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# CONTROL DESIGN APPLIED TO THE STABILIZATION OF A NONLINEAR MAGNETIC LEVITATION SYSTEM

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***Abstract.*** *Processes magnetic levitation systems are engineered to attract the attention of the scientific community and often arouse curiosity in general. Various studies on ferromagnetic levitation of bodies through electromagnetism have been developed worldwide in several areas, such as magnetic bearings for electric motors, transportation systems and high-speed magnetic levitation turbines. Due to the increasing trend of population and consequent expansion of urban areas, the transport rail appears as an alternative mobility of the population in urban streams, suburban, regional and long haul, distinguishing itself from road, collective and individual, being a supply of large capacity led to large flows. Thus, the technology of magnetic levitation trains (MAGLEV) has proven to be a great alternative valid to replace the current mass transport. This technology has the main objective reducing the environmental and social impacts of existing motorized mobility, with adoption of sustainable transport compatible with human health and ecosystem, working effectively, reducing the consumption of non-renewable sources, reusing and recycling their compounds and reducing the levels of noises. A system of electromagnetic levitation (EML) is inherently unstable. This work, consider the simplified model system for German Transrapid that uses magnetic forces to counteract the gravitational forces. The simplified equations of state for the magnetic levitation vehicle and its track are being developed state variables* $x_1$, $x_2$, *e* $x_3$ *represented by the current in the coil magnet, the vertical velocity of the vehicle, and vertical gain between the path guide and vehicle, respectively. Therewith, the objective of this work is to study the dynamic behavior of the vehicle model of magnetic levitation and propose a controller design for stabilizing the electromagnetic levitation.*

*Keywords: Magnetic Levitation System, Stabilization, Control Design*

## 1. INTRODUCTION

Processes magnetic levitation systems are engineered to attract the attention of the scientific community and often arouse curiosity in general (Lee et al, 2006). Several bodies of research on ferromagnetic levitation through electromagnetism have been developed worldwide in several areas, such as, magnetic bearings for electric motors, transportation systems and high-speed magnetic levitation turbines.

The growing demand for mobility points to the need for mass transit that will meet current and future needs providing speeds over a 500 km/h, adding safety, reduce environmental impacts and maintenance costs.

The technology of magnetic levitation trains (MAGLEV) has proven to be a valuable alternative to replace the current mass transit. This technology aims to reduce the main environmental and social impacts of existing motorized mobility, with adoption of sustainable transport compatible with the health of humans and ecosystems, working effectively, reducing the consumption of non-renewable resources, reusing and recycling their compounds and reducing noise levels. Compared with conventional trains, the superiority of MAGLEV train lies in that the friction, mechanical losses, vibration and noise are reduced substantially since it replaces the wheels by electromagnets and levitates on the guideway and avoids mechanical contact with the rail (Michail et al, 2008).

The concept of magnetic levitation trains per MAGLEV, was introduced in the last century by two Americans, Robert Goddard and Emile Bachelet. By mid-1930 Hermann Kemper of Germany developed the concept and demonstrated the use of magnetic fields applied to rail and air transport.

In 1968, Americans James R. Powell and Gordon T. Danby's Brookhaven National Laboratory have patented a magnetic levitation using superconducting coils to produce a magnetic field that levitate trains.

In 1987, Georg Bednorz French and German KAMuller produced ceramic superconducting electricity by mixing barium, lanthanum, copper and oxygen (McConnell, 2008). The superconductivity, a phenomenon displayed by certain substances such as metals and ceramics special, characterized by the drastic decrease in electrical resistance at very low temperatures. Therewith, the current flows through the material without losing energy (David, Eduardo, 2009). To demonstrate the practical importance of the phenomenon, scientists opened the field for various applications, including use in monorails, designed to be the mass transport of the XXI century. Among these projects is the MAGLEV.

With current technology, it is becoming increasingly feasible to build trains MAGLEVs for the operation of public transportation in large cities, providing a means for fast, comfortable and safe, comparable to small distances, air transport.

In Germany and Japan, the tests pointed to MAGLEVs speeds up to 550 km/h. These high speeds are possible because there is no contact between the guideway and the vehicle while it is moving, air being the primary source of resistance, which problem can be solved by aerodynamic adjustments (David, Eduardo, 2009). Unlike conventional trains, the MAGLEVs not carrying propelling units, which are located in the guideway. Therewith eliminates the need for wheels, brakes, engines and devices for capturing, converting and transmitting electricity. Consequently, MAGLEVs are lighter, quieter and less prone to wear than traditional trains.

Significant results are observed in systems using electromagnetic levitation to reduce energy losses through friction. The phenomenon of levitation is observed through the use of electromagnetic force generated by magnetic field susceptible to variation of voltage, current, frequency and characteristics of the environment and the body levitated. Because of this, the Maglev is a system unstable and difficult to control. References (Li, Meng, 2003; Li, Wensen, 1999; Shi et al, 2004) profoundly discussed the stability of the maglev train.

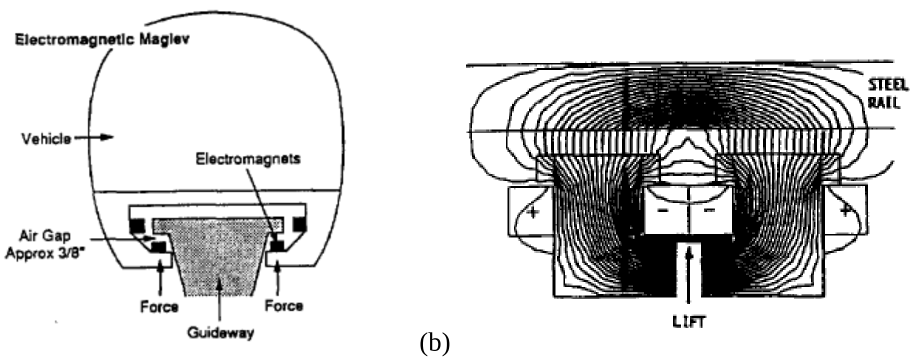
Generally, two approaches exist for generating magnetic levitation. The first is by using eddy current magnetic repulsive force to push a levitated object and the second is to use electromagnetic attractive force to suspend a ferromagnetic medium. Electromagnetic attraction is more efficient in energy consumption and is much more widely used. In this work, consider the simplified model system for German Transrapid that uses magnetic forces to counteract the gravitational forces (Zhao, Thornton, 1992). Details of the derivation of the model are discussed in (Thornton, 1991). Therewith, the objective of this work is to study the dynamic behavior of the vehicle model of magnetic levitation and propose a controller design for stabilizing the electromagnetic levitation.

In the last years, a significant interest in control of nonlinear systems exhibiting unstable behavior has been observed and many of the techniques have been discussed in the literature (Ott et al, 1990; Sinhá et al 200; Rafikov, Balthazar, 2008) and a number of control approaches for MAGLEV systems have been researched throughout the last two decades (Michail et al, 2008; Michail 2009; Sinha, Pechev, 2004; Sinha, Pechev, 1999; Park et al, 2001; Cho et al, 1993; Attilio, 2006). Recently, a technique was proposed by Rafikov and Balthazar in (2008): The linear feedback control problem for nonlinear systems has been formulated, under the optimal control theory viewpoint. Asymptotic stability of the closed-loop nonlinear system is guaranteed by means of a Lyapunov function, which can clearly be seen as the solution of the Hamiton-Jacobi-Bellman equation, thus guaranteeing both stability and optimality. The formulated theorem (Rafikov, Balthazar, 2008) expresses explicitly the form of the minimized functional and gives the sufficient conditions which allow using the linear feedback control for nonlinear systems.

The paper is organized as follows: in Section 2, we demonstrated the mathematical model for a magnetic levitation system. In Section 3, we discuss and include the Optimal Linear Control design problem for stabilization of the problem .In Section 4, we make the concluding remarks of this paper. In Section 5, we make some acknowledgements. Finally, we list out the bibliographic references

## 2. DYNAMIC SYSTEM

A system of electromagnetic levitation (EML) is inherently unstable (Zhao, Thornton, 1992). In this work, we consider the simplified model for German Transrapid system, illustrated in Figure 1. This model uses magnetic forces to counteract gravitational forces. Details of the derivation of the model are discussed in (Thornton, 1991).




(a) (b)

Figure 1. MagLev system, representing a simplified model of the German Transrapid. (a) Electromagnetic suspension; (b) Suspension magnet (Zhao, Thornton, 1992)

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The simplified state equations for the magnetically levitated vehicle and the guideway illustrates in Figure 1 are defined by (Zhao, Thornton, 1992):

$$\begin{aligned} \dot{x}_1 &= \frac{x_3\left(V_i - Rx_1\right)}{L_0 z_0} + \frac{x_1 x_2}{x_3} \\ \dot{x}_2 &= g - \frac{L_0 z_0 x_1^2}{2m x_3^2} \\ \dot{x}_3 &= x_2 \end{aligned} \tag{1}$$

where the state variables $x_1$, $x_2$, and $x_3$ represent coil current in magnet, vertical velocity of the vehicle, and vertical gap between the guide way and the vehicle. The parameter $m$ is the mass of the vehicle, $R$ is the coil resistance, $L_o$ is the coil inductance, $z_o$ is the vertical gap at the equilibrium, $g$ is gravitational acceleration, and $V_i$ is the coil input voltage.

The system has one equilibrium state at which the magnetic force exactly counterbalances the force due to gravity and the vehicle has no vertical velocity and acceleration, the equilibrium is a saddle node which is not stable.

Figure 2 shows the stability condition of the system where the parameters adopted for the numerical simulations were $L_o=0.1h$, $z_o=0.01m$, $R=1\Omega$, $m=10000kg$ and $g=9.8m/sec^2$, typical of a vehicle lift magnet (Zhao, Thornton, 1992). We further assume that $V_i$ is produced by a buck converter capable of delivering any voltage from 0 to 300 volts. The $V_i$ is varied demonstrating that its variation causes system instability

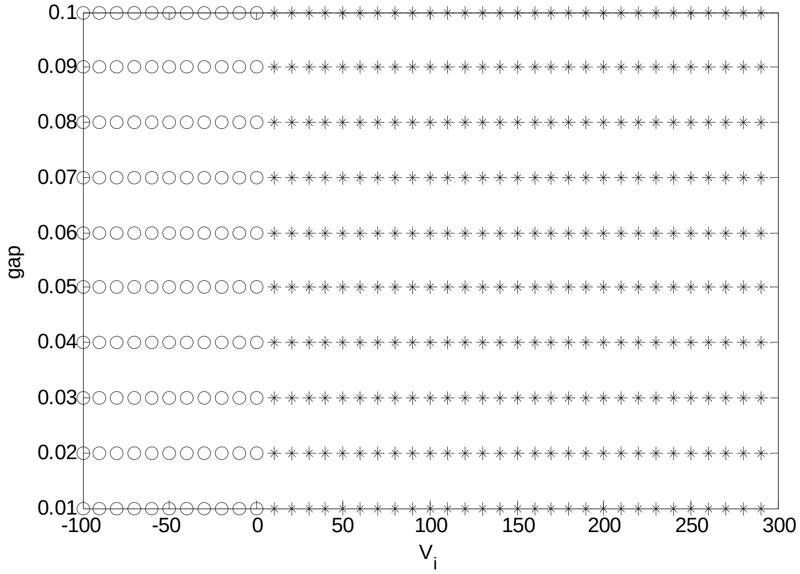


Figure 2. Stability diagram, (o) stable and (*) unstable equilibrium.

Figure 3 shows various Lyapunov exponents for different values of the $V_i$, seen that above the 128V system presents a chaotic behavior.

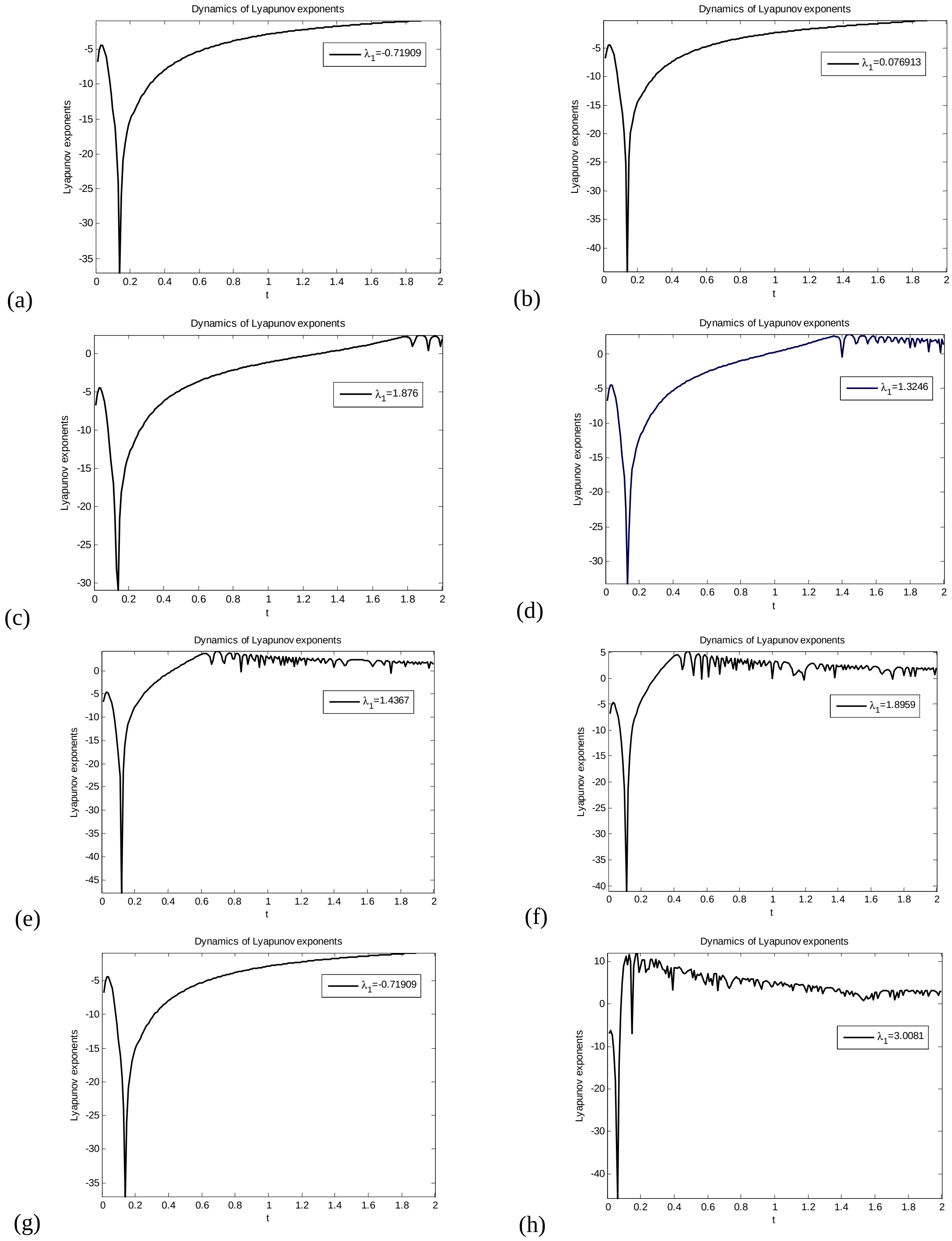


Figure 3. Dynamics of Lyapunov exponents (a) $V_i$ = *125* and λ=-0,719, (b) $V_i$ = *126* and λ=-0,076, (c) $V_i$ = *128* and λ=1,876 - Chaos, (d) $V_i$ = *130* and λ=1,324 - Chaos, (e) $V_i$ = *140* and λ=1,436 – Chaos, (f) $V_i$ = *150* and λ=1,895 – Chaos, (g) $V_i$ = *200* and λ=1,895 – Chaos, and (h) (f) $V_i$ = *2.240* and λ= 3.008 - Chaos.

Figure 4 shows shows the dynamic behavior of the model (1) for the parameters $L_o=0.1h$, $z_o=0.01m$, $R=1\Omega$, $m=10000kg$, $g=9.8m/sec^2$ and $V_i=300$ with chaotic behavior.

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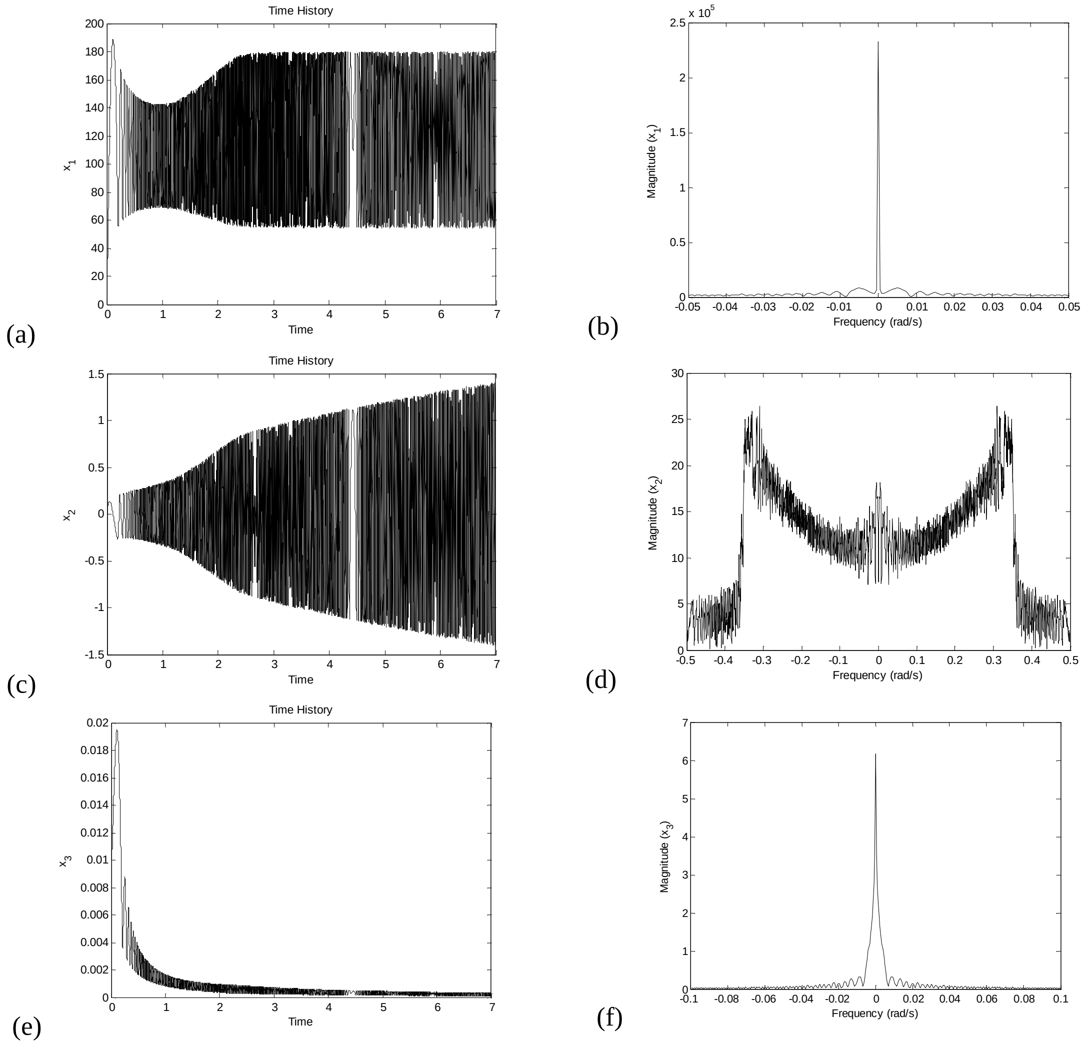


Figure 4. Maglev system. (a) Time History ($x_1$), (b) Frequency Spectrum ($x_1$) ,(c) Time History ($x_2$),(d) Frequency Spectrum ($x_2$), (e) Time History ($x_3$) and, (f) Frequency Spectrum ($x_3$).

Figure 5 shows the phase ($x_1$,$x_2$) and ($x_2$,$x_3$) with chaotic behavior.

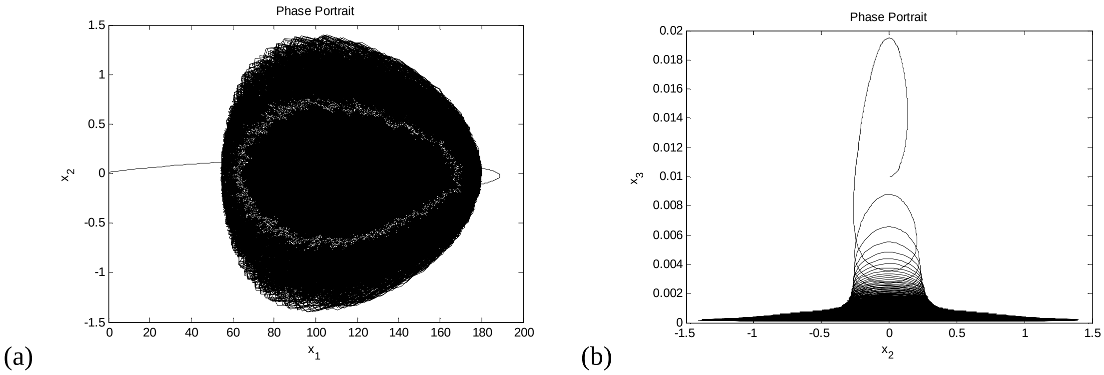


Figure 5. Phase Portrait. (a) ($x_1$,$x_2$) and, (b) ($x_2$,$x_3$).

In the next section, we proposed the optimal linear control design with the goal to stabilize the vehicle traveling down the guideway and to maintain a constant distance between the vehicle and the guideway despite any roughness in the guideway.

## 3. THE CONTROL DESIGN

In this section, we applied optimal linear control design for the magnetic levitation system (figure 1), reducing the oscillatory movement to a stable point. Next, we present the theory of the used methodology.

Due to the simplicity in configuration and implementation, the linear state feedback control, it is especially attractive (Rafikov, Balthazar, 2008; Chavarette, 2013; Chavarette *et al*, 2012).

We remarked that this approach is analytical, and it may use without dropping any non-linear term.

Let the governing equations of motion (1), re-written in a state form

$$\dot{x} = Ax + g(x)\,. \tag{2}$$

If one considers a vector function $\tilde{x}$, that characterizes the desired trajectory, and taken the control *U* vector consisting of two parts: $\tilde{u}$ being the feed forward and $u_f$ is a linear feedback, in such way that

$$u_t = Bu \tag{3}$$

where *B* is a constant matrix. Next, one taking the deviation of the trajectory of system (2) to the desired one $y = x - \tilde{x}$, may written as being

$$\dot{y} = Ay + g(x) - g(\tilde{x}) + Bu \tag{4}$$

where $G(y,\tilde{x})$ is limited matrix we proved the important result (Rafikov, Balthazar, 2008).

If there exit matrices *Q(t)* and *R(t)*, positive definite, being *Q* symmetric, such that the matrix $\tilde{Q} = Q - G^T(y,\tilde{x})P(t) - P(t)G(y,\tilde{x})$ is positive definite for the limited matrix *G*, then the linear feedback control is

$$u = -R^{-1}B^T P\,y\,. \tag{5}$$

It is optimal, in order to transfer the non-linear system (6) from any initial to final state $y(t_f)=0$, minimizing the functional $\tilde{J} = \int_0^\infty (y^T \tilde{Q}\, y + u^T R\, u)dt$, where the symmetric matrix *P(t)* is evaluated through the solution of the matrix Ricatti differential equation

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{6}$$

satisfying the final condition $P(t_f)=0$.

In addition, with the feedback control (6), there exists a neighborhood $\Gamma_0 \subset \Gamma$, $\Gamma \subset \Re^n$, of the origin such that if $x_0 \in \Gamma_0$, the solution $x(t) = 0,\ \ t \ge 0,$ of the controlled system (4) is locally asymptotically stable, and $J_{\min} = x_0^T P(0)\,x_0$. Finally, if $\Gamma = \Re^n$ then the solution $y(t)=0,\ t>0,$ of the controlled system (4) is globally asymptotically stable.

Using the theorem by Rafikov and Balthazar the dynamic error ***y*** can be minimized ( $y \to 0$ ) (Rafikov, Balthazar, 2008; Chavarette, 2013; Chavarette *et al*, 2012).

### 3.1 Theorem.

If there is matrixes *Q* and *R*, positive definite, *Q* symmetric, such that the matrix

$$\tilde{Q} = Q - G^T(x,\tilde{x})P - PG(x,\tilde{x}) \tag{7}$$

is positive definite for the limited matrix $G$, then the linear feedback control

$$u = -R^{-1}B^T P\,y \tag{8}$$

is optimal, in order to drive the non-linear system (6) of any initial state to the terminal state

$$y(\infty) = 0 \tag{9}$$

minimizing the functional

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$$J = \int_0^\infty (y^T \tilde{Q} y + u^T R u)dt \tag{10}$$

where the symmetric matrix $P$ is calculated from the nonlinear Riccati equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{11}$$

Next, we will apply this methodology in the magnetic levitation system (1).

### 3.2 Application of the Linear Optimal Control to the magnetic levitation system.

The equations (1) describing the magnetic levitation system controlled:

$$\begin{aligned}
\dot{x}_1 &= \frac{x_3(V_i - Rx_1)}{L_0 z_0} + \frac{x_1 x_2}{x_3} + U \\
\dot{x}_2 &= g - \frac{L_0 z_0 x_1^2}{2m x_3^2} \\
\dot{x}_3 &= x_2
\end{aligned} \tag{12}$$

where the function of control U is defined in the equation (1).

We will obtain $B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $y = \begin{bmatrix} x_1 - \tilde{x}_1 \\ x_2 - \tilde{x}_2 \\ x_3 - \tilde{x}_3 \end{bmatrix}$, $\tilde{x} = \begin{bmatrix} 100 \\ 0.01 \\ 0.01 \end{bmatrix}$, $Q = I_3$, $A = 10^6 \begin{bmatrix} 0 & 0.2 & -0.1 \\ -0.00002 & 0 & 3.02 \\ 0 & 0.000001 & 0 \end{bmatrix}$,

where the controllability matrix R of the system to the pair [A,B] is obtained by $R = [B \mid AB \mid A^2B \ldots \mid A^{2n-1}B]$.

Thus, $R = \begin{bmatrix} 1 & 0 & -4000000 \\ 0 & -20 & 0 \\ 0 & 0 & -20 \end{bmatrix}$.

Then the Matrix *P(t)* is done by $P = \begin{bmatrix} 8.2595 & -1.6805 & -1528288 \\ -1.6805 & 6180.834 & 336093.84 \\ -1528288 & -33609.38455 & 2.8688 * 10^{11} \end{bmatrix}$ and (an optimal control) $u = 8.259x_1 + 1.680x_2 + 1.5282x_3$. The trajectories of the system with control may be seen, through Figure 7. According to the optimal control verification (Rafikov, Balthazar, 2008), the function (4) is numerically calculated across $L(t) = y^T \tilde{Q} y$, where *L(t)* is defined positive and it is show in Fig. 6.

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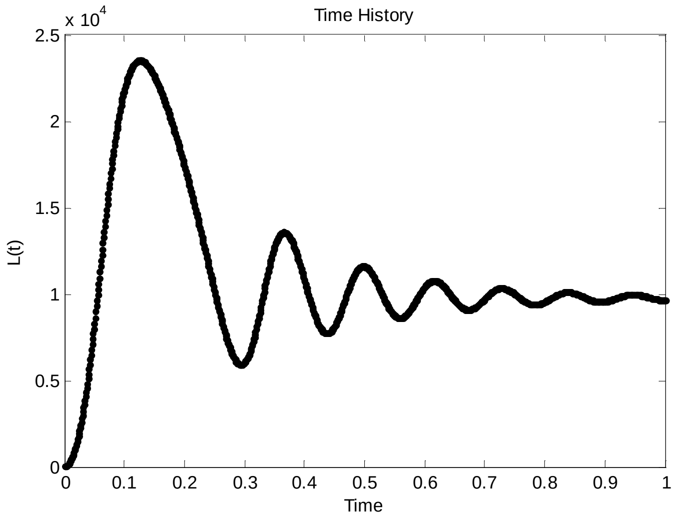



Figure 6. The function $L(t) = y^T \tilde{Q} y$ .

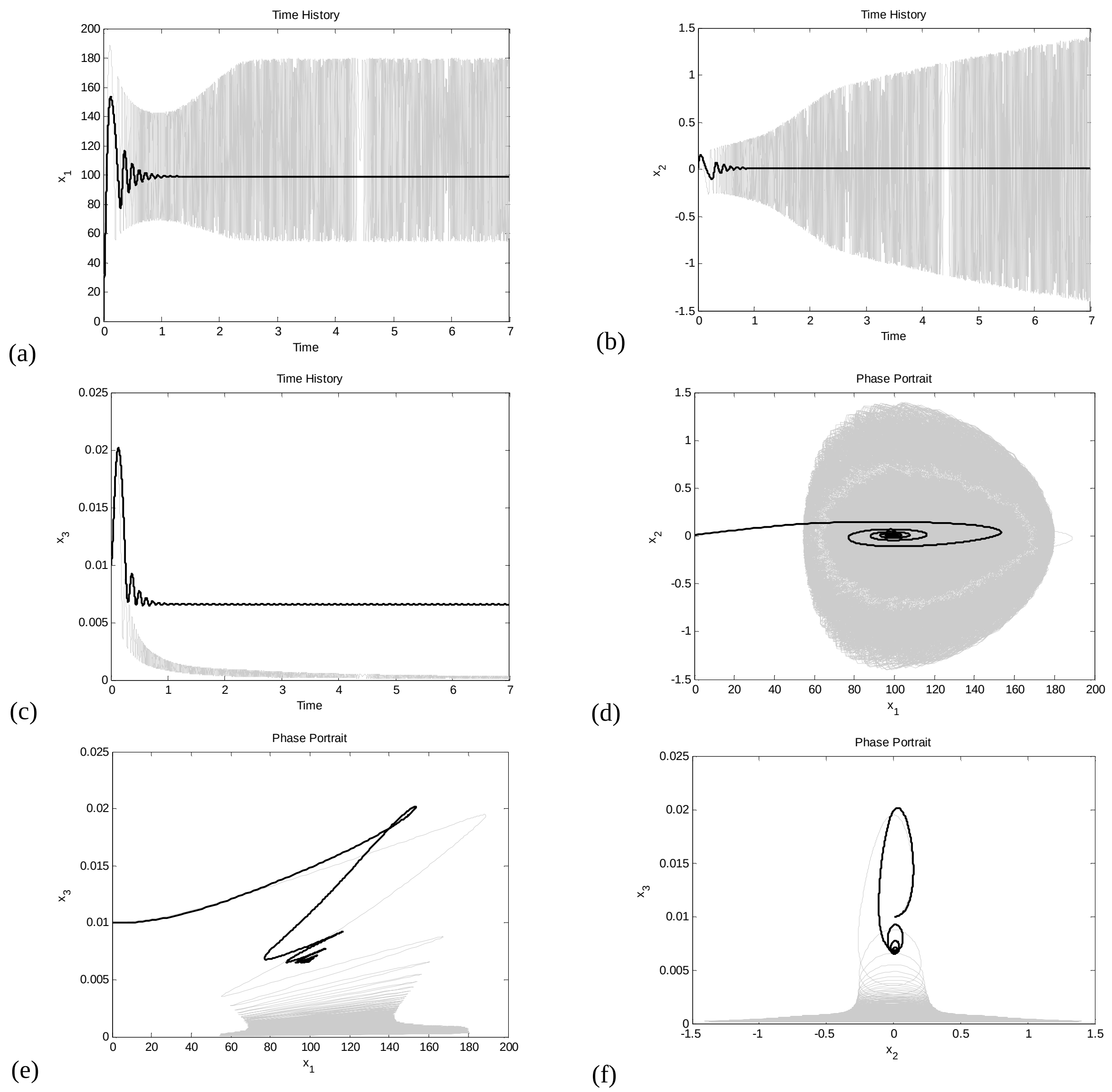
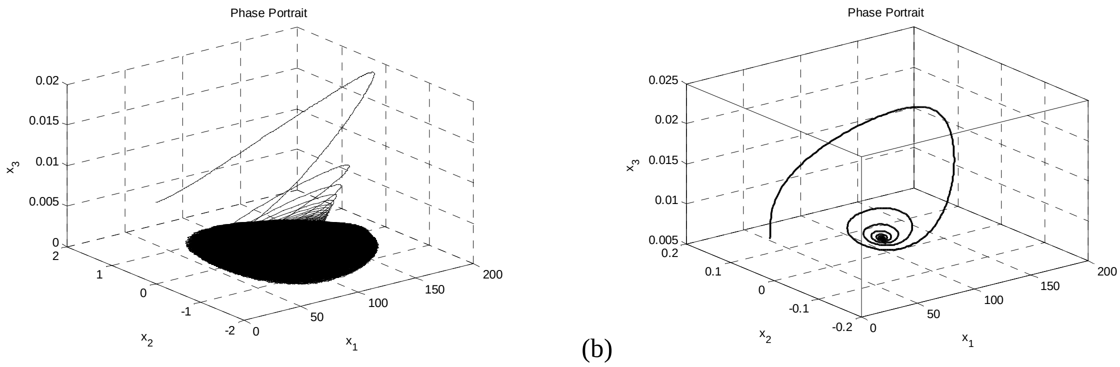



Figure 7. Uncontrolled (gray) and Controlled (black) Maglev system. (a) Time History ($x_1$); (b) Time History ($x_2$); (c) Time History ($x_3$); (d) Phase Portrait ($x_1$,$x_2$); (e) Phase Portrait ($x_1$,$x_3$) and, (f) ($x_2$,$x_3$).




(a) (b)

Figure 8. Portrait Phase ($x_1$, $x_2$, $x_3$); (a) Uncontrolled and (b) Controlled.

We observe in Figures 7 and 8 that the performance of the proposed controller reduced the amplitude of oscillation of the system, eliminating the chaotic behavior and consequence of the proposed controller illustrating the efficiency of this.

## 4. CONCLUSION

Processes magnetic levitation systems are engineered to attract the attention of the scientific community and often arouse curiosity in general. In this work, a dynamics of the Magnetic Levitation System proposed (Zhao, Thornton, 1992) is investigated through numerical simulations using the software Matlab 6.5®, consider the simplified model system for German Transrapid that uses magnetic forces to counteract the gravitational forces. The simplified equations of state for the magnetic levitation vehicle and its track are being developed state variables $x_1$, $x_2$, e $x_3$ represented by the current in the coil magnet, the vertical velocity of the vehicle, and vertical gain between the path guide and vehicle, respectively.

As mentioned in (Zhao, Thornton, 1992), the parameters adopted showed that the system has an unstable behavior, see Figures 2-5, when the input voltage, $V_i$, is applied. Figure 3 shows various Lyapunov exponents for different values of the $V_i$, seen that above the 128V system presents a chaotic behavior.

We also proposed to use of an optimal linear feedback control strategy, applied to mathematical model proposed (1) with the goal to stabilize the vehicle traveling down the guideway and to maintain a constant distance between the vehicle and the guideway despite any roughness in the guideway.

This kind control strategy reducing the chaotic movement of this system to a stable point, with show Figure 9, causing the coil current in magnet, the vertical velocity of the vehicle and vertical gap between the guide way were stabilized. The Figures 8-9 illustrated the effectiveness of the control strategy taken to these problems.

This technology has the main objective reducing the environmental and social impacts of existing motorized mobility, with adoption of sustainable transport compatible with human health and ecosystem, working effectively, reducing the consumption of non-renewable sources, reusing and recycling their compounds and reducing the levels of noises.

## 5. ACKNOWLEDGEMENTS

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