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RANDOM PROCESS MODELS FOR CORROSION GROWTH ON PIPELINES

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Abstract. *Linear random variable corrosion growth models are extensively employed in the reliability analysis of pipelines subject to corrosion. Such models are generally employed due to their simplicity, and due to the limited availability of corrosion data. However, such models grossly neglect well-known characteristics of the corrosion process. The present paper shows that the same, limited corrosion data can be employed to construct non-linear random process models of corrosion growth. The study compares linear and non-linear random variable, and linear and non-linear random process corrosion growth models. All models are adjusted to the same, limited set of actual corrosion data, which includes corrosion depth measures at two times. It is shown that non-linear random process corrosion growth models lead to the best fit to the data and represent the known physics of the problem much better. Results presented herein emphasize the importance of using appropriate uncertainty models for corrosion growth, and of calibrating such prediction models to the appropriate amount of corrosion data.*

Keywords: *pipeline, corrosion, corrosion growth model, random process, pipeline safety*

1. INTRODUCTION

Corrosion threatens the integrity of oil and gas pipelines. Prediction of corrosion growth and targeted inspection and maintenance actions are required in order to safely manage the operational life of pipeline systems.

Deterministic methods for evaluation of remaining life of corroded pipelines have been proposed (ASME, 2009; Bubenik *et al.*, 1992; CSA, 2007; DNV, 2004). Due to inherent uncertainties in the corrosion process and in operating conditions, it is today widely acknowledged that probabilistic methods of life prediction are more robust than their deterministic counterparts. This is especially true in the life-cycle cost optimization of pipeline systems, as the expected costs of failure should also be taken into account. In life-cycle cost optimization, the costs of inspection, maintenance and repair are balanced with expected costs of failure. Life-cycle cost optimization yields a proper point of compromise between the competing goals of economy and safety.

Probabilistic approaches to pipeline life prediction have been reported in the literature (Ahammed and Melchers, 1996; Ahammed, 1998; Hong, 1999; Pandey, 1998; De Leon and Macías, 2005; Pandey *et al.*, 2009; Caleyo *et al.*, 2009a, 2009b; Velázquez *et al.*, 2009; Zhou, 2010; Zhou *et al.*, 2012; Valor *et al.*, 2013). Many of these approaches represent problem uncertainty in terms of random variables. This includes assuming a random variable model for the corrosion rate (Ahammed and Melchers, 1996; Ahammed, 1998; Pandey, 1998; De Leon and Macías, 2005; Zhou, 2010), and results in linear growth of mean corrosion depths. Linear random variable models are employed because they are simple and can be easily adjusted to limited corrosion data (e.g., corrosion measures at only two times). However, a number of issues can be raised with respect to such simple random variable models:

1. they are (arguably overly) conservative models, since there is ample evidence that corrosion growth follows a smaller-than-one power of time (Caleyo *et al.*, 2009a, 2009b; Velázquez *et al.*, 2009; Valor *et al.*, 2013; Melchers, 2003, 2004);
2. they result in highly non-linear growth of the standard deviations of corrosion defects;
3. they strongly defy physics, since they cannot be employed to grow the defect back to time zero (i.e., if you try to grow the defect back, you find out that the defect would have started growing at negative times…!);
4. they do not represent inherent variability (Pandey *et al.*, 2009) of corrosion growth in time.

As an alternative to random variable corrosion models, Hong (1999) modeled corrosion growth by a Markov random process. Caleyo *et al.* (2009b) developed a model for pitting corrosion using a non-homogenous linear growth Markov process. In this model, the pipe wall thickness is divided in discrete states and the corrosion damage size, at any point in time, is represented by a discrete random variable. The authors focused on the estimation of future pit depth and growth rate distributions from actual corrosion data and measured values of soil characteristics reported in previous articles by the authors (Velázquez *et al.*, 2009; Caleyo *et al.*, 2009a). Later, Valor *et al.* (2013) compared different corrosion growth rate models in terms of reliability results and concluded that their Markov model yields the best performance.

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Pandey *et al*. (2009) used a Gamma process with stationary increments to represent the cumulative temporal deterioration in combination with a simplified failure model, which assumed initial resistance and load effects as deterministic constants. One drawback of the model by Pandey *et al*. (2009) is that the sample paths of deterioration growth are discontinuous in time.

Also using random variable corrosion models, other authors addressed multiple corrosion defects. De Leon and Macías (2005) showed that spatial correlation between initial depths of multiple corrosion defects significantly affects system reliability, especially in pipelines with large number of segments. Zhou (2010) evaluated the system reliability of a pipeline segment with multiple corrosion defects. The impact of spatial correlation between various uncertain parameters of different defects on system failure probability was investigated.

More recently, Zhou *et al*. (2012) compared the random variable corrosion rate model (Zhou, 2010) with a corrosion model based on a Gamma process, similar to the one presented in Pandey *et al*. (2009), in terms of reliability analysis. Focusing on modeling the defect growth over time, Zhou *et al*. (2012) ignored the time-dependency of the internal pressure, assumed most of the parameters involved as deterministic quantities, and assumed the defect length not to grow over time. The authors characterized the spatial correlation between depth growths for different defects using a copula function, and investigated the impact of the spatially correlated (or dependent) defect depths, the uncertainties in the initial defect sizes and the model error associated with the burst pressure on the system failure probability.

Paik and Kim (2012) build a corrosion model for ship seawater ballast tanks by employing a 2-parameter Weibull probability distribution. The authors propose an approach for one to evaluate the parameters of this distribution such as to comply with observable corrosion data. A similar approach is adopted to represent growth of corrosion pits in offshore subsea oil well tubes (Hairil Mohd and Paik, 2013). Papers handling damage assessment in pipelines (Yang *et al*., 2007) and reliability of subsea pipelines subject to mechanisms other than corrosion (Elsayed *et al*., 2012) may also be referred to.

In general, corrosion models can be classified in two types: empirical and phenomenological models. Phenomenological models are built based on and such as to reflect the fundamental physics of the problem. Empirical models, on the other hand, are simply built such as to comply with observable corrosion data, without providing any physical interpretation of the causes of the underlying phenomena. Clearly, empirical models can and should reflect problem physics to some extent, even when not built from phenomenological reasoning. This article addresses empirical models, and shows how they can be built such as to better reflect problem physics.

Phenomenological models for corrosion in marine environments have been developed by Melchers and co-workers (Melchers, 2003, 2004, 2008). To the authors' best knowledge, no such phenomenological model exists for corrosion in (buried) soil conditions, which can be applied to pipeline safety management. Empirical models have been extensively used in the literature (Caleyo *et al*., 2009a, 2009b; Zhou *et al*., 2012; Paik and Kim, 2012).

The present paper addresses empirical corrosion models to predict the time evolution of corrosion in buried pipelines. Comprehensive stochastic models of corrosion growth are proposed and constructed. A power law function is used to represent defect growth in time (Velázquez *et al*., 2009; Caleyo *et al*., 2009a). However, instead of building a parameterized random process by considering parameters of the power law function as random variables, the proportionality factor of the power law function is modeled as a Poisson square wave process. This allows temporal uncertainty in defect growth to be represented, but yields continuous growth of defects in time, contrary to other approaches (Pandey *et al*., 2009; Caleyo *et al*., 2009b; Valor *et al*., 2013; Zhou *et al*., 2012). A Gamma distribution is used to represent the intensity of independent corrosion growth pulses, which is justified by the principle of maximum entropy (Kapur and Kesavan, 1992). A simpler variant of this model, using a linear function rather than a power law, is also constructed by modeling the corrosion rate as a Poisson square wave process with Gamma-independent pulse heights. Parameters of both corrosion growth models are calibrated to actual (observed) corrosion data in buried pipelines (Caleyo *et al*., 2009b) and used to investigate the time-varying reliability of an example pipeline. The developed random process corrosion growth models can be employed in the life-cycle cost optimization of pipelines, and this can be the subject of future research.

The article is laid out as follows. Sections 2 to 5 describe four different models for corrosion growth, which result from combination of linear or non-linear corrosion growth models, and random variable (RV) or random process (RP) corrosion growth models. The four models are adjusted to the same set of actual corrosion data, as described in Section 6. These models are employed in Section 7 in the reliability analysis of an example pipeline, and results for the different corrosion growth models are compared. Concluding remarks are presented in Section 8.

2. LINEAR RANDOM VARIABLE MODEL

Random variable corrosion growth models, commonly employed in the literature (Ahammed and Melchers, 1996; Ahammed, 1998; Pandey, 1998; De Leon and Macías, 2005; Zhou, 2010), simply represent the rate of defect growth, v_D, as a random variable. As a consequence, defect depth at any time $t > t_0$ is simply obtained as:

$$D_{\max}(t) = D_0 + v_D\, t \qquad (1)$$

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where t_0 is the time of the first inspection (corrosion depth measure), and D_0 represents the random variable describing "initial" defect depth, at the time of the first inspection.

Note that $D_{max}(t)$ explicitly appears in the limit state functions (Section 7.1), rather than D_0 and v_D. Strictly speaking, defect depths obtained using Eq. (1) are called parameterized random processes. However, since only random variables are used in describing random corrosion growth, models like Eq. (1) are generally referred to as random variable corrosion growth models. Given one realization of random variables D_0 and v_D, defect growth in time ($D_{max}(t)$) is deterministic. Hence, random variable corrosion growth models do not represent the inherent variability of corrosion growth in time (Pandey *et al.*, 2009).

Random variable corrosion rate models result in linear growth of mean defect size, as applying the expected value operator E[·], one obtains:

$$\mathrm{E}[D_{\max}(t)] = \mathrm{E}[D_0] + \mathrm{E}[v_D\, t] \tag{2}$$

However, random variable models result in highly non-linear variation of the standard deviation of defect size, as applying the variance operator Var[·], one obtains:

$$\sigma_{D_{\max}}(t) = \sqrt{\mathrm{Var}[D_{\max}(t)]} = \sqrt{(\sigma_{D_0})^2 + (\sigma_{D_0})^2\, t^2} \tag{3}$$

When corrosion growth is represented as a power law function of time, it is observed that the exponent is always smaller than unity (Caleyo *et al.*, 2009a, 2009b; Velázquez *et al.*, 2009; Valor *et al.*, 2013; Melchers, 2003, 2004). Hence, when an unitary exponent is employed in linear models such as Eq. (1), these become necessarily conservative when used for extrapolation, i.e., for computing defect sizes for times smaller than t_0 or larger than the time of an hypothetical second inspection. Indeed, if linear models are projected backwards, time t_0 is often found to be negative, as will be shown herein. Hence, linear corrosion growth models seriously offend the underlying physics of the problem.

For purposes of comparison with other models, the defect growth rate (v_D) in this paper is assumed to follow a Gamma distribution. The Gamma distribution is employed herein by virtue of the Principle of Maximum Entropy (Kapur and Kesavan, 1992): for a strictly positive variable whose mean and standard deviation are known, the maximum information entropy is obtained by assuming a Gamma distribution.

3. LINEAR RANDOM PROCESS MODEL

In order to account for the temporal variability of corrosion growth in time, Pandey *et al.* (2009) proposed representing cumulative temporal deterioration using a Gamma process with stationary increments. One drawback of such model is that sample paths of deterioration are discontinuous in time, which is not realistic unless at the microscopic level.

Building on the idea of Pandey et al. (2009), but with the purpose of constructing a more realistic model, it is proposed herein to model defect growth rate as a Poisson square wave process with stationary and independent increments (pulse heights). This leads to continuous defect growth in time, and allows temporal variability to be properly taken into account, as follows.

The rate of defect growth, v_D, is characterized herein by a so-called Poisson square wave process. A typical realization of a Poisson square wave process is illustrated in Fig. 1. In this type of process, both pulse heights (Y_i) and durations ($t_{bi} = t_{i+1} - t_i$) are represented as random variables. Following the theory of Poisson processes (Melchers, 1999), pulse durations are represented as exponential random variables with parameter λ. Following Pandey *et al.* (2009), pulse heights are represented as independent and identically distributed Gamma random variables with parameters μ_Y and σ_Y. In principle, any other strictly positive random variable distribution could be considered for pulse heights; however, use of Gamma distribution is justified by the application of the Principle of Maximum Entropy (Kapur and Kesavan, 1992).

Figure 2 illustrates the construction of defect size random processes. From the initial (random) defect size (D_0) and from defect growth parameters (λ, μ_Y and σ_Y), random processes characterizing defect size ($D_{max}(t)$) are constructed as follows. For each pulse of the defect growth rate (say, a pulse which is sampled by Monte Carlo simulation), the increment in defect size is given by:

$$D_{\max}(t_{i+1}) = D_{\max}(t_i) + Y_i\,(t_{i+1} - t_i) \tag{4}$$

where n is the total number of pulses of the defect growth rate process at a given sample. Thus, the generated sample paths of defect size are continuous functions of time, as illustrated in Fig. 2. This approach is more realistic than directly representing defect size as a Gamma process with stationary increments (Pandey *et al.*, 2009). However, this linear random process corrosion growth model also suffers from the limitation that it cannot be used to grow the defect back to t_0, since this time may become negative.

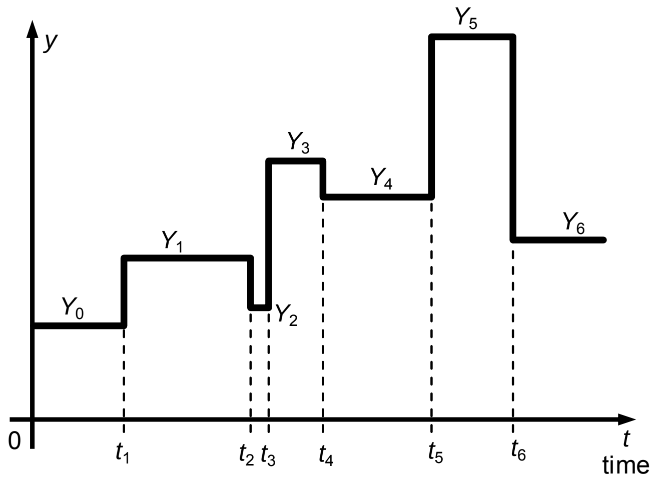

Figure 1. Realization of a Poisson square wave process (adapted from Melchers, 1999)

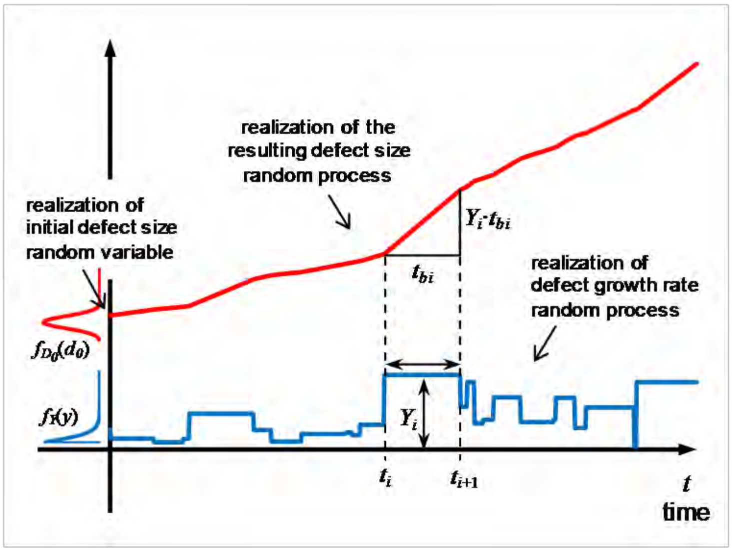

Figure 2. Construction of defect size “linear” random process, from random (variable) initial defect and Poisson square wave process defect growth rate

4. NON-LINEAR RANDOM VARIABLE MODEL

Velázquez *et al*. (2009) and Caleyo *et al*. (2009a) employed an empirical, non-linear power law function to model growth of corrosion pits in low-carbon steel. Such function relates the maximum defect depth, D_{max}, to the exposure time:

$$D_{\max}(t) = k\,(t - t_0)^{\alpha} \tag{5}$$

where t_0 is the corrosion starting (initiation) time, and k and α are the proportionality and exponent factors, respectively. Velázquez *et al*. (2009) and Caleyo *et al*. (2009a) constructed a regression model in order to relate the proportionality and exponent factors to a number of random variables characterizing soil and pipe properties. Therefore, proportionality and exponent factors, k and α, are also modeled as random variables. The corrosion initiation time, t_0, was determined for each soil category considered in the study (Velázquez *et al*., 2009). For a 'generic' soil class, t_0 = 2.88 years was encountered.

Because of the non-linearity of Eq. (5), obtaining exact closed form expressions for the mean and standard deviation of defect size (similar to Eqs. (2) and (3)) is not trivial. However, it is clear that the resulting defect size, $D_{max}(t)$, will be

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a parameterized random process, as it is a function of (two) random variables. Because this non-linear model depends on two random variables only, it will be referred herein as non-linear random variable corrosion growth model.

For the purpose of comparison with other models, the defect size proportionality factor (k) is assumed to follow a Gamma distribution. The exponent factor of the defect size (α) is assumed to follow a Lognormal distribution. The mean and standard deviation of random variables k and α are denoted by (μ_k, σ_k) and ($\mu_\alpha, \sigma_\alpha$), respectively.

5. NON-LINEAR RANDOM PROCESS MODEL

Combining the non-linear random variable with the random process corrosion growth models presented in Sections 3 and 4, one obtains a non-linear random process corrosion growth model, as proposed in this section.

In this model, the proportionality factor (k) of the defect size function (Eq. (5)) is characterized by a Poisson square wave process, with pulse heights (Y_i) and durations ($t_{bi} = t_{i+1} - t_i$) represented as random variables. As explained in Section 3, exponential and Gamma distributions are used to represent the arrival of new pulses and pulse intensities, respectively. On the other hand, the exponent factor (α) is represented by a Lognormal (assumed) random variable, with moments μ_α and σ_α.

Figure 3 illustrates the construction of defect size random processes for this model. From parameters (λ, μ_Y and σ_Y) characterizing the proportionality factor of the defect size (k in Eq. (5)), defect size random processes ($D_{max}(t)$) are constructed as follows. For each pulse of the proportionality factor k (say, a pulse which is sampled by Monte Carlo simulation), the increment in defect size is given by:

$$D_{\max}(t_{i+1}) = D_{\max}(t_i) + Y_i\,[(t_{i+1} - t_0)^{\alpha} - (t_i - t_0)^{\alpha}] \qquad \text{for} \quad i = 0, 1, \ldots, n \tag{6}$$

where n is the total number of pulses of the proportionality factor process at a given sample, and α is a realization of the exponent factor random variable.

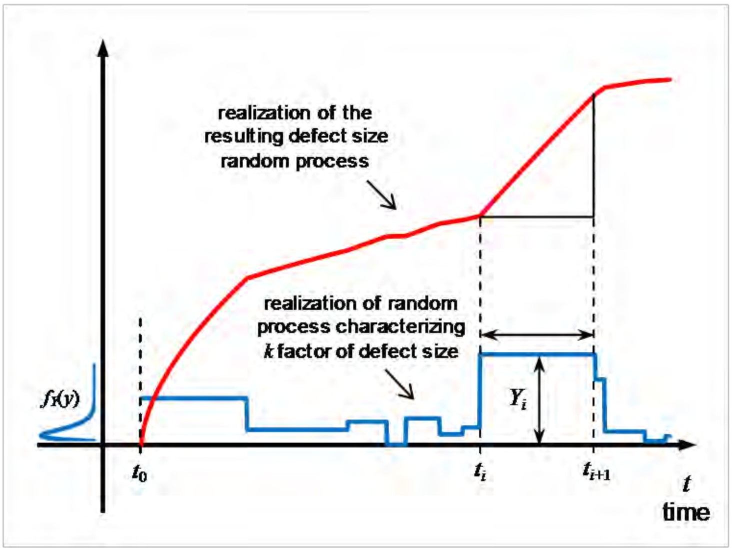

Figure 3. Construction of defect size "non-linear" random process, from Poisson square wave process characterizing proportionality factor of defect size

Again, the generated defect size sample paths are continuous functions of time, as illustrated in Fig. 3. This non-linear random process corrosion growth model is capable of reproducing the variability of corrosion process in time, and does not defy problem physics as $t_0 > 0$.

In the following section, parameters of the four models described in Sections 2 to 5 are adjusted to represent the same set of limited actual corrosion data.

6. CALIBRATION OF CORROSION MODELS TO ACTUAL DATA

In order to explore the differences between the four models presented in the previous sections, these models are calibrated to the same set of (limited) corrosion data.

Caleyo *et al.* (2009b) presented histograms of pitting corrosion depths in an 82 km long operating pipeline, coal-tar coated with wall thickness of 9.52 mm, used to transport sweet gas. This pipeline, commissioned in 1981, was inspected at the beginning of 2002 and in mid-2007, using magnetic flux leakage in-line inspection (ILI). From these histograms,

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sample mean and standard deviation of defect depths were calculated for the 2002 and 2007 inspections. Table 1 summarizes these results.

Table 1. Parameters of corrosion depths, calculated from actual data (Caleyo *et al*., 2009b).

Inspection	Total number of defects	Mean (mm)	Standard deviation (mm)
beginning of 2002	3577	2.64	0.83
mid-2007	3851	3.09	0.88

Importantly, the set of results contains actual corrosion data in a pipeline collected for only two inspections (two time points). This may appear to be less than ideal, but on the other hand, this is the best that one can expect to have in practice, due to difficulties and constraints related to in-line inspections, defect matching, etc.

For linear random variable and linear random process models, results of the 2002 inspection were used to represent the "initial" defect depth distribution, using the method of moments. A Gaussian distribution was assumed, hence $D_0 \sim N(2.64,0.83)$ mm. From this common initial defect depth, parameters of the linear random variable and random process corrosion growth models were calibrated in order to match the actual mean and standard deviation from the 2007 inspection (t = 5.5 years, where t = 0 corresponds to the 2002 inspection). For the random variable model, this is accomplished by solving Eqs. (2) and (3) for $\mathrm{E}[v_D] = \mu_{v_D}$ and σ_{v_D} , for t = 5.5 years. For the random process model, a data-fitting optimization algorithm was used. A routine from IMSL (2006) libraries was employed for this end, namely BCPOL, which uses a direct search complex optimization algorithm. The optimization variables in this search are the model parameters λ, μ_Y and σ_Y.

For non-linear random variable and random process models, on the other hand, no assumption about an "initial" defect size distribution is needed, because the purpose is to represent defect growth since the time at which corrosion starts. Actual corrosion data collected by Caleyo et al. (2009b) gives corrosion initiation time for a generic soil class as t_0 = 2.88 years, which is used for both non-linear models. Parameters of non-linear corrosion growth models were calibrated in order to match the actual mean and standard deviation from the 2002 and 2007 inspections (t = 21 and t = 26.5 years, respectively, where t = 0 corresponds to the time of pipeline commissioning). For the two non-linear models, the BCPOL optimization routine (IMSL, 2006) was also employed. For the non-linear random variable model, the aim is to find parameters μ_k, σ_k, μ_α and σ_α. For the non-linear random process model, the aim is to solve for parameters λ, μ_Y, σ_Y, μ_α and σ_α. During the course of these optimization analyses, it was found that parameter λ (the arrival rate of pulses in the Poisson model) has small influence in the optimization process. Hence, in order to render the optimization process simpler, and after performing an initial investigation, this parameter was set to λ = 0.5 years^{-1}, which corresponds to a mean time between pulse arrivals of 2.0 years.

Table 2 shows the parameters of the four corrosion growth models, calibrated to actual data (mean and standard deviation), as described above. The four calibrated corrosion growth models are compared in Fig. 4, in terms of the time evolution of mean and standard deviation of defect depth. In the figure, the calibration points corresponding to the two inspection times (t_1 and t_2) are identified and corresponding values are indicated.

Table 2. Parameters of corrosion growth models, calibrated to actual data.

Model		Unit	Distribution type	Mean	Standard deviation
Linear random variable	Growth rate	mm/year	Gamma	0.082	0.053
Linear random process	Growth rate pulse heights	mm/year	Gamma	0.082	0.076
	Growth rate pulse durations	year	Exponential	2.0	2.0
Non-linear random variable	Proportionality factor	mm/year$^\alpha$	Gamma	0.475	0.130
	Exponent factor	---	Lognormal	0.592	0.023
Non-linear random process	Proportionality factor pulse heights	mm/year$^\alpha$	Gamma	0.474	0.326
	Proportionality factor pulse durations	year	Exponential	2.0	2.0
	Exponent factor	---	Lognormal	0.593	0.003

In Fig. 4a it can be observed that mean defect growth is indistinguishable between the two linear (RV/RP) models, and also between the two non-linear (RV/RP) models. However, while in linear models defect means grow linearly in

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time, in non-linear models this growth is non-linear due to the underlying corrosion growth power law function. It is also emphasized that linear models describe corrosion growth only between t_1 and t_2. Back-extrapolation reveals that linear models defy problem physics by predicting a "negative" corrosion initiation time $t^* \approx -11.3$ years. Forward extrapolation shows that linear models are more conservative than their non-linear counterparts.

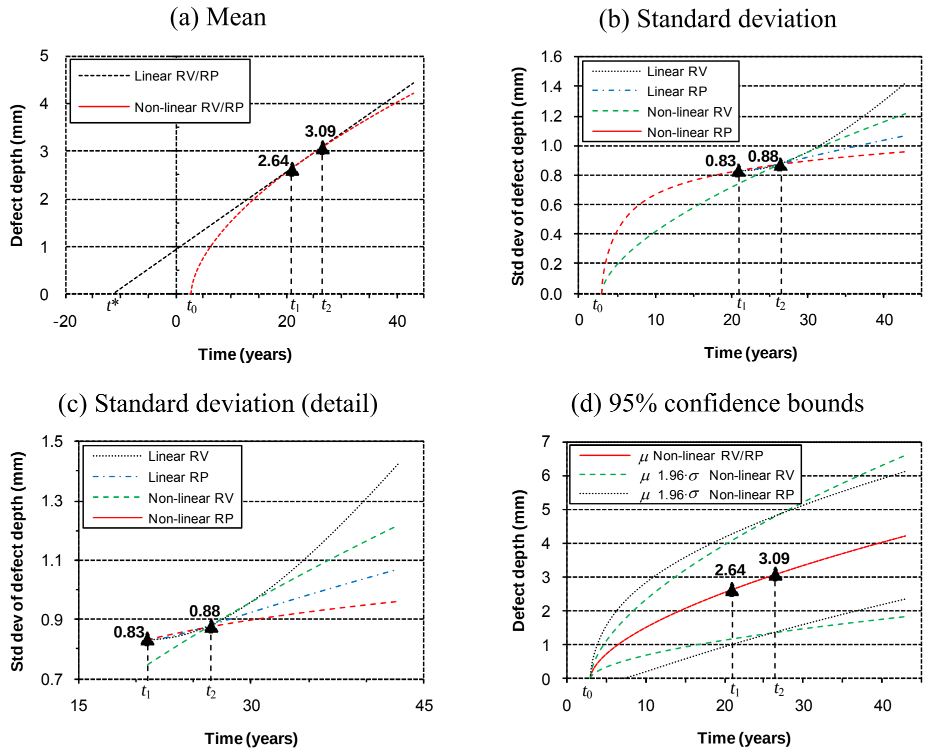

Figure 4. Time evolution of corrosion depths for different corrosion growth models, calibrated to actual data. RV: random variable; RP: random process

Figure 4b shows the standard deviation of defect depths. Figure 4c is the same as the former, but plotting only the standard deviations starting from t_1, in order to highlight the differences between different models. Figure 4d shows the 95% confidence bounds ($\mu \pm 1.96 \cdot \sigma$) for defect growth for the two non-linear models.

An important observation from Fig. 4b-c is that, for the non-linear random variable model, the optimization algorithm was not able to match the actual standard deviation at t_1, due to a lack of "degrees of freedom" of this model. Hence, it is possible to match the other model parameters (i.e. actual mean at t_1 and t_2, and standard deviation at t_2), but there is an intrinsic lack-of-fit to the non-linear random variable model. The consequence of this lack of fit is that the forward-projected standard deviations are larger than for the non-linear random process model. The non-linear random process model, on the other hand, precisely fits the actual means and standard deviations at both times.

For both linear models, it is possible to match actual corrosion parameters. However, mean and standard deviation at t_1 are matched a priori, as the moments of the "initial" defect depth. Only the moments at t_2 are, in fact, matched in the calibration process.

It can be observed (Fig. 4b-c) that, after the second inspection, the popular linear random variable model exhibits the largest standard deviation, following Eq. (3). The standard deviation of defect depth for the non-linear random process model is the largest among the four models between the two inspections, but is the smallest for times larger than the second inspection. It is at these extrapolated times that the projections of future pipeline reliability and planning of future inspection times occur. Hence, the non-linear random process model proposed herein is the least conservative amongst the studied models; however, it is the most accurate in terms of representing problem physics and fitting actual corrosion results for two inspections. These differences in behavior of the mean and standard deviation have an impact on evaluated failure probabilities, as shown in Section 7.

Figure 4 illustrates the difficulties of extrapolating corrosion predictions from models calibrated to limited data (only two inspections). In order to compare the different models in terms of their extrapolation capabilities, corrosion data for a third time point would be required. Unfortunately, the authors do not know any published set of actual, correlated corrosion data in buried pipelines for three or more times. Hence, we are in no position to evaluate the prediction capabilities of the four models. The evidence shown herein, however, is that the proposed non-linear random

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process model represents problem physics much better, and precisely matches actual corrosion data for two inspection times. Accordingly, practical decisions often have to be made based on limited data as presented herein.

7. RELIABILITY ANALYSIS OF AN EXAMPLE PIPELINE

This section illustrates the impact of the different corrosion growth models described herein on the reliability of an example pipeline. Apart from the random corrosion models, this example involves other uncertain quantities, such as internal pressure loading, pipe diameter, pipe wall thickness, pipe ultimate tensile strength and model error for the burst pressure. Probabilistic characterization of involved quantities is summarized in Tab. 3.

Table 3. Probabilistic characteristics of quantities involved in pipeline numerical example.

Quantity	Unit	Distribution type	Mean	Coefficient of variation
Diameter(1)	mm	Deterministic	610	---
Wall thickness(2)	mm	Normal	9.52	0.015
Tensile strength(1)	MPa	Normal	496	0.03
Model error for burst pressure model(1)	---	Lognormal	0.97	0.108
Defect length(3)	mm	Deterministic	90	---
Annual maximum internal pressure(4)	MPa	Gumbel (maxima)	7.056	0.05

(1) Based on Zhou (2010).
(2) The mean is based on Caleyo *et al.* (2009b). The distribution type and coefficient of variation are based on Zhou (2010).
(3) Based on Zhou *et al.* (2012).
(4) Internal pressure is modeled by a Borges process, according to Section 7.2. The distribution type, mean and coefficient of variation are based on Zhou (2010).

Since actual corrosion data for growth of defect lengths were not found by the authors for purposes of calibration, it is assumed, for simplicity, that the defect length does not grow over time and that it is a deterministic constant. This assumption does not affect comparisons between results of the different studied models, but should be reconsidered in actual pipeline reliability analysis. In addition, it has been shown (Caleyo *et al.*, 2002) that changes in the defect length have little to no influence on the estimation of the failure probability associated with individual corrosion defects.

7.1 Failure modes and limit state functions for pipelines containing corrosion defects

The object of the prese osion defect, which can be characterized by maximum d

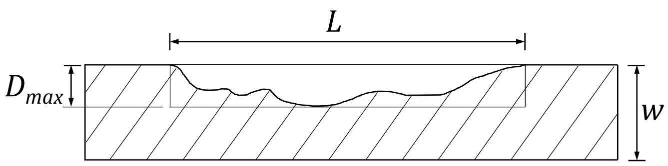

Figure 5. Corrosion defect dimensions

For each possible failure mode of a corroded pipeline, one limit state function is written so as to define the boundaries of the failure and safety domains. In this article, an approach presented elsewhere (Zhou, 2010; Zhou *et al.*, 2012) is adopted, where three different failure modes (hence limit state functions) are considered for a given corrosion defect. Following Zhou (2010) and Zhou *et al.* (2012), at an active corrosion defect, a pressurized steel pipe may fail by small leak or burst. Small leak occurs when the defect grows through the pipe wall, and a burst occurs when the pipe wall undergoes plastic collapse due to internal pressure at the defect location prior to the defect growing through the pipe wall. A burst is further classified as a rupture (a failure where the through-wall defect resulting from a burst is long enough to undergo unstable axial extension) or a large leak (a burst without unstable axial extension of the resulting through-wall defect). It is assumed that a small leak can be detected and repaired in a timely manner such that the likelihood of a burst following a small leak is considered negligible. In pipeline reliability analysis it is important to distinguish between small leaks, large leaks and ruptures because the consequences associated with these distinct failure modes can be drastically different, with the consequences of small leaks being the least severe and those of ruptures being the most severe.

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The limit state function for small leak at a given corrosion defect is defined as (Zhou, 2010; Zhou *et al.*, 2012):

$$g_1 = w - D_{\max} \tag{7}$$

where w is pipe wall thickness, and D_{max} is maximum defect depth in the through-wall thickness direction. A small leak is considered to occur if $g_1 \leq 0$.

The limit state function for burst is given by (Zhou, 2010; Zhou *et al.*, 2012):

$$g_2 = r_b - p \tag{8}$$

where r_b is burst pressure of a pipe containing a part-through wall corrosion defect, which is generally a function of pipe geometry, pipe material and defect geometry, and p is pipe internal pressure. The limit state $g_2 \leq 0$ represents a burst failure. The PCORRC model developed by Leis and Stephens (1997) is used herein to estimate the burst pressure, as follows:

$$r_b = X_M \frac{2\,\sigma_u\, w}{d}\left[1 - \frac{D_{\max}}{w}\left(1 - \exp\left(-\frac{0.157\,L}{\sqrt{\dfrac{d(w-D_{\max})}{2}}}\right)\right)\right] \tag{9}$$

where σ_u is ultimate tensile strength of pipe material; d is pipe diameter; L is defect length in pipe longitudinal direction, and X_M is a multiplicative model error term.

The limit state function for rupture is defined as (Zhou, 2010; Zhou *et al.*, 2012):

$$g_3 = r_{rp} - p \tag{10}$$

where r_{rp} is rupture pressure, that is, the pressure resistance of a pipe containing a through-wall flaw that results from the burst of the pipe at the corrosion defect. The through-wall flaw will undergo unstable axial extension and lead to a rupture if $g_3 \leq 0$.

The rupture pressure prediction model suggested by CSA (2007) is adopted. The rupture pressure, r_{rp}, is given by:

$$r_{rp} = \frac{2\,w\,\sigma_f}{M\,d} \tag{11}$$

where σ_f is the flow stress, defined as 0.9 σ_u, and M is the Folias factor:

$$M = \begin{cases} \sqrt{1 + 0.6275\dfrac{L^2}{d\,w} - 0.003375\dfrac{L^4}{d^2\,w^2}} & \text{for } \dfrac{L^2}{d\,w} \leq 50 \\ 0.032\dfrac{L^2}{d\,w} + 3.293 & \text{for } \dfrac{L^2}{d\,w} > 50 \end{cases} \tag{12}$$

The three limit states described by g_1, g_2 and g_3, do not single-handedly represent the failure modes for a corroded pipe segment. Failure modes for small leak, large leak and rupture are described by combinations of g_1, g_2 and g_3, as follows (Zhou, 2010; Zhou *et al.*, 2012). A small leak is assumed to take place when $g_1 \leq 0$ and $g_2 > 0$. Hence, a small leak only happens if the pipeline does not burst. A burst is assumed to take place when $g_1 > 0$ and $g_2 \leq 0$. A large leak occurs when $g_1 > 0$, $g_2 \leq 0$ and $g_3 > 0$, and a rupture occurs when $g_1 > 0$, $g_2 \leq 0$ and $g_3 \leq 0$. The limit state functions, g_1, g_2 and g_3, hence the three failure modes, depend on time, because the defect grows over time and causes deterioration of pipeline strength, and also because the internal pressure may vary with time.

7.2 Random load model

The pipeline internal pressure, p, is the only load considered in limit state functions (Eqs. (7) to (12)). Pipeline pressure fluctuates randomly in time due to changing operating conditions. In all detail, pipeline pressure should be represented as a continuous process of time. However, instantaneous fluctuations of pipeline pressure are of no interest in reliability analysis, since failure generally occurs under peak (extreme) pressures. Hence, daily, weekly, monthly or annual maxima should be of interest. Following this logic, Zhou (2010) characterized internal pressure by a discrete random process of pulses representing annual peaks (extremes) of internal pressure. The discrete Borges random process (see Fig. 6) is generated by a sequence of independent and identically distributed random variables Y_i (the annual maxima of internal pressure), each acting over a deterministic time interval t_b, the so-called holding time (Melchers, 1999), which in this case is equal to one year. It is clear that a smaller discretization time could also be considered (daily/weekly maxima of internal pressure acting over pulses of a day/week duration), as long as measured data is available to characterize these maxima. In order to allow comparisons with the literature, annual maxima of internal pressure are also considered in the present article.

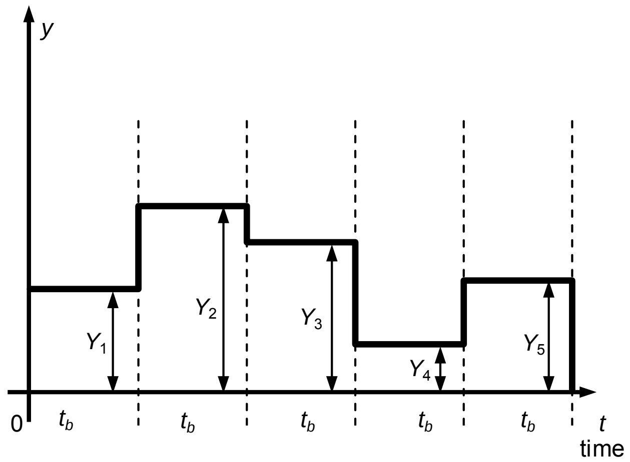

Figure 6. Realization of a Borges internal pressure load process (adapted from Melchers, 1999)

Use of a random process load (internal pressure) model, however, is a significant improvement over random variable load models considered in the literature (Ahammed and Melchers, 1996; Ahammed, 1998; Pandey, 1998; Valor *et al.*, 2013; De Leon and Macías, 2005; Zhou *et al.*, 2012). To be consistent with extreme value and structural reliability theories (Melchers, 1999; Gumbel, 2004), a random variable load would have to represent the extreme (maximum) loading for the lifetime of the structure, or at least for the evaluation period. However, this type of extreme value (random variable) load models are strictly not valid (Marley and Moan, 1994; Beck and Melchers, 2004) for problems where resistance is reduced over time, for instance, due to corrosion. In comparison, combining annual loading extremes with resistance degradation due to corrosion is an approximation, but an acceptable one, considering the small amount of degradation in one year (with respect to usual pipeline life of a couple of decades).

7.3 Failure probability results

The four corrosion growth models previously described are considered in the solution of this example pipeline problem. Also, the three failure modes for the example pipeline, i.e. small leak, large leak and rupture, are considered. All reliability analyses were performed by Monte Carlo simulation, using 10^6 samples of each random quantity. Results are presented in Fig. 7, for probabilities of failure of the three failure modes and for the system failure probability (i.e. any failure mode).

It should be noted that, similarly to Fig. 4, failure probabilities for linear random variable and random process models were calculated from t = 21 years, i.e. from the first inspection time, while failure probabilities for non-linear random variable and random process models were calculated from t = 0, i.e. from the pipeline commissioning time. Resulting probabilities of large leak and rupture (Fig. 7b-c) are, for this example, very low, say less than 10^{-4} or 10^{-5} for most of the times considered. It should also be remembered that the non-linear random variable model did not match actual data exactly at the first inspection time.

It is observed (Fig. 7a) that probability of small leak for the non-linear random process model is greater than for the other three models, until about t = 34 years. After that time, the two random variable (linear and non-linear) models exhibit greater probabilities of small leak than the non-linear random process model. The linear random process model presents probabilities of small leak which are less than those for the non-linear random process model for almost the entire simulated time. So, in general, random process models are less conservative in estimating future failure probabilities. Similar observations can be made for the system probability of failure (Fig. 7d).

At last, it is observed that the linear corrosion growth models are so overly conservative that, even for a reliability analysis “starting 21 years later”, failure probabilities more than 20 years after are already virtually the same as for the less conservative non-linear models.

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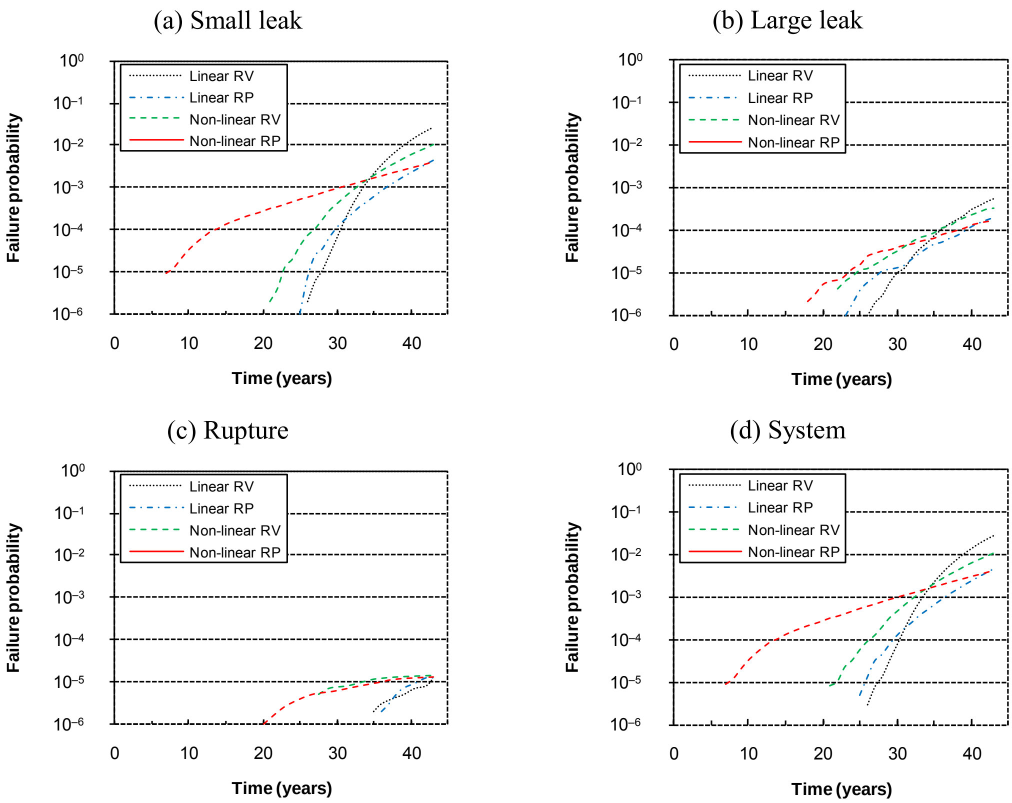

Figure 7. Time-varying failure probabilities for the example pipeline. Linear and non-linear, random variable and random process models. RV: random variable; RP: random process.

8. CONCLUDING REMARKS

This article proposed two novel random process corrosion growth models for buried pipelines. These models represent corrosion rate as pulses of a Poisson square wave process, and hence are able to represent the inherent time-variability of corrosion growth and produce continuous corrosion growth histories. A linear and a non-linear (power-law) version of the Poisson square wave corrosion rate model have been presented. The linear model suffers from the same limitation of linear random variable models, that is, the model is overly conservative and cannot be extrapolated back in time. The non-linear random process corrosion growth model was shown to precisely fit actual corrosion data for two inspections, and to represent problem physics much better (time to corrosion initiation and inherent time-variability of corrosion growth properly represented).

In application to an example pipeline problem, it was shown that, on the long run, linear models present conservative predictions of future failure probabilities, especially the random variable one. Conservativeness is fair to design and operate pipeline systems, but adversely affects solutions as the cost-effective optimization of inspection intervals. Results obtained herein emphasize the importance of calibrating prediction models to the appropriate amount of corrosion data, and also the importance of using appropriate uncertainty models for the corrosion growth process.

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