ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

THEORETICAL AND EXPERIMENTAL STUDY OF A HYPERELASTIC POLYMER UNDER PURE SHEAR

Fernando S. Araújo, fernandoaraujo083@gmail.com
Postgraduate Program of Mechanical Engineering, PGMEC
Universidade Federal Fluminense, UFF
Rua Passo da Patria 156, 24210-240 Niterói, RJ, Brazil
Luiz C. S. Nunes, luizcsn@mec.uff.br
Laboratory of Opto-Mechanics (LOM/LMTA)
Department of Mechanical Engineering, TEM/PGMEC
Universidade Federal Fluminense, UFF
Rua Passo da Patria 156, 24210-240 Niterói, RJ, Brazil

Abstract. *The aim of this study is to investigate the mechanical behavior of a hyperelastic polymer under pure shear condition obtained through the planar tension testing. The material used for manufacturing the rectangular thin sheet specimens was an adhesive based on silane modified polymer (FlextecFT101). The full field displacement was measured by means of the Digital Image Correlation (DIC) method. In addition, classical hyperelastic constitutive models based on strain-energy functions that are available in the literature were used. The stress-stretch relations were experimentally achieved. Finally, the experimental results indicated a nonlinear stress–stretch behavior and theoretical models were fitted to the measured data of stress versus amount of shear in order to determine the initial shear modulus.*

Keywords*: Hyperelastic material, Pure shear, Digital image correlation method*

1. INTRODUCTION

Elastomeric materials have been applied in many engineering areas such as automotive parts like tires, engine and transmission mounts, center bearing supports, and exhaust rubber parts, seals systems and so on. Nowadays, the design of these highly technical parts necessitates the use of simulation tools such as finite element softwares and the mechanical proprieties knowledge. In this context, an appropriate constitutive model is an essential prerequisite for good numerical predictions (Marckmann and Verron, 2006).

The mechanical properties of a perfectly elastic material may conveniently be represented in terms of the strain energy W per unit volume for a pure homogeneous strain which must be a function of two strain invariants I_1 and I_2. These invariants are expressible in terms of the principal extension rations λ_1, λ_2 e λ_3 (Treloar,1943; Jones and Treloar,1975; Rivlin and Saunders,1951).

For material characterization, different types of mechanical tests may be performed on rubbers. In order to provide pure shear on a thin sheet of rubber-like material, a planar shear test was carried out by Treloar (1943). Rivlin and Saunders (1951) developed an experimental and theoretical investigation on pure shear. The characterization of hyperelastic rubber-like materials by means of planar testing has also been performed by Sasso, *et al*.,2008. Initial shear modulus of a hyperelastic material has been estimated by means of simple shear test (Nunes, 2010). Besides, standard ISO 1827 specifies the method for the determination of the shear modulus for rubber.

The scope of the present work focuses on mechanical behavior of a hyperelastic polymer under pure shear condition obtained through the planar tension testing. Moreover, the stretch values were measured by means of DIC method. The material parameters, taking into account experimental data and the most well-known models in the literature for rubber such as Mooney-Rivlin, Yeoh, Arruda-Boyce, Ogden and Vargas, were estimated using Levenberg-Marquardt method implemented into Matlab routines.

2. EXPERIMENTAL PROCEDURE

In order to provide pure shear, a planar shear test was carried out (Treloar, 1943; Treloar, 2005; Rivlin and Saunders, 1951). This test was based on a rectangular sheet of FT 101 adhesive under tension in its plane normal to the clamped edges. Figure 1 illustrates the rectangular specimen under tension. Thin sheets with dimensions of 150x70x3.4 mm^3 were employed, the effective area being 150x10 mm^2 due to the clamped edges. It is important to emphasize that the width of the effective area was at least 10 times greater than the length in the stretching direction. As a result, the specimen must remain perfectly constrained in the lateral direction while specimen thinning occurs only in the thickness direction. Experimental tests were performed under quasi-static loading conditions (without time effects) and at room temperature, i.e., 25 ° C.

The digital image correlation (DIC) method was employed for measuring the displacements of the polymer. DIC is a powerful optical-numerical method developed to estimate full-field surface displacements, being well documented in

ISSN 2176-5480

the literature (Dally and Riley, 2005; Sutton, *et al*., 2009). The basic principle of DIC is to match maximum correlation between small zones (or subsets) of the specimen in the undeformed and deformed states. The specimens were sprayed with black paint to obtain a random black and white speckle pattern in order to perform the correlation procedure. A CCD camera (Sony XCD-SX910) set perpendicularly to the specimen was used for capturing the images. All images were acquired using a 10 Zoom C-Mount lens. The images of the undeformed and deformed specimen were captured and processed using a DIC program (home-made DIC code), in order to estimate the displacement fields. The size of the measurement field was 1280x960 pixels and the reference and target subsets equal to 31 x 31 and 71 x 71 pixels, respectively. The accuracy of this method is approximately equal to 0.01 pixels.

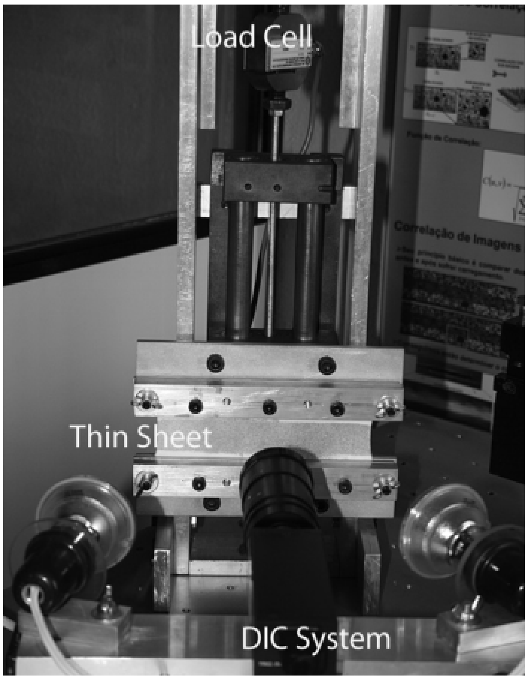

Figure 1. Experimental arrangement for pure shear.

3. HYPERELASTIC CONSTITUTIVE RELATIONSHIPS

Consider the central region of the adhesive from a rectangular thin sheet specimen, as illustrated in Fig. 2. The pure shear occurs only in the central part of the sheet. Assuming that the material is incompressible, i.e. $\lambda_1\lambda_2\lambda_3 = 1,$ the principal stretches can be expressed as $\lambda_1 = \lambda,\ \lambda_2 = 1$ and $\lambda_3 = \lambda^{-1}$. Thus, the deformed configuration as a function of a reference configuration can be written as

$$x_1 = \lambda_1 X_1 \ , x_2 = 1 \text{ and } x_3 = \lambda_1^{-1} X_3 \tag{1}$$

The associated principal stretches are defined as a function of initial and final lengths (L_0 and L) in the stretched direction. These expressions are given as

$$\lambda_1 = \frac{L}{L_0}\ , \lambda_2 = 1 \ and \ \lambda_3 = \frac{L_0}{L} \tag{2}$$

Using Eq. (2) , the deformation gradient tensor for pure shear **F** can be expressed as

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/\lambda \end{bmatrix} \tag{3}$$

ISSN 2176-5480

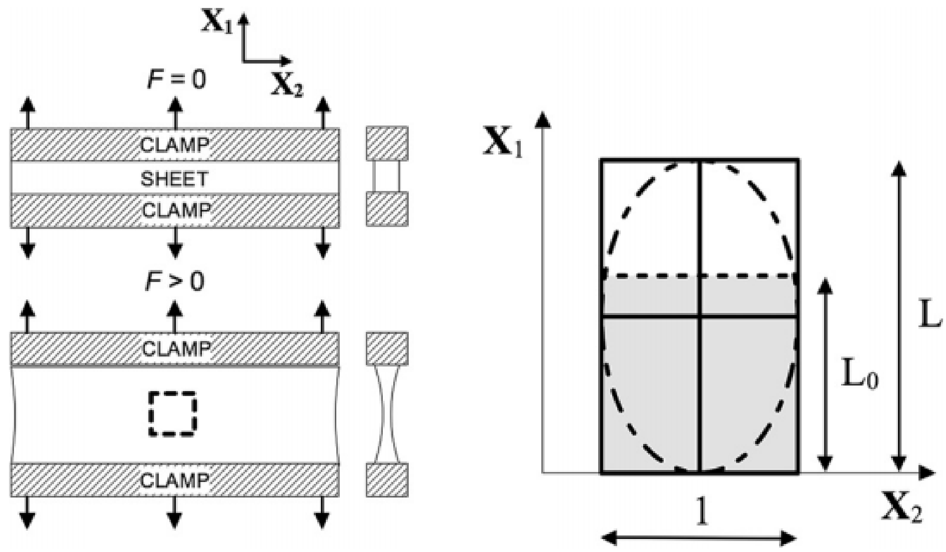

Figure 2. Element under pure shear.

The left Cauchy-Green tensor, which is also known as Green's deformation tensor, is an important strain measure in material coordinates. The deformation tensor **B**, can be written as functions of the deformation gradient tensor,

$$\mathbf{B} = \mathbf{F}\mathbf{F}^{\mathbf{T}} = \begin{bmatrix} \lambda^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/\lambda^2 \end{bmatrix} \quad (4)$$

There are many forms for expressing the three scalar invariants of a second-order tensor. They can be written in terms of the metric tensor in the undeformed and deformed states, as well as the relative stretches. The principal scalar invariants of the left Cauchy-Green deformation tensor can be determined as

$$I_1 = tr\boldsymbol{B} = \lambda^2 + 1 + 1/\lambda^2 \quad (5)$$

$$I_2 = \frac{\mathbf{1}}{\mathbf{2}}[(\boldsymbol{tbB})^{\mathbf{2}} - \boldsymbol{trB^2}] = \lambda^2 + 1 + 1/\lambda^2 \quad (6)$$

$$I_3 = det\boldsymbol{B} = J = 1 \quad (7)$$

Numerous stress tensors have been defined in the literature. The first Piola-Kirchhoff stress tensor **P** is called the engineering stress (or nominal stress) because it is defined as the force per unit unstrained area. This tensor is in general not symmetric. The Cauchy stress tensor, also known as true stress, is defined as the force per unit strained area. The relation between these stress tensors is given by

$$\boldsymbol{\sigma} = J^{-1}\boldsymbol{P}\boldsymbol{F}^{\boldsymbol{T}} \quad (8)$$

As illustrated in Holzapfel (2000) the Cauchy stress can be written in terms of the left Cauchy-Green tensor **B**

$$\boldsymbol{\sigma} = -pI + 2\left(\frac{\partial W}{\partial I_1} + I_1\frac{\partial W}{\partial I_2}\right)\boldsymbol{B} - 2\frac{\partial W}{\partial I_2}\boldsymbol{B}^{\mathbf{2}} \quad (9)$$

Where $\boldsymbol{p}$ is the arbitrary hydrostatic pressure.

Substituting Eqs. (4) and (5) into (9), the principal stress component can be given by

$$\boldsymbol{\sigma_1} = 2\left[\left(\lambda^2 - \frac{1}{\lambda^2}\right)\left(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2}\right)\right] \quad (10)$$

On the other hand, the principal Cauchy stresses in terms of stretches are given by (Destrade and Ogden, 2005)

$$\boldsymbol{\sigma_i} = \lambda_i\frac{\partial W}{\partial \lambda_i} - p \quad (11)$$

or in the present case, being $\sigma_3 = 0$

$$\boldsymbol{\sigma_1} = \boldsymbol{\sigma_{11}} = \lambda_1 P_{11} \tag{12}$$

Considering a thin sheet under tensile load, as described by Jones and Treloar (1945) , it is possible to calculate the work done by the external applied force taking into account the unstrained block, which gives

$$dW = P_{11} d\lambda_1, with\ P_{11} = \frac{F}{A} \tag{13}$$

Where F and A are the applied load and the unstrained area, respectively.

A suitable strain–energy function for incompressible isotropic hyperelastic materials, i.e. $I_3 = 1$ can be expressed as a set of independent strain invariant of the left Cauchy-Green tensor B, given by

$$W = W[I_1(\boldsymbol{B}), I_2(\boldsymbol{B})] \tag{14}$$

There are several forms of strain-energy function in the literature to describe the elastic properties of hyperelastic materials (Marckmann and Verron, 2006; Holzapfel, 2000). This work focuses on five of the most common ones, i.e. Mooney–Rivlin (first order), Ogden, Yeoh, Vargas and Arruda–Boyce that are implemented in several FEM commercial codes.

These models have been chosen because they are representative, but not exhaustive, of widely used hyperelastic constitutive equations. The Mooney model is the most used in rubber industrial development because of its simplicity and its good representation of moderate deformations (Meunier, *et al*.,, 2008) and the Ogden model has been chosen for its high versatility which allows it to fit almost any experimental data. All models have only two parameters except Vargas model that use one parameter.

3.1 Mooney–Rivlin model

Mooney-Rivlin observed that rubber response is linear under simple shear loading conditions. The original first order model is represented by the equation:

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) \tag{15}$$

Parameters C_{10} and C_{01} are constants of the material and are determined from experimental data. They are related to the ground state shear modulus by the equation

$$\mu = 2(C_{10} + C_{01}) \tag{16}$$

Substituting expression of W into Eq. (10) we obtain the Cauchy stress component.

$$\boldsymbol{\sigma_1} = 2(\lambda^2 - \frac{1}{\lambda^2})(C_{10+}C_{01}) \tag{17}$$

3.2 Ogden model

This model associates the strain-energy with the principal stretches according to Eq. (18) (Marckmann and Verron, 2006; Ogden, 1997).

$$W = \sum_{n=1}^{N} \frac{\mu_n}{a_n} ({\lambda_1}^{a_n} + {\lambda_2}^{a_n} + {\lambda_3}^{a_n} - 3) \tag{18}$$

$$\mu = \frac{1}{2} \sum_{n=1}^{N} \mu_n a_n \tag{19}$$

Where μ is still the ground state shear modulus and the material parameters should fulfilled the following stability condition $\mu_n a_n > 0\ \forall n = 1, N$.

ISSN 2176-5480

3.3 Yeoh model

Yeoh model for a hyperelastic material is based on a representation of the strain energy density in a 3-term expansion of the first strain invariant, I_1 (Yeoh, 1993).

$$W = \sum_{i=1}^{3} C_{i0}(I_1 - 3)^i \tag{20}$$

Where C_{i0} are material parameters and the shear modulus can be considered as $\mu = 2C_{10}$

3.4 Arruda-Boyce model

This model is based on statistical mechanics which account for the non-Gaussian nature of the molecular chain stretch with an effective or representative network structure (Boyce and Arruda, 2000). Furthemore, physically based foundation of the non-Gaussian statistical mechanics netwok models provides a constitutive law that requires only two material properties- the network chain density, N, and the limiting chain extensibility, $\sqrt{n}$. The continuum mechanics invariant-based constitutive models is equivalent phenomenological representation of the microstruturally based statistical models. The first invariant, I_1, has correlate with the average chain stretch in the network model. Thus, an alternative expression of the "8-chain" strain energy model of Arruda and Boyce can be obtained by expanded series polynomial, which models a rubber chain segment between chemical crosslinks as a number N of rigid links of equal length *l*. The parameter N is related to the locking stretch λ_m, the stretch at which the chains reach their full extended state, $\lambda_m = \sqrt{N}$.

Their proposed strain-energy is a truncated Taylor series of the inverse Langevin function. A formulation that retains the first five terms of this function takes on the following form:

$$W = \mu \sum_{i=1}^{5} \frac{\alpha_i}{\lambda_m^{2i-2}} \left(I_1^i - 3^i\right) \tag{21}$$

Where μ is a shear-modulus like parameter and the coefficients α_i are

$$\alpha_1 = \frac{1}{2},\ \alpha_2 = \frac{1}{20},\ \alpha_3 = \frac{11}{1050},\ \alpha_4 = \frac{19}{7000},\ \alpha_5 = \frac{519}{673750} \tag{22}$$

3.5 Vargas model

It is a special case of Ogden model. It result from Eq. (18) by setting N=1, $\alpha_1 = 1$ (Holzapfel, 2000).

$$W = C_1(\lambda_1 + \lambda_2 + \lambda_3 - 3) \tag{23}$$

Where $\mu = C_1/2$ is shear modulus

4. RESULTS AND DISCUSSION

For evaluating the principal stretches, the initial and final sizes of the small area at the central region of thin sheet were taken into account. The size variations were determined by the DIC program. Using these results and Eq. (2), the principal stretches were achieved. For each applied load, an image was captured and the procedure was repeated.

Experimental tests were performed in a very slow strain rate (quasi-static). This condition ensures that no effects of viscosity on rubber are influencing stress data.

Figure 3 shows the results of full-field displacements, u(x, y) and v(x, y), achieved when a tensile load equal to 455 N is applied using the DIC method. These maps were obtained on surface area at central region of the thin sheet i.e., analysis region illustrated in Fig. 2. The displacement field u(x, y) has no significant variation, and v(x, y) displacement field varies linearly with the vertical direction (X_1), that is direction of applied load.

This example was chosen to show that the u(x, y) displacement field can be neglected when compared with the v(x, y) displacement field. Therefore, it can be considered that the deformation occurs only in the vertical direction.

ISSN 2176-5480

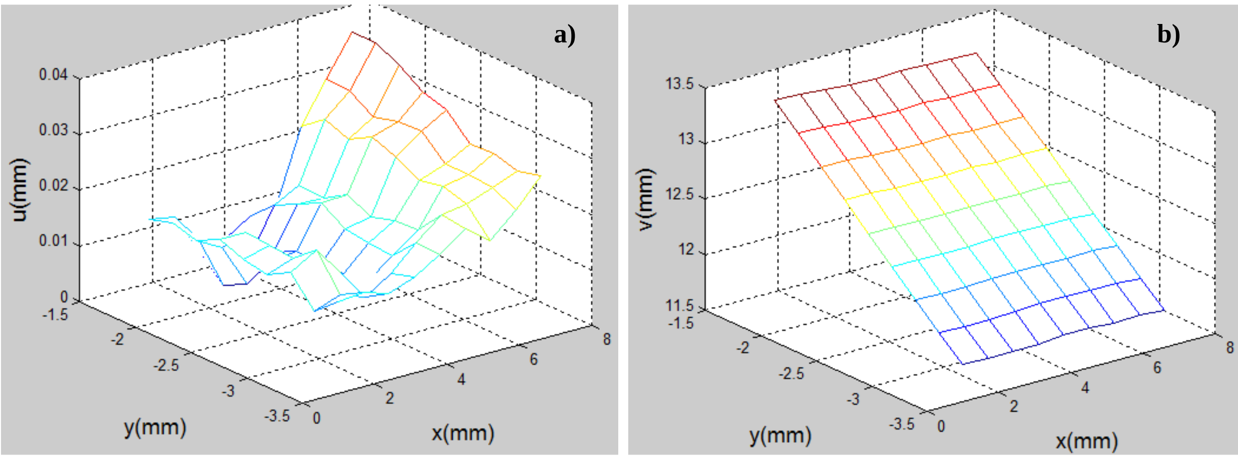

Figure 3. Full-field displacements obtained on surface area at central region of adhesive with applied load of 455N: (a) u(x, y) displacement and (b) v(x, y) displacement.

Figure 4 illustrates the nominal or engineering stress as a function of stretch. Three tests were performed by means of experimental setup. The data obtained from repeated measurements show suitable repeatability. For deformations with stretch values higher than 1.5 it is possible to notice a little discrepancy among the stresses values. One can see that the relationship between stress and stretch is nonlinear.

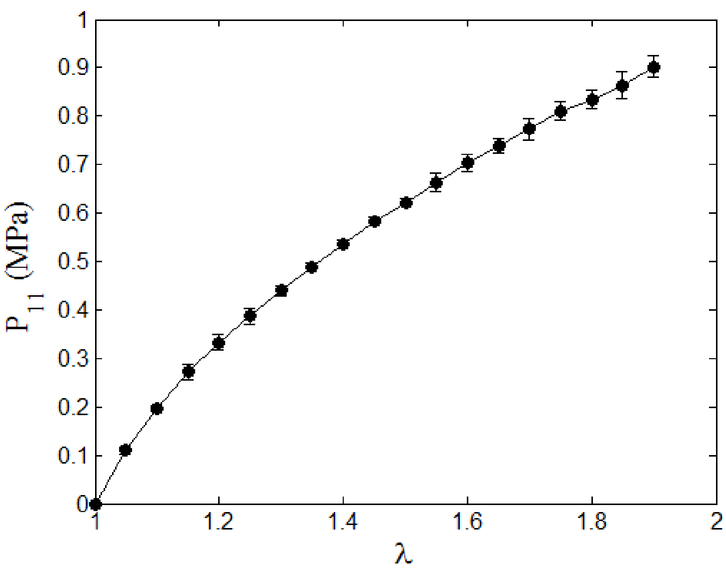

Figure 4. Diagram of nominal stress versus stretch for three tests.

In order to compare both nominal and true stresses, Eqs. 13 and 12 were used. Considering values of loads applied to the specimen and the initial area, the nominal stress (P_{11}) is obtained according to Eq. 13. Thus, the Cauchy stress (σ_{11}) is determined using the principal stretch and nominal stress as stated in Eq. 12. The mean results are depicted in Fig.5. It is possible to observe the linear relationship between true stress and principal stretch. For small deformations both nominal and true stresses coincide. On the other hand, for large deformations Cauchy stress is higher than nominal stress. This is due to Cauchy stress takes into account the strained area.

ISSN 2176-5480

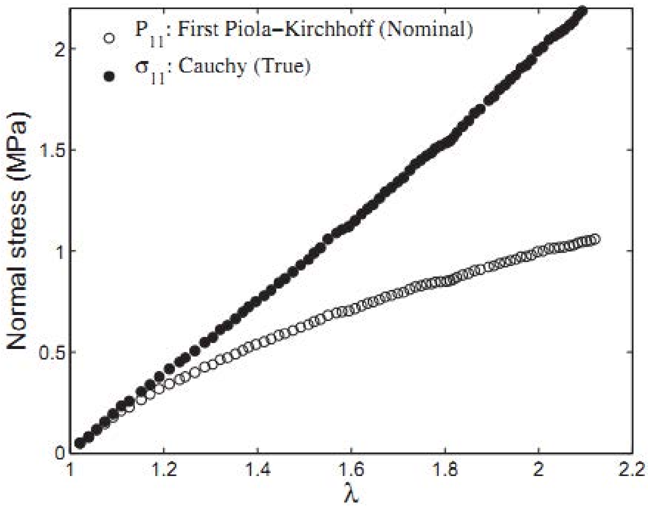

Figura 5. Nominal and true stresses versus stretch.

In order to compare the predictions of the resulting theoretical stress-stretch relations with experimental data over pure shear, the strain-energy function was taken into account. It is clear that the material properties must be invariant under changes of observer and load conditions. For that reason, the values of shear modulus (μ) were estimated.

Five hyperelastic constitutive equations have been chosen: Mooney, Yeoh, Arruda-Boyce, Ogden and Vargas. The first three depend on the invariants of the left Cauchy–Green tensor B, whereas the two last one is written in terms of principal elongations. The material parameters from each model are estimated using Levenberg-Marquardt method, which is a standard technique used to solve nonlinear least squares problems (Levenberg, 1944; Marquardt, 1963; Gill, *et al*., 1981).

A comparison between fittings obtained from different models is illustrated in Figs. 6 and Figure 7 for Cauchy stress and nominal stress, respectively. The final values of shear modulus and RMS for all the fitted models are summarized in Table 1. Parameters C_{ij} are constants of the material and were determined from experimental data. They are related to the ground state shear modulus by the equations reported in section 3.

Among the evaluated models, the most accurate forms result from the Ogden, Yeoh and Arruda Boyce. These models are substantially equivalent in this study and they give a good approximation of experimental data.

Fitting for the Ogden model gives a slightly better fitting with an RMS of 0.008107 MPa and a μ equal to 0.5251 MPa.

Considering the first order Mooney potential form, its fitting gives the result of 0.5164 MPa shear modulus and RMS of 0.01085MPa and thus it is able to describe with a good approximation. On the other hand, is evident that Vargas model fitting is less accurate than those ones. Results are an RMS of 0.04583 MPa and a value of μ equal to 0.5875 MPa.

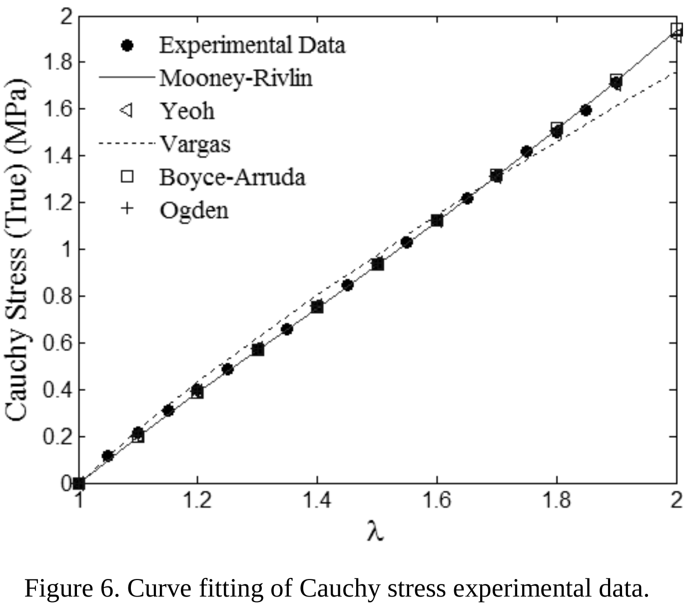

Figure 6. Curve fitting of Cauchy stress experimental data.

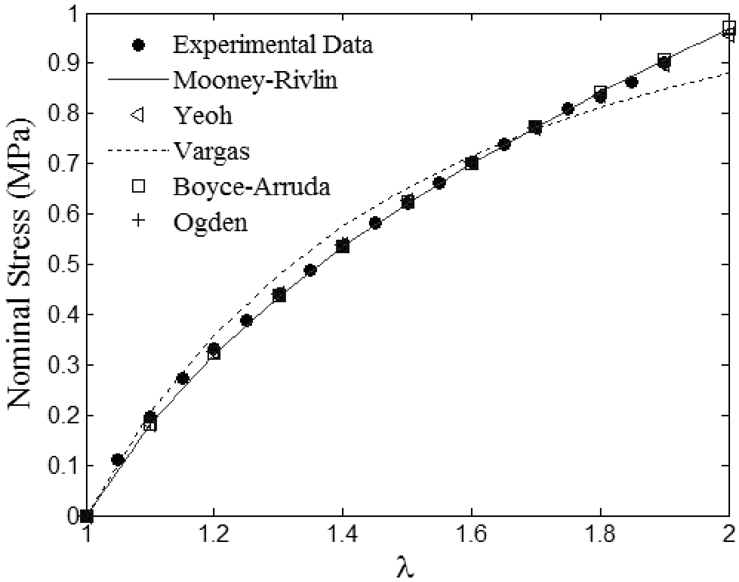

Figure 7. Fittings of Nominal stress experimental data

Table 1. Values of the hyperelastic energy parameters fitted on experimental data from all homogeneous tests.

Modelo	Constants	Shear modulus (μ) [MPa]	RMS [MPa]
Mooney	C_{10} = 0.04802, C_{01} = 0.2102	0.5164	0.01085
Yeoh	C_{10} = 0.262, C_{20} = -0.001568	0.524	0.008167
Arruda-Boyce	N = 2830, μ = 0.5176	0.5176	0.008202
Ogden 1 term	α_1 = 1.9, μ_1 = 0.5527	0.5251	0.008107
Vargas	C1 = 1.175	0.5875	0.04583

5. CONCLUSIONS

The aim this work was to investigate the mechanical behavior of a hyperelastic material under pure shear and theoretical models were fitted to the measured data of stress versus amount of shear in order to determine the initial shear modulus. It has been observed from experimental data that stress and stretch relationship is nonlinear. In order to find a best model to the experimental data, five models hyperelastic energy density models was evaluated in this work and implemented into Matlab routines. The Ogden and Yeoh models were found to be the most accurate. Both of them give a good description of the material with concordance of the ground state shear modulus and with low RMS. The initial shear modulus varied from 0.5164 MPa through 0.5875 MPa obtained from Mooney and Vargas models, respectively.

6. ACKNOWLEDGEMENTS

The financial support of Rio de Janeiro State Funding, FAPERJ, and Research and Teaching National Council, CNPq, are gratefully and acknowledged.

7. REFERENCES

Boyce, Mary C, Arruda, Ellen M. “Constitutive Models of Rubber Elasticity: A Review”. Rubber Chemistry and Technology: March 2000, pp. 504-522.

Dally, J.W., Riley, W.F. Experimental Stress Analysis, fourth ed.,McGraw Hill, 2005.

Destrade, M., Ogden, R.W, “Surface waves in a stretched and sheared incompressible elastic material”, International Journal of Non-Linear Mechanics 40 (2005) 241 – 253.

Gill, P.R., Murray, W., Wright, M.H. Practical Optimization, Academic Press, London, 1981, pp. 136–137.

Holzapfel, Gerhard.A, “Nonlinear Solid Mechanics – A Continuum Approach for Engineering”. John Wiley & Sons, Ltd, Ed. 2000.

ISO, 1827. Rubber, vulcanized or thermoplastic - Determination of shear modulus and adhesion to rigid plates - Quadruple-shear methods. Third edition, 2007.

ISSN 2176-5480

Jones, D F, Treloar, LRG. "The properties of rubber in pure homogeneous strain". J. Phys. D: Appl. Phys., Vol. 8, 1975.

Levenberg, K. "A method for the Solution of Certain Problems in Least Squares". Quart. Appl. Math. 2 (1944) 164–168.

Marckmann, G., Verron, E. Comparison of Hyperelastic Models for Rubberlike Materials. Rubber Chemistry and Technology: November 2006, Vol. 79, No. 5, pp. 835-858.

Marquardt, D. SIAM Journal on Applied Mathematics. 11 (1963) 431–441.

Meunier, L., Chagnon, G., Favier, D., Orgéas, L. and Vacher, P., 2008, "Mechanical experimental characterisation and numerical modelling of an unfilled silicone rubber". Polymer Testing, Vol. 27, pp 765–777.

Nunes, L.C.S. "Mechanical characterization of hyperelastic polydimethylsiloxane by simple shear test", Mat. Sci. Eng. A 528 (2011) 1799–1804.

Ogden, R.W. Non-linear Elastic Deformations, Dover Publications, Mineola, NY, 1997.

Rivlin, R.S, Saunders, D.W. "Large Elastic Deformations of Isotropic Materials. VII. Experiments on the Deformation of Rubber". Phil.Trans. R. Soc. Lond. A 243 (1951) 251–288.

Sasso, M., Palmieri, G., Chiappini, G., Amodio, D. "Characterization of hyperelastic rubber-like materials by biaxial and uniaxial stretching tests based on optical methods". Polymer Testing 27 (2008) 995–1004.

Sutton, M.A., Orteu, J.J., and Schreier, H.W. Image Correlation for Shape, Motion and Deformation Measurements, Springer Science and Business Media LCC, 2009.

Treloar, L.R.G, "The Physics of Rubber Elasticity". Third edition, Clarendon Press, Oxford, 2005.

Treloar, L.R.G. "Stress-Strain Data for Vulcanised Rubber Under Various Types of Deformation". Trans. Faraday Soc. 39 (1943) 241.

Yeoh, O. H., "Some Forms of the Strain Energy Function for Rubber," Rubber Chem. Technol., Vol. 66, 1993, pp. 754-771.

8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.