



MECHANICAL RESPONSE CHARACTERIZATION OF LIQUID-GAS MIXTURES UNDERGOING GASEOUS AND VAPOROUS CAVITATION

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Abstract. *This paper presents a consistent thermodynamics model to describe the occurrence of gaseous and vaporous cavitation in isothermal flows of compressible liquids. The fluid carries an inert gas in suspension as well as dissolved in the liquid. It is regarded as an homogeneous mixture of liquid, vapor and inert gas, which share the same fields of velocity and temperature. The governing equations are formed by the mass conservation equations for each constituent in both the liquid and gaseous phases along with a momentum conservation equation for the mixture as a whole. The void fraction is treated as a state variable and its restriction is properly incorporated in the constitutive relationships. The constitutive equations are derived within the framework of the Thermodynamics of Irreversible Processes by specifying two thermodynamical potentials: the Helmholtz free energy and a pseudo-potential of dissipation. The cavitation processes are consistently described as irreversible processes. By means of a simple numerical simulation, the influence of the gaseous and vaporous cavitation (together or isolatedly) on the mechanical response of the fluid is investigated when the mixture is subjected to a single expansion motion. The obtained results show that quite distinct physical behaviors are observed, when compared to the situation in which both cavitation mechanisms are accounted for.*

Keywords: *Vaporous cavitation, gaseous cavitation, thermodynamics of irreversible processes, internal variable constitutive theory, compressible fluids.*

1. INTRODUCTION

Liquids are known to be able to absorb certain amount of gas in contact with their free surfaces. Through a diffusive process, the gas dissolves in the liquid and stays there, unless its temperature is raised beyond the boiling point or its pressure is lowered below the saturation pressure of the dissolved gas. In this last process, usually referred to as gas release or, alternatively, gaseous cavitation, the gas comes out of solution as small bubbles that either are carried by the fluid flow stream as free gas or migrate towards the free surface or the upper parts of the systems where the liquid is confined, forming small and discrete cavities (Swaffield, 1972).

On the other hand, vaporous cavitation is the formation of the vapor phase in a liquid. It is distinguished from boiling in the sense that the former is generated by pressure reduction, whereas the latter refers to vapor formation as result of heat addition (Arndt, 1981).

Internal as well as external liquid flows may be subjected to vaporous and gaseous cavitation under steady or unsteady regimes. Whatever the cavitation process takes place, the cavitation region of the fluid flow is in general restricted to localized sites and may be induced by different aspects related to the fluid flow (Rood, 1991; Arndt, 1981). Among them, boundary curvature of solid surfaces in contact with the liquids (e.g. the runner blades in a turbine or pump), vortices, turbulence and transient expansion waves such as column separation in piping systems conveying liquids (Wylie and Streeter, 1993). Typical examples include water distribution systems, sewage networks, nuclear reactor piping systems and aviation fuel pipelines (Graham, 1969; Swaffield, 1972; Wylie and Streeter, 1993; Bergant *et al.*, 2006). Also, it is a relevant feature in structural failure prediction of pipelines transporting liquids (Freitas Rachid and Costa Mattos, 1998) and in the analysis of the response of submerged structures excited by transient acoustic waves (Fellipa and Deruntz, 1984). Due to its importance, cavitation has been the subject of extensive research in the past years.

Since the gaseous cavitation in general precedes the vaporous cavitation these phenomena should be analyzed simultaneously, in spite of having quite distinct time scales. However, in the great majority of these problems the cavitation phenomena are approached separately. Moreover, the mechanical models used to describe these physical phenomena are derived without appealing to any thermodynamic theory. For instance, in vaporous cavitation the pressure is assumed to remain constant at the saturated vapor pressure and the phase change process is considered to take place without presenting any kind of dissipation (Sumam *et al.*, 2010; Freitas Rachid, 2003). The same problem takes place with respect to the gaseous cavitation in which the Henry's law is extended to non-equilibrium states in order to derive the rate of mass transfer of gas (Chaudry *et al.*, 1990; Kessal and Bennacer, 2005).

To circumvent such a deficiency, it is proposed in this paper a logically consistent thermodynamic model to describe both the gaseous and the vaporous cavitation phenomena by means of an internal variable theory Freitas Rachid (2003); Borges and Freitas Rachid (2013). The fluid is regarded as a continuum mixture of a liquid, its vapor and an inert gas which coexist at a same material point and time instant. The no-slip condition between the phases is assumed as a basic assumption, so that only one balance equation of momentum is considered for the mixture as a whole. The temperature is supposed to be the same for both phases and the cavitation processes are assumed to be an isothermal irreversible transformation. To properly account for the irreversible character of the cavitation phenomena, the constitutive equations are derived within the framework of the Thermodynamics of Irreversible Processes. By means of a simple numerical simulation, the influence of the gaseous and vaporous cavitation (together or isolatedly) on the mechanical response of the fluid is investigated when the mixture is subjected to a single expansion motion.

2. BALANCE EQUATIONS AND THE SECOND LAW OF THERMODYNAMICS

In contrast to general two-phase fluid flows where the phases can assume very different geometrical configurations throughout the flow region (Graham, 1969; Ishii and Hibiki, 2006), vaporous as well as gaseous cavitation are localized phenomenon which takes place at discrete and small regions of the fluid flow. Therefore, it is reasonable to admit that the constituents in these phases have the same velocities. Also, by considering that both phases always share the same temperature during these phase transformations, it suffices to consider the balance equations of mass, momentum along with one version of the second law for the mixture as a whole, instead of doing it for each phase separately. Such approach is equivalent to assume that the fluid can be regarded as a pseudo-mixture of the liquid and gaseous phases with average properties. Under suitable regularity assumptions, and also by restricting the analysis to isothermal transformations ($\dot{\theta} = 0$ and $\nabla\theta = 0$; θ being the absolute temperature), only the following classical forms of the balance equations and a local version of the second law are required to describe the problem (Germain and Muller, 1995):

$$\dot{\rho} + \rho \nabla \cdot \mathbf{v} = 0, \quad (1)$$

$$\rho \dot{\mathbf{v}} = -\nabla p + \nabla \cdot \mathbf{S} + \rho \mathbf{g}, \quad (2)$$

$$d = -(p + \Psi) \nabla \cdot \mathbf{v} + \mathbf{S} \cdot \mathbf{D} - \dot{\Psi} \geq 0. \quad (3)$$

The above equations, in Eulerian coordinates, represent the balance of mass, the balance of linear momentum and the Clausius-Duhem inequality, respectively. As usual, the superimposed dot stands for the material time derivative, ρ is the mass density of the mixture, \mathbf{v} is the spatial velocity field, $\mathbf{S} = \mathbf{S}^T$ is the extra stress tensor due to motion, \mathbf{g} is the external body force per unit mass and $\mathbf{D} = \frac{1}{2}[\nabla \mathbf{v} + (\nabla \mathbf{v})^T]$ is the rate of deformation tensor. Since all of the constituents are assumed to be compressible, p stands for the thermodynamic pressure and, finally, Ψ represents the Helmholtz free energy of the mixture per unit volume. Equation (3) is a local version of the Second Law of the Thermodynamics (SLT) and requires that the rate of energy dissipation, d , be nonnegative. It establishes a distinction between possible ($d \geq 0$) processes and impossible ($d < 0$) ones. Processes that do not violate the second law are classed as reversible or non-dissipative ($d = 0$) and irreversible or dissipative ($d > 0$).

The balance equations presented so far are valid for the mixture as whole. From now on, it is assumed that the mixture is a two-phase, two-constituent homogeneous mixture. The gaseous phase is formed by the vapor of a liquid and a inert free gas, whereas the liquid phase is constituted of pure liquid along with dissolved gas. The concentration of dissolved gas in the liquid c is defined as being the quantity of mass of dissolved gas per unit volume of liquid. The liquid and gaseous phases are assumed to coexist at every material point and time instant. To take it into account, we consider an internal variable α which denotes the void volume fraction. The void volume fraction α , $\alpha \in (0, 1)$, is defined as being the ratio of the volume occupied by the vapor along with the inert gas in suspension and the total volume of the mixture. Thus, the mass density of the mixture can be expressed as:

$$\rho := (1 - \alpha)(\rho_l + c) + \alpha(\rho_v + \rho_g), \quad (4)$$

in which ρ_l , ρ_v and ρ_g stand for the mass densities of the liquid, vapor and the free gas, respectively, all of them assumed as being compressible fluids.

By taking into account the interchange of mass of gas as well as vapor between the liquid and gaseous phases along with Eq. (4), Eq. (1) can be splitted as follows:

$$(1 - \alpha)\dot{\rho}_l - \rho_l \dot{\alpha} + (1 - \alpha)\rho_l \nabla \cdot \mathbf{v} + \Gamma_v = 0, \quad (5)$$

$$(1 - \alpha)\dot{c} - c\dot{\alpha} + (1 - \alpha)c \nabla \cdot \mathbf{v} + \Gamma_g = 0, \quad (6)$$

$$\alpha \dot{\rho}_v + \rho_v \dot{\alpha} + \alpha \rho_v \nabla \cdot \mathbf{v} - \Gamma_v = 0, \quad (7)$$

$$\alpha \dot{\rho}_g + \rho_g \dot{\alpha} + \alpha \rho_g \nabla \cdot \mathbf{v} - \Gamma_g = 0, \quad (8)$$

in which Γ_g and Γ_v are associated with the gaseous and vaporous cavitation phenomena and represent the rate of mass transfer per unit volume of fluid of gas and vapor, respectively.

When $\Gamma_g > 0$, the gas dissolved in the liquid evolves in the fluid as free gas in suspension. On the other hand, if $\Gamma_g < 0$, then the free gas dissolves in the liquid. Finally, if $\Gamma_g = 0$, then there is no interchange of mass of gas between the liquid and gaseous phases and, consequently, the mass of dissolved gas in the liquid and the mass of free gas are conserved independent from each other. Analogously, when $\Gamma_v > 0$, the liquid transforms into vapor. On the other hand, if $\Gamma_v < 0$, then vapor is converted into liquid. However, if $\Gamma_v = 0$, then there is no interchange of mass of vapor between the liquid and gaseous phases and, consequently, the mass of liquid and the mass of vapor are conserved independent from each other.

Equations (2), (5-8) along with (3) form the fundamental balance equations to describe the gaseous and vaporous cavitation problem in homogeneous flows under isothermal transformations. To complete the problem description, we must add the constitutive relationships for the mixture. They encompass expressions for p , \mathbf{S} as well as for Γ_g and Γ_v , which must satisfy the SLT given by Eq. (3) no matter the external actions, the initial and the boundary conditions.

3. CONSTITUTIVE THEORY

The constitutive relations describing the macroscopic mechanical behavior of the mixture are derived in the framework of the Thermodynamics of Irreversible Processes. In this theory, once the local state of the material has been characterized by means of an appropriate choice of a set of state variables, two thermodynamical potentials - the Helmholtz free energy and a pseudo-potential of dissipation - are sufficient to derive a complete set of constitutive equations.

For this particular problem, we choose as state variables the local mass densities of the liquid ρ_l , the vapor ρ_v , the free gas ρ_g , the concentration of dissolved gas in the liquid c , the void fraction α and the absolute temperature θ . This specific choice restricts the validity of the forthcoming analysis to a particular class of Newtonian and generalized Newtonian fluids, since the list of state variables might incorporate other variables not considered herein as in the case of many non-Newtonian fluids.

As we shall see, the restriction associated with α is treated in this work as a physical property in the constitutive equations. This approach has already been used in the modeling of other problems in continuum mechanics.

3.1 Helmholtz Free Energy

Following the classic assumption of the Thermodynamics of Irreversible Processes, the free energy per total unit volume Ψ is supposed to be a function of the state variables. Since the fluid is regarded as a mixture of the two phases, its behavior is supposed to comprise a combination of the liquid and vapor thermo mechanical properties. Thus, the following form is proposed:

$$\Psi(\rho_l, \rho_v, \rho_g, c, \alpha, \theta) := \Psi'(\rho_l, \rho_v, \rho_g, c, \alpha, \theta) + \bar{I}(\alpha), \quad (9)$$

in which

$$\Psi'(\rho_l, \rho_v, \rho_g, c, \alpha, \theta) := (1 - \alpha)\rho_l\Psi_l(\rho_l, \theta) + (1 - \alpha)c\Psi_c(\theta) + \alpha\rho_v\Psi_v(\rho_v, \theta) + \alpha\rho_g\Psi_g(\rho_g, \theta), \quad (10)$$

$$\Psi_l(\rho_l, \theta) := -C_l\theta\log\theta + a_l^2(\log\rho_l + \rho_l^o/\rho_l), \quad (11)$$

$$\Psi_c(c, \theta) := -C_g\theta\log\theta + a_g^2\log(p_s/a_g^2) + a_g^2, \quad (12)$$

$$\Psi_v(\rho_v, \theta) := -C_v\theta\log\theta + a_v^2\log\rho_v + L, \quad (13)$$

$$\Psi_g(\rho_g, \theta) := -C_g\theta\log\theta + a_g^2\log\rho_g, \quad (14)$$

$$\bar{I}(\alpha) := \begin{cases} 0 & , \text{if } \alpha \in (0, 1) \\ +\infty & , \text{otherwise.} \end{cases} \quad (15)$$

In the above expressions, Ψ_i , with $i \in \{l, c, v, g\}$, represent the free energies per unit mass of the liquid, dissolved gas, vapor and free gas, respectively. The terms C_i , with $i \in \{l, v, g\}$, are the specific heats at constant volume of the liquid, the vapor and the free gas, respectively. The parameters $a_i^2 = \hat{a}_i^2(\theta)$, with $i \in \{l, v, g\}$, are the isothermal sonic speeds, whereas $\rho_l^o = \hat{\rho}_l^o(\theta)$ and $L = \hat{L}(\theta)$ are positive material functions which depend on the temperature. The function \hat{L} is associated with the phase change latent heat. The choice of the free energies Ψ_l and Ψ_v were done in order to approximate, for a fixed θ far below from the critical temperature, the liquid and vapor behaviors, respectively, in the subcooled-liquid and superheated-vapor regions of a simple compressible substance.

The term $\bar{I}(\alpha)$ is the indicator function of the convex set $(0, 1) \subset \mathbb{R}$. It is included in the free energy potential to take the internal constraint of the mixture into account as a constitutive assumption. In other words, the term $\bar{I}(\alpha)$ prevents α from getting out of its admissible interval since it would be required an infinite amount of energy to do this.

The state laws, relating the reversible components of the thermodynamic forces to the state variables, are obtained from the free energy potential as follows:

$$B^{\rho_l} := \frac{\partial\Psi'}{\partial\rho_l} = (1 - \alpha)(\Psi_l + \frac{p_l}{\rho_l}) = (1 - \alpha)g_l, \quad (16)$$

$$B^c := \frac{\partial \Psi'}{\partial c} = (1 - \alpha)\Psi_c, \quad (17)$$

$$B^{\rho_v} := \frac{\partial \Psi'}{\partial \rho_v} = \alpha\left(\Psi_v + \frac{p_v}{\rho_v}\right) = \alpha g_v, \quad (18)$$

$$B^{\rho_g} := \frac{\partial \Psi'}{\partial \rho_g} = \alpha\left(\Psi_v + \frac{p_g}{\rho_g}\right) = \alpha g_g, \quad (19)$$

$$B^\alpha := \frac{\partial \Psi'}{\partial \alpha} + h, \text{ with } h \in \partial \bar{I}(\alpha) \quad (20)$$

in which

$$p_l := \rho_l^2 \frac{\partial \Psi_l}{\partial \rho_l}, \quad p_v := \rho_v^2 \frac{\partial \Psi_v}{\partial \rho_v}, \quad p_g := \rho_g^2 \frac{\partial \Psi_g}{\partial \rho_g}, \quad (21)$$

$$\frac{\partial \Psi'}{\partial \alpha} = \rho_v \Psi_v + \rho_g \Psi_g - \rho_l \Psi_l - c \Psi_c. \quad (22)$$

In the above equations, p_i and g_i , with $i \in \{l, v, g\}$ stand for the partial pressures and Gibbs specific free energies of the constituents in the mixture, respectively. The term $\partial \bar{I}(\alpha)$ in Eq. (20) is the subdifferential of the indicator function $\bar{I}(\alpha)$ at α , which is given by the set (Moreau *et al.*, 1988; Ekeland and Teman, 1976);

$$\partial \bar{I}(\alpha) = \{h \in \mathbb{R} \mid \bar{I}(\alpha^*) - \bar{I}(\alpha) \geq h(\alpha^* - \alpha); \quad \forall \alpha^* \in (0, 1)\}. \quad (23)$$

From the above definition, we can see that $\partial \bar{I}(0 < \alpha < 1) = \{0\}$ and $\partial \bar{I}(\alpha) = \emptyset$ if $\alpha \ni (0, 1)$.

Once we have chosen a specific form for Ψ we are able to compute its material time derivative $\dot{\Psi}$ which appears in Eq. (3):

$$\dot{\Psi} = \frac{\partial \Psi_l}{\partial \rho_l} \dot{\rho}_l + \frac{\partial \Psi_c}{\partial c} \dot{c} + \frac{\partial \Psi_v}{\partial \rho_v} \dot{\rho}_v + \frac{\partial \Psi_g}{\partial \rho_g} \dot{\rho}_g + (\rho_v \Psi_v + \rho_g \Psi_g - \rho_l \Psi_l - c \Psi_c) \dot{\alpha} + \lim_{\substack{\Delta t \rightarrow 0 \\ \Delta t > 0}} \frac{\bar{I}(\alpha(t)) - \bar{I}(\alpha(t - \Delta t))}{\Delta t}. \quad (24)$$

In computing $\dot{\Psi}$ care should be taken since Ψ is not a smooth function. By using the definition of the subdifferential Eq. (23) it can be shown that:

$$\lim_{\substack{\Delta t \rightarrow 0 \\ \Delta t > 0}} \frac{\bar{I}(\alpha(t)) - \bar{I}(\alpha(t - \Delta t))}{\Delta t} \leq h \dot{\alpha} = 0, \quad \forall h \in \partial \bar{I}(\alpha(t)). \quad (25)$$

When the above result is used in Eq. (3) along with the state laws Eqs. (16-20), Eq. (3) can be written as:

$$d := -(p + \Psi) \nabla \cdot \mathbf{v} + \mathbf{S} \cdot \mathbf{D} - B^{\rho_l} \dot{\rho}_l - B^c \dot{c} - B^{\rho_v} \dot{\rho}_v - B^{\rho_g} \dot{\rho}_g - B^\alpha \dot{\alpha}. \quad (26)$$

To obtain a complete set of constitutive equations, it suffices to specify a pseudo-potential of dissipation from which complementary laws are derived in such a way that the local version of the second law Eq. (3) or Eq. (26) is always verified, regardless the initial and boundary conditions.

3.2 Pseudo-Potential of Dissipation

To assign a dissipative character to the mixture, and also to ensure that the SLT is always satisfied, we assume that the pseudo-potential of dissipation Φ is an objective, convex and differentiable function of \mathbf{D} , $\dot{\alpha}$, B^{Γ_g} and B^{Γ_v} , i.e. $\Phi(\mathbf{D}, \dot{\alpha}, B^{\Gamma_g}, B^{\Gamma_v}; \alpha, \theta)$, where α and θ are considered to be parameters, with the following properties:

$$\Phi(\mathbf{D}, \dot{\alpha}, B^{\Gamma_g}, B^{\Gamma_v}; \alpha, \theta) \geq 0, \quad \forall (\mathbf{D}, \dot{\alpha}, B^{\Gamma_g}, B^{\Gamma_v}) \quad \text{and} \quad \Phi(\mathbf{0}, 0, 0, 0; \alpha, \theta) = 0. \quad (27)$$

The additional information associated with the dissipative behavior can be obtained from Φ through the following complementary laws:

$$\mathbf{S} := \frac{\partial \Phi}{\partial \mathbf{D}}, \quad B^{\dot{\alpha}} := \frac{\partial \Phi}{\partial \dot{\alpha}}, \quad \Gamma_g := \frac{\partial \Phi}{\partial B^{\Gamma_g}}, \quad \Gamma_v := \frac{\partial \Phi}{\partial B^{\Gamma_v}}, \quad (28)$$

in which B^{Γ_g} and B^{Γ_v} are the irreversible thermodynamic forces associated with the gaseous and vaporous cavitation and $B^{\dot{\alpha}}$ the irreversible thermodynamic force associated with the evolution of the gaseous volume fraction.

In addition, if we choose the rate of the energy dissipation d as being given by;

$$d := \mathbf{S} \cdot \mathbf{D} + B^{\dot{\alpha}} \dot{\alpha} + B^{\Gamma_g} \Gamma_g + B^{\Gamma_v} \Gamma_v - \bar{I}(\alpha) \mathbf{D} \cdot \mathbf{I}, \quad (29)$$

for any actual evolution, with \mathbf{I} being the second-order identity tensor, then we get from the convexity property of Φ that:

$$d := \mathbf{S} \cdot \mathbf{D} + B^{\dot{\alpha}} \dot{\alpha} + B^{\Gamma_g} \Gamma_g + B^{\Gamma_v} \Gamma_v \geq \Phi(\mathbf{D}, \dot{\alpha}, B^{\Gamma_g}, B^{\Gamma_v}; \alpha, \theta) - \Phi(\mathbf{0}, 0, 0, 0; \alpha, \theta) \geq 0, \quad (30)$$

for all $(\mathbf{D}, \dot{\alpha}, B^{\Gamma_g}, B^{\Gamma_v})$ and $\alpha \in (0, 1)$.

In view of Eq. (30), it is easy to see that $d \geq 0$ for any actual evolution and so the SLT expressed by Eq. (3) is always satisfied. It is worth noting that the last term in the right-hand side of Eq. (29) was introduced to give the model a thermodynamically consistent character. If α could be out of the interval $(0, 1)$, then the dissipation would become negative or infinite what is not possible. It is important to remark that Φ does not need to be a convex function. The convexity property is only a sufficient condition in order to satisfy Eq. (30).

From the mechanical viewpoint, Eq. (29) establishes that the rate of energy dissipation is the sum of four parcels. The first parcel in the right-hand side of Eq. (29) is associated with viscous effects due to motion of the mixture as whole. The other three parcels reflect internal changes in the mixture. The second and third terms express the dissipative mechanisms associated with the gaseous and vaporous cavitation and with viscous dissipation in the liquid, as a result of the expansional-compressional motion of the gaseous phase.

3.3 Constitutive Laws

The set of constitutive equations is completed provided a specific form of Φ is chosen. Before specifying a form for Φ , however, it should be noticed that the thermodynamic forces B^{ρ_l} , B^c , B^{ρ_v} , B^{ρ_g} , B^α , B^{Γ_g} , B^{Γ_v} and $B^{\dot{\alpha}}$ are not independent. There exists an intrinsic relationship among them which is a consequence of the mass balance conservation equations and the choice of the rate of energy dissipation given by Eq. (29). As we shall see, there exists an expression for the thermodynamic pressure of the mixture which is a consequence of the constitutive assumptions made so far.

In effect, for any actual evolution of the mixture with mass density given by Eq. (4), the mass balance expressed by Eqs. (5-8) defines a subspace of the linear space spanned by \mathbf{v} , $\dot{\rho}_l$, \dot{c} , $\dot{\rho}_v$, $\dot{\rho}_g$ and $\dot{\alpha}$. So, in order that Eq. (3) be equal to Eq. (29) one must have:

$$\begin{aligned} -(p + \Psi)\nabla \cdot \mathbf{v} - B^{\rho_l}\dot{\rho}_l - B^c\dot{c} - B^{\rho_v}\dot{\rho}_v - B^{\rho_g}\dot{\rho}_g - B^\alpha\dot{\alpha} &= B^{\Gamma_v}(\dot{\alpha}\rho_v + \alpha\dot{\rho}_v + \alpha\rho_v\nabla \cdot \mathbf{v}) + \\ & B^{\Gamma_g}(\alpha\dot{\rho}_g + \rho_g\dot{\alpha} + \alpha\rho_g\nabla \cdot \mathbf{v}) + B^{\dot{\alpha}}\dot{\alpha} - \bar{I}(\alpha)\nabla \cdot \mathbf{v} + \\ & \eta((\rho_v - \rho_l)\dot{\alpha} + (1 - \alpha)\dot{\rho}_l + \alpha\dot{\rho}_v + ((1 - \alpha)\rho_l + \alpha\rho_v)\nabla \cdot \mathbf{v}) + \\ & \xi((\rho_g - c)\dot{\alpha} + (1 - \alpha)\dot{c} + \alpha\dot{\rho}_g + ((1 - \alpha)c + \alpha\rho_g)\nabla \cdot \mathbf{v}), \end{aligned} \quad (31)$$

in which η and ξ are Lagrange multipliers. Since Eq. (31) holds for any actual independent evolution of \mathbf{v} , $\dot{\rho}_l$, \dot{c} , $\dot{\rho}_v$, $\dot{\rho}_g$ and $\dot{\alpha}$ and $\alpha \in (0, 1)$, the Lagrange multipliers η and ξ can be eliminated from the above equations giving rise to the following relationships:

$$p = (1 - \alpha)p_l + \alpha(p_v + p_g), \quad (32)$$

$$B^{\dot{\alpha}} = p_v + p_g - p_l, \quad (33)$$

$$B^{\Gamma_v} = g_l - g_v, \quad (34)$$

$$B^{\Gamma_g} = \Psi_c - g_g. \quad (35)$$

Equations(32-35) state that the thermodynamic pressure of the mixture is the mean value between the liquid, the gas and vapor pressures, the thermodynamic force associated with the void fraction is the difference between the sum of the gas and vapor pressures and the liquid pressure, the thermodynamic driving force associated with vaporous cavitation is the Gibbs free energy difference between the liquid and vapor and, finally, the the thermodynamic driving force associated with gaseous cavitation is the difference between the Helmholtz free energy of the dissolved gas and the Gibbs free energy of the free gas.

Without loosing generality, if we assume the mixture behaves as a Newtonian fluid one possible choice for Φ is:

$$\Phi(\mathbf{D}, \dot{\alpha}, B^{\Gamma_g}, B^{\Gamma_v}; \alpha, \theta) = \frac{\lambda}{2}(\mathbf{D} \cdot \mathbf{I})^2 + \mu \mathbf{D} \cdot \mathbf{D} + \frac{\beta_v}{2}(B^{\Gamma_v})^2 + \frac{\beta_g}{2n_g}\langle B^{\Gamma_g} \rangle^{2n_g} + \frac{\epsilon}{2}\dot{\alpha}^2 \quad (36)$$

where $\beta_v = \hat{\beta}_v(\alpha, \theta)$, $\beta_g = \hat{\beta}_g(\alpha, \theta)$, $n_g = \hat{n}_g(\alpha, \theta)$, with $n_g > 1/2$, and $\epsilon = \hat{\epsilon}(\alpha, \theta)$ are positive generic functions of α and θ . The term $\langle \chi \rangle$ stands for the maximum value between zero and χ , i. e., $\langle \chi \rangle := \max\{0, \chi\}$.

The material parameters $\lambda = \hat{\lambda}(\alpha, \theta)$ and $\mu = \hat{\mu}(\alpha, \theta)$ are average properties between, respectively, $\lambda_l(\theta)$ and $\lambda_v(\theta)$ and $\mu_l(\theta)$ and $\mu_v(\theta)$ having α as a weighting factor, such that $\hat{\lambda}(\alpha = 0, \theta) = \lambda_l(\theta)$, $\hat{\lambda}(\alpha = 1, \theta) = \lambda_v(\theta)$, $\hat{\mu}(\alpha = 0, \theta) = \mu_l(\theta)$ and $\hat{\mu}(\alpha = 1, \theta) = \mu_v(\theta)$. These material parameters must be such that $\mu \geq 0$ and $\lambda + 2/3\mu \geq 0$ in order to satisfy the SLT given by Eq. (3). For this specific choice, it is possible to porve that the state and complementary laws are reduced to:

$$\mathbf{T} = -p\mathbf{I} + \lambda(\mathbf{D} \cdot \mathbf{I}) + 2\mu\mathbf{D}, \quad (37)$$

$$p = (1 - \alpha)p_l + \alpha(p_v + p_g), \quad (38)$$

$$\Gamma_v = \beta_v (g_l - g_v), \quad (39)$$

$$\Gamma_g = \beta_r (a_g^2)^{2n_g-1} \left(\log \frac{p_s}{p} \right)^{2n_g-1}, \quad (40)$$

$$\epsilon \dot{\alpha} = p_v + p_g - p_l, \quad (41)$$

in which

$$p_l = a_l^2 (\rho_l - \rho_l^o), \quad p_v = a_v^2 \rho_v, \quad p_g = a_g^2 \rho_g, \quad (42)$$

$$g_l - g_v = a_l^2 \log \left(\frac{p_l + a_l^2 \rho_l^o}{p_{sv} + a_l^2 \rho_l^o} \right) + a_v^2 \log \frac{p_{sv}}{p_v} + a_l^4 \rho_l^o \left(\frac{1}{p_{sv} + a_l^2 \rho_l^o} - \frac{1}{p_l + a_l^2 \rho_l^o} \right), \quad (43)$$

$$\beta_g = (\text{Re})^{0.86} (a_g^2)^{-(2n_g-1)} \beta_g', \quad (44)$$

where $p_{sv} = \hat{p}_{sv}(\theta)$ stands for the saturated vapor pressure, $p_s = \hat{p}_s(\theta)$ represents the saturated pressure of the dissolved gas into the liquid, $\text{Re} = UL/\nu_l$ (with U and L being, respectively, one characteristic velocity and length of the fluid flow and ν_l the kinematic viscosity of the liquid) the Reynolds number of reference associated with the liquid flow and β_g' a material parameter.

Equations (2), (5-8) and (37-42) completed by suitable boundary and initial conditions describe the isothermal flow of a compressible Newtonian fluid subjected to the occurrence of both gaseous and vaporous cavitation. Since the constitutive relation for a Newtonian fluid is well-known, we shall from now on focus attention on influence of the the gaseous and vaporous cavitation on the mechanical response of the fluid.

4. MECHANICAL RESPONSE OF THE FLUID UNDER GASEOUS AND/OR VAPOROUS CAVITATION

To investigate the influence of the vaporous and gaseous cavitation on the mechanical response of the fluid, numerical simulations are carried out with the proposed model by considering the occurrence of these phenomena together or separately. To simplify the interpretation of the results and to reduce the number of material parameters to be specified, we shall consider that the vaporous and gaseous cavitation are the only dissipative mechanisms present. In other words, we admit that $\epsilon = 0$, so that there is no dissipation associated with the expansion-compression of the void fraction. Therefore, by virtue of Eqs. (38) and (41), $p = p_l = p_v + p_g$ for all time instants and $\alpha \in (0, 1)$.

For this purpose, we consider a piston-cylinder system, whose cross-sectional area is A and initial volume is V_o . It is filled with a sample liquid-vapor-gas of water-air with dissolved gas at at $293K$. The mixture can have its original volume expanded and contracted along the time by means of external actions. It is further assumed that, at any time instant the mixture is homogeneous throughout its volume, so that the convective terms of the balance equations of mass can be neglected. Under these circumstances, the time rate of the volume change per unit volume of the whole mixture within the system is described by the term $\nabla \cdot \mathbf{v}$, which is assumed to be a prescribed input function of the time (see Fig. (1)). Under these assumptions, the balance equations of mass given by Eqs. (5-8) can be reduced to a set of ordinary differential equations in terms of the time instant t for the mixture pressure $p = p_l$, the vapor pressure p_v , the void fraction α and the concentration of dissolved gas c . This problem is numerically solved by using the Gear's method; an adaptive numerical method up to fifth-order suitable for solving stiff ordinary differential equations. Time-steps of the order of 1 ms were used with an accuracy of 10^{-6} .

Table 1. Material parameters of the mixture of water-air at $\theta = 293K$.

| Material coefficients | Water | Air |
|---|----------------------|------------------------|
| a_l (m/s) | 1477 | - |
| a_v (m/s) | 367 | - |
| a_g (m/s) | - | 290 |
| ρ_l^o (kg/m ³) | 998.0029 | - |
| p_{sv} (kPa) | 2.34 | - |
| p_s (kPa) | - | 125.0 |
| β_v (kg ² /(m ³ W)) | 1.0×10^{-8} | - |
| β_g' (kgm ³ /s) | - | 8.77×10^{-12} |
| n_g (-) | - | 2.21 |
| ϵ (kPa/s) | 0 | 0 |

The sample of liquid-vapor-gas mixture of water-air is in equilibrium at a total initial pressure $p(t = 0) = p_o$ and initial vapor pressure $p_v(t = 0) = p_{vo}$, such that $p_o > p_{sv}$ and also $p_o > p_s$ (at $\theta = 293$ K) having, at $t = 0$, a void volume fraction of α_o . The motion induced by the piston generates a triangular single expansion stroke of duration $2\Delta t$ (see Fig. (1)), rendering a total volumetric deformation $\Delta V = AU\Delta t$, being U the maximum velocity reached by

the piston. The constitutive constants of the model for the mixture liquid-vapor-gas of water-air at 293 K are shown in Tab. 1 (Freitas Rachid, 2003; Borges and Freitas Rachid, 2013) and the initial values and geometrical parameters of the piston-cylinder system are presented in Tab. 2.

Table 2. Geometrical parameters of the system and initial conditions.

| Parameters | Values |
|----------------------------|----------------------|
| A (m ²) | 0.1964 |
| V_o (m ³) | 0.589 |
| Δt (s) | 0.5 |
| U (m/s) | 0.3 |
| Re (-) | 1.49×10^5 |
| $\Delta V/V_o$ (%) | 4.9 |
| p_o (kPa) | 506.625 |
| p_{vo} (kPa) | 2.3566 |
| α_o (-) | 1.0×10^{-7} |
| c_o (kg/m ³) | 0.050 |

The void fraction response as a function of the time to the input function $\nabla \cdot \mathbf{v}$ is shown, along with this input function, in Fig. 1 for the four different situations regarding the combination of gaseous and vaporous cavitation models: gaseous and vaporous cavitation, vaporous cavitation only, gaseous cavitation only and no cavitation at all. The expansion promoted by the volumetric deformation induced by $\nabla \cdot \mathbf{v}$ lowers pressure p to a value that both mechanisms of cavitation (the vaporous and the gaseous) are activated. Since $c \ll \rho_l$, as well as $\rho_v \ll \rho_l$ and $\rho_g \ll \rho_l$ during the time span of the simulation for all the four cases, the void fraction evolution is the same in all cases and is solely impacted by the input function $\nabla \cdot \mathbf{v}$. At the end of the first second, the input function has caused an overall volume deformation $\Delta V/V_o$ of 5% approximately which is kept constant from that instant on. This volume deformation causes the void fraction to jump down abruptly from its initial value of 1.0×10^{-7} to 5.0×10^{-2} , as it can be seen in Fig. 1.

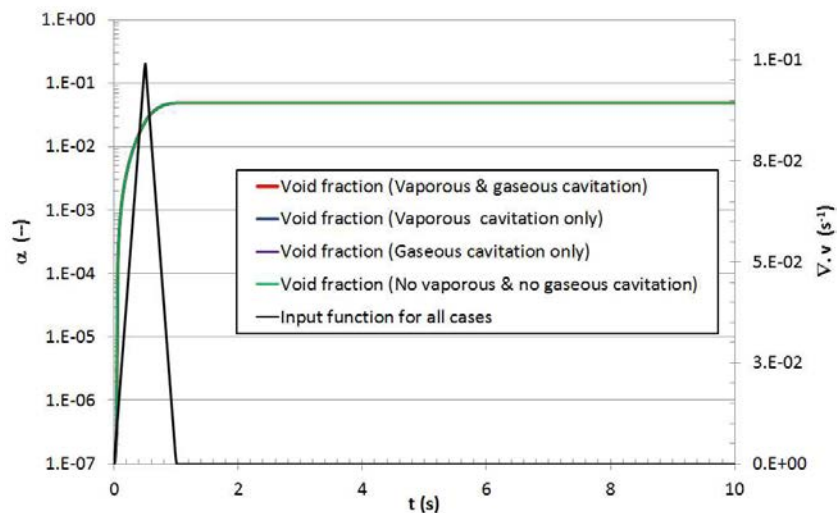


Figure 1. Input function $\nabla \cdot \mathbf{v}$ and void volume fraction vs time for all cavitation models.

The dimensionless pressure histories for the mixture and vapor are presented in Fig. 2 for both models with and without gaseous and vaporous cavitation. As it can be seen in Fig. 2, the absence of both mechanisms of cavitation is responsible for quite different responses of the mixture pressure as well as the vapor pressure. When no cavitation is considered, the pressures p and p_v reach values of the order of 10^{-6} of their reference values p_o and p_{vo} (see Tab. 2). On the other hand, when the gaseous and vaporous cavitation are taken into account the vapor pressure reaches a minimum of the order of 0.6% of p_{sv} around $t = 1$ s and soon after $t = 4$ s the vapor pressure has practically restored to a value close to its initial value p_{vo} (see Tab. 2). The mixture pressure also attains a minimum around $t = 1$ s. However, in contrast to the vapor pressure the mixture pressure does not evolve to a constant value but keep growing steadily little by little as a result of the gaseous cavitation. This behavior is a direct consequence of the quite distinct time scales of the vaporous and gaseous cavitation. Whereas the former takes place in a time scale of the order of the input excitation (in this case of the order of one second) the other phenomenon can last up to hundreds or thousands of seconds.

In order to better identify the influence of both cavitation phenomena on the fluid response, the dimensionless pressure

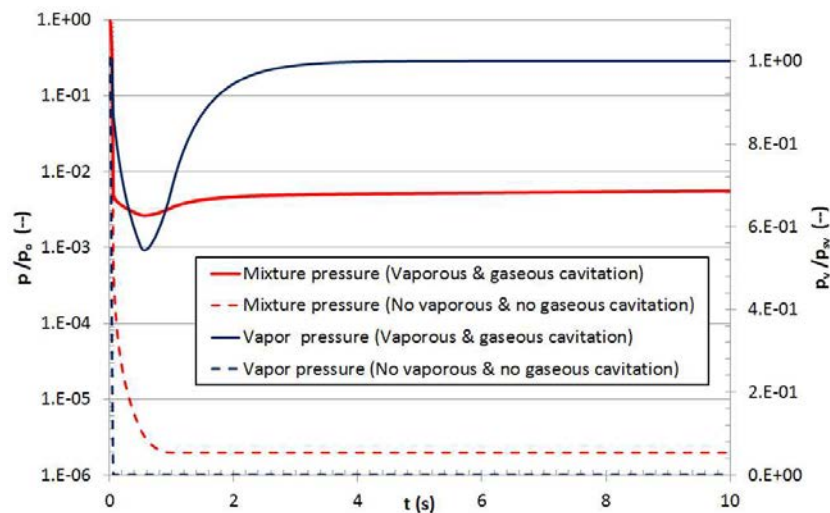


Figure 2. Mixture and vapor pressure vs time for the models with and without vaporous and gaseous cavitation.

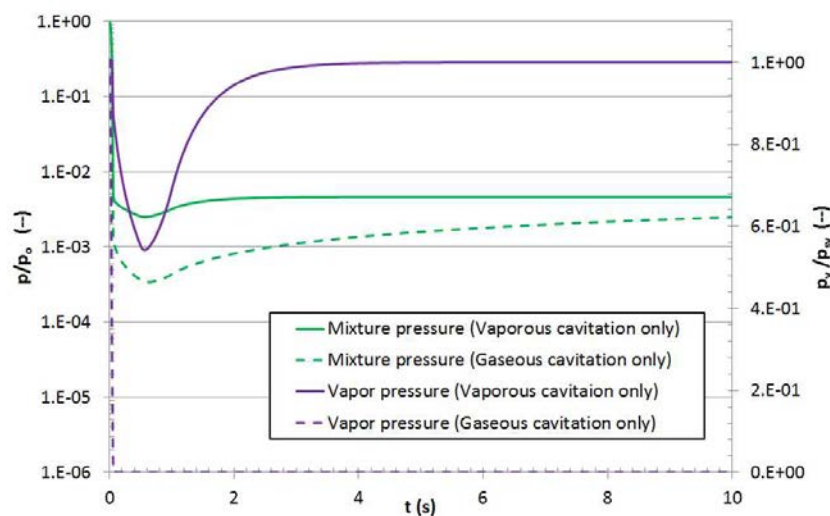


Figure 3. Mixture and vapor pressure vs time for the models either vaporous or gaseous cavitation.

histories associated with the other situations in which the gaseous and vaporous cavitation are considered isolatedly are presented in Fig. 3. By comparing the responses of the mixture pressure and the vapor pressure when only the vaporous cavitation is considered with those when both cavitation phenomena exist (see Fig. 2), it can be seen that almost no difference is observed in these pressure responses. In other words, the effect of the gaseous cavitation on the pressure histories for the mixture and the vapor is practically irrelevant. The same can not be said with respect to the vaporous cavitation, inasmuch as the pressure histories when only the gaseous cavitation phenomenon exists is completely different from the most real situation when both cavitation phenomena are susceptible to take place. Although the prediction of the mixture pressure is not far away from the most real situation in the model with gaseous cavitation only, the predicted behavior of the vapor pressure is severely compromised, as it can be noticed in Fig. 3. By looking at the mixture pressure responses when only one cavitation phenomenon is considered, it becomes clear that the pressure recovery phenomenon is an intrinsically feature associated with solely the gaseous cavitation (see Fig. 3).

Figure 4 exhibits the time histories of the rate of mass transfer of vapor and gas for the four cases: with both gaseous and vaporous cavitation, with gaseous cavitation only, with vaporous cavitation only and without any kind of cavitation. These plots corroborate the assertive made in the previous paragraph regarding the influence of the vaporous cavitation on the gaseous cavitation. Since the responses of the rate of mass transfer of vapor are practically superimposed when both cavitation phenomena are considered and when only the vaporous cavitation process is admitted, it can be inferred that the gaseous cavitation does not affect the mass transfer process associated with the vaporous cavitation. The reciprocal is not true. Quite different behaviors are observed for the time histories of mass transfer of gas when both phenomena are considered and when only the gaseous cavitation is taken into account. In the absence of vaporous cavitation, the predicted rate of gas mass transfer is overestimated by a factor of approximately ten times. It is worthwhile to note that since the mixture pressure remains above the saturation pressure of the dissolved gas in the liquid, i. e. $p(t) > p_s$ for

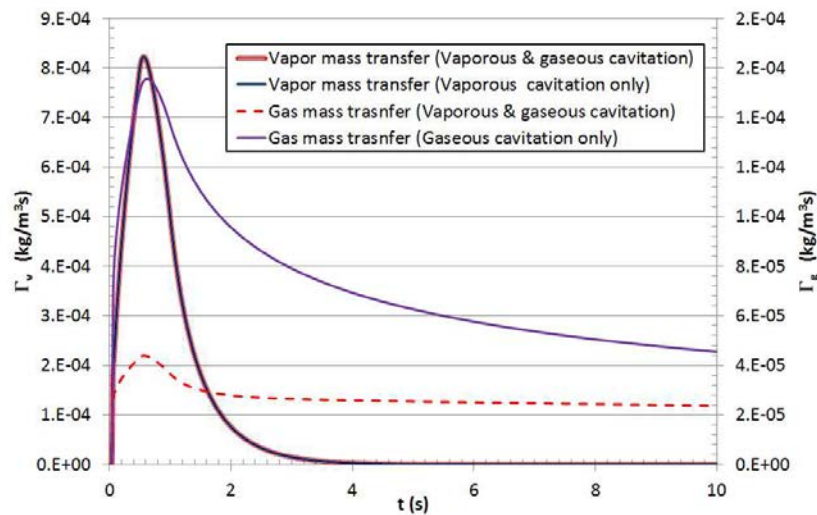


Figure 4. Rate of vapor and gas mass transfer for all cavitation models.

$t > 0$, the gas dissolved in the liquid is continuously transformed in free gas since Γ_g does not come to zero. In contrast, the vapor production ceases soon after $t = 4$ s approximately. At the end of this time period, a total amount corresponding to $8.55 \times 10^{-4} \text{ kg/m}^3$ of liquid has been transformed into vapor.

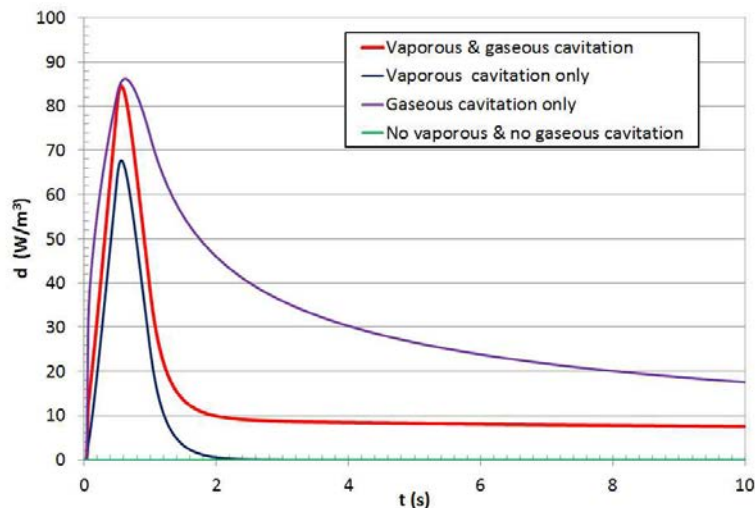


Figure 5. Rate of energy dissipation for all cavitation models.

Finally, it is displayed in Fig. 5 the plot of the rate of energy dissipation, $d = B^{\Gamma_g}\Gamma_g + B^{\Gamma_v}\Gamma_v$, as a function of time for every combination of the gaseous and vaporous cavitation models. As it is expected, the situation with no gaseous nor vaporous cavitation does not present any dissipation at all. By taking as reference the situation in which both cavitation models are accounted for, one can clearly see that the rate of energy dissipated is underestimated when the vaporous cavitation is considered alone and overestimated when only the gaseous cavitation is admitted. Since the energy dissipation represents the area below the curve, a simple calculation reveals that at the end of the period of simulation ($t = 10$ s) amounts energy equivalent to 46 J/m^3 , 136 J/m^3 and 333 J/m^3 have been dissipated when only the vaporous cavitation is considered, both gaseous and vaporous cavitation are admitted and only gaseous cavitation is taken into account, respectively.

5. CONCLUSIONS

A coherent thermodynamic model for describing gaseous and vaporous cavitation in isothermal fluid flows has been proposed in this paper. The fluid is regarded as being a mixture of liquid, vapor and an inert gas, all of them treated as compressible constituents, which coexist at a same material point and time instant. The void fraction is considered an internal variable and its constraint is treated as a material property. Dissipative effects associated with both gaseous and vaporous cavitation as well as with the void fraction evolution are taken into account in the model within the context of the

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Thermodynamics of Irreversible Processes. By means of simple numerical simulations, the influence of each cavitation mechanism on the mechanical response of the fluid is assessed when the fluid is subjected to a single expansion motion. The obtained results show that if no cavitation or if only the gaseous cavitation is considered, quite distinct behaviors of the fluid response are observed in comparison to the case in which both gaseous and vaporous cavitation are taken into account. Moreover, it is shown that, in spite of the time scales of the cavitation processes be quite different, the vaporous cavitation significantly alter the evolution of the gaseous cavitation, even though the reciprocate is no true. Since only the gaseous cavitation considerably overestimates the energy dissipation, the use of solely this mechanism in simulations should be avoided whenever accurate simulations are sought. The model simplicity and its capability to properly describe real situations enable its usage as a promising tool in the physical situations in which both the vaporous cavitation and the gaseous cavitation are likely to occur.

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