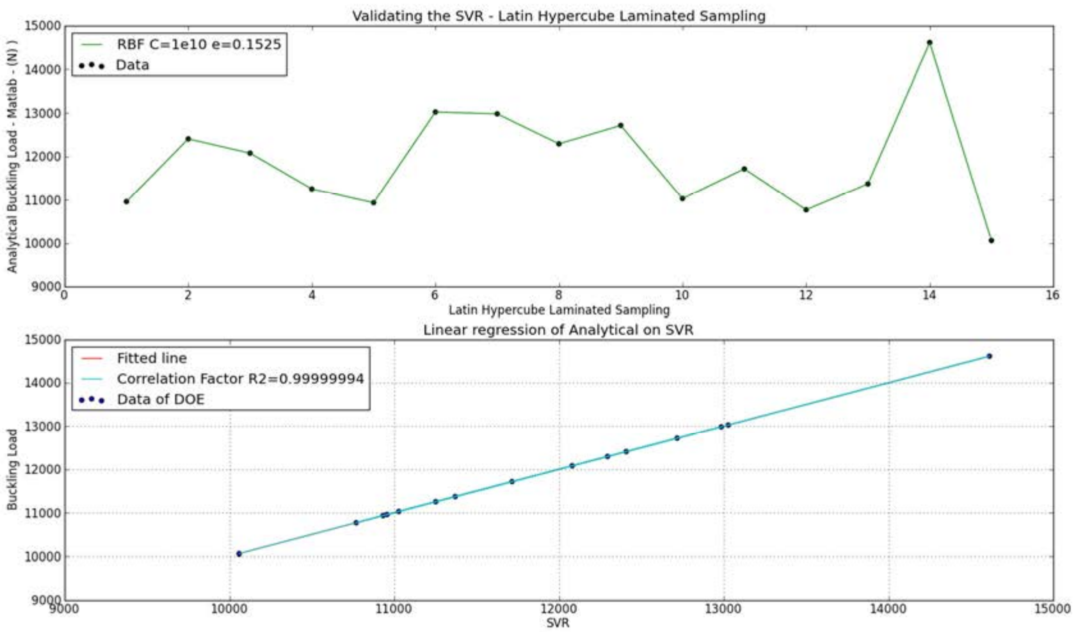

Figure 7. Latin hypercube and SVR for validating buckling critical load sampling

6. CONCLUSIONS

Statistics theories of DOE and SVR are used in the proposed work. The design of experiments is applied to generate a sampling set to compute the buckling load. Latin hypercube, the design of experiment used in this work, explored the wide range of variable or extending in entire region of design space. The second theory, the statistic learning theory, is implicit in the artificial intelligence proposed by Vapinik in their support vector machine. A variant support vector regression was tested in buckling load for laminated composite plate. Both theories are reported. Few works are found applying the support vector regression to laminate composite, however, the first approach of studies was simulated buckling load in analytical way and by finite element method considering the angle varying in 5 to 5 degree. The latin hypercube was coded in Matlab® and Python and SVR was developed in Python script. The tests were simulated and achieved the similar results in analytical and finite element method which were considered to follow the analysis applying the latin hypercube and support vector regression. In order to verify the proposed method some sampling was generated by latin hypercube based on the angle variables for stacking sequence of laminated composite plate. The input sampling matrix with the number of sample and buckling critical load was computed with support vector regression. The correlation factor of the sampling in the regression fitting was very good, especially, considering that the regression function was simulated with a small data set. A metamodel worked better with radial basis function or Gaussian function either linear or polynomial function for kernel function. Output vector from support vector regression in training processes and in a validated analyses was quite closed confirming their good supervised learning. The numerical results confirm that SVR method is appropriated to analyze the buckling load for laminated composites finding an estimating function that can be used in an optimization process. The results analyzed with the correlation factor of sampling regression proved that latin hypercube associated with SVR is an efficient methodology to metamodel applied for laminated composites. Implementing this approximation model the computational time can be substantially reduced.

7. ACKNOWLEDGEMENTS

The authors are grateful to Post Graduation Program in Mechanical and Materials Engineering (PPGEM-UTFPR) and the first author acknowledges the Brazilian funding CAPES for the financial support.

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