ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

ON THE NON-LINEAR BEHAVIOR OF VIBRATION-BASED ENERGY HARVESTERS

Luciana L. Silva
Universidade Federal do Rio de Janeiro, COPPE - Departament of Ocean Engineering, 21.945.970, Rio de Janeiro -RJ, Brazil
e-mail: lucianals@lts.coppe.ufrj.br

Marcelo A. Savi
Universidade Federal do Rio de Janeiro, COPPE - Departament of Mechanical Engineering, 21.945.972, Rio de Janeiro - RJ, Brazil
e-mail: savi@mecanica.ufrj.br

Paulo C. C. Monteiro Jr
Universidade Federal do Rio de Janeiro, COPPE - Departament of Ocean Engineering, 21.945.970, Rio de Janeiro -RJ, Brazil
e-mail: camara@lts.coppe.ufrj.br

Theodoro A. Netto
Universidade Federal do Rio de Janeiro, COPPE - Departament of Ocean Engineering, 21.945.970, Rio de Janeiro -RJ, Brazil
e-mail: tanetto@lts.coppe.ufrj.br

Abstract. *Vibration-based energy harvesting has received great attention over the last years. A wide range of nonlinear effects is observed in energy harvesting devices and the analysis of the power generated suggests that they have considerable influence on the results. Linear constitutive models for piezoelectric materials can provide inconsistencies in the prediction of the power output of the energy harvester, mainly close to resonant conditions. The main goal of this contribution is to investigate the effect of the nonlinear behavior in the piezoelectric coupling. A one-degree of freedom mechanical system is coupled to an electrical circuit by a piezoelectric element and different behaviors of piezoelectric coupling are investigated. Results of the experimental tests available in the literature are compared with those obtained for nonlinear and linear piezoelectric behaviors showing their influence on system behavior.*

Keywords: *Energy harvesting, vibration, piezoelectricity, nonlinear dynamics.*

1. INTRODUCTION

Energy harvesting has an increasing importance nowadays being the objective of several research efforts (Roundy et al., 2004). Vibration-based energy harvesting is a promising are of the use of the piezoelectric materials (Ertuk & Inman, 2011; Liao et al., 2001). Mechanical vibration energy can be converted into electrical energy through piezoelectric elements used for power harvesting in various forms of structure (Sodano et al., 2004). As a result, electrical power can be stored or used to directly run and maintain low-power devices.

In recent years, there are several theoretical and experimental studies that investigate the design and performance optimization of vibration-based energy harvesters (du Toit & Wardle, 2006; Sodano et al., 2004; Erturk & Inman, 2008; Roundy, 2004; Triplett & Quinn, 2009). In order to estimate the amount of power output of energy harvesting devices, several mathematical models have been developed to describe electro-mechanical coupling mechanisms. In this regard, the description of the piezoelectric electro-mechanical behavior points to a non-linear constitutive behavior with hysteretic characteristics. Nevertheless, it is usual to adopt a linear relation between strain and electrical field. Under this assumption, there is a single constant for all values of strain-electrical field, known as the coupling coefficient.

The linear description of the piezoelectric materials can provide inconsistencies in the power output prediction of the energy harvester, as shown in Kim et al. (2010). du Toit & Wardle (2006) showed that the use of linear constitutive relations under-predicted the experimental voltage produced from energy harvesting devices. Non-linear effects have been identified in several situations. Crawley & Anderson (1990) presented experimental results by considering non-linear behavior of the strain-electrical field, providing the evidence that a linear model is not valid for large strains. Triplet & Quinn (2009) treated a dynamical system with non-linear stiffness considering non-linear piezoelectric constitutive relation. The analysis of the power generated by the harvesting system suggests that non-linear effects have considerable influence on the results, and coincides with the inconsistencies in predicting the amount of power generated from the harvesting systems found in previous researches (du Toit & Wardle, 2006).

This contribution deals with the vibration-based energy harvesting considering quadratic nonlinear influences on the behavior of the piezoelectric element. An archetypal system composed of a one-degree of freedom mechanical system connected to an electrical circuit by a piezoelectric element is adopted. The main goal is to investigate the non-linear effect in energy harvesting. Therefore, a comparison of numerical results obtained from linear and non-linear models with experimental results available in Kim et al. (2010) is conducted. Results show that the proper inclusion of nonlinear term in the physical model can be used to adequately capture the experimental observed behavior, reducing inconsistencies of piezoelectric energy harvesters operating near resonance conditions obtained from the linear description.

2. VIBRATION-BASED ENERGY HARVESTING

An archetypal model to describe the vibration-based energy harvesting system is shown in Figure 1. It consists of a mechanical system connected to an electrical circuit by a piezoelectric element, responsible for the electro-mechanical conversion. A mass-spring-damper oscillator with mass, *m*, stiffness *k*, and a linear viscous coefficient *b*, represents the mechanical system. This system is subjected to a base excitation *u=u(t)*, and the mass displacement is represented by *y*; *z* is the mass displacement relative to the base. An electrical resistance, R_l, represents the electrical circuit and *v* denotes the voltage across the piezoelectric element. The electro-mechanical coupling is provided by the piezoelectric element being represented by θ.

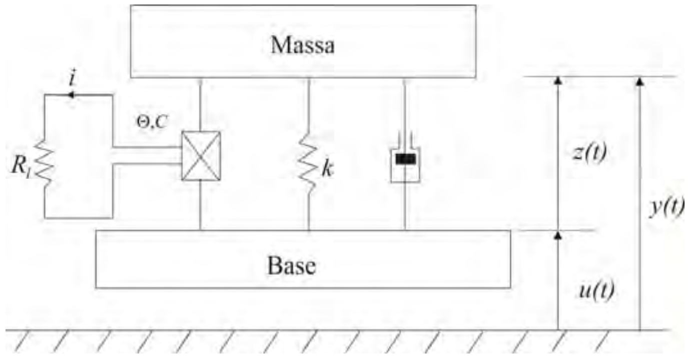

Figure 1. Archetypal model of the vibration-based energy harvesting system (Triplett & Quinn, 2009).

Therefore, the system dynamics may be described by the following equation:

$$m\ddot{z} + b\ddot{z} + kz - \Theta v = -m\ddot{u} \tag{1}$$

$$\Theta\dot{z} + C\dot{v} + \frac{1}{R_l}v = 0 \tag{2}$$

where $(\dot{\blacksquare}) \equiv d(\blacksquare)/dt$

The electro-mechanical coupling provided by the piezoelectric element, Θ, needs to be properly described by some constitutive equation. The next section treats this modeling.

2.1 Piezoelectric Constitutive Equations

The description of the 3-D behavior of piezoelectric materials is now in focus. Hence, consider that S_i is the strain, T_i is the stress, D_i is the electric displacement, and E_i is the applied field. The elastic compliance, piezoelectric coupling and permittivity matrices are denoted respectively by s_{ij}, d_{ij} and ε_{ij}. The superscript '*E*' stands for measurements at zero or constant electric field and '*T*' denotes measurements that are taken at zero or constant stress. Therefore, the 3-D constitutive equations are given by:

$$S_i = s_{ij}^E T_j + d_{mi}E_m \quad \text{(inverse effect)} \tag{3}$$
$$D_m = d_{mi}T_i + \varepsilon_{mk}^T E_k \quad \text{(direct effect)} \tag{4}$$

The inverse effect is associated with the generation of strain/stress in response to an applied electrical field; the direct effect is related to electrical charge that is a response to an applied strain/stress. It is assumed that 3-axis is associated with the poling direction, perpendicular to directions 1 and 2. On the other hand, the shear planes are indicated by the subscripts 4, 5 and 6.

Some physical situations as cantilever beams can be modeled as 1-D media and therefore, the piezoelectric element can be designed to operate in either 31 or 33 modes of vibration depending on the arrangement of the electrodes. The left side of Figure 2 shows the 33 mode where stress is applied along the 3-axis (in the direction of polarization) and the charge is collected in the same direction. The 31 mode, on the hand, has stress applied along the 1-axis (perpendicular to the direction of polarization) and charge is collected on the same surface as before.

ISSN 2176-5480

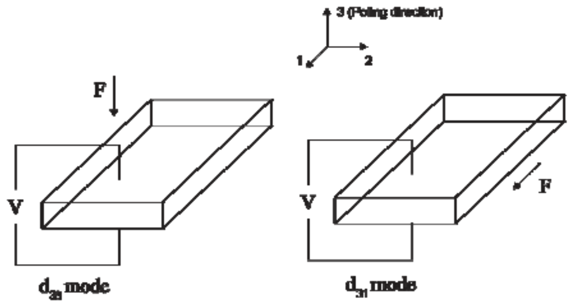

Figure 2. Operating mode of piezoelectric energy harvester transducer with 3-axis as a poling direction.

Coefficients d_{ij} establish the relation between electric displacement and stress, or strain and electric field. The linear model considers this piezoelectric coefficient as the slope of the linear fit of the strain–voltage curve (Kang et al., 2011). Nevertheless, experimental data shows that this electro-mechanical behavior is strongly dependent on the electric field intensity above certain threshold values, as shown in Figure 3a from experimental data of Crawley & Anderson (1990). From this curve it is possible to use a secant method in order to determine the value of d_{31} as a function of strain, see Figure 3b. Therefore, linear constitutive relation can be represented by a single constant value for all strain-electric fields (dashed curve in Figure 3b). A non-linear approach of the piezoelectric coupling coefficient can be established by assuming a linear dependence on the induced strain, as proposed by Triplett & Quinn (2009) (dotted curve in Figure 3b) or more specifically assuming a quadratic approximation (Stanton et al., 2010). Stanton et al. (2010) showed that the quadratic approximation of the piezoelectric coupling coefficient provides more accurate response with experimental data. This argues is employed here and therefore, quadratic approximation is used for the constitutive description of the piezoelectric behavior, obtaining the following equation.

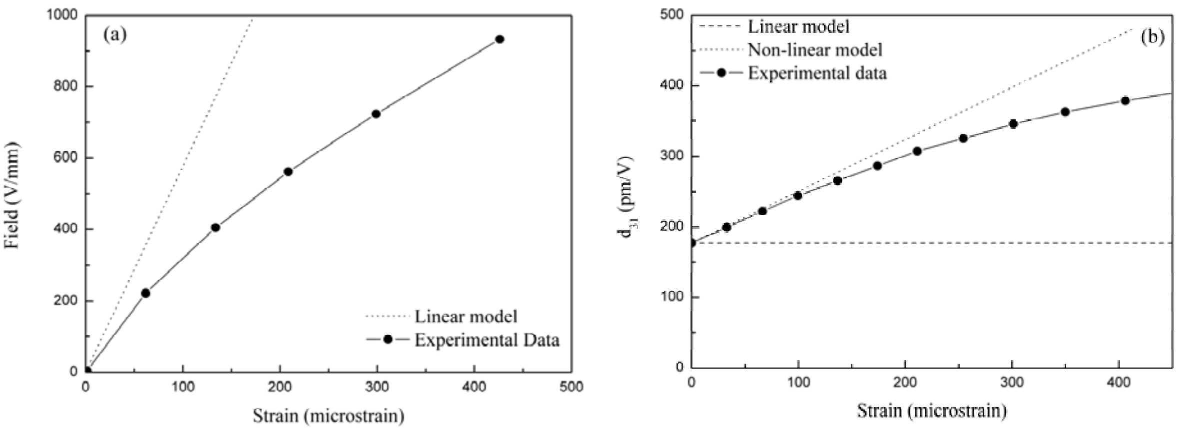

Figure 3. Electro-mechanical behavior of piezoelectric materials (Crawley & Anderson, 1990). Electric field-strain curve (left) and d_{31}-strain curve (right).

$$\Theta(z) = \theta + \beta z^2 \tag{5}$$

where θ and β are respectively, linear and non-linear piezoelectric coupling coefficients. Note that the definition of the linear model assumes β = 0. Based on this constitutive model, the energy harvesting archetypal system is modeled as follows.

$$m\ddot{z} + b\ddot{z} + kz - (\theta + \beta z^2)v = -B_f \ddot{u} \tag{6}$$

$$(\theta + \beta z^2)\dot{z} + \mathrm{C}\dot{v} + \frac{1}{R_l} v = 0 \tag{7}$$

where B_f is the forcing function that related to base excitations, as show in Kim et al. (2010).

3. NUMERICAL SIMULATIONS

This section discusses numerical simulations of the energy harvesting system. The main goal is to establish a comparison between linear and non-linear models, using experimental data obtained from Kim et al. (2010) as a reference. The experimental data is obtained using a brass reinforced bending actuator, T226-A4-503X, from Piezo

Systems Inc., which the resonant frequency is 109.5Hz without a proof mass. It should be highlighted that the coupling coefficient is assumed to be $(\theta + \beta z^2)$, and the definition of the linear model assumes $\beta = 0$. This approach is valid for systems with small induced strains. On the other hand, the non-linear model considers $\beta \neq 0$. Crawley & Anderson (1990) points that non-linear model seems to be more appropriate to represent systems with large induced strains.

For all simulations, the following parameters are employed (Kim et al., 2010): M=0.00878Kg, K=4150Nm^{-1}, θ=-0.004688NV^{-1}, C=4.194e^{-8}F, B_f=0.006872 and the base acceleration at 2.5ms^{-2}. Non-linear model also needs to define the non-linear coupling coefficient, β. A non-linear least-squares algorithm is employed to determine an optimal fit to the experimental data for the resonant frequency. The value β= -2.36e8C/m^3 provides the best theoretical agreement in the mechanical, voltage across the resistive load and power response in comparison with experimental data.

Figures 4-6 show the general behavior of energy harvesting system for different frequency excitations: (a) 95 Hz, (b) 109.5 Hz (resonance frequency), (c) 135 Hz and (d) 160 Hz. All cases establish a comparison of linear, non-linear and experimental data. Figure 4 presents tip displacement versus electrical load. Figure 5 presents the voltage-electrical load curve, while Figure 6 presents the power versus the electrical load.

It should be observed that the linear model is in good agreement with experimental data at various electrical loadings and different operating frequencies, but as the operating frequency gets closer to resonance condition, there is a large discrepancy between the displacement, voltage and power results obtained from linear model when compared with experimental measurement. This effect occurs because under resonant conditions, there are large induced strains, and the contribution of the nonlinearity in piezoelectric coupling becomes more important. In other frequencies, results of both models are quite similar because the induced strain is small compared with resonance frequency.

This general behavior is even more evident observing power curve. Figure 6(b) shows that the peak power measured at electrical load 10kΩ has the highest discrepancy compared to the linear model, 40.7%. The use of the non-linear model provides a deviation of 11.3%. Since many piezoelectric harvesters are designed to operate under resonant conditions, it is extremely important to consider non-linear models to obtain better prediction power .

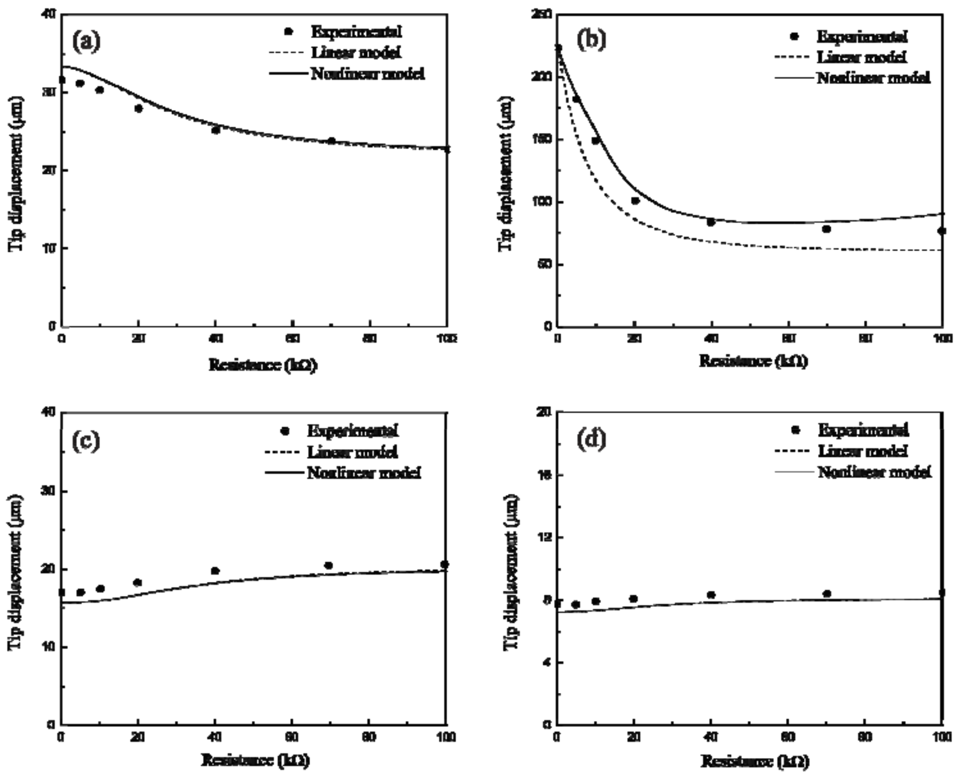

Figure 4. Results of linear (solid line), non-linear (dashed line) models and experimental data (scatter line) of tip displacement versus the electrical resistive load at different frequencies (a) 95 Hz, (b) 109.5 Hz (resonance frequency), (c) 135 Hz and (d) 160 Hz.

ISSN 2176-5480

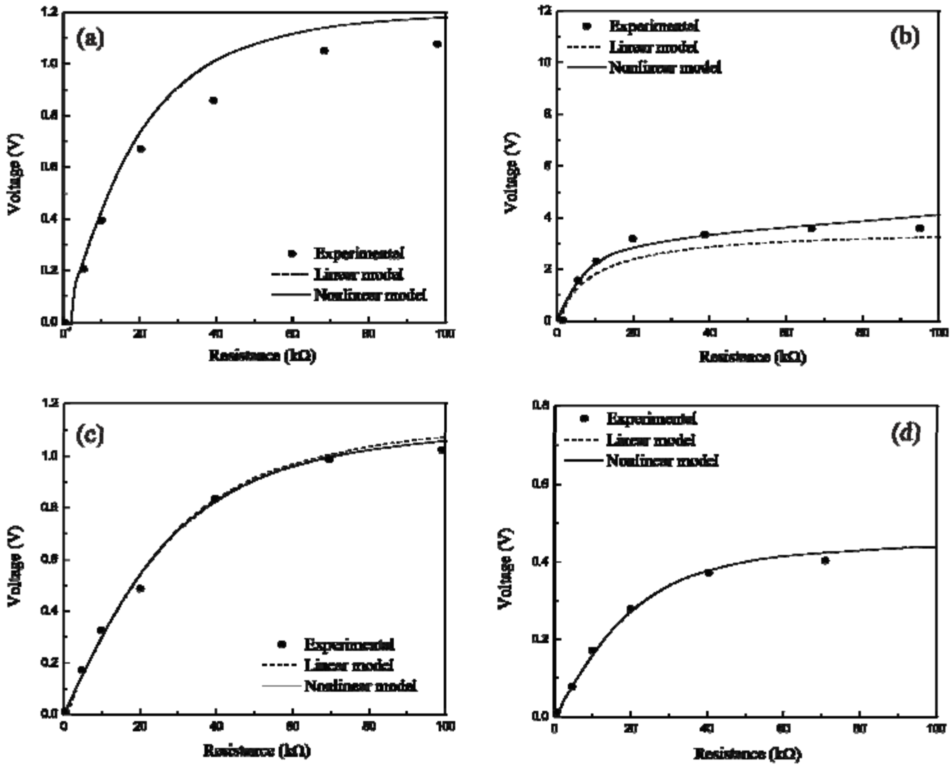

Figure 5. Results of linear (solid line), non-linear (dashed line) models and experimental data (scatter line) of voltage versus the electrical resistive load at different frequencies (a) 95 Hz, (b) 109.5 Hz (resonance frequency), (c) 135 Hz and (d) 160 Hz.

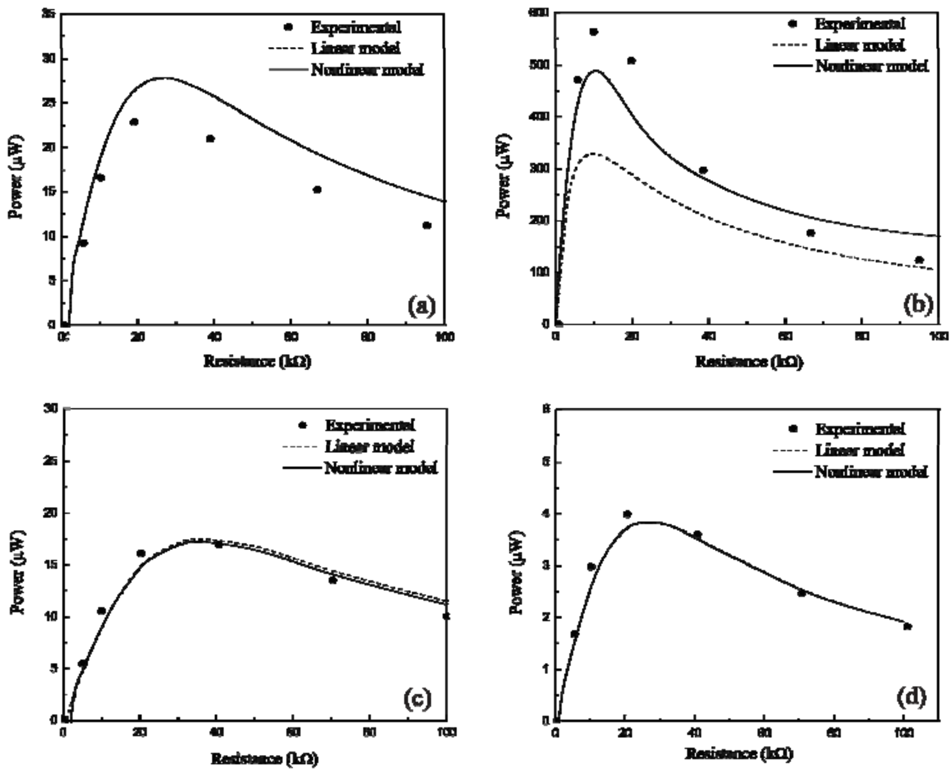

Figure 6. Results of linear (solid line), non-linear (dashed line) models and experimental data (scatter line) of power versus the electrical resistive load at different frequencies (a) 95 Hz, (b) 109.5 Hz (resonance frequency), (c) 135 Hz and (d) 160 Hz.

ISSN 2176-5480

Figure 7-9 shows similar analysis by assuming an electrical resistive load, varying the frequency. Each Figure shows tip displacement, voltage and power as a function of frequency. Figure 7 considers electrical resistive load of 10kΩ; Figure 8 considers 40kΩ, while Figure 9 considers 100kΩ. Once again, linear and non-linear models are compared with experimental data. When comparing results of the non-linear model with those of the linear model, it is possible to see that the maximum tip displacement, voltage and power are always greater for the linear model. Note that the increase of the electrical resistive loads tends to shift maximum value of the tip displacement, voltage and power curves to the left. Besides, both non-linear and linear models are not able to reproduce the value of displacement, voltage and power for frequency 115.25Hz, and this disagreement increases with the increase of the electrical resistive load. This is probably due to the anti-resonance frequency (Kim et al, 2010).

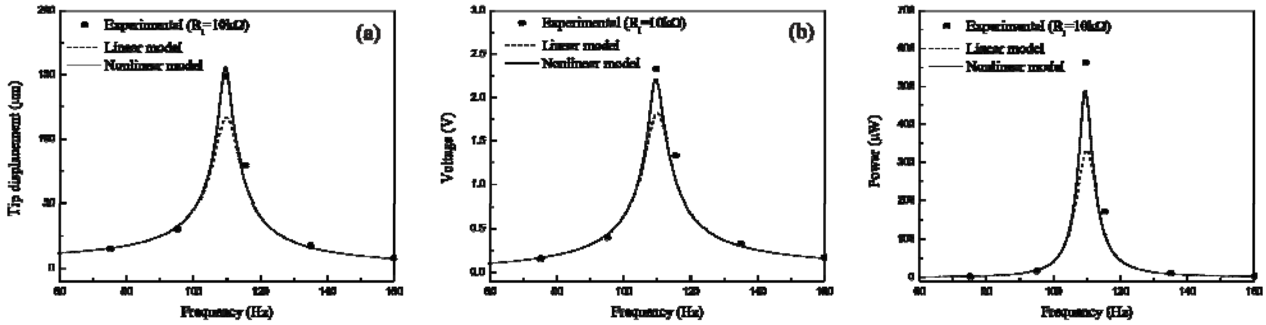

Figure 7. Results of linear (solid line), non-linear (dashed line) models and experimental data (scatter line) of (a) tip displacement, (b) voltage and (c) power versus frequency at electrical resistive load of 10kΩ.

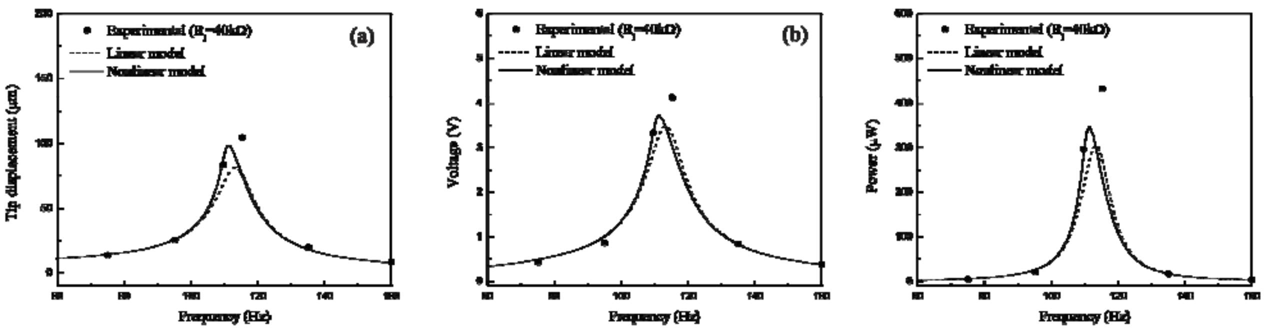

Figure 8. Results of linear (solid line), non-linear (dashed line) models and experimental data (scatter line) of (a) tip displacement, (b) voltage and (c) power versus frequency at electrical resistive load of 40kΩ.

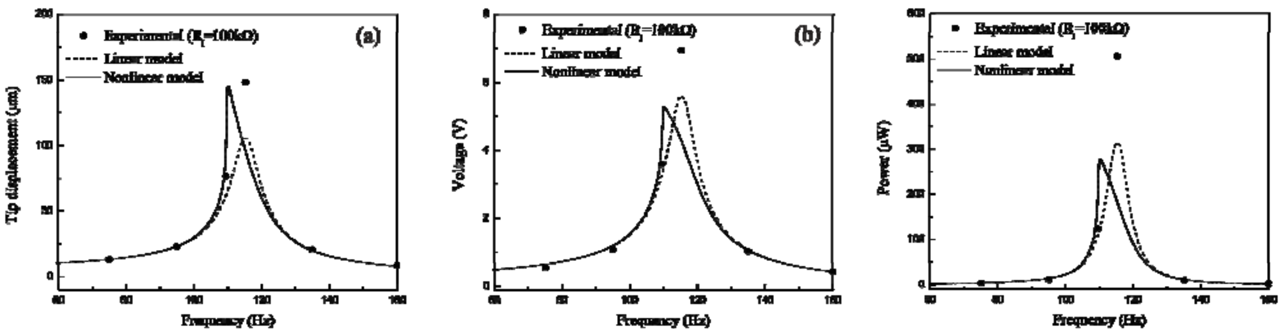

Figure 9. Results of linear (solid line), non-linear (dashed line) models and experimental data (scatter line) of (a) tip displacement, (b) voltage and (c) power versus frequency at electrical resistive load of 100kΩ.

ISSN 2176-5480

4. CONCLUSIONS

This paper deals with the analysis of the influence of the non-linear behavior of the piezoelectric element in vibration-based energy harvesting systems. Numerical simulations are carried out considering linear and non-linear behaviors of piezoelectric coupling showing their influence on system dynamics. Quadratic non-linearity is assumed and experimental data from Kim et al. (2010) are used as a reference. Results show that piezoelectric non-linearity can significantly influence the performance of the system in terms of the harvested power especially under resonant conditions. Non-linear model capture the general behavior of the energy harvester, presenting good theoretical agreement with experimental data close to resonant conditions. Moreover, it is important to observe that results suggest that the inclusion of non-linear terms in the energy harvester models can be used to reduce discrepancies predicted by linear models.

5. ACKNOWLEDGEMENTS

The authors would like to thank the Brazilian Research Agencies CNPq, CAPES and FAPERJ and through the INCT-EIE (National Institute of Science and Technology - Smart Structures in Engineering) the CNPq and FAPEMIG for their support. The Air Force Office of Scientific Research (AFOSR) is also acknowledged.

6. REFERENCES

Crawley E. F. and Anderson E. H., 1990, “Detailed models of piezoceramic actuation of beams”, Journal of Intelligent Material Systems and Structures, Vol. 1, pp. 4-25.

Crawley E. F., de Luis, J., 1987, “Use of piezoelectric actuators as elements of intelligent structures”, AIAA Journal, Vol. 25 (10), pp. 1373–1385.

duToit N. E. and Wardle B. L., 2006, “Performance of microfabricated piezoelectric vibration energy harvesters”, Integrated Ferroelectrics, Vol. 83, pp. 13–32.

Erturk A. and Inman D. J., 2011, “Piezoelectric energy harvesting”, John Wiley & Sons Ltd., Chichester, UK.

Hu Y. R. and Ng A., 2005, “Active robust vibration control of flexible structures”, Journal Sound Vibration, Vol. 288 (1-2), pp. 43–56.

Kang L. H., Lee D. O. and Han J. H., 2011, “A measurement method for piezoelectric material properties under longitudinal compressive stress a compression test method for thin piezoelectric materials”, Measurement Science Technology, Vol. 22: 065701.

Kim M., Hoegen M., Dugundji J. and Wardle B. L., 2010, “Modeling and experimental verification of proof mass effects on vibration energy harvester performance”, Smart Materials and Structures, Vol. 19: 045023.

Liao W. H., Wang D. H. and Huang S. L., 2001, “Wireless monitoring of cable tension of cable-stayed bridges using PVDF piezoelectric films”, Journal of Intelligent Material Systems and Structures, Vol. 12, pp. 331–339.

Roundy S., Steingart D., Frechette L., Wright P. and Rabaey J., 2004, “Power sources for wireless sensor networks”, Lect. Notes Computer Science, Vol. 2920, pp. 1–17.

Sodano H. A., Inman D. J. and Park G., 2004, “A review of power harvesting from vibration using piezoelectric materials”, The Shock and Vibration Digest, Vol. 36, No. 3, pp.197–205.

Stanton S. C., Erturk A., Mann B. P. and Inman D. J.,2010, “Nonlinear piezoelectricity in electroelastic energy harvesters: Modeling and experimental identification”, Journal of Applied Physics Vol. 108, pp.0749031-0749039.

Triplett A. and Quinn D. D., 2009, “The effect of non-linear piezoelectric coupling on vibration-based energy harvesting”, Journal of Intelligent Material Systems and Structures, Vol. 20, pp.1959-1967.

7. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.