ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, RibeirãoPreto, SP, Brazil

STOCHASTIC MODELING OF A SIMPLIFIED WIND TURBINE DYNAMICS

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Abstract. *Due to the reduction of available fossil energy resources and also due to the safety problems and high cost of nuclear energy, the number of researches related to renewable energies, especially wind energy is increasing. This paper presents a simplified wind turbine model that takes into account the blade dynamics. In general, the vibration of the wind turbine blades affects the behavior and the efficiency of the whole system. The vibration of the blade tip is analyzed considering uncertainties in several parameters, such as wind speed, material properties and dimensional properties. A probabilistic approach together with the Monte Carlo method is used to propagate these uncertainties throughout the numerical model and to perform a sensitivity analysis.*

Keywords: *wind turbine, nonlinear dynamics, blade dynamics, stochastic analysis, uncertainty*

1. INTRODUCTION

Windmills have existed for more than 3,000 years and have greatly facilitated agricultural development. The windmill is the oldest device for exploiting the energy of the wind. The first electricity-producing wind turbine was constructed by Charles F. Brush in the United States in 1887. Brush's machine had a 17-m-diameter rotor, consisting of 144 blades, and a 12-kW generator (Sørensen Jens Nørkær, 2011). In 1918 approximately 3% of the Danish electricity consumption was covered by wind turbines. Because of supply crises, renewed interest was paid to wind energy during World War II (Sørensen Jens Nørkær, 2011). Wind power has established itself as a major source of non-polluting, inexhaustible renewable energy. During the last decades wind power has evolved as a strong alternative to fossil fuels in the electricity generation industry. The World Wind Energy Association's Estimate for the future is a global capacity of 600,000 MW by 2015 and more than 1,500,000 MW by 2020 (Crozier Aina, 2011).

Today, state-of-the-art wind turbines have rotor diameters of up to 120 m and 5-MW installed power, and these are often placed in large wind farms with a production size corresponding to a nuclear power plant. Nowadays in researches, wind is found as one of the possible and powerful energy sources. Although research in wind energy has taken place for more than a century now, there is no doubt that wind-energy competitiveness can be improved through further cost reductions, collaboration with complementary technologies, and new innovative aerodynamic design (Sørensen Jens Nørkær, 2011). Figure 1 shows the past record and projection of the accumulated capacity of wind energy from 1990 to 2012 (Sangyun Shim, 2007).

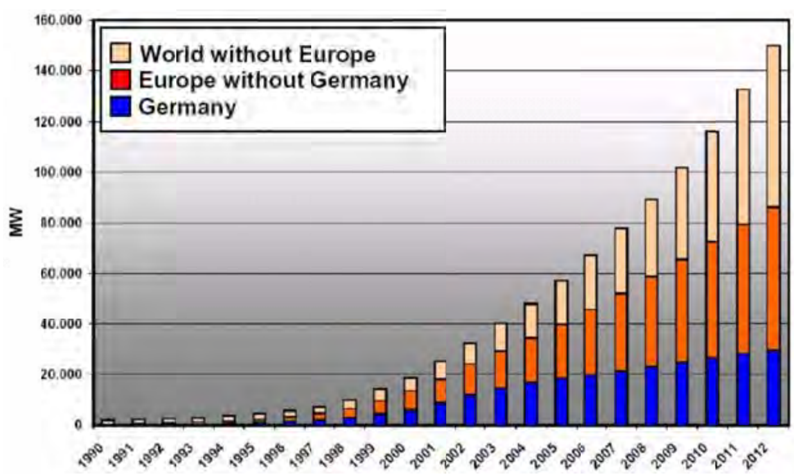

Figure 1. Accumulated capacity of wind energy (Sangyun Shim, 2007)

In general case, there are many parameters contributing to the behavior of a wind turbine, such as: low level jet, turbulent wind, wake turbulence, tidal and storm surge depth variation, buoyancy of support, extreme wave and scour. A scheme of these effects is shown in Figure 2.

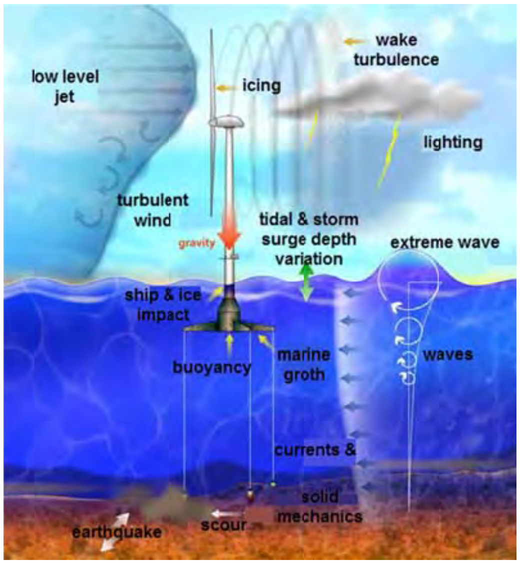

Figure 2. General parameters affecting wind turbine model (Matha Denis, 2009)

To study wind turbine behavior, we consider wind turbine as a mechanical system that has characteristics such as blade airfoil properties, material properties, geometry, mass and inertia properties, transmission system behavior and structural stiffness and damping. These parameters together define the response of the system to its inputs. Environmental effects such as wind power, mooring or support excitation and electric network loads are inputs for the system and any response of system to inputs due to system characteristics, are outputs of the system. In this paper deflections of blade of wind turbine and rotational velocity of rotor are considered as outputs of system.
In section 2, a mathematical model of a rotating flexible blade and the equations of the deflection of its tip are presented. In section 3 uncertain parameters are introduced and in section 4, numeric results for the deterministic model and the stochastic model are offered and compared.

2. MATHEMATICAL MODEL

The deflections of blade tip have three components, one in radial direction or in direction of the longitudinal axis of the blade that is called *radial* or *centrifugal* deflection. The other one is the transversal deflection of the blade tip due to the lift force in the plane of rotation and is called *in-plane* deflection and is in the direction of a vector in the plane of motion that is perpendicular to radius. The last one is in the direction of rotation axis and is perpendicular to the plane of rotation and is called *out of plane* deflection.

All of these deflections are caused by centrifugal forces, aerodynamic forces and the gravitational force. Figure 3 shows the directions of these components. The moving frame that is connected to the blade is the reference frame to derive the equations of deflections of the blade's tip.

To obtain the mathematical model, some simplifying assumptions are made. First we assume that the rotation is about a fixed axis normal to the rotor disk, which means that the tower of wind turbine and its nacelle are fixed. Then the only moving parts are flexible blades that are rotating about a fixed axis and all blades at rest lay on the same plane that is perpendicular to the axis of rotation.

Second assumption is that the material and the geometrical properties of blades are constant and they do not change when the body of blades deforms due to the dynamic forces. It means that the strain is assumed to be small.

Third assumption is that the material properties of the blade such as module of elasticity and density are constant and also the cross sectional area of the blade is constant along the longitudinal axis.

Fourth assumption is that we can neglect the structural damping of the blade vibrations.

As the fifth assumption, we assume that the wind direction is always perpendicular to the rotation plane of the blade and it is constant at any time. Also we assume that when the rotor of wind turbine rotates, we can neglect the drag that is produced by the relative motion of the blade with respect to the air that surrounds it.

And finally we assume that the angle of attack of the blade is constant. It means that wind and the chord of the cross section of the blade always have a constant angle.

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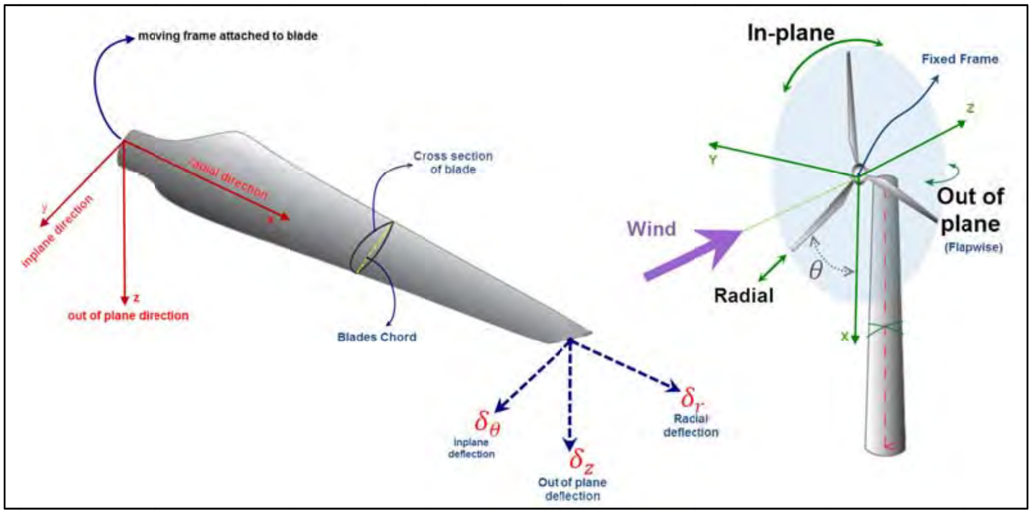

Figure 3. Components of deflection of blade tip

2.1 Rotor motion

The motion of the rotor is caused by the wind speed, therefore the wind speed is the source term and the total lift force on the blade that is in the direction of y axis of the blade's reference frame is given by:

$$F_L = \frac{1}{2}\int_A \rho_{air}.v_{wind}^2.C_L.dA, \tag{1}$$

Where, A is the reference area of the blade that is the area of projection the blade on the plane of the rotation. Due to our assumptions, the chord h of the blade is constant along the longitudinal axis of the blade, we say that $A = hR$ where, R is the length of each blade that is the radius of rotation of wind turbine as well. The direction of the lift force is always in the direction of rotation, which means that it is perpendicular to radius and in the plane.

In Eq. (1), ρ_{air} is the density of air, v_{wind} is wind speed normal to the rotation plane of blades, C_L is the coefficient of lift on the blade which we assume that is constant along the longitudinal axis of blade. With these definitions and considering assumptions that we made, Eq. (1) becomes:

$$F_L = \frac{1}{2}\rho_{air}.v_{wind}^2 C_L.h.R \tag{2}$$

Now for the rotational equation of the rotor with n blades, and due to the assumption of uniform distribution of the lift along the longitudinal axis of the blade (Bagbanci Hasan, 2011; KÜHN Martin Johannsen. 2001; Perdana Abram, 2008), we have:

$$n\int_0^R x\frac{F_L}{R}dx - T_G(\dot{\theta}) = J\ddot{\theta} + C\dot{\theta} + K\theta \tag{3}$$

Where, T_G, is the resisting torque against the rotation of the rotor, J is polar moment of inertia of the set of rotor and blades about the rotation axis and finally, C and K are the rotational damping and the rotational stiffness of combined system of the rotor, transmission system and the electric generator. We assume that a plane that contains the axis of tower and the axis of rotation, is the reference to measure the angle θ and each blade will have the same response when passes a specific angle θ. The importance of Eq. (1) is that we need the solution of this equation to solve the Eqs. (7), (10) and (12) related to the blade deflections.

Our model is a simplified model of the wind turbine blade dynamics and also we have chosen the parameters of a real wind turbine that are introduced in section 4. As we knew , the form of angular velocity response to wind speed (Jah A.R, 2011; Manwell J.F, et al., 2002; KÜHN Martin Johannsen. 2001), we had to introduce the resistance torque in Eq. (3) such that the results follow the real form of response. Therefore the form of resistance torque $T_G(\dot{\theta}) = K_G\dot{\theta}^2$ with constant K_G, is chosen from the results of solving Eq. (3) with different types of functions for the resistance torque.

2.2 Radial deflection of tip

Radial deflection is caused by the centrifugal forces and the gravity force component along the x axis of the blade; then the normal force in each cross section of the blade in the distance r of rotating axis is:

$$F(r) = g\,cos(\theta)\int_r^R \rho_b A_c dx + \omega^2 \int_r^R \rho_b A_c x dx \tag{4}$$

Where, in this equation, negative direction is the direction toward the center of the rotation, A_c is the cross sectional area of the blade, ρ_b is the material density of the blade and ω is the rotational velocity of rotor, which is the time derivative of θ. Due to assumption of constancy for the parameters along the longitudinal axis of the blade, we have:

$$F(r) = \rho_b A_c \left(g.cos(\theta)(R-r) + \frac{1}{2}\omega^2 (R^2 - r^2) \right) \tag{5}$$

Therefore the radial deflection of the blade tip with $r = R$ is given by:

$$\delta_r = \int_0^R \frac{F(r)}{EA_c} dr = \frac{\rho_b g}{E}.cos(\theta)\int_0^R (R-r)dr + \frac{\rho_b}{2E}\omega^2 \int_0^R (R^2 - r^2)dr \tag{6}$$

Then:

$$\delta_r = \frac{\rho_b g}{2E} cos(\theta) R^2 + \frac{\rho_b \dot{\theta}^2}{3E} R^3 \tag{7}$$

2.3 Flapwise (out of plane) deflection

To derive the out of plane and also the in-plane deflection equations, we assume a cantilever beam with the Euler-Bernoulli model as the blade of the wind turbine.

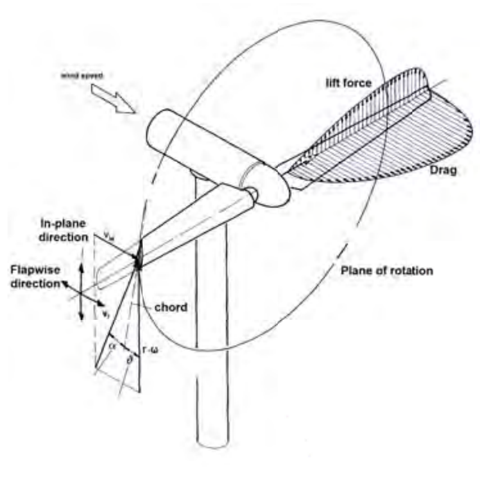

Figure 4. Lift and drag sample distribution on blade (Crozier Aina, 2011)

Figure 4 shows a general distribution of the *lift* and *drag* forces on a blade and we assume that the distribution is uniform, then the drag force F_D is given by:

$$F_D = \frac{1}{2}\rho_{air} v_{wind}^2 A.C_D = \frac{1}{2}\rho_{air} v_{wind}^2.R.h.C_D \tag{8}$$

Where the positive direction is the direction of the wind, C_D is the drag coefficient of blade which is assumed to be constant along the axis of blade and $A = hR$ is the reference area of the blade. To calculate the deflection of an Euler-Bernoulli beam , we use Eq. (9).

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$$\frac{d^2}{dx^2}\left(EI\frac{d^2\delta(x)}{dx^2}\right) = q(x) \tag{9}$$

Where, E is the elasticity modulus, I is the area moment of inertia, $\delta(x)$ is the deflection of beam and $q(x)$ is the load distribution along the beam. Therefore, the equation of blade tip deflection δ_z is given by:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2\delta_z}{dx^2}\right) = \frac{F_D}{R} = \frac{1}{2}\rho_{air}v_{wind}^2.h.C_D \tag{10}$$

2.4 Inplane deflection

Forces that cause the inplane deflection of the blade are the gravitational force component which is normal to the blade and the lift force that both of them have uniform distribution on the blade. Therefore, the load distribution due to the gravity and the lift is:

$$f_\theta = \frac{F_L}{R} - \frac{\rho_b}{R}A_c.R.g.sin(\theta) = \frac{1}{2}\rho_{air}.v_{wind}^2C_L.h - \frac{\rho_b}{R}A_c.R.g.sin(\theta) \tag{11}$$

Again, using the Euler-Bernoulli equation, we have the equation for the inplane deflection δ_θ:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2\delta_\theta}{dx^2}\right) = \frac{1}{2}\rho_{air}.v_{wind}^2C_L.h + \rho_b\, g.A_c.\sin\theta \tag{12}$$

3. PROBABILISTIC MODEL

In deterministic solution we assume that all of parameters have certain values. One of the parameters that for sure has uncertainty is the wind power. Speed and direction of the wind changes time by time and to have a good estimation of system behavior, we will model it as a random variable.

Many researchers have measured wind behavior to obtain a probability distribution of wind energy and they have introduced different models. Figure 5 illustrates a graph of a typical segment of wind data.

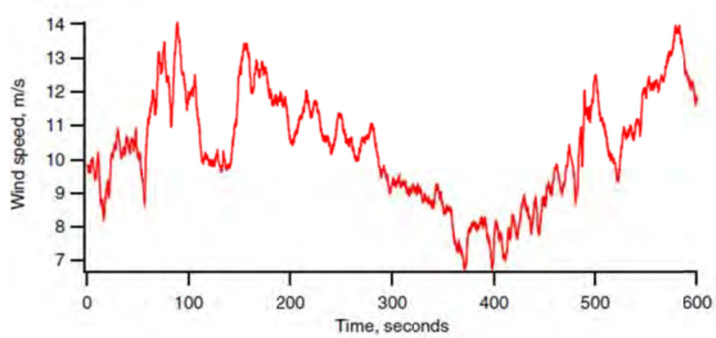

Figure 5. Sample wind data (Manwell J.F, *et al*., 2002)

The likelihood that the wind speed has a particular value can be described in terms of a probability density function (pdf). Experience has shown that the wind speed is more likely to be close to the mean value than far from it, and that it is nearly as likely to be below the mean as above it. The probability density function that best describes this type of behavior for turbulence is the Gaussian or normal distribution (Manwell J.F, *et al*., 2002). The normal probability density function for continuous data in terms of the variables used here is given by:

$$p(v) = \frac{1}{\sigma_v\sqrt{2\pi}}exp\left(-\frac{(v_{vind} - \mu_v)^2}{2\sigma_v^2}\right) \tag{13}$$

Where, v is wind speed, μ_v is its mean value and σ_v is its standard deviation from the mean value.

Figure 6 illustrates a histogram of the wind speed about the mean wind speed in the sample data of Figure 5. The Gaussian probability density function that represents the data is superimposed on the histogram.

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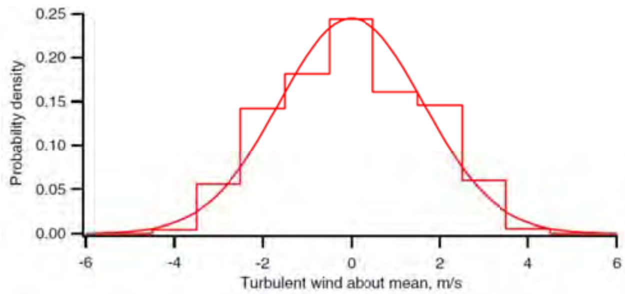

Figure 6. Gaussian probability density function and histogram of wind data (Manwell J.F, *et al*., 2002)

In a system like wind turbine, there are many parameters which are not certain, for example because of uncertainty of production, the properties of material such as module of elasticity, density and geometry are not exact. As we don't have data of the distribution of these parameters, it is better to assume a uniform distribution for them.

4. NUMERICAL RESULTS

4.1 Deterministic

To obtain numeric approximations of the equations, we use the data of a sample turbine, whose parameters shown in Table 1 (Cheney M.C., 1999; Park Joon-Young, *et al*., 2010). Using a MATLAB code, the results of the deterministic model are shown in Figure 7 and Figure 8.

Figure 7 shows the angular velocity of the wind turbine rotor which is the solution of Eq. (3). As described before, we assume that the wind direction is normal to the plane of rotation of the blades and it has constant value with respect to time. In this case because of the resisting moments against rotation that grows with quadratic ratio of the wind speed, the angular velocity of rotor reaches the value of 10.7 rad/s and becomes stable after 150 seconds.

In Figure 8-a, the radial deflection of blade is shown. As shown in Eq. (7), it is the effect of the gravity and the centrifugal forces in the direction of the longitudinal axis of the blade and therefore it is combination of the oscillatory motion due to the term $cos(\theta)$ and the quadratic form $\dot{\theta}^2$. At time $t = 0$ the angle $= 0$, therefore the blade is downward and we see that at time $t = 0$ there is an initial value for the radial deflection of the blade tip that is because of its elongation due to its weight. To make this more clear, the deflection of two blades that at the time $t = 0$, one of them is in $\theta = 0$ and the other one is in $\theta = \frac{2\pi}{3}$ are shown in Figure 9.

Constant value of wind speed with Eqs. (8) and (10) make the out of plane deflection independent of time and this is shown in Figure 8-b. Also the assumption of constant wind speed causes initial value for the lift force in Eq. (2) at $t = 0$ and therefore we see an initial value for the inplane deflection of the tip in Figure 8-c.

In Figure 8 we see that most important deflection is the In-plane deflection that is because of the large dimensions of turbine and the effect of the gravity and the centrifugal forces and also small amount of the moment *I*.

Table 1. Sample wind turbine parameters

Parameter	Abv.	Value	Unit
Wind speed	v_{wind}	12	*m/s*
Air density	ρ_{air}	1.15	Kg/m^3
Number of blades	n	3	-
Chord of blade	h	1.85	m
Blade length	R	45	m
Lift coefficient	C_L	1.20	-
Drag coefficient	C_D	0.08	-
Moment of inertia of blade	J_b	$7.291e6$	$Kg.m^2$
Moment of inertia of Rotor	J	$3 \times J_b$	$Kg.m^2$
Rotational damping of rotor	C	500	Nms/rad
Rotational stiffness of rotor	K	0	Nm/rad
Coefficient of rotational resistance	K_G	$4.4e5$	Nms^2/rad^2
Density of blade material	ρ_b	1600	Kg/m^3
Young modulus of blade material	E	$1.45e11$	N/m^3
Moment of cross section of blade	I	1	m^4
Cross section area of blade	A_c	0.15	m^2

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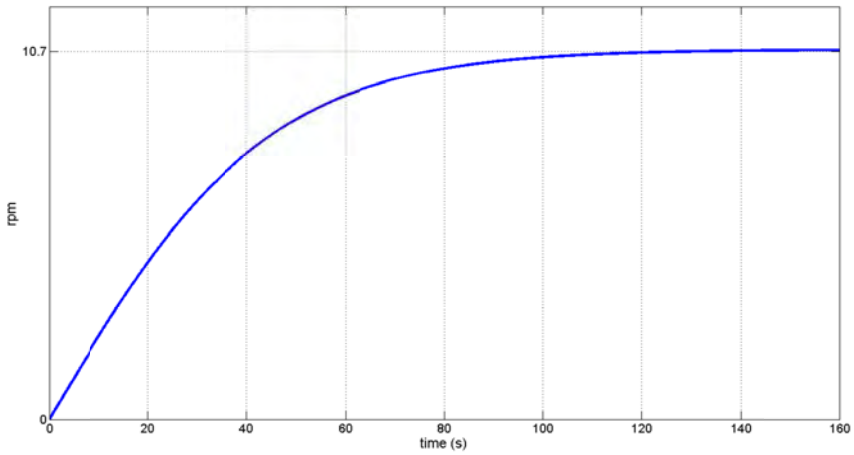

Figure 7. Deterministic solution for angular velocity of turbine rotor

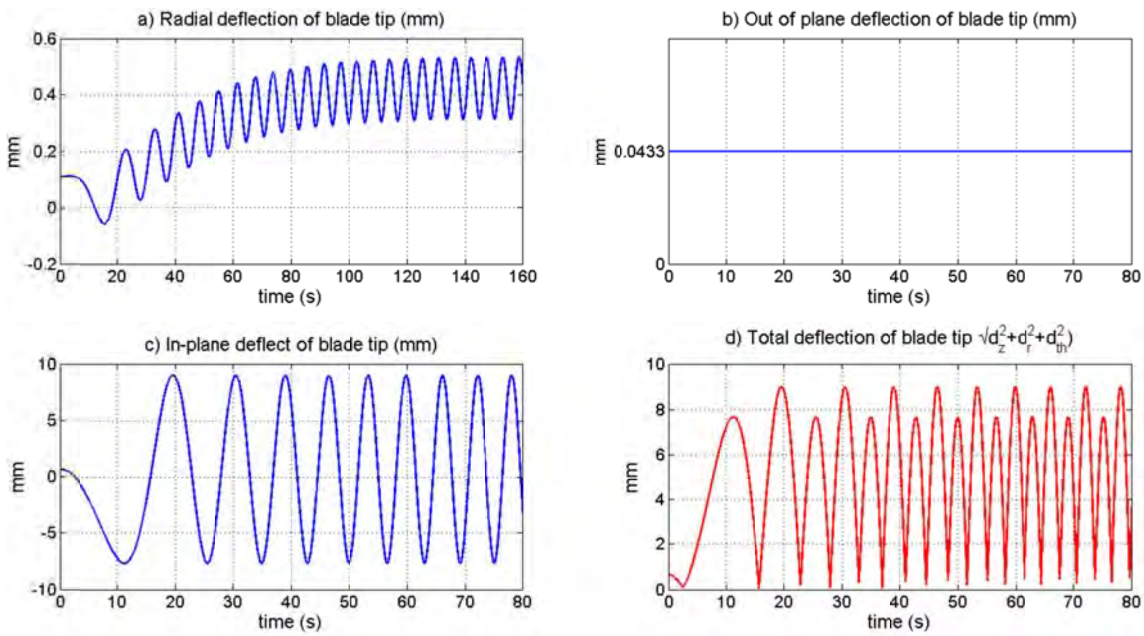

Figure 8. Deterministic solutions of deflection of blade tip

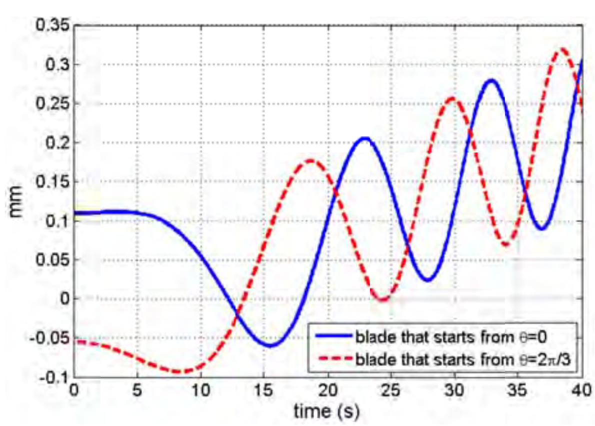

Figure 9. Comparison of the radial deflection of two different blades of wind turbine

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4.2 Stochastic analysis with one random variable

First we will consider only the uncertainties related to the wind speed. We assume a random distribution for wind speed with the mean value equal to its nominal value from Table 1 which is 12m/s and the 90% confidence limits of standard deviations of $\pm 10\%$. This means that with a probability of 90%, the wind speed will be in the interval of $[10.8\,,13.2]\ m/s$. Due to this distribution of the wind speed, the mean values, upper limit and lower limit of 90% confidence of the angular velocity of the wind turbine in stochastic process of 1000 observations are shown in Figure 10-a. With these results we see that with the probability of 90%, the final value of the angular velocity of wind turbine will be within the interval $[9.7\,,11.8]\ rpm$, which is equal to $\pm 10\%$ change of the mean value of $10.7\ rpm$.

The coefficient of variation (CV), which is defined as the standard deviation over the mean, is now analyzed at each time of simulation. Figure 10-a shows that the standard deviation increases with time (larger statistical envelope). But, Fig. 10-b shows that the CV always decreases, which is not evident from Fig. 10-a. This result is due to the increasing of the mean value of the velocity.

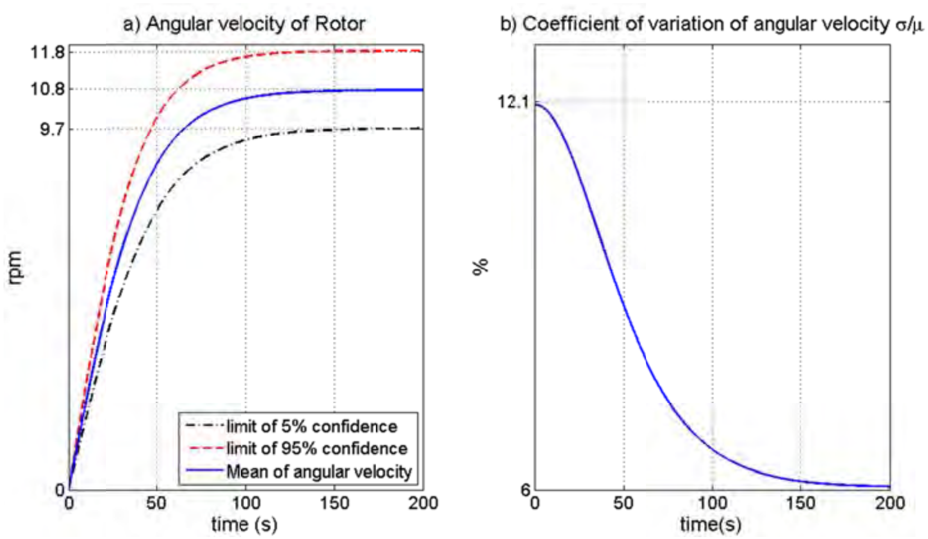

Figure 10. Stochastic analysis results for the angular velocity with wind as random variable

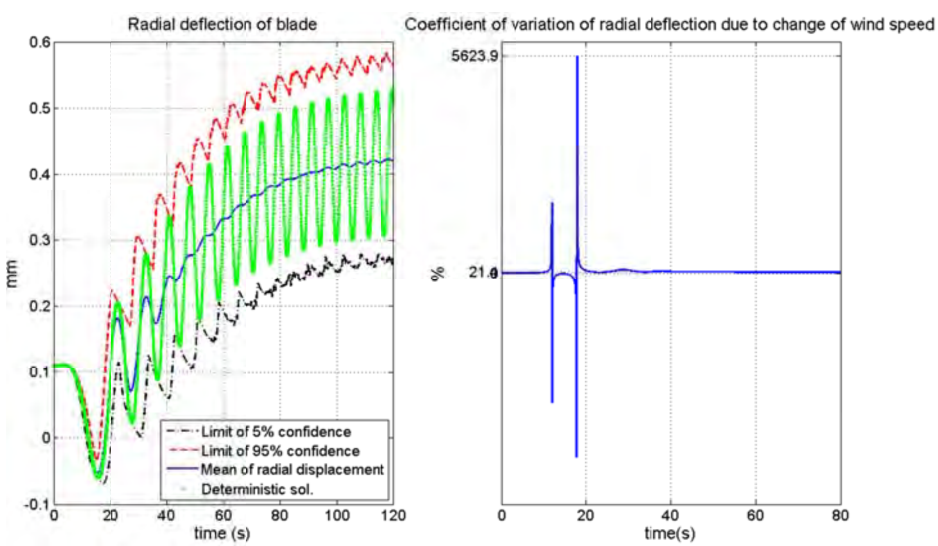

Figure 11. Stochastic analysis of the radial deflection of blade tip with wind speed as random variable

In Figure 11, we see that if the probability that the wind speed has the value within the interval $[9.8\,,13.2]m/s$ be 90%, then with probability of 90%, the value of the radial deflection of the blade tip is within the interval

ISSN 2176-5480

$[0.35\,, 0.55]mm$; But because of the oscillations, the interval limits are not precise. Using the CV we see that the ratio of the standard deviation of the radial deflection of the blade tip 21.6% w is two times the CV of the angular velocity. When the value of wind speed is not exact, the radial deflection is more sensitive to changes of wind speed than the sensitivity of the angular velocity of wind turbine to the wind speed. Note that extreme peaks in the graph of CV, belong to times that mean value has zero value.

Figure 12 shows that with the same probability distribution for wind speed, the probability that the value of the inplane deflection of blade tip be in the interval $[-7.5\,, 9]mm$ is 90%. Again from the graph of the CV it is clear that the sensitivity of the inplane deflection to the changes of the wind speed, in comparison with the radial deflection, is very high. Since the mean value passes zero many times, this graph has many extreme peaks.

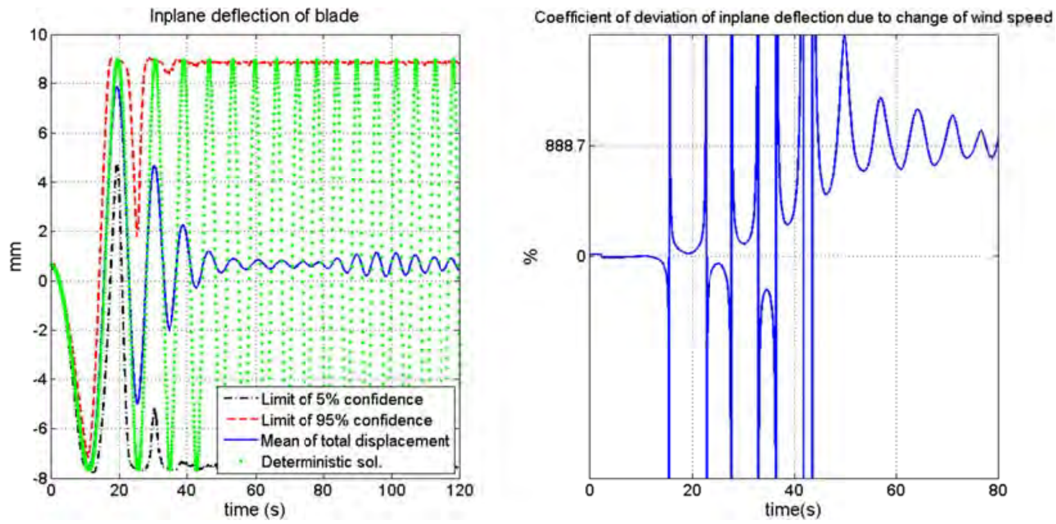

Figure 12. Stochastic process of total deflection of blade tip with wind speed as random variable

4.3 Stochastic analysis with more than one parameter as random variable

Now to bring uncertainty of other parameters of the system into account, we give a uniform random distribution to parameters of our simple model with 10% changes with respect their exact values, including $I, \rho_{air}, C_L, C_D, \rho_b, E$. We see that if each uncertain parameter of the system, with 90% probability be in the interval $[0.9\mu\,, 1.1\mu]$, the final value of the angular velocity of wind turbine will be in the interval $[9.5, 12]rpm$ with confidence of 90%. In Figure 13 we can see that, taking into the account many sources of uncertainty, confidence limits of the angular velocity only have about 3% of increase in comparison to the system with only the wind as the only source of uncertainty. We can say that our model of wind turbine is more sensitive to the changes of wind speed than other parameters.

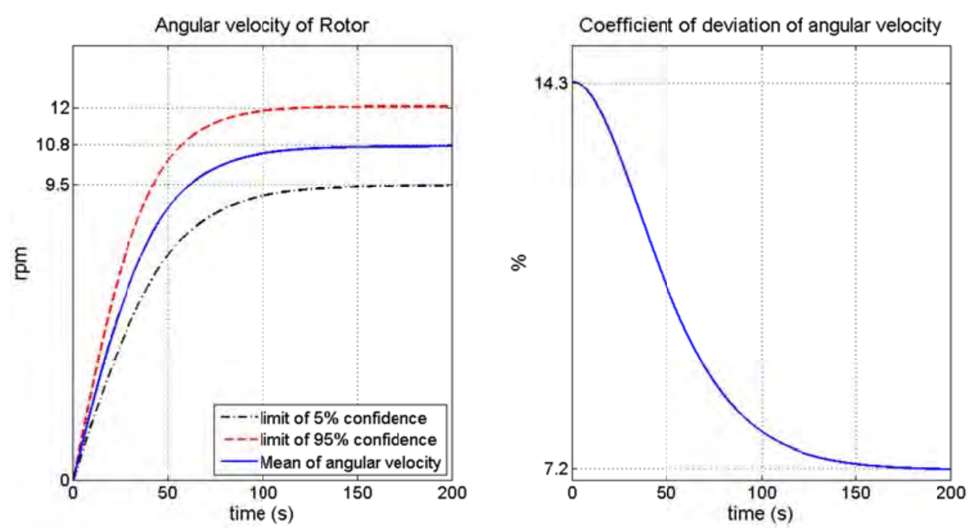

Figure 13. Stochastic process of angular velocity with all parameters as random variables, 1000 observations

ISSN 2176-5480

5. CONCLUSION

A simple model for a three bladed wind turbine with a fixed support is introduced in this paper. With this simplified model, results of stochastic analysis show that, the role of wind speed as a random variable as the most important term in the response of the dynamical system of the blades. This model is a basic model to understand the behavior of a wind turbine to bring more details into account for future works. In the next step, the authors are working on a flexible model of a wind turbine combined with multibody dynamics in Msc.ADAMS as a part of future works to model and study an offshore wind turbine that has a big role to use renewable energy sources within next few years.

6. ACKNOWLEDGEMENTS

The authors acknowledge the financial support of the Brazilian agencies CNPQ, CAPES and FAPERJ.

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