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ANALYSIS OF NONPARAMETRIC UNCERTAINTY QUANTIFICATION OF STRUCTURAL DYNAMIC MODELS

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Abstract. *The study of uncertainties has been the subject of constant research due to its importance in the increasing the reliability of numerical models especially when it's representing complex structural systems, thus its quantification becomes inevitable. In this work, the initial focus will be given to epistemic uncertainties, also called model uncertainty or non-parametric uncertainties that occur due to lack of knowledge or approximations made in the system under study. Analyses are performed from a structural dynamic system composed of a steel beam with boundary conditions fixed-free. The studies are possible when one have on hand the results of mean modelling corresponding to real model studied, of stochastic simulations obtained by non-parametric approach, and of experiments performed with the application of impulse excitation technique. With the tests it is possible to obtain the dynamic response as a function of frequency (FRF). In this article, the mass matrix will have its uncertainty propagated with different dispersion parameters in each case and considering additions of mass in original system in order to bring about changes in the behaviour of the response and a consequent increase of uncertainty in the system. Finally, conclusions are shown about the dispersion parameters used and its influence in the response of the system, about changes in the response due to increase of mass in the original system and about the reliability of the mean model corresponding to the real model studied.*

Keywords: *Nonparametric Approach, Epistemic Uncertainty, Model Uncertainty, Uncertainty Quantification, Dynamic Models.*

1. INTRODUCTION

The study of uncertainty quantification has been the subject of out constant research because of the importance in increasing the reliability of numerical models. According to Brandão (2007), the uncertainty is related to the variability of the variables that describe the system, and the variability is presented in structural systems in the form of uncertainty. The quantification of uncertainties in this case should be considered especially in models of complex systems, since such systems need to be tested so that details of complexity can be revealed and analysed, and even then only some information or conclusions can be deduced from them.

However, mathematical models are constructed so that it is possible simulate real situations into appropriate software considering that the simulation can replace real experiments that would be performed by systems manufactured from designed systems. In this case the system under study is represented by a steel beam and one can distinguish three types of models considered in the analysis. They are: (a) Designed model, which corresponds to the system designed by design engineers. (b) Real model, that refers to the system manufactured from the system designed. In this case, one have differences in geometric parameters, in boundary conditions, materials etc. between the two systems, designed and real. (c) Mean model, concerning the modelled system from the designed system, it represents the real system and it is also called predictive model. It is modelled by the finite element method with boundary conditions fixed - free, 80 Euler – Bernoulli, 81 nodes and 160 degrees of freedom.

The purpose therefore is to make the mean model can faithfully represent the real model, and in this process of modelling the mean model of the designed system, uncertainties are introduced and should be quantified so that it can increase the reliability of the numerical model constructed.

In the process of modelling, errors can also be present in the model. But it is very important that a distinction be made between errors and uncertainties. According to Soize (2005a)

> […] the errors are related to the construction of an approximate output $\underline{v}^n$ of output $\underline{v}$ of the mean model for a given input f and for a given parameter s. For instance, if the mean model is a boundary value problem (BVP) defined on a bounded domain, the use of the finite element method for constructing a n–dimensional space approximation of the BVP solution, introduces an error $\|\underline{v} - \underline{v}^n\|$ related to the finite element mesh size, where $\|\cdot\|$ is an appropriate norm (Soize, 2005a).

Then, specifically the model error must not be confused with the model uncertainty. According to Lian and Mahadevan (2011), the model error depends "[…] on whether the selected model correctly represents the real phenomenon". By the way the model uncertainty arises mainly due to lack of knowledge of the system, due the errors in

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the estimation of the theoretical models used in the analysis and not depends on their parameters. In the case of this paper, only model uncertainty are considered and quantified.

Nevertheless, in this paper one will realized studies comparing the modelled system, called mean model, the 95% confidence limits (CL) of the uncertainties propagated in the mass matrix, and the real system that corresponds to the manufactured system. These studies are possible because one have on hand the FRF of mean model, the FRF of stochastic simulations obtained by non-parametric approach, and the FRF of real model obtained by experiments performed with the application of impulse excitation technique. Some variations in the mass matrix also will be induced considering additions of mass in original system in order to bring about changes in the behaviour of the response and a consequent increase of uncertainty in the system. In all this studies, different dispersion parameters will be used in each case. Finally conclusions will be showed about the influence of the dispersion parameter in the response of the system, about changes in the response due to the increase of mass in the original system and about the reliability of the mean model.

2. DYNAMIC SYSTEM TO BE ANALYZED

The equation of motion of a structural linear dynamic system with n degrees of freedom can be represented in frequency domain as follows:

$$-\omega^2\mathbf{M}\,\ddot{\mathbf{u}}(\omega) + i\omega\mathbf{C}\,\dot{\mathbf{u}}(\omega) + \mathbf{K}\,\mathbf{u}(\omega) = \mathbf{f}(\omega) \tag{1}$$

in which the vectors $\mathbf{u}(\omega)$, $\dot{\mathbf{u}}(\omega)$, $\ddot{\mathbf{u}}(\omega)$ respectively represent the vector of displacement, velocity and the acceleration of the mass of the system considered. $\mathbf{f}(\omega)$ is the vector of the external force applied to the system. $\mathrm{i} = \sqrt{-1}$. $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are respectively the $n \times n$ random matrices of mass, damping and stiffness. Such matrices are real, symmetric and positive-definite; they belong to positive-definite ensemble proposed by Soize (2000) and studied in Soize (2003a) and Soize (2005a), whose details can also be found in Justino (2012).
The external force vector can be represented by:

$$\mathbf{f} = (f_1, f_2, \cdots, f_n)^t \tag{2}$$

The signal obtained on the analysis is a velocity signal, then:

$$\dot{\mathbf{u}} = (\dot{u}_1, \dot{u}_2, \cdots, \dot{u}_n)^t \tag{3}$$

Particularly in this paper the Eq. (2) and Eq. (3) can be represented respectively by:

$$\mathbf{f} = (0, \cdots, f_{79}, \cdots, 0)^t \tag{4}$$

$$\dot{\mathbf{u}} = (0, \cdots, \dot{u}_{159}, \cdots, 0)^t \tag{5}$$

The points f_{79} and $\dot{u}_{159}$ are considered because the response of the mean model, of the real system and of the simulated model need to have the same excitation point and the same reading point. In this case the vibration is caused by the excitation in the central position of the beam that corresponding to the length of 100 mm from their ends, and this point corresponds to the value of 79 on the finite element method. In the case of reading results, represented by the speed signal caused by the movement of the beam, it is captured by the laser vibrometer at the end of the beam and this point corresponds to the value of 159 on the finite element method.

The response on the frequency domain is:

$$\mathbf{H}(\omega) = (-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K})^{-1} \tag{6}$$

where $\mathbf{H}(\omega)$ is the frequency response function (FRF) of the studied system obtained for a defined frequency band, that in this case is given by:

$$\mathbb{B} = [0,1200] \quad [Hz] \tag{7}$$

In this paper the structural linear dynamic system is constituted by a steel beam with boundary conditions fixed-free, and it have dimensions, mass and properties as following: length: 200.000 mm; height: 23.064 mm; thickness: 2.736 mm; mass: 137.28 g; modulus of elasticity: 183.000 GPa; density: 7.830 g/cm^3. This dynamic system will be used to obtain the FRF of the mean model, of the experiment and of the simulated system.

ISSN 2176-5480

3. MEAN MODEL

The mean model was constructed by the mean matrix of mass, damping and stiffness. The mean matrix of mass and stiffness is obtained by finite element method. The dynamic system constituted by a steel beam was modelled with boundary conditions fixed - free, 80 Euler – Bernoulli elements, 81 nodes and 160 degrees of freedom. The damped matrix was considered as following:

$$\bar{C} = coef_1 \times \bar{\boldsymbol{M}} + coef_2 \times \bar{\boldsymbol{K}} \tag{8}$$

where $coef_1 = 1$ e $coef_2 = 10^{-7}$.

The FRF of the mean model is:

$$-\omega^2 \bar{\boldsymbol{M}}\, \ddot{\bar{\boldsymbol{u}}}(\omega) + i\omega \bar{\boldsymbol{C}}\, \dot{\bar{\boldsymbol{u}}}(\omega) + \bar{\boldsymbol{K}}\, \bar{\boldsymbol{u}}(\omega) = \bar{\boldsymbol{f}}(\omega) \tag{9}$$

$$\bar{\boldsymbol{H}}(\omega) = (-\omega^2 \bar{\boldsymbol{M}} + i\omega \bar{\boldsymbol{C}} + \bar{\boldsymbol{K}})^{-1} \tag{10}$$

The bars over the symbols represent the mean values. The vectors and the matrices are deterministic, and the matrices are real, symmetric and positive-definite.

The FRF of the mean model is obtained to be analysed with the FRF of the experiment and the FRF of the stochastic simulation of the same system. These responses will be showed on section 8.

4. STOCHASTIC MODELING

The stochastic modelling is the first step in order to proceed to the uncertainty quantification. First, one must choose the matrices that will be randomizes. In case of this procedure the mass, damping and stiffness matrices will be called $\mathbf{G}$ in order to make a general procedure. This may be considered because, according to Adhikari (2007), the random matrices of the dynamic system have similar probabilistic characteristics. Thereafter, the sample space must be defined. It identifies the values that can be assumed by the random matrices and corresponds to the construction of the probability density function (PDF) for each of the matrices considered earlier. It should also say that at this stage of modelling, the success of the process depends of the use of the appropriate PDF for each of the random matrices, so that are eliminated on the analysis errors resulting from the use of an incorrect PDF. Fortunately, in the case of a damped linear dynamic structural system one have the Wishart distribution studied in detail by Adhikari (2007), Adhikari (2008) and Adhikari (2009). Some details of these studies can also be seen in Justino (2012).

The nonparametric uncertainty quantification is possible when one apply the nonparametric approach that considers the Random Matrix Theory (RMT) and de Maximum Entropy Principle (MEP). This theory was proposed by Soize (1998) and Soize (2000), its validation was realizes on Soize (2001), Soize (2003a), Soize (2003b) e Chebli *et al* (2004), and an overview about the approach was made by Soize (2005b). Some details about the nonparametric approach, including RMT and MEP, can also be seen in Justino (2012).

In addition, the modelling of uncertainty includes vibration problems in the range from low to high frequency. But an important information should be remembered here, the parametric and nonparametric approaches should be considered taking into account, besides the type of uncertainty that need to be quantified, also the vibration frequency range considered for the system. In case of low frequency bands of vibration, parametric uncertainties are considered in detail, already in mid frequency ranges, parametric and non-parametric uncertainties should be quantified. Finally, as regards the high-frequency bands of vibration, nonparametric uncertainties need to be quantified.

Details about how to obtain the Wishart distribution for uncertainty quantification in structural dynamics models are not the subject of this article, but it can be seen on the references cited above.

However it's of great importance to know that the Wishart distribution is represented by the follow equation with its optimum parameters by:

$$\begin{cases} p_{\mathbf{G}}(\mathbf{G}) = \left\{(2)^{\frac{1}{2}np}\Gamma_n\left(\frac{1}{2}p\right)|\boldsymbol{\Sigma}|^{\frac{1}{2}p}\right\}^{-1} |\mathbf{G}|^{\frac{1}{2}(p-n-1)} exp\ \left\{tr\left(-\frac{1}{2}\boldsymbol{\Sigma}^{-1}\,\mathbf{G}\right)\right\} \\ \qquad\qquad with\ parameters \\ \qquad\qquad p = \theta + n + 1 \\ \qquad\qquad \boldsymbol{\Sigma} = \frac{\bar{\mathbf{G}}}{\theta} \\ \qquad\qquad in\ which \\ \qquad \theta = \frac{1}{\tilde{\delta}_{\mathbf{G}}^2}\left\{1 + \frac{\{tr(\bar{\mathbf{G}})\}^2}{tr\left(\bar{\mathbf{G}}^2\right)}\right\} - (n+1) \end{cases} \tag{11}$$

ISSN 2176-5480

in which $p_G(\mathbf{G})$ is the Wishart PDF of $\mathbf{G}$, Γ_n is the Gamma Function, p is a scalar parameter of Wishart PDF and $\mathbf{\Sigma}$ is the matrix parameter of Wishart PDF.

5. DISPERSION PARAMETER

The dispersion parameter δ is the information about the uncertainty on the system. It's very important information for uncertainty quantification and according to Soize (2003a), its calculation can be done by the following equation:

$$\delta = \left\{\frac{E\{\|\mathbf{G}-E[\mathbf{G}]\|_F^2\}}{(\|E[\mathbf{G}]\|_F^2)}\right\}^{1/2} \tag{12}$$

in which $E[\mathbf{G}]$ is the mathematical expectation of random matrix $\mathbf{G}$ and $\|\cdot\|_F$ is the Frobenius norm of $(\cdot)$.

The dispersion parameter must also be calculated within an interval of possible values considering the nonparametric approach. According to Soize (2003a) the interval that must be considered for the calculation of this parameter is given by:

$$0 < \delta < \left\{\frac{(n_0+1)}{(n_0+5)}\right\}^{1/2} \tag{13}$$

in which $n_0 > 1$ is an integer that is given and fixed, and in this case, δ is independent of n. It can be stated that, in general, the dimensions of the model in question are large and sometimes above 100. If an example, if consider $n = 10$ then will be $n_0 = 10$ which, when applied in Eq. (12) correspond to a high uncertainty (0.856) which generally is not achieved in applications (Soize, 2003a).

One aim of this article is the analysis of the dispersion parameter in the response of the system then is used the following values of dispersion parameters:

$$\begin{aligned} \delta_{1\mathbf{M}} &= 0{,}11 \\ \delta_{2\mathbf{M}} &= 0{,}2 \end{aligned} \tag{14}$$

wherein $\delta_{1\mathbf{M}}$ and $\delta_{2\mathbf{M}}$ are de dispersion parameters of the mass random matrix considered in the analysis.

The range of valid values for the dispersion parameter is given in Eq. (13), and its maximum value when one consider $n_0 = 10$ on this same equation is given by $\delta_{\mathbf{M}} = 0{,}856$. Nevertheless, it was chosen values smaller than the maximum value because it is not expected that a simple system as discussed in this paper has higher values of dispersion parameters. The values shown in Eq. (14) were chosen arbitrarily within the allowed range.

6. STOCHASTIC SIMULATION

The Monte Carlo method is used to perform simulations and obtain statistical response of the simulated system.

It is necessary to generate a sufficient number of samples so that one can obtain statistics of the response or to determine a number of moments (mean and dispersion). The main problem now is to determine how many simulations are needed to construct an approximation of the response to pre-defined error. For this one used the quadratic convergence method.

According to Soize (2005b), the convergence in accordance with the size of the random matrix and the number of realizations required in Monte Carlo simulation is given by:

$$conv(n_s, n) = \left\{\frac{1}{n_s}\sum_{k=1}^{n_s}\int_{\omega\in\mathbb{B}}\|Q^n(\omega;\theta_k)\|^2 d\omega\right\}^{1/2} \tag{15}$$

in which n is the order of random matrices; n_s corresponds to the number of Monte Carlo simulation, ω is the frequency on band $\mathbb{B}$, $Q^n(\omega;\theta_k)$ corresponds to the response of the stochastic system calculated for each simulation k with corresponding result θ_k.

The simulations are repeated until a convergence criterion is confirmed. A deviation value needs to be considered for this convergence and in this case the value is 5%.

In case of this paper, the stochastic simulation was made by a computational program develop by the author. It was performed for a dynamic system described on section 2. The propagation of uncertainty occurs in the random matrix of mass. 95% confidence limits are calculated in this case.

The results of these simulations will be showed and analysed with the FRF of the mean model and the FRF of the experiment of the same system and they will be showed on section 8.

ISSN 2176-5480

7. EXPERIMENT

In this section is shown details about the experiment performed with a dynamic system constituted by a steel beam as previously explained.

The tests were performed at the Mechanical Vibrations Laboratory of Mechanical Engineering Institute, Federal University of Engineering (UNIFEI) – Itajubá – and it aims at obtaining the FRF of the system tested, which in this case is represented by the steel beam already described.

7.1 Configuration of experiment

The experiment was performed for the frequency range 0-1200 Hz and the propagation of uncertainty occurs in the random matrix of mass. In addition to test of the beam, two other configurations were tested. These configurations consist of the addition of masses (represented by magnets) on the beam. These variations in the mass were performed in order to demonstrate that any change in the project, or in this case, on the beam under study, be it an increase in reinforcing or fixing equipment in the beam for instance gives rise to even more uncertainty that cause changes on the system response. The FRF in this case becomes different of the FRF of original design. Thus the configurations are:

Configuration 1: beam without the addition of magnets. It is showed in Fig. (1a).

Configuration 2: beam with addition of two magnets at the positions 95 mm and 150 mm from the beam. It is showed in Fig. (1b).

Configuration 3: beam with addition of four magnets at the positions 20 mm, 60 mm, 95 mm and 150 mm from the beam. It is showed in Fig. (1c).

Measurements were taken from the left end to the right end of the beam.

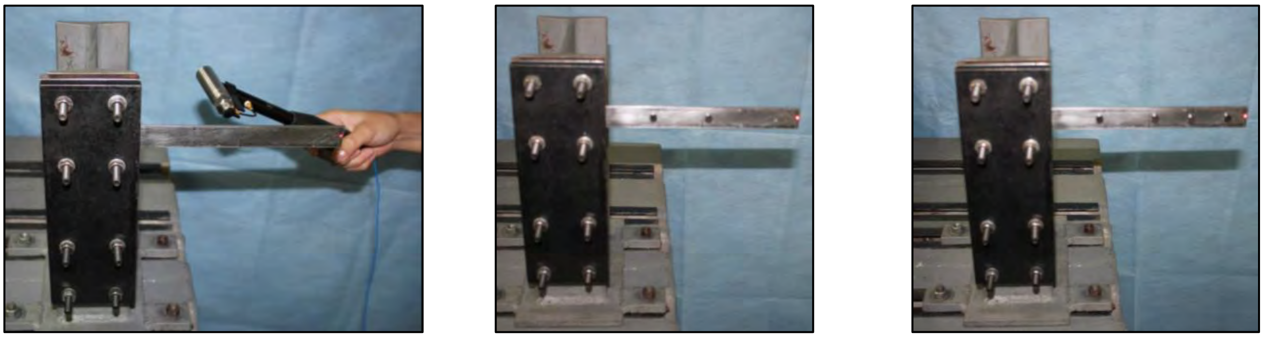

Figure 1 – Configuration of the experiment. (a) Configuration 1. (b) Configuration 2. (c) Configuration 3.

The impulse was caused by an instrumented impact hammer and the excitation point was in the central position of the beam corresponding to the length of 100 mm from their ends. Already reading results, represented by the speed signal caused by the movement of the beam is picked up by the laser vibrometer at its end that corresponds at the position 200 mm in length. The signals from the load cell of the impact hammer and of laser vibrometer is captured to a signal analyser which then shows the FRF curves obtained in the testing. A general configuration of the experiment can be seen in Fig. (2).

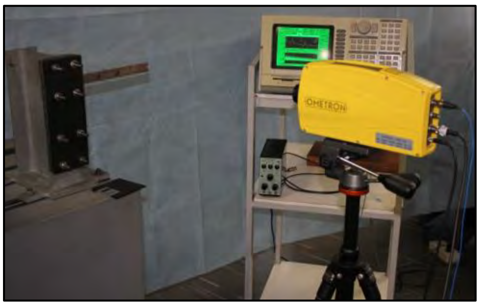

Figure 2 – General configuration of the experiment.

8. SIMULATED AND EXPERIMENT RESULTS

This section presents the results (FRF) obtained in stochastic simulations that corresponds to 95% confidence limits for uncertainty, the FRF for the mean model and the FRF obtained by the experiment. The frequency band of 0 – 1200

ISSN 2176-5480

Hz was divided into two frequency bands: 0 – 800 Hz and 800 – 1200 Hz. So, was considered the two dispersion parameters shown on Eq. (14) for each of the two frequency band defined. The results (FRF) are shown as following.

8.1 Frequency band: 0 – 800 Hz

- ✓ Dispersion parameter = 0.11

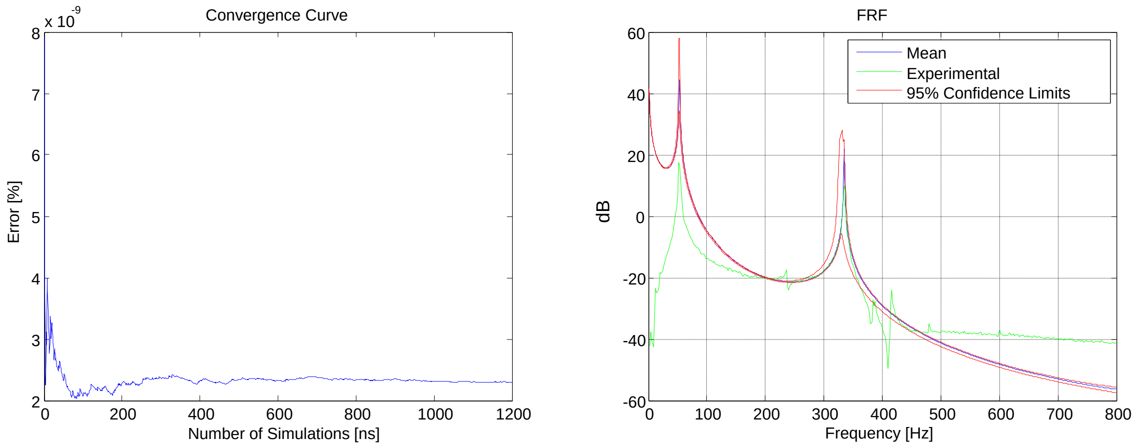

Figure 3 – (a) Convergence. (b) 95% confidence limits. $\delta_K = 0{,}11$. Without additional magnets.

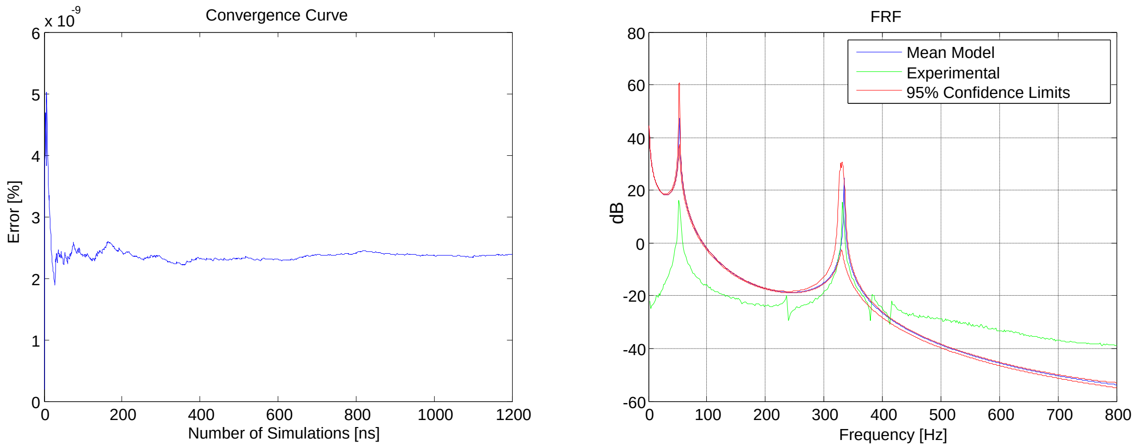

Figure 4 – (a) Convergence and (b) 95% confidence limits. $\delta_K = 0{,}11$. With addition of two magnets.

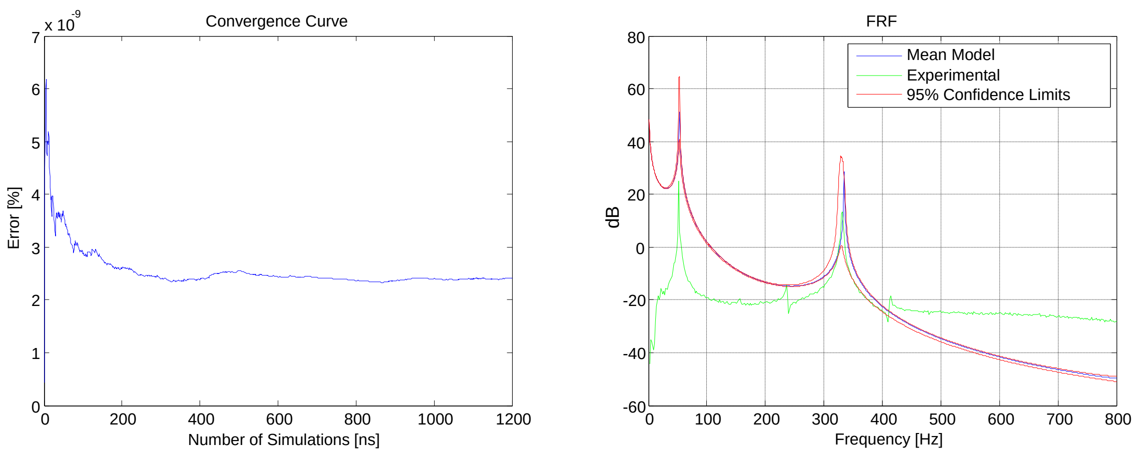

Figure 5 – (a) Convergence and (b) 95% confidence limits. $\delta_K = 0{,}11$. With addition of four magnets.

ISSN 2176-5480

Observing the Fig. (3a), Figure (4a) and the Figure (5a), it can be said that a good convergence is reached to a value of number of simulations $n_s = 1000$.

Considering now the Fig (4b) and the Fig (5b), there are no agreement between the FRF of the mean model and the FRF of the experimental test and the two curves are in large part outside of the 95% confidence limits. Moreover when observing the Fig. (3b), although there is disagreement of the FRF also in the frequency range 0-200 Hz due the gain to the FRF experimental, it can be said that the FRF of the mean model and the FRF of the test agreed well for a frequency range approximately 200 to 450 Hz, remaining within the 95% confidence limits calculated.

- ✓ Dispersion parameter = 0.2

By observing the Fig. (6a), Fig. (7a) and the Fig. (8a), one can say that as in the previous case, good convergence is attained for a value of number of simulations $n_s = 1000$.

Taking into account the Fig. (7b) and the Fig. (8b), there are no agreement between the FRF of the mean model and the FRF of the test. On the other hand, in the case of the Fig. (6b), although there is disagreement of the FRF also in the frequency range 0-200 Hz due the gain to the experimental FRF. The FRF of the mean model and the FRF of the experimental test continued agreeing for frequency range approximately 200-450 Hz, but one can see the output of these curves of 95% confidence limits calculated.

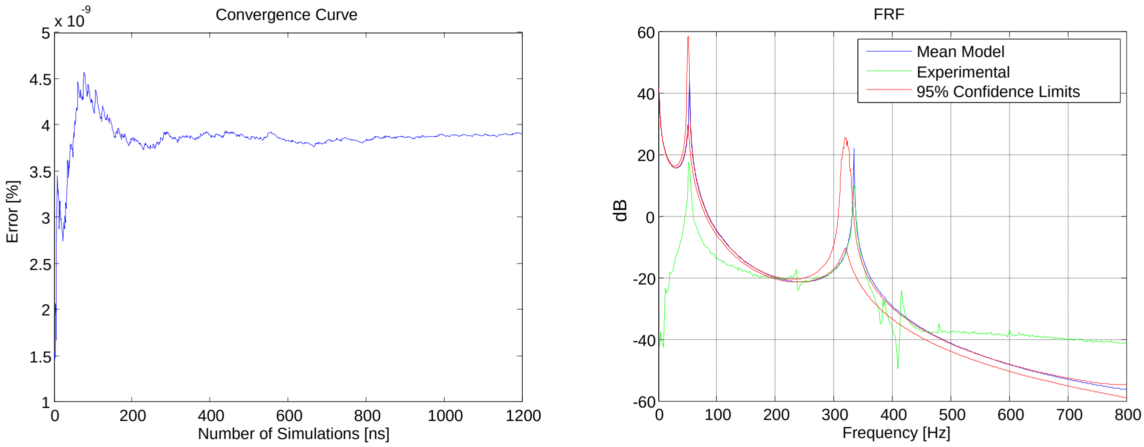

Figure 6 – (a) Convergence and (b) 95% confidence limits. $\delta_{\mathbf{K}} = 0{,}2$. Without additional magnets.

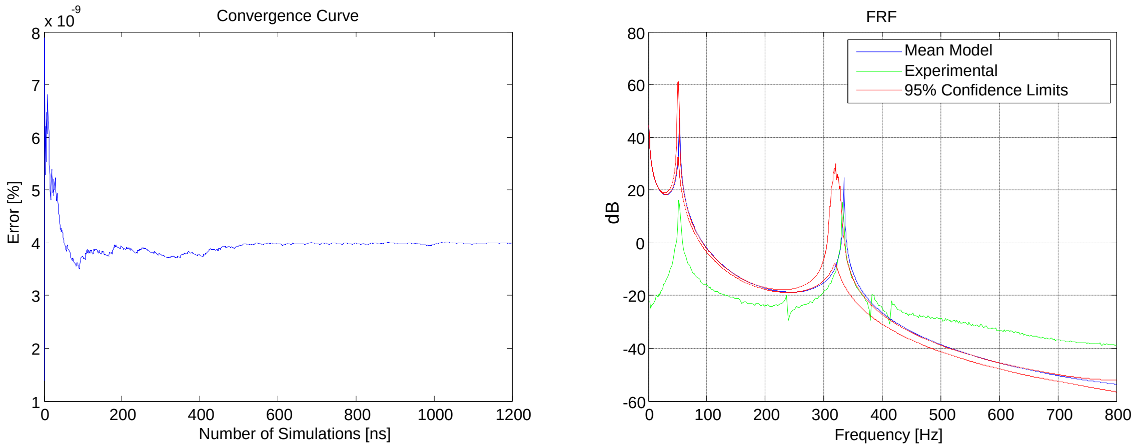

Figure 7 – (a) Convergence and (b) 95% confidence limits. $\delta_{\mathbf{K}} = 0{,}2$. With addition of two magnets.

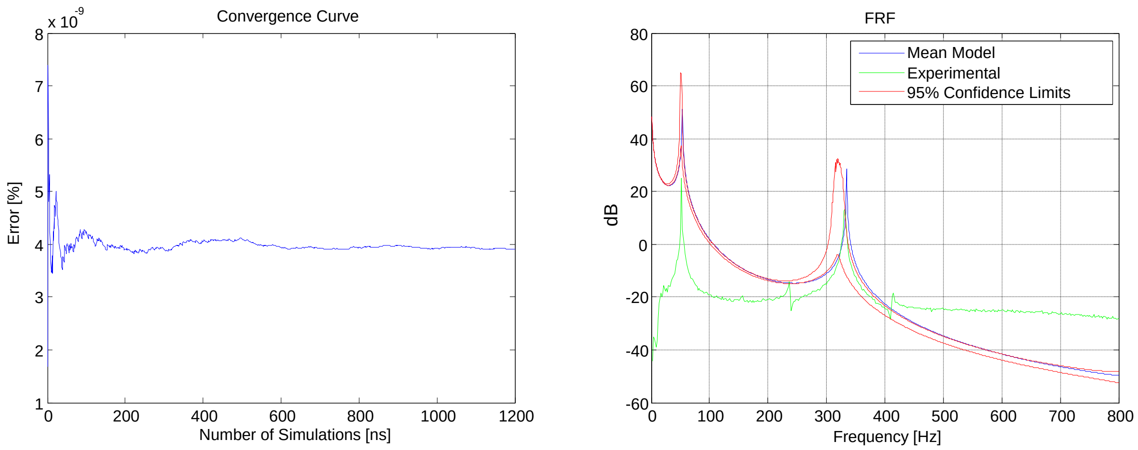

Figure 8 – (a) Convergence and (b) 95% confidence limits. $\delta_K = 0{,}2$. With addition of four magnets.

8.2 Frequency band: 800 –1200 Hz

✓ Dispersion parameter = 0.11

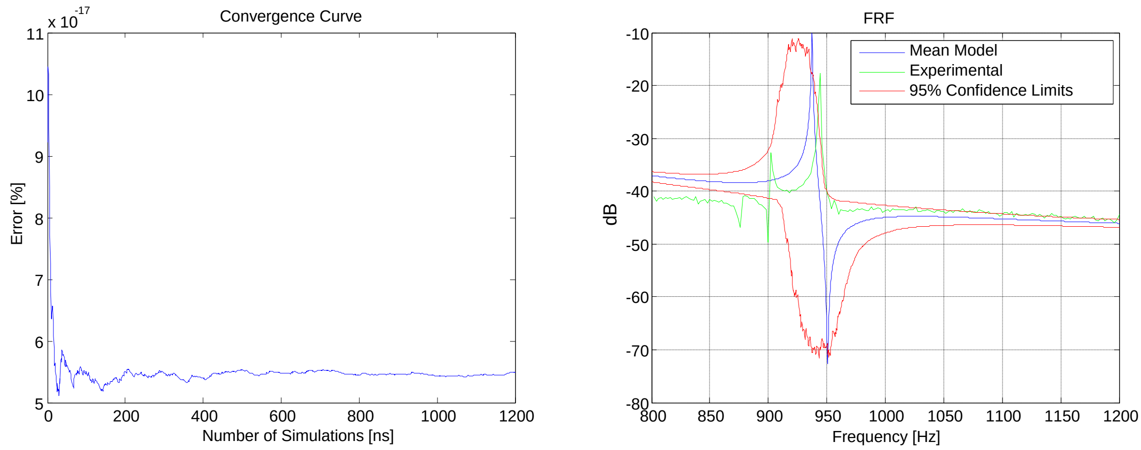

Figure 9 – (a) Convergence and (b) 95% confidence limits. $\delta_K = 0{,}11$. Without additional magnets.

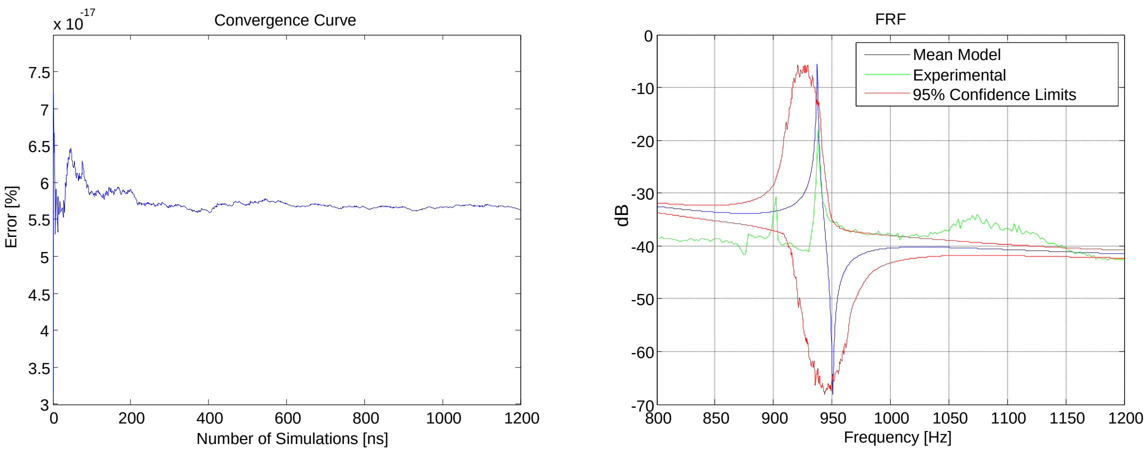

Figure 10 – (a) Convergence and (b) 95% confidence limits. $\delta_K = 0{,}11$. With addition of two magnets.

ISSN 2176-5480

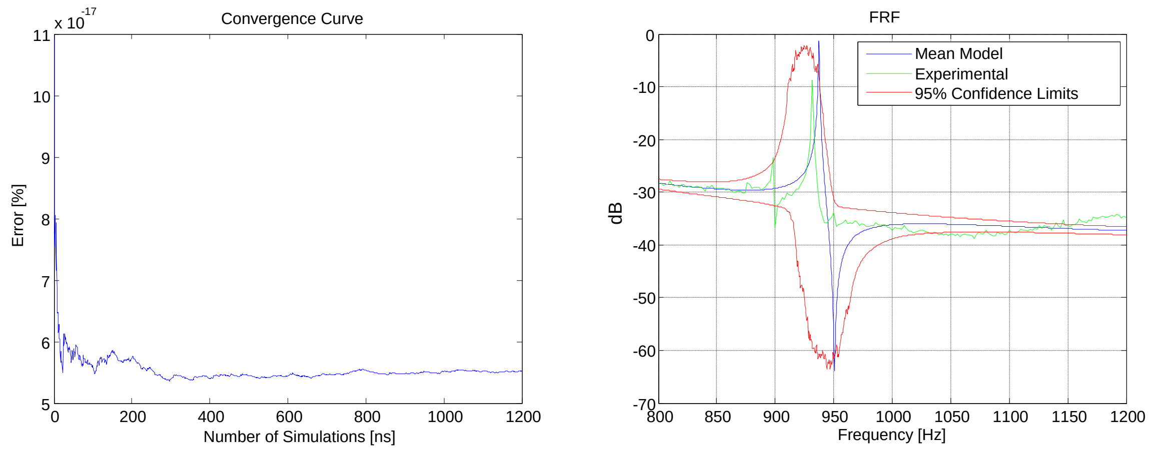

Figure 11 – (a) Convergence and (b) 95% confidence limits. $\delta_{\mathbf{K}} = 0{,}11$. With addition of four magnets.

The value of the suitable number of simulations for these results remains $n_s = 1000$ that to be seen in Fig. (9a), Fig. (10a) and Fig. (11a).

When verifying the Fig. (9b), Fig. (10b) and the Fig. (11b), it is possible to say that there are no correlation between the FRF of the mean model and the FRF obtained in the test. One can also observe that these results remain partly within the 95% confidence limits obtained by stochastic simulation. However, there is a perceived greater proximity of the FRF curves of the mean model and of the experiment for frequency range approximately 980 Hz and 1200 Hz – Fig. (9b), and the FRF of the mean model remains entirely within the IC while the FRF of the test lies partly within this same range.

- ✓ Dispersion parameter = 0.2

When analysing the Fig. (12a), Figure (13a) and the Figure (14a), it is clear that the value of the suitable number of simulations for this case corresponds to $n_s = 1000$.

In all graphs – Fig. (12b), Fig. (13b) and Fig. (14b), for the frequency range below approximately 980 Hz the results are not satisfactory. Verifying the Fig. (12b), one senses a greater proximity of FRF curves of the mean model and the FRF of the test for values of frequencies approximately from 980 Hz to 1200 Hz. Moreover, on this interval, the FRF curves are inserted in 95% confidence limits obtained in the simulation.

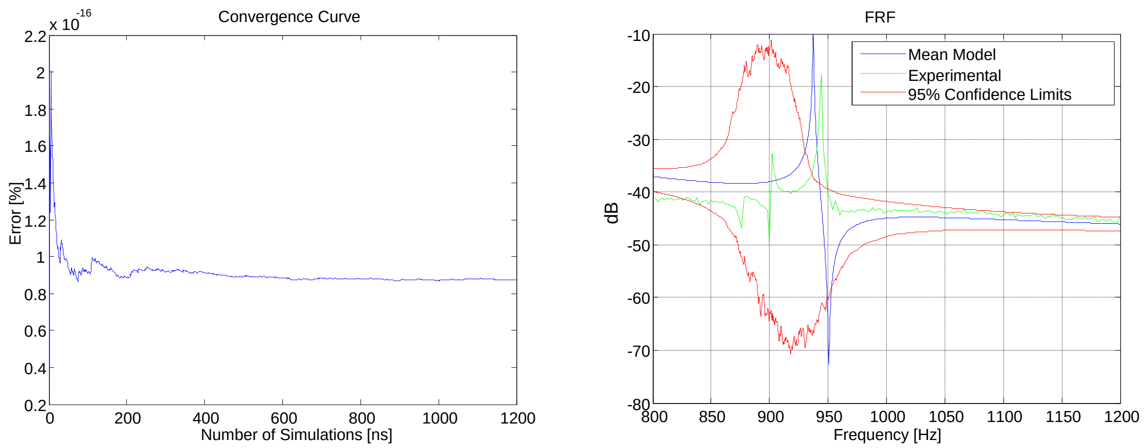

Figure 12 – (a) Convergence and (b) 95% confidence limits. $\delta_{\mathbf{K}} = 0{,}2$. Without additional magnets.

ISSN 2176-5480

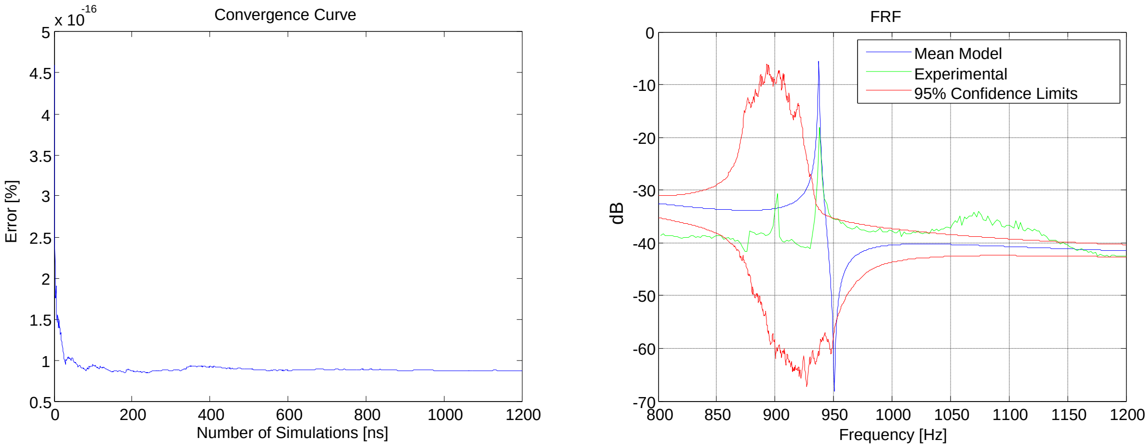

Figure 13 – (a) Convergence and (b) 95% confidence limits. $\delta_{\mathbf{M}} = 0{,}2$ With addition of two magnets.

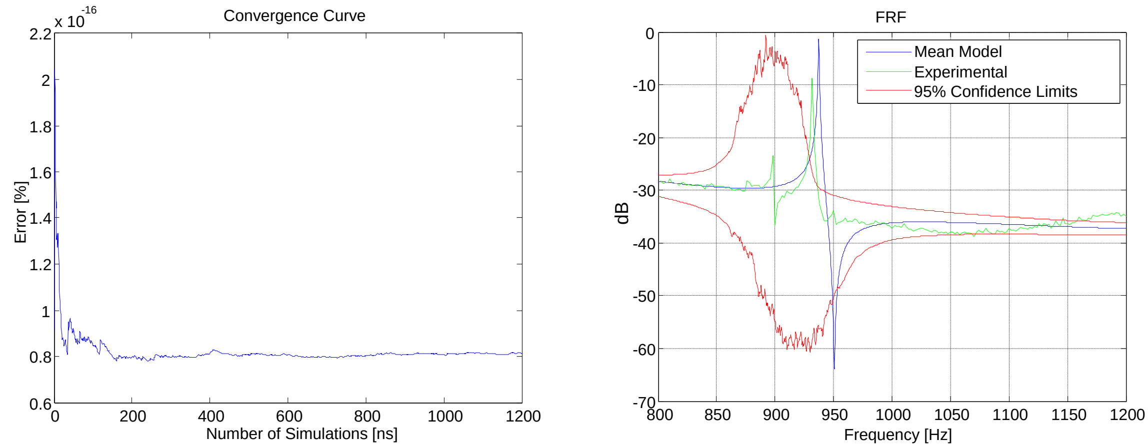

Figure 14 – (a) Convergence and (b) 95% confidence limits. $\delta_{\mathbf{K}} = 0{,}2$. With addition of four magnets.

9. CONCLUSIONS

First, it can be concluded about the results that when analysing the curves of convergence of the simulations, there are a good convergence to a value around $n_s = 1000$ of Monte Carlo simulations, regardless of dispersion parameter used, which means that this amount is sufficient so that one can obtain reliable results for the system under study.

In case of the Fig. (3b), relating to obtaining a 95% confidence limits for $\delta_{\mathbf{K}} = 0{,}11$, frequency range 0 – 800 Hz and without addition of magnets, it could be seen a good agreement between the results of the FRF of the mean model and the experimental FRF for a frequency range approximately 200 – 450 Hz. This result is not seen in the Fig. (4b) and in the Fig. (5b), which were obtained under the same conditions of the results presented in the Fig. (3b), but in this case with the addition of two and four magnets respectively. What one want to show with this study (by adding magnets in the system) is that the response of the analysed problem changes when adding more mass to the initial system, as this causes an increase of model uncertainty that was not considered in the original design of that system.

In case of the Fig. (6b), Fig. (7b) and the Fig. (8b) relating to obtaining a 95% confidence limits for $\delta_{\mathbf{K}} = 0{,}2$, frequency range 0 – 800 Hz one can conclude from the results presented that, in general, the FRF of the mean model obtained does not represent the real system in none of configurations. The exception is the first configuration, in the frequency range 200 – 450 Hz (Fig. 6b). With regard to the behaviour of the two curves (the FRF of the mean model and the FRF of the experiment) are outside of the 95% IC calculated. This fact can be justified by the value of the dispersion parameter be high to such a study, since the simulated system corresponds to a simple structure.

In case of the Fig. (9b) considered for frequency range 800 – 1200 Hz, $\delta_{\mathbf{K}} = 0{,}11$ and without addition of two and four magnets respectively, it can be observed that there are a good proximity between the FRF of the mean model and the FRF of the test for a frequency range approximately from 980 Hz to 1200 Hz. In addition, in this same figure, the FRF curves are inserted in the 95% confidence limits on this frequency range. In case of the Fig. (10b) and the Fig (11b), one can consider that a greater part of the FRF curves are included in 95% confidence limits on the frequency range from 980 Hz to 1200 Hz but on the other results, when one considered the addition of the magnets, the mean model does not represent well the real system tested. This is due because is not expected addition of mass on the system originally calculated, it increases the model uncertainty in the system and causes changes in the responses thereof.

When one considered the Fig. (12b) for frequency range 800 - 1200 Hz, $\delta_{\mathbf{K}} = 0{,}20$ and without addition of magnets, in general, one can say that the FRF of the mean model can satisfactorily represents the real system for the frequency range 980 - 1200 Hz, which does not occur on the other results in the Fig. (13b) and in the Fig. (14b). The justification here is that the quantification of nonparametric uncertainty becomes increasingly dominant with increasing frequency range analyzed. With regard to the two curves of FRF (mean model and experiment), they remain partially outside of the 95% confidence limits for the frequency range approximately 980 - 1200 Hz in the Fig. (13b) and in the Fig. (14b). This fact can be justified by the value of the dispersion parameter be high to such study, since the simulated system corresponds to a simple structure.

On the other hand, some observations about the behaviour of the response need to be done in view of the different dispersion parameters considered in the analysis. Regarding the results obtained for the FRF of the mean model, the FRF of the test and a 95% confidence limits in the frequency range 0 – 1200 Hz and without adding mass, which corresponds to the real system – Fig. (3b), Fig. (6b), Fig. (9b) and Fig. (12b), and taking as reference the vertical axis of these graphs, it can be seen that the confidence region increases as the dispersion parameter values become higher.

One can observe in the same graph results, but now with reference to the horizontal axis that as the frequency increases the width of the 95% confidence limits also increases on the natural frequencies region. This means that the uncertainty in the system increases with increasing frequency thus hampering its predictability.

10. ACKNOWLEDGEMENTS

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12. RESPONSIBILITY NOTICE