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FRICTION DYNAMICS MATHEMATICAL MODELING IN SPECIAL PNEUMATIC CYLINDER

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Abstract: *This work deals with the friction dynamics mathematical modeling in a special pneumatic cylinder from the improved LuGre model. The friction it's one of their main nonlinearities that disturb the control of pneumatic actuators and affects the precision of the system position response. In pneumatic positioning systems the friction forces between the slider-piston system surfaces in contact are strongly dependent on the physical characteristics of the contacting surfaces, such as their material properties and geometry, and their lubrication conditions. Including such factors could cause instability on system performance described through a 5th order model. The parameters of the friction, to Static friction, Coulumb friction, Stribeck friction and viscous friction are obtained from the experimental tests adjusted with the aid of the nlinfit function from the Matlab® software. The prototype and design are been developed at the in NIMeP/UNIJUÍ with financial support by ANEEL P&D Program and CELPE (partner company). The knowledge of the friction force in pneumatic actuators is an important step to obtain the precise control and its appropriate design.*

Keywords: *Friction modeling, pneumatic actuators, parameters identification.*

1. INTRODUCTION

This work presents the friction dynamics mathematical modeling in a special pneumatic cylinder that used the LuGre model, likewise the identification of friction parameters through of experimental tests realized in a bench of the Nucleus of Mechanization Prune and Innovation (NIMeP), belonging to Department Exact Sciences and Engineering (DCEEng) in Campus Panambi of the Regional University of Northwestern Rio Grande do Sul State (UNIJUÍ).

Pneumatics actuators are very attractive system for various applications, especially in robotic, because there are advantages i.e. low cost, lightness, durability and it's clean when compared with the hydraulic actuators, also it owns easy maintenance, self-cooling properties, good power density (power/dimension rate), fast acting with high accelerations and installation flexibility and besides the compressed air is available in almost all industry plants. These characteristics become pneumatic actuators competitive in a large band of applications in motion control to materials and parts handling, packing machines, machine tools, robotics, food processing and process industry (Guenther et al., 2006; Weickgenannt et al., 2010; Qiong et al., 2011). The pneumatics servo positioning also presented low risk of environmental contamination and the operation in relation the hydraulic systems, since, if occur in hydraulic a leak oil system, it can generate serious environmental damages, even more if this oil can be flammable, point out Suzuki (2010).

Unlike of these advantages, pneumatic positioning systems have some undesirable characteristics which limit their use in applications that require a fast and precise response (Guenther et al., 2006; Allgayer, 2011). These undesirable characteristics derive from the high compressibility of the air (Weickgenannt et al., 2010) and from the nonlinearities present in pneumatic systems, such as comportment nonlinear of the air flow-pressure relationship through valve orifice and their servo valve dead zone (Valdiero et al., 2011), besides friction in linear cylinder seal (Andrighetto et al., 2006).

Friction is effect caused by contacting surfaces direct between both surfaces which present relative movement and affect behavior of the pneumatics systems causing difficulties of control and degradation in system performance, it brings on instability for it presents nonlinear characteristics of difficult modeling.

In section 2 is shown the work purpose. Next in section 3 the definition of methodology. A brief description of the pneumatic actuator is present in section 4. In section 5 occur the mathematical modeling of the system considered nonlinear characteristics. Section 6 shows the obtained results in mathematical modeling and in computer simulations, proven in experimental tests. At last the conclusions.

2. WORK PURPOSE

This work is one of the results of the research and development project in scope of the Research and Technological Development Program of the Electrical Energy Sector regulate by ANEEL (ANEEL code PD-0043-0311/2011) and deal of the pneumatic in research for application in prune of trees equipment. The study and mathematical modeling of a pneumatic actuator along the course of movement allows adequate removal of worker or fixed base of the robotic manipulator in relation when branch what being cut, this averting the serious accident, especially in high voltage network line, this way the dynamic behavior these equipment need to be modeled.

The pruning problem is that same needs the use of reliable equipment in required movements. However, there are several factors that difficult to obtain good performance with repetitions, friction in the pneumatic cylinder seal is just one (RICHTER, 2013).

In doing so this work presents some results of the friction parameters identification a pneumatic actuator over the given application in forest equipment for arrange working about 2,5meters of course.

3. METHODOLOGY

To obtain the static parameters of the friction dynamic model are realized different experiment in open mesh and steady-state (constant velocity). This way between each value of the signal control, in rung, to obtain a velocity in steady-state and is calculate possible the corresponding friction force. Like this each experiment represents one point in static map of friction, and adjusts the nonlinear mathematical model of the friction force in steady-state the experimental points with the aid of the *nlinfit* function from the MatLab *software* and obtained the dynamic friction parameters values.

For analysis of the results it used computer simulations realized in MatLab/Simulink software with Runge Kutta integration method, step fixed of 0.0001s, with data of the experimental tests realized in a bench of the Nucleus of Prune Mechanization and Innovation (NIMeP), belonging to Department Exact Sciences and Engineering (DCEEng) in Campus Panambi of the Regional University of Northwestern Rio Grande do Sul State (UNIJUÍ).

In experimental work bench, where it was realized the tests, there is a system of control and data acquisition compound over the plate dSPACE 1104 responsible for the capture and storage data of the tests bench, the what use integration of the software's MatLab/Simulink and ControlDesk permitted capture, control and manipulation of the data in real time through a construction of a graphic interface, that possibility the detail analysis of the obtain results.

4. DESCRIPTION OF THE PNEUMATIC ACTUATOR

The servo pneumatic positioning linear is a system dynamic composed over the servovalve of directional control, a special pneumatic cylinder linear of action pair and simple piston, and control system. This servo system allows positioning one load in desired position of the actuator curse or following a variable trajectory in time function. Figure 1 illustrated through a schematic drawing the servo pneumatic positioning.

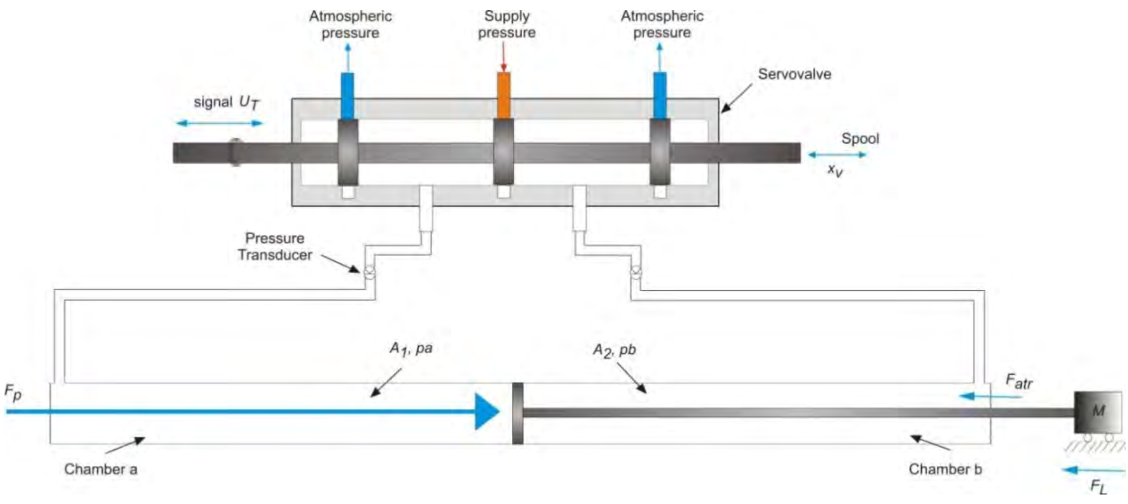

Figure 1.Schematic drawing of a servo pneumatic positioning linear.
Source: Author herself

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The servo pneumatic positioning works with air compressibility that is provided servo valve a given of supply's pressure (p_s) previously regulate. During the operation, the control signal U_T energizes valve's solenoid so that a resulting magnetic force is applied in the valve's spool, producing the spool displacement. The spool displacement opens control orifices so that one port is connected to the supply's pressure line and the other is connected to the atmosphere. Consequently, there is the pressure difference between cylinder chambers, resulting in a force that moves the mass Min a positive or negative displacement y, depending on the control signal input. This generate force in pneumatic actuator is related to the pressure difference between the two sides of the piston, call pneumatic force (F_p).

5. MATHEMATICAL MODELING OF THE PNEUMATIC ACTUATOR WITH FRICTION DYNAMIC

The model nonlinear of 5th order also is describe in the way of variable state, considering $y_1 = y$, $y_2 = \dot{y}$, $y_3 = p_a$, $y_4 = p_b$ and $y_5 = z$:

$$\dot{y}_1 = y_2 \tag{1}$$

$$\dot{y}_2 = -\frac{F_{atr}(y_5, y_2)}{M} + \frac{A_1}{M} y_3 - \frac{A_2}{M} y_4 \tag{2}$$

$$\dot{y}_3 = -\frac{\gamma A_1}{V_{a0} + A_1 y_1} y_2 y_3 + \frac{\gamma RT}{V_{a0} + A_1 y_1} q_{ma}(U_T, y_3) \tag{3}$$

$$\dot{y}_4 = \frac{\gamma A_2}{V_{b0} - A_2 y_1} y_2 y_4 - \frac{\gamma RT}{V_{b0} - A_2 y_1} q_{mb}(U_T, y_4) \tag{4}$$

$$\dot{y}_5 = y_2 - \alpha(y_5, y_2) \frac{\sigma_0}{g_{ss}(y_2)} sgn(y_2) y_5 \tag{5}$$

where y_1 is position of the piston, y_2 is velocity, y_3 and y_4 the pressure in the chambers a e b cylinder, and y_5 is dynamic of the microscopic deformations, A_1 and A_2 are the cylinder cross-sectional area of the chambers a and b, T is the air supply temperature, R is the universal gas constant, $\gamma = C_p / C_v$ is the ratio between the specific heat values of the air, where C_p and C_v are the specific heat of the air at constant pressure and Constant volume, respectively, V_{a0} and V_{b0} are the initial volume of air in the line and at the chambers a and b extremity, include the pipeline, p_a and p_b are the pressures in the chambers a and b, respectively, q_{ma} and q_{mb} are the air mass flow rate into chambers a and b of the cylinder, given by equations (6) and (7) respectively.

$$q_{ma}(U_T, p_a) = g_1\big(p_a, sgn(U_T)\big) arctg(2U_T) \tag{6}$$

$$q_{mb}(U_T, p_b) = g_2\big(p_b, sgn(U_T)\big) arctg(2U_T) \tag{7}$$

where g_1 and g_2 are signal function signal given by equations (8) and (9).

$$g_1\big(p_a, sgn(U_T)\big) = \beta \Delta p_a = \begin{cases} (p_s - p_a)\beta^{ench} & if \quad U_T \geq 0 \\ (p_a - p_{atm})\beta^{esv} & if \quad U_T < 0 \end{cases} \tag{8}$$

$$g_2\big(p_b, sgn(U_T)\big) = \beta \Delta p_b = \begin{cases} (p_s - p_b)\beta^{ench} & if \quad U_T < 0 \\ (p_b - p_{atm})\beta^{esv} & if \quad U_T \geq 0 \end{cases} \tag{9}$$

ISSN 2176-5480

Where p_s is the supply pressure, p_{atm} is the atmospheric pressure and β^{ench} and β^{env} are the constant coefficients characteristics respectively of the to fill up and to deflate cylinder chambers, obtained experimental according Endler (2009).

The friction force produced by contact between surfaces it expresses in the equation (2), is given by friction LuGre model, purpose by Canudas-de-Wit *et al.* (1995 apud Ritter, 2010), by adapted instead of a viscous friction of drag, which is caused by resistance when movement of a rigid body through a flow, proportional when velocity square and a lot time result of agitated out flow Ge et al. (1999 apud Valdiero, 2005, p.35), as show in Fig. 2.

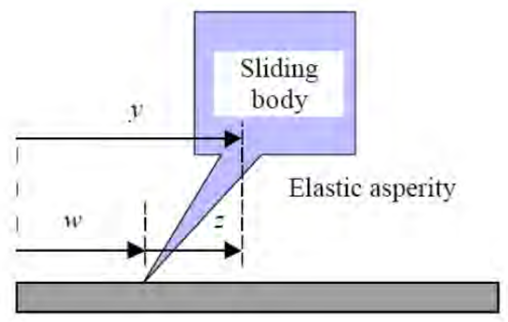

Figure 2. Model of subject to friction force showing elastic (z) and inelastic (w) displacement components.
Source: Valdiero *et al*. (2010).

$$F_{atr} = \sigma_0 z + \sigma_1 \dot{z} + C_d\big(\dot{y}(t)\big)^2 sgn\big(\dot{y}(t)\big) \tag{10}$$

where σ_0 is the stiffness coefficient of the microscopic deformations z during the presliding displacement, z is a friction internal state that describes the average elastic deflection of the contact surfaces during the stiction phases, σ_1 is a damping coefficient associated for variation rate of z, C_d is the drag coefficient, $\dot{y}$ is the velocity relative between surfaces and the function $sgn\big(\dot{y}(t)\big)$ which maintain an element characteristic.

The microscopic deformations in z (Eq. 5) is present by Eq. (11):

$$\frac{dz}{dt} = \dot{y} - \alpha(z,\dot{y})\frac{\sigma_0}{g_{ss}(\dot{y})}|\dot{y}|z \tag{11}$$

where $g_{ss}(\dot{y})$ is a positive function that describes the steady-state characteristics of the model for Constant velocity motions and given by:

$$g_{ss}(\dot{y}) = F_c + (F_s - F_c)e^{-\left(\frac{\dot{y}}{\dot{y}_S}\right)^2} \tag{12}$$

where F_c is the Coulomb friction force, F_s is the static friction force and $\dot{y}_s$ is the *Stribeck* velocity established of a reduction quick of the friction force resulted instant in that tear body the static friction until to reach Coulomb friction. Figure 3 illustrate the behavior of the friction force as a function of velocity in steady-state.

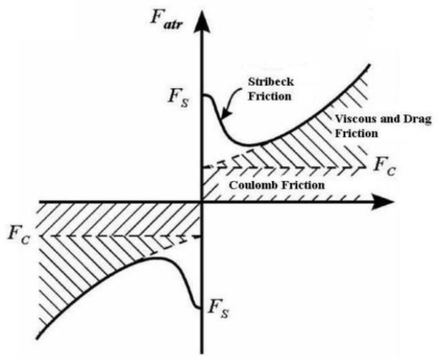

Figure 3. Friction force characteristics combined in steady-state.
Source: Valdiero *et al*. (2010).

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The function $\alpha(z,\dot{y})$ in accordance with (Dupont *et al.*, 2000; Ritter, 2010) go incorporate the LuGre model and is used to represent the *stiction,* in other words, the static regime in very low velocities. This function is defined by Eqs. (13) and (14).

$$\alpha(z,\dot{y}) = \begin{cases} \left.\begin{array}{ll} 0, & if\ |z| \le z_{ba} \\ 0 < \frac{1}{2} sen\left(\pi \frac{z - \left(\frac{z_{max}(\dot{y}) + z_{ba}}{2}\right)}{z_{max}(\dot{y}) - z_{ba}}\right) + \frac{1}{2} < 1, & if\ z_{ba} < |z| < z_{max}(\dot{y}) \\ 1, & if\ |z| \ge z_{max}(\dot{y}) \end{array}\right\} \begin{array}{c} sgn(\dot{y}) \\ = \\ sgn(z) \end{array} \\ \quad 0, \qquad if\ sgn(\dot{y}) \neq sgn(z) \end{cases}$$

$$0 < z_{ba} < z_{max}(\dot{y}) = \frac{g_{ss}(\dot{y})}{\sigma_0}\ to\ \forall \dot{y} \in \Re \tag{14}$$

where z_{ba} is a breakaway displacement, such that to $z \le z_{ba}$, all movements is friction interface consists in elastic displacement only and z_{max} is the maximum value of microscopic deformations and is velocity dependent. In detail modeling are can get in Richter (2013).

6. RESULTS

This section presents the results of the pneumatic actuator dynamic mathematical modeling, where stand out experimental identification of the friction dynamics characteristics.

Values of the pneumatic actuator parameters utilized in the work bench of experimental tests are presented in Table 1.

Table 1. Parameters values of the pneumatic actuator used in the numerical simulations.

Parameters description	Simbology	Value	Observations
Right limit of dead zone	zmd	$0{,}78\ volts$	Experimentally obtained
Left limit of dead zone	zme	$0{,}78\ volts$	
Right slope of output	md	1	
Left slope of output	me	1	
Constant coefficient to fill up	β^{ench}	0.69501×10^{-8}	Experimentally obtained according Endler (2009)
Constant coefficient to deflate	β^{esv}	0.898105×10^{-8}	
Supply pressure	p_s	$7 \times 10^5\ Pa$	Measured
Atmospheric pressure	p_{atm}	$1 \times 10^5\ Pa$	Literature
Universal gas constant	R	$287\ Jkg/K$	
Temperature of supply air	T	$293\ K$	
Specific heat ratio, dimension less	γ	1.4 $Adimensional$	
Forcer area	A_1	$4.1 \times 10^{-3}\ m^2$	Calculated
Deducted piston rod area	A_2	$3.0 \times 10^{-3}\ m^2$	
Initial volume at chamber *a*	V_{a0}	$5.1 \times 10^{-3}\ m^3$	
Initial volume at chamber *b*	V_{b0}	$4.0 \times 10^{-4}\ m^3$	
Breakaway displacement	z_{ba}	$0.9\ z_{max}$	Calculated
Acoplate mass	M	$5\ kg$	Measured

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The static parameters identification of the friction is from various experiments in open mesh, it has a signal input control signal in tension, varied of low velocities until high velocity system work, data capture of the experimental work bench. Moreover, each one of the realized experiments represents a graphic point presented in Figure 4.

Experimental tests was realized in work bench with a special double action pneumatic cylinder and simple piston with different area ($A_1 \neq A_2$) available in the NIMeP/UNIJUÍ laboratory. For each experiment determined the best time interval in that actuator position in this stretch representing a line (combined with the pressures and pneumatic force when constants). Moreover the constant velocity, the acceleration is canceled and force produces by difference of chambers pressures equaled the actuator friction force, in accordance with the equation movement express by 2th Newton of law ($M\ddot{y} + F_{atr} = F_p$), i.e. in steady state $F_{atr} = F_p$. With the friction force in steady state, obtained experimental curve represents a static map where the statics parameters friction four, C_d, F_c, F_s e $\dot{y}_s$, it could easily identifies. In this procedure is used the algorithm *nlinfit* of the MatLab *software*.

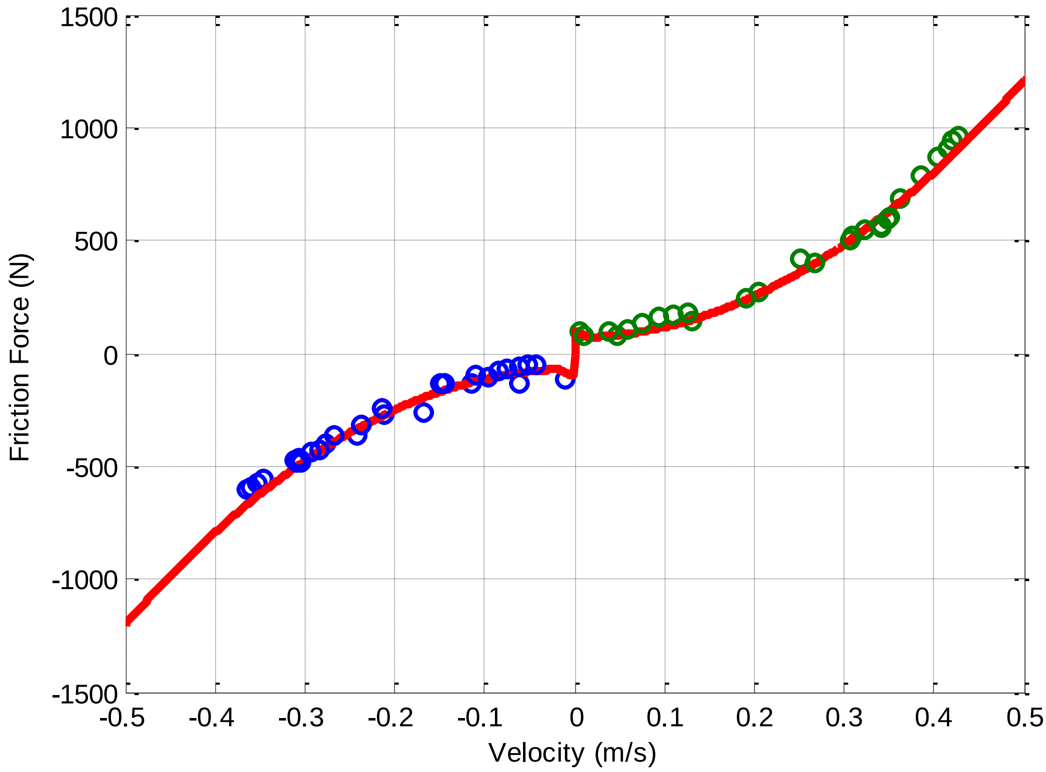

Figure 4. Determination of the friction static map in a special double action and simple piston pneumatic cylinder.

This way, from the algorithm *nlinfit* and computer simulations the best adjust for statics parameters of the friction is describe in Table 2 and with acquisition this static parameters it is possible to obtain the parameters friction dynamics, σ_0 e σ_1.

The assessment of the friction dynamic parameters of a pneumatic actuator is describe according Perondi (2002) which obtained from the realization of measures micro displacements of until 1 μm $(1.10^{-6}m)$ in regime of pre-sliding, used an equipment accurately (optical rosette), while for bigger displacements it needs to carry incremental measured.

According to Perondi (2002) the measures of the displacement forcer cylinder in regime pre-sliding were hampered by presence of dynamics components because of transmission of mechanics vibrations of the working environment for bench through the base. In this work also used the purpose methodology by Valdiero (2005) *apud* Miotto (2009), in that parameters σ_0 it has adjusted value from simulations, followed the premise of that order of the micro deformations z in region of pre-sliding be acceptable values between of $1\, e\, 50\, \mu m$. This way the friction dynamic parameter σ_0 is represented by equation (15).

$$\sigma_0 = \frac{F_c}{|1a50|.10^{-6}} \tag{15}$$

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Valdiero (2005) asserts that the dynamic parameter value σ_1, called damping coefficient proportional to the pre-sliding displacement change rate, can be obtained by the following expression,

$$\sigma_1 \leq \frac{\sigma_2}{\left(\frac{F_s}{F_c} - 1\right)} \tag{16}$$

Table 2. Lugre friction model parameters.

Parameters description	**Simbology**	**Value**
Static friction force	F_s	$100\ N$
Coulomb friction force	F_c	$70\ N$
Drag coefficient	C_d	$4546.7\ Ns^2/m^2$
Stribeck velocity	$\dot{y}_s$	$0.01\ m/s$
Stiffness coefficient	σ_0	$1.4 \times 10^6\ N/m$
Damping coefficient	σ_1	$50\ Ns/m$

With the pneumatic actuator parameters determined and identified, it made the simulation through implementation of the mathematical model purpose in way of blocks diagram shown in Figure 5.

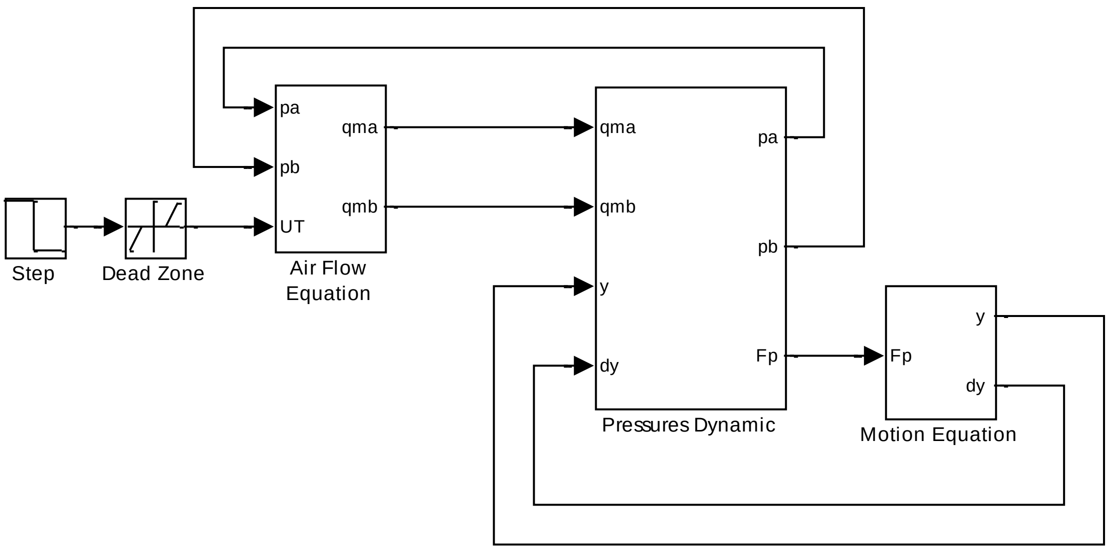

Figure 5. Blocks diagram of the pneumatic actuator nonlinear mathematical model used in computer simulations.

The Figure 6 present results of the developed mathematical model validation, in the advance with step into of *7 Volts* and initial condition to with draw of the experimental tests realized bench, it comparing the position of the pneumatic actuator obtained in computer simulations without friction dynamic and with identification of the parameters friction dynamic the realized experimental tests.

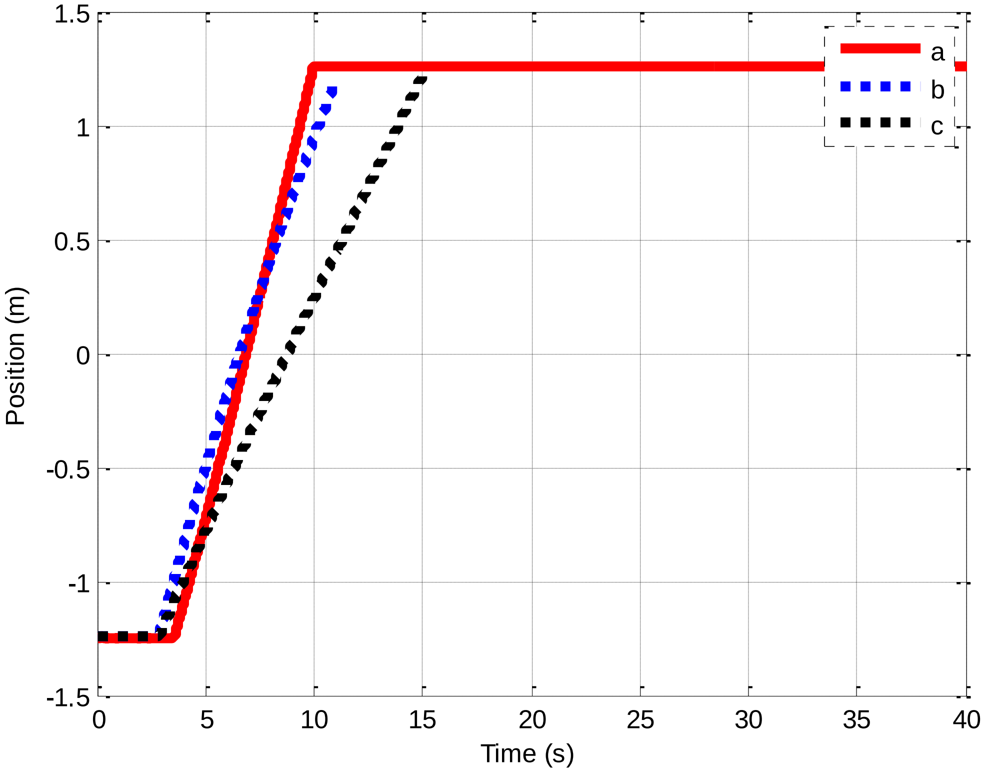

Figure 6. Comparative graphic of the experimental tests with the simulations for advance movement of the piston cylinder (for control signal input of 7Volts) in which (a) experimental, (b) with identified dynamic friction and (c) without dynamic friction.

7. CONCLUSION

It presented the development of a mathematical model which describes the dynamic comportment of a special pneumatic cylinder from the improved LuGre model, for application in with forest equipment positioning motion, including main nonlinear characteristics. The comparison of the computer and experimental results illustrates the validation of the mathematical model purposed for study of the dynamic pneumatic actuator with nonlinear friction, as well as validation of the methodology for parameters identification. Results obtained will are used in improvement for the P&D project with title “Development of Mechanized Solution for the Tree Pruning with Incidence on Energized Air Lines of Electrical Energy Distribution”.

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