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A SIMPLE WAY TO MODEL SKELETAL MUSCLES BY FEM

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Abstract. *The contribution of this study is a simple strategy to model skeletal muscles associated with bones. The simplicity of our approach is related to the introduction of active muscular fibers inside the soft tissue without increasing the number of degrees of freedom of the whole continuum. The skeletal system is modeled (to present the idea) as a two dimensional reinforced elastic solids developing both small and large deformations. Fibers are freely spread over de domain without necessity of node coincidence and without increasing the number of degrees of freedom. Active and passive fibers are stated before the application and a non-linear approach is used to solve the desired mechanical problem. Numerical examples are employed to demonstrate the potential of the proposed methodology and future developments are discussed.*

Keywords: *skeletal muscles, positional FEM, fibers, geometrical nonlinearity.*

1. INTRODUCTION

Various important and high standard works related to the mechanical behavior and analysis of skeletal muscles are present in literature. Selecting some recent works is a hard task; however we should mention references Tang, *et al*., (2009), Böl and Sturmat (2011), Böl (2010) and Lu, *et al*., (2010), in which one can find a very good state of art related to the subject. Following Tang, *et al*., (2009), there are two basic ideas from which skeletal muscle models come from: a phenomenological model originated from Hill (1938) – *apud* Tang, *et al*., (2009) and a biophysical cross-bridge model derived by Huxley (1957) – *apud* Tang, *et al*., (2009). Hills-based model are related to the macro behavior of muscles while Huxley-based models were built upon biochemical, thermodynamic and mechanical experiments for describing muscle at the molecular level. Huxley-based models have been mainly used to understand the properties of the microscopic contractile elements. The intention of this work is to modestly contribute with the macro-mechanical modeling of skeletal muscles, therefore Huxley-based models are not the focus of our work.

The challenge is to generalize the pioneer one-dimensional Hill model to general 3D models and applying them in FE models to analyze complex geometries. Following Böl and Sturmat (2011), to incorporate further more complex geometrical aspects of skeletal muscles, planimetric and 3D models were designed (see, e.g. Blemker and Pinsky (2005) and Tsui, *et al*., (2004)). Most of these continuum-based models use a macroscopic description of the passive muscle behavior (soft tissue) combined with a 1D, possibly micromechanically motivated, modeling of the active muscle fibres.

The way followed here is similar to the one proposed by (Böl and Sturmat (2011) and Lu, *et al*., (2010)), that is, as earlier mentioned by Van Leeuwen (1992), to split the muscle behavior into a passive and an active part. The proposed concept is based on the idea of representing the passive part by means of an assembly of non-linear fiber elements. In each fiber element, the force-stretch behavior of a certain group of collagen fibres is implemented. To incorporate muscle activation, similarly to passive fibers, active muscle fibers are also modeled as non-linear one dimensional elements and embedded in the continuous.

The novelty of our work is the strategy followed to embed these fibers into the soft tissue (continuum). Differently from Tang, *et al*., (2009) and Böl and Sturmat (2011), our fiber elements do not contribute to unit cells (solid finite elements) but directly to the whole continuum, that is, the internal force developed by fibers does not depends on the homogeneous muscle discretization to be transmitted, but are directly transmitted from fibers to the soft tissue.

The 2D solid finite element applied here to discretize the continuum is isoparametric of third order. Curved high order fiber elements are developed to be embed in the continuum in to simulate muscle fibers. The adopted nodal parameters are positions and the Saint-Venant-Kirchhoff constitutive law is chosen to model, in a simplified way, the soft tissue behavior (Bonet, *et al*., (2000), Coda and Paccola (2007), Ciarlet (1993) and Ogden (1993)).

Muscular fiber elements are introduced in soft tissue by means of nodal kinematic relations. This strategy directly ensures the adhesion of fibers nodes to the continuum without increasing the number of degrees of freedom and without the need of nodal matching (Vanalli, *et al*., (2008) and Sampaio, *et al*., (2013)). To solve the resulting geometrical nonlinear problem we adopt the Principle of Stationary Total Potential Energy, Tauchert (1974). From this principle we find the nonlinear equilibrium equations. The Newton-Raphson iterative procedure, Luenberger (1989), is used to solve the nonlinear system. Our level of application is limited to 2D models and two examples are shown to demonstrate the possibilities of the technique.

2. COUPLING KINEMATICAL RELATION

As mentioned at introduction it is the intention of this paper to collaborate in the analysis of muscles by the presentation of a general and simple way to introduce fibers inside an elastic continuum. The requisite to start this procedure for curved fibers and curved (2D for instance) solid elements is to know the solid dimensionless coordinates related to fiber nodes coordinates, see Fig. 1.

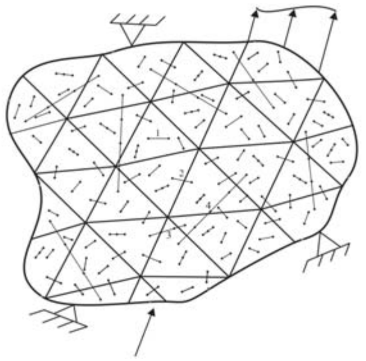

Figure 1. simple illustration of a 2D domain with general immersed fibers

This is done solving the pair of dimensionless solid variable (ξ_1^p, ξ_2^p) associated to the physical initial fiber node position in the following nonlinear system,

$$X_i^P = \phi_l(\xi_1^P, \xi_2^P) X_i^l \tag{1}$$

where ϕ_l are the shape functions of the solid element, X_i^P are the known (initial) coordinates of fiber nodes (generated independently of solid mesh) and X_i^l are the know solid nodes coordinates. To solve Equation (1) one expands it in Taylor series until the first order and starts with a trial dimensionless coordinate, (ξ_1^{pt}, ξ_2^{pt}), i.e.:

$$X_i^P \cong \phi_l(\xi_1^{Pt}, \xi_2^{Pt}) X_i^l + \left. \frac{\partial \phi_l(\xi_1, \xi_2)}{\partial \xi_j} \right|_{(\xi_1^{Pt}, \xi_2^{Pt})} \Delta\xi_j \quad \text{or} \quad X_i^P = X_i^{Pt} + H_{ij} \Delta\xi_j \tag{2}$$

in which X_i^{Pt} is a trial position of the fiber node calculated from the solid element geometry and the trial dimensionless coordinates and H_{ij} is a two dimensional matrix. The correction of the trial dimensionless coordinates $\Delta\xi_i$ is calculated solving the following linear system of equation:

$$H_{ij} \Delta\xi_j = X_i^P - X_i^{Pt} \tag{3}$$

The procedure is a simple and fast Newton-Raphson nonlinear solver that relates all fiber nodes to the connected solid element revealing the pair of dimensionless variables (ξ_1^p, ξ_2^p). From this information one also knows the current position of fiber nodes as a function of solid nodes positions, i.e.,

$$Y_i^P = \phi_l(\xi_1^P, \xi_2^P) Y_i^l \tag{4}$$

where Y_i^l are the current positions (unknown) of solid nodes. Equation (4) ensures the connection among nodes of fibers to the matrix. Moreover, as Equation (4) writes fiber nodes as function of solid nodes what enables to write the Helmholtz free energy of a reinforced solid as a function of solid node only, that is without the additional fiber degrees of freedom.

3. TOTAL HELMHOLTZ FREE ENERGY AND ITS DERIVATIVES

The Helmholtz free energy stored in a reinforced body is the sum of the strain energy stored in the matrix and the fibers:

$$U = U_{mat} + U_f \tag{5}$$

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where U_{mat} is the strain energy stored in the 2D solid finite elements used to discretize the matrix and U_f is the strain energy stored in the fiber finite elements. Therefore, the internal force at a node β in direction α, considering both the fiber and matrix contributions, is determined using the conjugate energy concept:

$$F_\alpha^{\beta int} = \frac{\partial(U_{mat}+U_f)}{\partial Y_\alpha^\beta} = F_\alpha^{\beta int/mat} + \frac{\partial U_f}{\partial Y_\alpha^\beta} \quad (6)$$

in which Y_α^β is a degree of freedom related to the solid discretization.

In our formulation U_f is written as a function of fiber degrees of freedom Y_i^P, see Vanalli, *et al.*, (2008) and Sampaio, *et al.*, (2013)). Therefore, using the chain rule via Equation (4), one writes:

$$\frac{\partial Y_i^P}{\partial Y_\alpha^\beta} = \frac{\partial Y_i^l}{\partial Y_\alpha^\beta}\phi_l(\xi_1^P,\xi_2^P) = \delta_{\alpha i}\delta_{\beta l}\phi_l(\xi_1^P,\xi_2^P) = \delta_{\alpha i}\phi_\beta(\xi_1^P,\xi_2^P) \quad (7)$$

If the fiber node belongs to the solid element and direction α (solid) is equal to direction i (fiber), expression (39) results $\partial Y_i^P / \partial Y_\alpha^\beta = \phi_\beta(\xi_1^P,\xi_2^P)$, otherwise it results zero. Therefore for a fiber node belonging to a general element results:

$$\frac{\partial U_f}{\partial Y_\alpha^\beta} = \frac{\partial U_f}{\partial Y_i^P}\frac{\partial Y_i^P}{\partial Y_\alpha^\beta} = \phi_\beta(\xi_1^P,\xi_2^P)F_\alpha^{P int/f} \quad (8)$$

Using (7) and (6) one writes:

$$F_\alpha^{\beta int} == F_\alpha^{\beta int/mat} + \phi_\beta(\xi_1^P,\xi_2^P)F_\alpha^{P int/f} \quad (9)$$

In the solution process the second derivative of the Helmholtz free energy is also important, the fiber contribution, following the chain rule, is given by:

$$\frac{\partial^2 U_f}{\partial Y_\alpha^\beta \partial Y_\gamma^\xi} = \frac{\partial^2 U_f}{\partial Y_\omega^{\rho f}\partial Y_w^{\rho f}}\frac{\partial Y_\omega^{\rho f}}{\partial Y_\alpha^\beta}\frac{\partial Y_\omega^{\rho f}}{\partial Y_\gamma^\xi} + \frac{\partial^2 U_f}{\partial Y_\omega^{\rho f}\partial Y_\pi^{\eta f}}\frac{\partial Y_\omega^{\rho f}}{\partial Y_\alpha^\beta}\frac{\partial Y_\pi^{\eta f}}{\partial Y_\gamma^\xi} + \frac{\partial^2 U_f}{\partial Y_\pi^{\eta f}\partial Y_\omega^{\rho f}}\frac{\partial Y_\pi^{\eta f}}{\partial Y_\alpha^\beta}\frac{\partial Y_\omega^{\rho f}}{\partial Y_\gamma^\xi} + \\ + \frac{\partial^2 U_f}{\partial Y_\pi^{\eta f}\partial Y_\pi^{\eta f}}\frac{\partial Y_\pi^{\eta f}}{\partial Y_\alpha^\beta}\frac{\partial Y_\pi^{\eta f}}{\partial Y_\gamma^\xi} \quad (10)$$

4. EQUILIBRIUM EQUATION AND SOLUTION PROCESS

In this section, the strategy adopted to solve the reinforced 2D solid geometrically nonlinear equilibrium is described.

The nonlinear analysis starts writing the total potential energy as follows:

$$\Pi(Y) = U_{mat}(Y) + U_{fib}(Y) - \Omega(Y) \quad (11)$$

where Π is the total potential energy of the system, U is the Helmholtz free energy including matrix and fiber contributions written regarding solid nodal positions and Ω is the potential energy of external conservative applied forces given by:

$$\Omega = F_j Y_j \quad (12)$$

where F_j is the vector of external forces and Y_j is the current position vector.

The Principle of Stationary Total Potential Energy, Tauchert (1974), is applied writing the equilibrium equations as the derivative of total energy regarding nodal positions (2D solid for instance), as:

$$g_j = \frac{\partial \Pi}{\partial Y_j} = \frac{\partial(U_{mat}+U_{fib})}{\partial Y_j} - F_j = F_j^{int} - F_j = 0 \quad (13)$$

where F_j^{int} is the internal force vector or the strain energy gradient vector calculated regarding solid nodal positions, Equation (6). The nodal current positions are the unknowns of the problem, so, when adopting a trial position in Equation (13) g_j is not null and becomes the unbalanced force vector of the Newton-Raphson procedure, Luenberger (1989). Expanding the unbalanced force vector around a trial solution $\boldsymbol{Y^0}$, one has:

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$$g_j(\mathbf{Y}) = g_j(\mathbf{Y^0}) + \left.\frac{\partial g_j}{\partial Y_k}\right|_{(\mathbf{Y^0})} \Delta Y_k + O_j^2 = 0 \tag{14}$$

which can be rewritten, neglecting higher order terms as:

$$\Delta Y_k = -\left(\left.\frac{\partial g_j}{\partial Y_k}\right|_{\mathbf{Y^0}}\right)^{-1} g_j(\mathbf{Y^0}) = -\left(\left.\frac{\partial^2 (U_{mat} + U_{fib})}{\partial Y_k \partial Y_j}\right|_{\mathbf{Y^0}}\right)^{-1} g_j(\mathbf{Y^0}) = -\left(H_{kj}\right)^{-1} g_j\left(\mathbf{Y^0}\right) \tag{15}$$

where ΔY_k is the correction of position and $H_{kj} = \left.\frac{\partial^2 U}{\partial Y_k \partial Y_j}\right|_{\mathbf{Y^0}}$ is the Hessian matrix or tangent stiffness matrix.

The trial solution is improved by:

$$Y_k = Y_k^0 + \Delta Y_k \tag{16}$$

until ΔY_k or g_j become sufficiently small. The load level is incrementally applied in order to describe the equilibrium path of the analyzed biomechanical structure.

5. EXAMPLES

As mentioned before, the proposed concept to model passive and active behavior of muscles is splitting the muscle in soft tissue and fibers. Fibers can be passive or active. Passive fibers have simple elastic behavior, for which a stretching results in stress response, while active fibers receive a cerebral impulse that indicates its’ shortening. These fibers are embedded in the continuous (soft supporting tissue) through the special strategy described in this paper. This technique does not increase the number of degrees of freedom.

5.1 Muscle contraction - active muscular fibers

This first example illustrates the behavior of active muscular fibers inside the supporting tissue (soft), simulating a muscle contraction. The supporting tissue is discretized by 5700 cubic triangular elements and a total of 30 muscular fibers (2790 cubic fiber elements), each one with 0.1 of area, are considered in the model, see Fig. 2. The elastic modulus of the tissue and the fibers are 15000 and 150000 respectively.

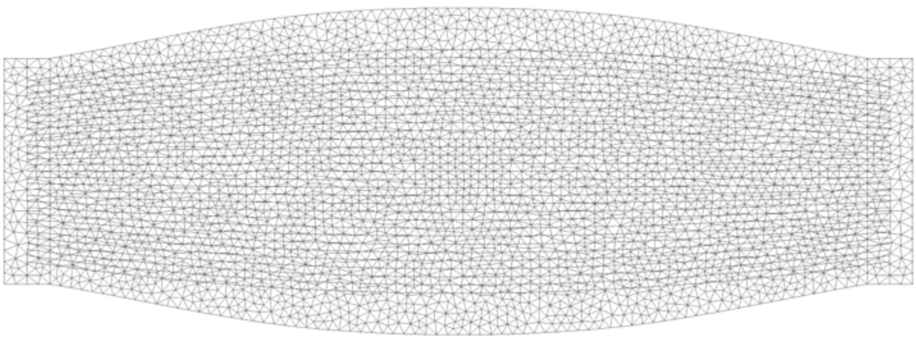

Figure 2. Supporting tissue and fibers

The horizontal and vertical displacements are restricted at the ends of the tissue. The results were obtained applying constant strain of 0.5% in each fiber of the muscle. The contraction of the fibers generates reaction forces at the restricted points, whose resultant value in horizontal direction is 2182.27.

Figure 3 shows the displacement distribution of tissue. As expected there is a shortening in transverse direction, related to the necessity of curved fibers become straight.

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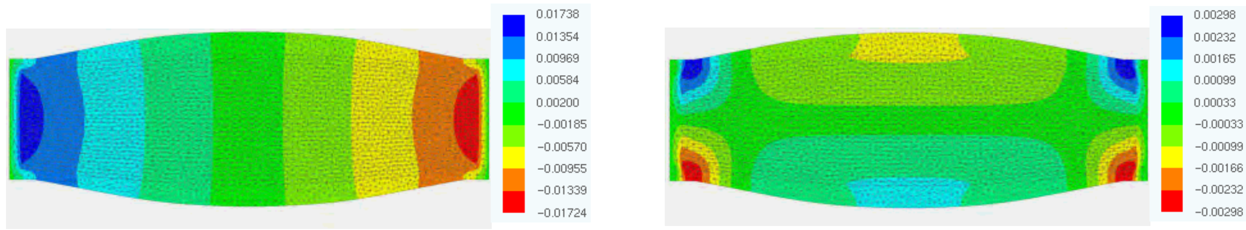
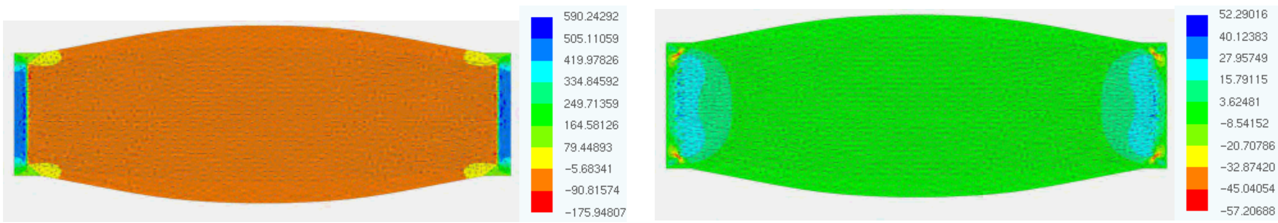


(a) (b)

Figure 3. Tissue displacements – (a) horizontal and (b) vertical directions

Fig. 4 shows the stress distribution in the soft tissue referred to the fiber contraction. As one can see, following horizontal direction, the extremities suffers severe tension while the center suffers moderate compression. Following transverse direction the central region suffers moderate compression while extremities suffer a general tension, reflecting the arch effects caused by initial geometry.

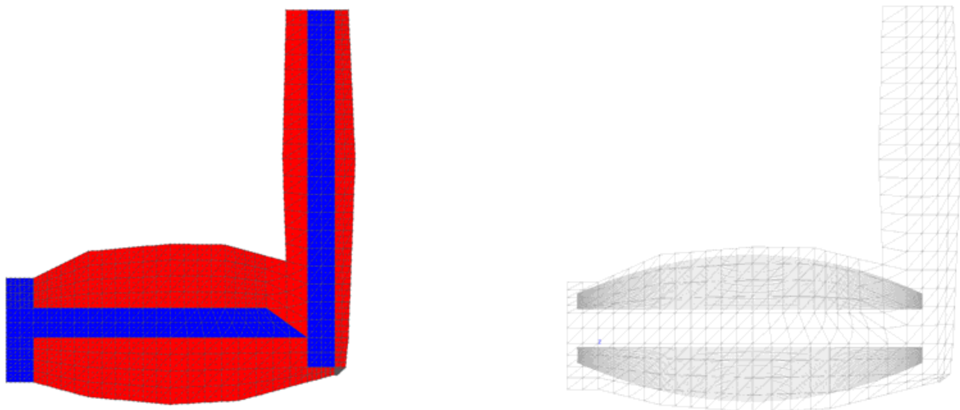


(a) (b)

Figure 4. Stress distribution – (a) horizontal and (b) vertical directions

5.2 2D Arm modeling

This example is an illustrative 2D arm modeling. The supporting tissue and bones are discretized by 640 cubic triangular elements, see Fig. 5a.



(a) Supporting tissue and bones mesh (b) Fibers inside the domain

Figure 5. Supporting tissue, bones and fibers

A total of 70 muscular fibers are considered for the biceps and triceps as depicted in Fig. 5b (35 each). The area of each fiber is 0.1.

The modeling is done applying a horizontal force (320) at the bone connection with the hand. The force is applied in two ways: from left to right (moving away the arm from the forearm) called here "distal force" and from right to left (approximating the arm and the forearm) called here "proximal force".

The first, "passive modeling", considers all tensioned fibers passive with a very high elastic modulus (300000) while compressed fibers have small elastic modulus. The second, "reactive modeling" considers a very low elastic modulus for all fibers (300); however fibers that are subject to tension (elongation) are considered actives, i.e., the muscle contracts in a way to cancel the detected strain (sensor and actuator). This contraction is always applied at the end of each step, all models employ 1000 steps.

The elastic modulus of the bone was considered as $15x10^7$.

Figure 6 shows a comparison between passive and reactive models for distal force and Fig. 7 shows a comparison for the proximal force.

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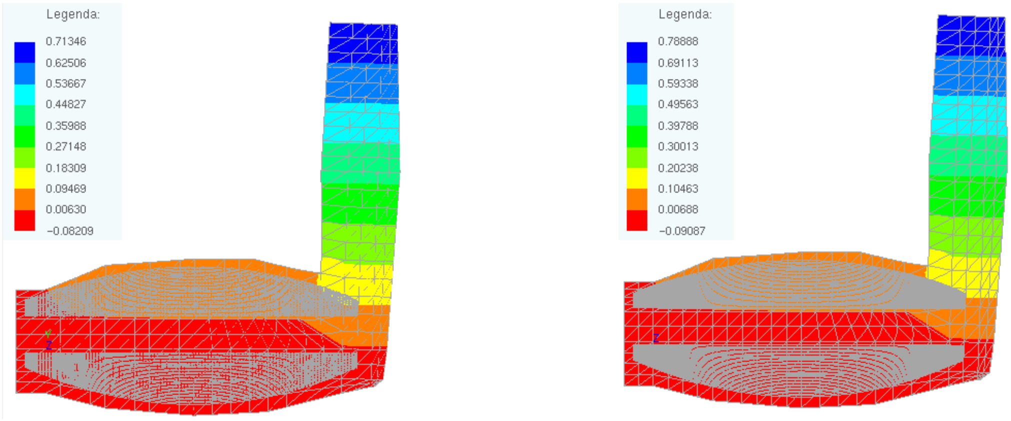
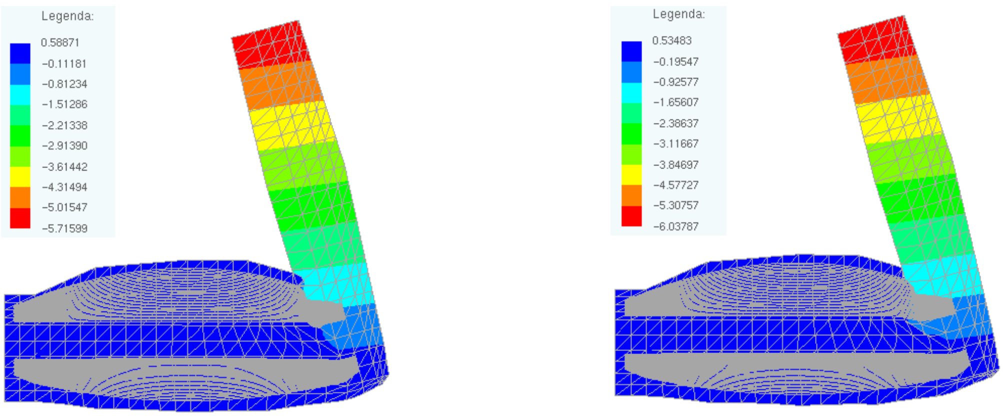


(a) Passive modeling (b) Reactive modeling

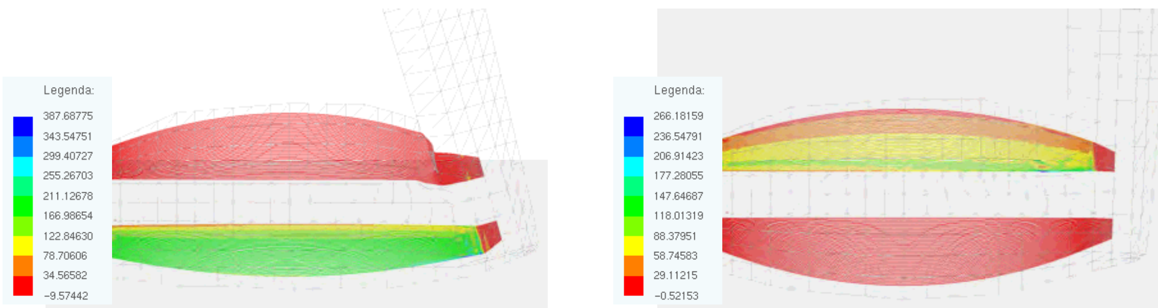
Figure 6. Distal force – (a) Passive, (b) reactive



(a) Passive modeling (b) Reactive modeling

Figure 7. Proximal force – (a) Passive, (b) reactive

As expected, muscles compensate the small elastic modulus by severe contraction. Figure 8 shows the normal force in fibers for proximal and distal reactive modeling. As expected, the signal of muscular fiber forces are in acoordance with the global achieved equilibrium.



(a) Proximal (b) Distal

Figure 8. Fibers forces in reactive models

6. CONCLUSIONS

A general strategy to model skeletal muscle has been proposed and successfully tested. This scheme is very promising due to its generality and easy way of using. The direct imposition of contraction in muscular fibers guarantees a high stiffness to reacting and active muscles when comparing with much stiffer passive fibers. The combination of contracting fibers, passive fibers and a passive supporting tissue reveals to be an efficient and simple

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way to model skeletal muscles. Further works will include: a 3D implementation, a better constitutive model for active and passive fibers as well as for the supporting tissue.

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