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TUBULAR DIFFUSION FLAMES

Helcio Francisco Villa Nova
Instituto de Engenharia Mecânica - Universidade Federal de Itajubá
helcio.villanova@gmail.com

Fernando Fachini Filho
Grupo de Mecânica de Fluidos Reativos - Laboratório de Combustão e Propulsão - Instituto Nacional de Pesquisas Espaciais
fachini@lcp.inpe.br

Abstract. *The aim of this work is to study the tubular diffusion flames. With this configuration, the influence of some characteristics of the flow, e.g. velocity, curvature and stretch, on the flame are analyzed. The work of pressure in the energy equation is considered. This term is associated with the pressure gradient in the axial component of the moment equation. It is responsible for accelerating the flow and decreasing its temperature, with a direct influence on the flame temperature. The present analytical model provides an estimative of flame properties, e.g. position and temperature. These properties are validated with a commercial CFD code.*

Keywords: *laminar flame, diffusion flame, tubular flame, aircraft propulsion*

1. INTRODUCTION

The interaction of flame with the flow is a problem with application to the turbulent combustion study that occur inside of combustion chamber of airbreathing gas turbine and other propulsion engines. The effect of the interaction of a vortex with the flame causes effects of stretch and curvature similar to that effect of the interaction of flame with the flow in the tubular flame. The structure and behavior of turbulent flames may be analysed from laminar flames Libby *et al.* (1989). The tubular flame is a interesting problem of interaction of flame with flow where there are stretch and curvature. The stretch affect the flame in the longitudinal direction. The curvature affect the flame in the tangential direction. Others effects can to affect the flame position, species distribution, heat transfer. The tubular flame has been studied in several conditions. In Takeno and Ishizuka (1986) was done a study of laminar premixed flame with tubular geometry and the flow of mixture have radial and tangential velocity component. Libby *et al.* (1989) analysed the cylindrical premixed flame formed from radial and tangential velocity components. The study of Takeno *et al.* (1986) analysed the extinction in tubular premixed flame without tangential velocity component. Wang *et al.* (2006) did an analytical study for stretch rate in tubular premixed flame. Wang *et al.* (2007) analysed the curvature effect on the tubular diffusion flame. Hu *et al.* (2007) did a experimental study of laminar non-premixed flame. The laminar tubular flame is formed from counterflow with the thermochemical process being controlled by velocity field. In the premixed tubular flame the velocity field establishes the tubular geometry for flame (Libby *et al.* (1989)). In the diffusion tubular flame the velocity field specifies the flame position. The theoretical works Takeno and Ishizuka (1986) and Libby *et al.* (1989) have solved the interaction problem from technique of similarity. Wang *et al.* (2007) has used the CHEMKIN commercial software to get results from the solution of the problem . In this work, the authors employed the generalized formulation of Shvab-Zel'dovich with velocity field from counterflow in cylindrical coordinates. The results are obtained with numerical solution of differential equations. In the Takeno and Ishizuka (1986) and Hu *et al.* (2007), the results are compared with experimental data obtained from cylindrical burner device. The actual work, the results are compared with numerical CFD technique, employing the commercial software CFX which is a finite volume code.

2. PHYSICAL MODEL

The physical model analysed is formed by two concentric semi-infinitely cylinders. The internal cylinder has radius r_1 and the external cylinder has radius r_2. The fuel is injected from external cylinder with radial velocity $-V_2$ and the oxidant is injected from internal cylinder with radial velocity V_1. The z and r represent the longitudinal distance and the radial distance, respectively, from origin. The diffusion flame is formed into the annular space $r_2 > r > r_1$ by the counter flow fuel-oxidant. Figure (1) shows the physical model of burner for tubular flame formation.

This physical model is associated to following hypothesis: the flow is laminar and steady, the properties of gases are constant and the chemical reaction is produced from the one-step process between Hydrogen and the Oxygen.

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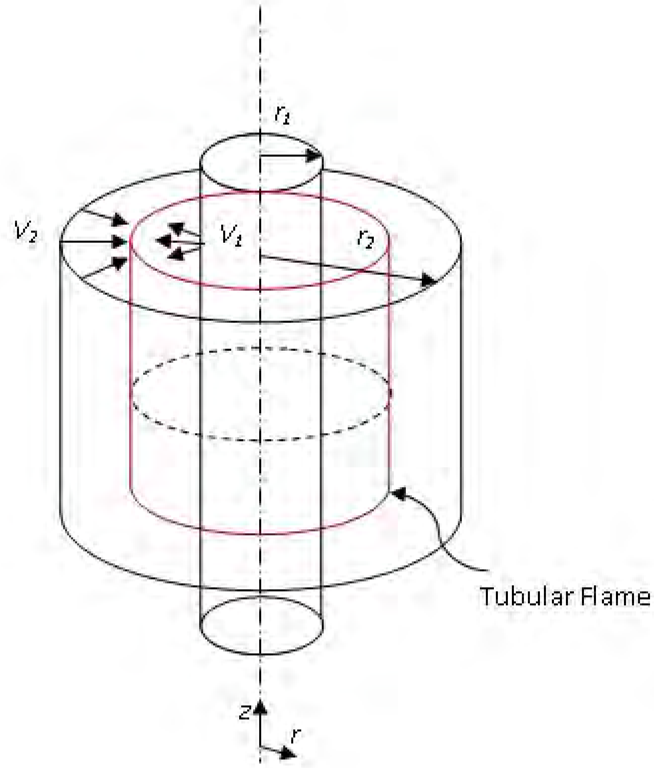

Figure 1. The burner model for tubular flame formation

2.1 Tubular counterflow

The counter flow in tubular geometry in non-reactive flow condition (cold condition) has been treated by Wang *et al.* (2006), which employed the analytical solution for velocity field of Yuan and Finkelstein (1956). Following Wang *et al.* (2006), the mass conservation equation and axial momentum equation in the non-dimensional forms are:

$$m + \frac{dn}{d\tilde{r}} = 0 \tag{1}$$

$$\rho[(\frac{m}{\tilde{r}})^2 + \frac{n}{\tilde{r}}\frac{d}{d\tilde{r}}\left(\frac{m}{\tilde{r}}\right)] = -\frac{1}{\tilde{z}}\frac{\partial P}{\partial \tilde{z}} \tag{2}$$

The variables n and m are related with the non-dimensional radial and axial flow velocities according to

$$\tilde{v}_r \equiv \frac{v_r}{V_2} = \frac{n}{\tilde{r}} \tag{3}$$

$$\tilde{v}_z \equiv \frac{v_z}{V_2} = \tilde{z}\frac{m}{\tilde{r}} \tag{4}$$

The non-dimensional pressure P is defined as $P \equiv p/\rho_2 V_2^2$. The independent variables $\tilde{r}$ and $\tilde{z}$ are defined as $\tilde{r} \equiv r/r_2$ and $\tilde{z} \equiv z/r_2$.

In this work it is considered that pressure gradient linear with the axial length, then

$$G \equiv -\frac{1}{\tilde{z}}\frac{\partial P}{\partial \tilde{z}}$$

Like n, m and P, the other variables will also be nondimensionalized with condition at $r = r_2$.

The combination of Eq. (1) and Eq. (2) produces the equation for n isolated, as follow:

$$\rho[(\frac{dn}{d\tilde{r}})^2 - \tilde{r}n\frac{d^2n}{d\tilde{r}^2} + n\frac{dn}{d\tilde{r}}] = G\tilde{r}^3 \tag{5}$$

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This ordinary differential equation is solved for two ensemble of boundary conditions:

$$\tilde{r} = 1, \ \ n = 1, \ \ \rho = 1, \ \ \frac{dn}{d\tilde{r}} = 0, \quad \text{in} \ \ r_1/r_2 \leq \tilde{r} \leq r_s/r_2 \tag{6}$$

and

$$\tilde{r} = \frac{r_1}{r_2}, \ \ n = \frac{V_1 r_1}{V_2 r_2}, \ \ \rho = \frac{\rho_1}{\rho_2}, \ \ \frac{dn}{d\tilde{r}} = 0, \quad \text{in} \ \ r_s/r_2 \leq \tilde{r} \leq 1 \tag{7}$$

In these conditions the subindex s represents stagnation condition of counterflow. As in Yuan and Finkelstein (1956) and Wang *et al.* (2006), the solution of Eq. (5) in $r_1/r_2 \leq \tilde{r} \leq r_s/r_2$ is

$$n = \frac{V_1 r_1}{V_2 r_2} \sin\left(\frac{V_2 r_2}{V_1 r_1}\sqrt{\frac{G}{4\rho}}\tilde{r}^2 + \frac{\pi}{2} - \frac{V_2 r_1}{V_1 r_2}\sqrt{\frac{G}{4\rho}}\right) \tag{8}$$

and in $r_s/r_2 \leq \tilde{r} \leq 1$ is

$$n = \sin(\sqrt{G}\,\tilde{r}^2/2 + \pi/2 - \sqrt{G}/2) \tag{9}$$

It is worth mentioning that the ratio $V_1 r_1/V_2 r_2$ represents the ratio of the Peclet number Pe_1/Pe_2, which is written as

$$A \equiv \begin{cases} Pe_1/Pe_2, & r_1/r_2 \leq \tilde{r} \leq r_s/r_2 \\ 1, & r_s/r_2 \leq \tilde{r} \leq 1 \end{cases}$$

in order to simplified Eqs. (8) and (9). Following the same procedure the term multiplying $\tilde{r}^2$ in Eq. (8) is represented as

$$B \equiv \begin{cases} \sqrt{G/4\rho}/A, & r_1/r_2 \leq \tilde{r} \leq r_s/r_2 \\ \sqrt{G}/2, & r_s/r_2 \leq \tilde{r} \leq 1 \end{cases}$$

The other constant in the solution (8) is

$$C \equiv \begin{cases} \pi/2 - B(r_1/r_2)^2, & r_1/r_2 \leq \tilde{r} \leq r_s/r_2 \\ \pi/2 - \sqrt{G}/2, & r_s/r_2 \leq \tilde{r} \leq 1 \end{cases}$$

Therefore, the solutions (8) and (9) can be written compactly as

$$n = A\sin(B\tilde{r}^2 + C) \tag{10}$$

The combination of Eq.(1) and Eq.(10) produces the equation for m, as follow:

$$m = 2AB\tilde{r}\cos(B\tilde{r}^2 + C) \tag{11}$$

It is worth mentioning that the function n is dependent on the ratio of Peclet numbers, $n \sim Pe_1/Pe_2$, meanwhile the function m is independent on the ratio of Peclet numbers.

Figures (2), (3) and (4) show the cases for typical values of ratio of radial injection velocities, ratio of injection temperatures and pressure gradient with axial lenght along the radius of the cylinders of the burner, respectively.

The present flow has characteristics of boundary layer. In the case where the chemical reaction is present, the flame is established within boundary layer. The order of thickness of stagnation layer is $1/\sqrt{Re}$.

2.2 Concentration and temperature field

In this problem, the energy and species conservation equations are in stationary condition and in radial direction, with the Lewis number done unity. Therefore, the conservation equations are given as

$$\frac{1}{Pe_2}\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left(\tilde{r}\frac{\partial Y_i}{\partial\tilde{r}}\right) - \tilde{v}_r\frac{\partial Y_i}{\partial\tilde{r}} = s_i\Omega \tag{12}$$

$$\frac{1}{Pe_2}\frac{1}{\tilde{r}}\frac{\partial}{\partial\tilde{r}}\left(\tilde{r}\frac{\partial\theta}{\partial\tilde{r}}\right) - \tilde{v}_r\frac{\partial\theta}{\partial\tilde{r}} = \tilde{v}_z\frac{\tilde{z}}{\rho}\frac{V_2^2}{c_p T_2}G - Q\Omega_f \tag{13}$$

The funtions θ and Y_i are the nondimensional temperature, $\theta \equiv T/T_2$, and the mass fraction of species i, where $i = F,\ O, F$ corresponds to fuel and O, oxidant, respectively. The parameter s_i is the stoichiometric coefficient in terms of mass of the reaction $F + sO_2 \rightarrow P$, then $s_F = 1$ and $s_O = s$. The nondimensional reaction rate Ω is defined as

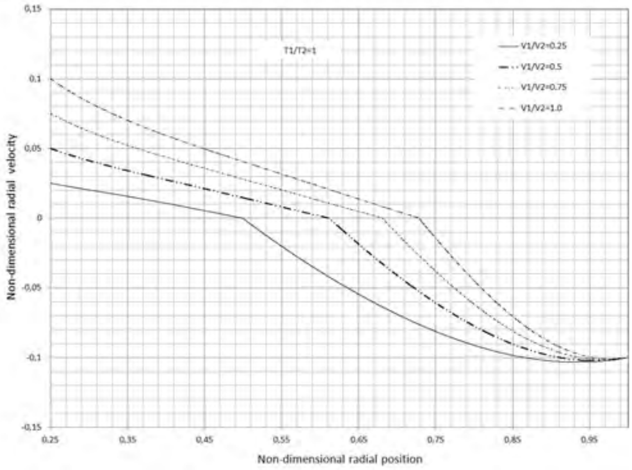

Figure 2. Radial velocity for V_1/V_2 =0.25, 0.5, 0.75 and 1.00, respectively.

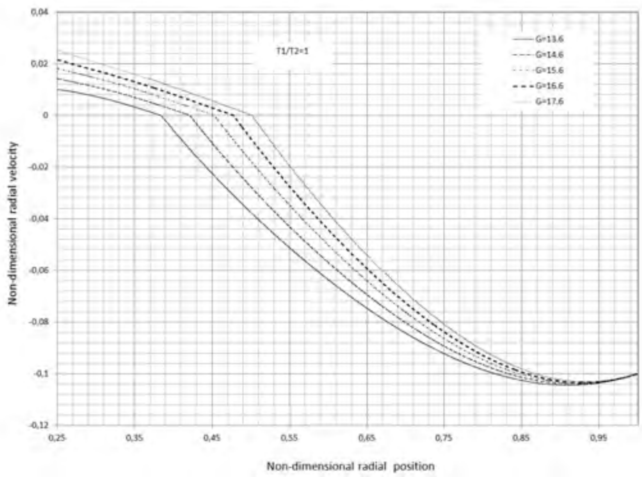

Figure 3. Radial velocity for G =11.85, 13.15, 14.52, 16.7 and 18.24, respectively.

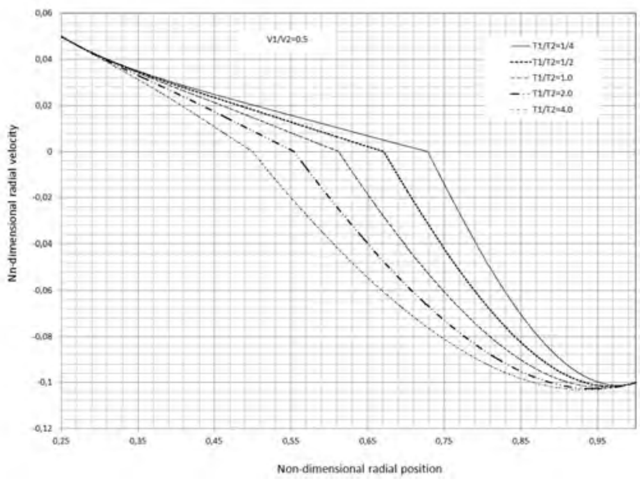

Figure 4. Radial velocity for T_1/T_2 =0.24, 0.48, 0.96, 1.45 and 1.93, respectively.

$\omega r_2^2/(\rho_2 \alpha)(1/\rho Pe_2)$, where α is the thermal diffusivity ($\alpha \equiv k/\rho_2 c_p$). Also, the parameter Q is the non-dimensional heat of combustion, defined according to $Q \equiv \Delta H_r/(c_p T_2)$.

The appropriate boundary conditions for equations of species are

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and

$$
\begin{aligned}
&Y_O - 1 = Y_F = \theta - T_1/T_2 = 0, && \text{at} \quad \tilde{r} = r_1/r_2 \\
&Y_O = Y_F = \theta - T_f/T_2 = 0, && \text{at} \quad \tilde{r} = r_f/r_2 \\
&Y_O = Y_F - 1 = \theta - 1 = 0, && \text{at} \quad \tilde{r} = 1
\end{aligned} \tag{14}
$$

The subindex f stands for the condition at the flame position.

Considering the Shvab-Zeldovich formulation, which leads to an equation for the mixture fraction $Z \equiv sY_F - Y_O + 1$ and an equation for the enthalpy $H \equiv [(1+s)/Q]\theta + Y_O + Y_F$,

$$\frac{1}{Pe_2}\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r}\frac{\partial Z}{\partial \tilde{r}}) - \tilde{v}_r\frac{\partial Z}{\partial \tilde{r}} = 0 \tag{15}$$

$$\frac{1}{Pe_2}\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r}\frac{\partial H}{\partial \tilde{r}}) - \tilde{v}_r\frac{\partial H}{\partial \tilde{r}} = \tilde{G}\tilde{z}\tilde{v}_z \tag{16}$$

in which the parameter $\tilde{G}$ is defined as

$$\tilde{G} \equiv \frac{V_2^2}{\rho c_p T_2}\frac{G(1+s)}{Q} = \frac{(\gamma-1)M_{a2}^2 G}{\rho}\frac{(1+s)}{Q}$$

and relates information about flow field (compressibity and pressure gradient) and exotermic chemical reaction (heat of combustion and stoichiometric coefficient). Note that γ is the specific heat ratio and M_a is the Mach number.

3. RESULTS

The two variables Z and H were obtained by numerical solution of Eqs. (15) and (16) with the boundary conditions (14). The numerical method employed was Runge-Kutta fourth order with step of 0.0001. In this work, was compared the 1-D radial modelling with the effect of the gradient of pressure in the axial direction to a complete Navier-Stokes commercial software CFX. The full 3-D modelling with CFX commercial software has employed an hexahedral spatial mesh for domain discretisation with 2400000 nodes and three open boundary conditions: botton, top and tubular wall. The chemical process is treated as single step reaction with finite rate chemistry. The kinetic parameters, i.e. the pre-exponential factor, the temperature exponent and the activation energy of reaction, are those usually employed in the chemical reaction literature (Perry and Green (1997)).

The numerical solution of the current model and of the full 3-D model are show in the Figs. (5) and (6). For this case, the stagnation flow position $\tilde{r}_s$ is 0.5 and the order of Re is 129, with this the flame should be established in the $0.41 < \tilde{r}_f < 0.5$, in the oxidant side. The current model shows the flame position in 0.48 and the full 3-D model shows the flame position in 0.52, respectively approximate. This finite volume complete Navier-Stokes commercial CFD code employs the finite rate as chemical model, which makes the flame position difficult to determining.

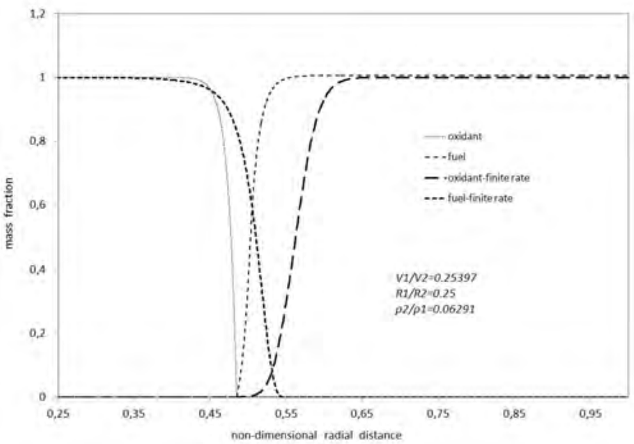

Figure 5. Mass fraction of oxidant and of fuel

4. CONCLUSIONS

The current analysis shows that the description of tubular diffusion flames considering the Burke-Schumann kinetics and constant density approximations represents well the position and temperature of the flame comparing to the finite

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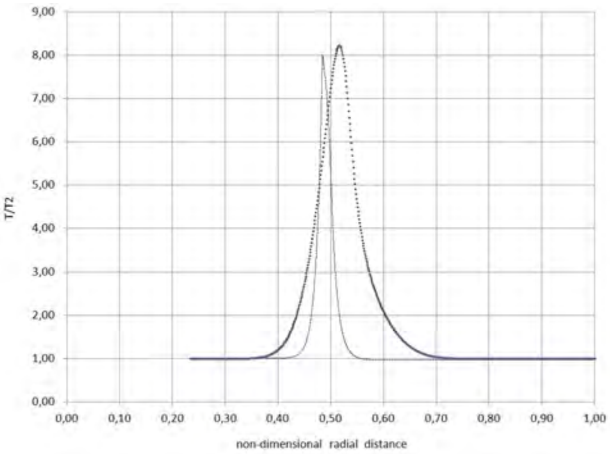

Figure 6. Non-dimensional temperature

volume numerical approach. Even with some drastic approximations, the current model is able to describe well the characteristics of tubular diffusion flames. An important advantage is presented by this model, it permits analytical solution which is costless when compared with those provided by full 3-D model. Therefore, in terms of computation time, this model has an incomparable advantage on the other model. The main aim of this work is achieved, to provide an accurate alternative to study tubular diffusion flames. In future, this model will be extented to turbulent regime.

5. REFERENCES

Hu, S., Wang, P., Pitz, R.W. and Smooke, M.D., 2007. "Experimental and numerical investigation of non-premixed tubular flames". *Proceedings of the Combustion institute*, Vol. 31, p. 1093.

Libby, P.A., Peters, N. and Williams, F.A., 1989. "Cylindrical premixed laminar flames". *Combustion and Flame*, Vol. 75, p. 265.

Perry, R.H. and Green, D.W., 1997. *Perry's Chemical Engineer's Handbook*. McGraw-Hill Professional, New York, 7th edition.

Takeno, T. and Ishizuka, S., 1986. "A tubular flame theory". *Combustion and Flame*, Vol. 64, p. 083.

Takeno, T., Nishioka, M. and Ishizuka, S., 1986. "A theoretical study of extinction of a tubular flame". *Combustion and Flame*, Vol. 66, p. 271.

Wang, P., Hu, S. and Pitz, R.W., 2007. "Numerical investigation of the curvature effects on diffusion flames". *Proceedings of the Combustion institute*, Vol. 31, p. 989.

Wang, P., j A Wehrmeyer and Pitz, R.W., 2006. "Stretch rate of tubular premixed flames". *Combustion and Flame*, Vol. 145, p. 401.

Yuan, S.W. and Finkelstein, A.B., 1956. "Laminar pipe flow with injection and section through a porous wall". *Transact. ASME*, Vol. 78, p. 719.

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