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DYNAMIC ANALYSIS OF PIPE AS EULER BERNOULLI BEAM WITH GFEM AND HFEM

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Abstract. *Pipe line is a widely used mean for natural gas and petroleum product transportation. Actually, petroleum is an essential way to provide energy and definitely plays an essential paper in economy. In the other hand, the leak of petroleum means financial and environment injure. Such occurrence may be caused by pipeline material degradation or external load, which leads mechanical stress to exceed material rupture limit. The material degradation could be caused by corrosion, in which the pipeline wall thickness is reduced, producing an irregular defect. Consequently, producing stress concentration effect and lead the pipe to a rupture pressure less than the original pressure. By the other hand, external loading could be instigated by earthquake, soil movement, or during the installation. For these cases, it is necessary to carry out an analysis a priori, to guarantee that the pipe will resist those loads. The pipe integrity assessment can be done by numerical calculation, by using Finite Element Method. Such methodology is widely used to evaluate solid behavior once it is subject to an external loading. This method is able to assess nodal displacement, and then calculate strain and stress. This work has the purpose to carry out a dynamic analysis considering pipeline subjected to external loading and free of corrosion. By this way, a local analysis is unnecessary, and only the global analysis will be done. From this point of view, in order to simplify the numerical solution, hypothesis of Euler Bernoulli beam will be adopted, and this beam model will be simulated by the use of a beam element of 2 nodes. For the dynamic solution method, implicit method will, together with HFEM and GFEM.*

Keywords: *GFEM;HFEM;EULER-BERNOULLI BEAM;IMPLICIT METHOD;IMPACT*

1. INTRODUCTION

Wave propagation in solid could be induced by different type of loads, which can be classified in practice as either conventional or impact loads. Impulsive loads are characterized by an instantaneous rise in magnitude followed by a rapid decrease during a very short interval of time. And most frequently, it is applied over a small area of the body. For conventional loads, such as harmonic or static loads, each part of body influences stresses and strain that occur in other parts of the body. However, under impact loads, stresses and strains are localized in the region at vicinity of the point where the impact load was applied.

When an elastic medium is deformed, usually, two types of waves may be propagated, wave of dilatation and waves of distortion. In addition to these two types of elastic waves, there are elastic waves which are propagated along the surface of a solid. These surface waves, known as Rayleigh waves have an important paper in seismic phenomena.

In 1972, one of the most pioneer works in the area of elastodynamics was presented by Goudreau and Taylor. In such work, several aspects of the differential equation governing the dynamic behavior were analyzed. As well as, different solution methods were analyzed, including implicit methods like Newmark and Houbolt, and explicit method. One of the merits of this study was to carried out an analysis of numerical error inherent in the method of discretization in time and in space discretization method. The stability condition has been intensively researched and tested in several examples. With the methods used by the authors, it was possible to capture the passage of wave using beam element. Wave profile was compared using different methods of solution. In 1977, Hilber et. al. have developed a method, later known as HHT, based on implicit Newmark method, in which they have introduced an adjustable parameter to control numerical dissipation. The method was developed with the purpose to provide an unconditionally stable solution for the equation of dynamic equilibrium. According to the authors, the Newmark method presents numerical oscillation for second and third order of derivative in time of displacement. In this case, it would be desirable to develop a method which aims to reduce numerical oscillation, but without affecting the vector of displacement. The HHT method is simple to program for those who already has Newmark algorithm implemented, only one parameter is introduced to calculate independent parameters of Newmark. According to stability study the authors, this parameter varies between 0 and $-1/3$. In 1983, Zienkiewicz et. al. have presented an alternative method of finite element solution, later known as hierarchical finite elements method (HFEM). The presented formulation was based on result enrichment by adding hierarchical polynomials without affecting the original shape function, deducted by Lagrange polynomial. In 2004, Solin et. al. have published a book dealing with HFEM using various categories of hierarchical polynomial as enrichment shape functions. They are functions of Legendre, of Lobatto, of Kernel, and among the others. In 2009, Arndt has developed a study on the performance analysis GFEM dynamic frame structure, more specifically, the natural

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frequency analysis of structure. In this study, several methods of refinement were analyzed and compared, such as h, and adaptive. The work began with the function analysis of enrichment for bar and later extended to Euler-Bernoulli beam. The main contribution of this research was the proposed enrichment sine functions, which have a similar curve shape in comparison to displacement curve. In 2009, Monteiro developed a new catcher to be introduced to discontinuous finite element method in time, in order to catch the wave passage in the solid. The time discontinuous finite element method shows to be efficient to identify the passing wave, even when the displacement or speed curve behaves as degree function in time. At the singular point, the conventional finite element method was showing results with numerical oscillation to, even using efficient implicit method. However, for time discontinuous finite element method, such numerical oscillation was not observed either in the beam element, or in the 2D quadrilateral element. In 2012, Torri has developed a study focused on the analysis of dynamic bar, beam Euler - Bernoulli, two-dimensional wave propagation and state of stress by using GFEM, HFEM and FEM with quadratic shape function. In this work, these methods were compared to determine the natural frequency of the structure, and GFEM was more accurate than other methods.

2. GENERALIZED FINITE ELEMENT METHOD AND HIERARCHICAL FINITE ELEMENT METHOD

The conventional FEM is based on polynomial shape function, such as Lagrange or Hermite function, for a field of nodal unknown variables. This type of formulation has some inherent disadvantages once it is desired to increase the order of approximation of the element. In this case, for addition of nodal shape function, all the other functions, already existents, should be changed completely. To avoid this problem, it is possible to define shape functions which once introduced into the approximation, doesn't have to change the shape functions previously defined, from the conventional formulation. By presenting this important feature, these shape functions are called hierarchical shape functions. Beside the conventional nodal shape function, other enrichment shape functions and their corresponding unknowns value no longer have the meaning of physical variable, as the conventional nodal function. The formulation of hierarchical finite element method (HFEM) differs from the conventional formulation due to the use of hierarchical shape functions of roder defined by user (Zienkiewicz, 1983). These functions are introduced into conventional finite element method in order to make the refine in solution. In elasticity problems, the hierarchical formulation consists in introducing new modes of deformation, by increasing the number of hierarchical shape function, as it is used in the interpolation of the physical variable (Zienkiewicz, 1983). The HFEM in its traditional formulation uses Lagrange polynomials as shape functions. This is because these polynomials are relatively easy to build and meet requirement of FEM, which can easily applied with boundary conditions. However, the Lagrange polynomials of lower order are all different from Lagrange polynomials of higher order. This provides difficulties once the approximation is improved by increasing the order of the polynomial approximation, in this case, all polynomials should be obtained again. The idea is to use HFEM space approximation that is hierarchical. That is, by increasing the order of approximation of n to $n+1$, where functions of order n don't have to be changed. The construction of hierarchical polynomial approximation spaces was described in detail by Solin et al. (2004). In this case, a rather simple way of building spaces hierarchical approach is to use Lobatto polynomials instead of Lagrange polynomial as local approximation functions.

For displacement in formulation of bar, Lobatto's function was used (Solin et. al. 2004), as given in below:

$$l_2(x) = \frac{1}{2}\sqrt{\frac{3}{2}}\left(x^2-1\right) \tag{2.1a}$$

$$l_3(x) = \frac{1}{2}\sqrt{\frac{5}{2}}\left(x^2-1\right)x \tag{2.1b}$$

$$l_4(x) = \frac{1}{8}\sqrt{\frac{7}{2}}\left(x^2-1\right)\left(5x^2-1\right) \tag{2.1c}$$

$$l_5(x) = \frac{1}{8}\sqrt{\frac{9}{2}}\left(x^2-1\right)\left(7x^2-3\right)x \tag{2.1d}$$

In the case with beam, that involves bending problem, Bardell's function was used (Bardell, 1991):

$$l_1 = \frac{1}{4}x^3 - \frac{3}{4}x + \frac{1}{2} \tag{2.2a}$$

$$l_2 = -\frac{1}{4}x^3 + \frac{3}{4}x + \frac{1}{2} \tag{2.2b}$$

ISSN 2176-5480

The Partition of unity method (PUM) presented by Melenk and Babuska (1997) was a foundation for other developed method in the solution of boundary value problem. The GFEM, or PUM, according to Barros (2002), was proposed independently by:

- Babuska and colleagues using the name of special finite element method, and latter as Partition of Unity Method (1997).
- Duarte and Oden (1996) *hp* cloud method, Oden et al. (1998) as a new option method of *hp* clouds.

The employment of the current name as the generalized finite element method was made by the first time in Melenk (1995, *ap ud* Barros 2002). The strategy used in the GFEM is to employ the shape functions of the FEM as PU. For two dimension elasticity problems, for example, are employed Lagrangian bilinear functions. The size of the original space of finite elements is then extended through the enriched function obtained by the *hp* clouds method. In other words, the GFEM is the method in which enriched functions are obtained from PUM and these functions are combined with the standard EFM polynomial in order to evaluate nodal unknown variable. The advantage by using PUM is the possibility to obtain enriched function that are representative of the phenomenon in question, these enriched function not necessarily have to be polynomial functions or other already know mathematical function. In the case of fracture, the function for enrichment could be a function already known for displacement evaluation. In the case of dynamic analysis, the enriched function can be adopted as a function of displacement in time, such as sine or cosine function. Such consideration allows a wider range of choose for local approximation spaces, and it can be used without changing the basic premises of FEM. An example for PU application is the shape functions, such as Lagrangian polynomial, where the clouds are formed by a set of finite elements that contribute to nodal values. This can be seen when analyzing these functions of Lagrange polynomials of order n=1. From figure 2.1 it can be noted that each function ϕ_i is defined inside two adjacent finite elements, except in the case of the function ϕ_1 and ϕ_{N_e+1}. The function, ϕ_2, for example, is defined at the junction of the first to the second finite element of figure 2.1. Therefore, in general, each subcover is given by the region defined by two neighboring finite elements. Consequently, the PU given by linear functions of the FEM based on Lagrange is as shown in figure 2.1, where each finite element is defined at the intersection of two subcover $\{\Omega_i\}$.

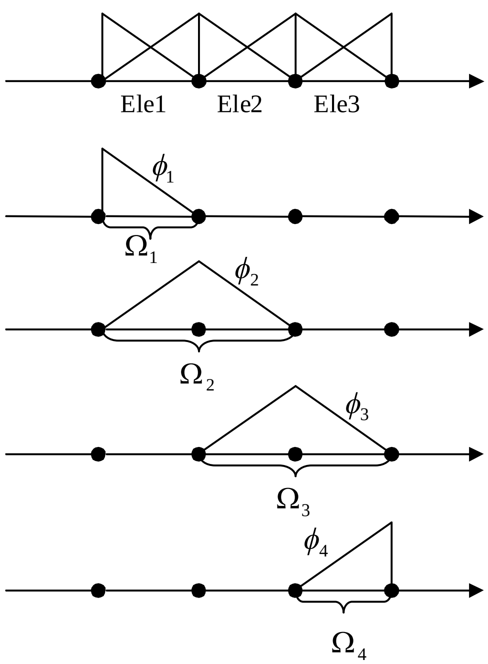

Figure 2.1

From above point of view, the equation for unknown nodal displacement is given:

$$\tilde{u}(x) = \sum_{j=1}^{N} \phi_j(x) \left\{ u_j + \sum_{i=1}^{qj} L_{ji}(x) b_{ji} \right\} = \Phi^T U \tag{2.3}$$

ISSN 2176-5480

Where,

$$U^T \overset{def}{=} \begin{bmatrix} u_1 & b_{11} & \dots & b_{1qj} & \dots & u_N & b_{N1} & \dots & b_{Nqj} \end{bmatrix} \tag{2.4}$$

Is the vector of nodal unknown variable, u_i, with unknown field variables, b_{ij}, which appears in the formulation due to enrichment function.

And,

$$\Phi^T \overset{def}{=} \begin{bmatrix} \phi_1 & L_{11}\phi_1 & \dots & L_{1qj}\phi_1 & \dots & \phi_N & L_{N1}\phi_N & \dots & L_{Nqj}\phi_N \end{bmatrix} \tag{2.5}$$

Is the vector with conventional FEM shape function ϕ_i, and enrichment function, L_{ij}, derived from PUM.

From previous works of Arndt (2010), some enrichment function was studied and analyzed for the case of dynamic analysis. These functions will be employed for this work and applied for several examples in comparison to HFEM and conventional FEM. The proposal of Arndt for bar formulation enrichment is:

$$v_{1j} = sen\left(\frac{\beta_j (x+1)}{2}\right) \tag{2.6a}$$

$$v_{2j} = \cos\left(\frac{\beta_j (x+1)}{2}\right) - 1 \tag{2.6b}$$

$$v_{3j} = sen\left(\frac{\beta_j (x-1)}{2}\right) \tag{2.6c}$$

$$v_{4j} = \cos\left(\frac{\beta_j (x-1)}{2}\right) - 1 \tag{2.6d}$$

And for beam formulation, including bending effect, Ardnt (2010) made proposal as:

$$v_{1j} = \cos\left(\beta_j \frac{\xi+1}{2}\right) - 1 \tag{2.7a}$$

$$v_{2j} = \cos\left(\beta_j \frac{\xi-1}{2}\right) - 1 \tag{2.7b}$$

3. HHT METHOD

In order to provide second order accuracy and unconditional stability, Hibert et. Al (1977) have developed a method based on Newmark method, by introducing new parameters into conventional parameters. Such new parameters provide the possibility to control numerical dissipation during the solution. The development made by Hilbert et. al. will be presented below.

Consider the original dynamic equation for vibration without damping. A new parameter named alfa was introduced into equation without affecting equation equilibrium.

$$[M]\{\ddot{u}\}_{n+1} + (1-\alpha)[K]\{u\}_{n+1} - \alpha[K]\{u\}_{n+1} = F(t_{n+1}) \tag{3.1}$$

The displacement and velocity vector will be evaluated according to conventional Newmark method. With initial condition, these equations will be:

$$u_{n+1} = u_n + \Delta t\, \dot{u}_n + \Delta t^2 \left(\left(\frac{1}{2} - \beta \right) \ddot{u}_n + \beta\, \ddot{u}_{n+1} \right) \tag{3.2}$$

$$\dot{u}_{n+1} = \dot{u}_n + \Delta t\, \left((1-\gamma) \ddot{u}_n + \gamma\, \ddot{u}_{n+1} \right) \tag{3.3}$$

$$u_0 = u$$

$$\dot{u}_0 = \dot{u}$$

$$\ddot{u}_0 = [M]^{-1} \left(F_0 - [K]\{u_0\} \right) \tag{3.4}$$

ISSN 2176-5480

According to study made by Hilbert et. al. (1977), when α assumes a positive value, then the numerical dissipation can not by efficient, and such parameter acts as a viscous linear damping ratio. For this reason, it was suggested the use of parameter α as a negative value and varies between 0 and -1/3. From the analysis of Hilbert et. al. (1977), α within this gamma of value will present numerical stability and dissipation. Once α is introduced into Newmark independents parameters, it will be:

$$\beta = (1-\alpha)^2 / 4 \text{ and } \gamma = 1/2 - \alpha \tag{3.10}$$

4. APPLICATIONS

Using enrichment function proposed by Arndt (2010) and HHT method with α equal to -0.1, several examples were analyzed with mechanical properties E = 207 GPa, density = 7830 kg/m^3.

4.1 Example: Bar subjected to static load

Consider a situation represented by figure 4.1, where the bar of 75 mm of external ratio and 70 mm of inner ratio, 1 m of length is subjected to a static load applied to free end of magnitude 1000 N. The analysis was carried out by using Newmark algorithm and HHT algorithm with parameter α equal to -0.1. During the analysis, 5000 time step was adopted with 1e-6 within each interval.

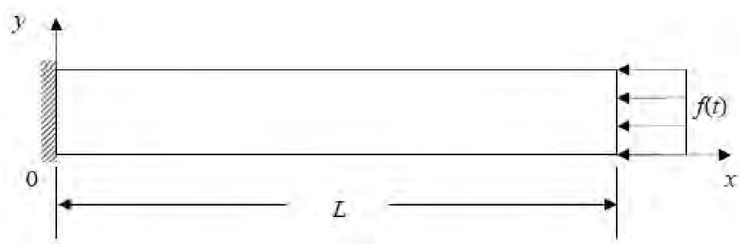

Figure 4.1 Bar subjected to static loading at the end.

The analytical solution for transversal displacement, velocity and acceleration, proposed by Nowacki, are shown at below:

$$u(x,t) = \frac{8 f_o L}{\pi^2 EA} \sum_{n=1}^{\infty} \left[\frac{(-1)^{n-1}}{(2n-1)^2} \sin\left(\frac{(2n-1)\pi x}{2L} \right) \left(1 - \cos\left(\frac{(2n-1)\pi ct}{2L} \right) \right) \right] \tag{4.1}$$

$$\frac{du(x,t)}{dt} = \frac{4 f_o c}{\pi EA} \sum_{n=1}^{\infty} \left[\frac{(-1)^{n-1}}{(2n-1)} \sin\left(\frac{(2n-1)\pi x}{2L} \right) \left(\sin\left(\frac{(2n-1)\pi ct}{2L} \right) \right) \right] \tag{4.2}$$

$$\frac{d^2 u(x,t)}{dt^2} = \frac{2 f_o c^2}{LEA} \sum_{n=1}^{\infty} \left[(-1)^{n-1} \sin\left(\frac{(2n-1)\pi x}{2L} \right) \left(\cos\left(\frac{(2n-1)\pi ct}{2L} \right) \right) \right] \tag{4.3}$$

Where $c = \sqrt{E/\rho}$.

In this analysis, different element formulation was put into comparison in order to study its behavior once it was used in different solution algorithm. The different element formulation is GFEM, HFEM and conventional finite element using first order (FEM) and second order (QFEM) Lagrange element. And for solution algorithm, it was considered the Newmark linear acceleration (L) and average acceleration (C), as well as the HHT method. The result of such comparison is shown at below.

From the figure 4.2, which is showing displacement curve, these methods have shown an acceptable results in comparison to analytical solution proposed by Nowacki. From observation made above, it is possible to say that the Newmark method doesn't present an acceptable behavior in first and second order of time derivative. But the next question was which element formulation has more stable behavior by using HHT method. In the figure 4.3, the velocity singularity was observed and all finite element formulation was capable to identify. But some of formulation presents numerical oscillation at the vicinity of singularity point. By making a zoom in velocity degree curve, figure 4.4, in which the analytical solution presents a mathematical phenomenon known as Gibb's effect at singular point. While finite element method enriched by hierarchical Lobatto function and first order Lagrange element present numerical oscillation at the same singular point. But, the same phenomenon wasn't observed in GFEM and second order Lagrange

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element, especially for GFEM, in which the numerical oscillation could be considered as negligible. By making a zoom in figure 4.2, the figure 4.5, only with HHT method results, it is possible to observe that GFEM and second order Lagrange element doesn't present discrepancy at time, while HFEM and FEM conventional have such behavior.

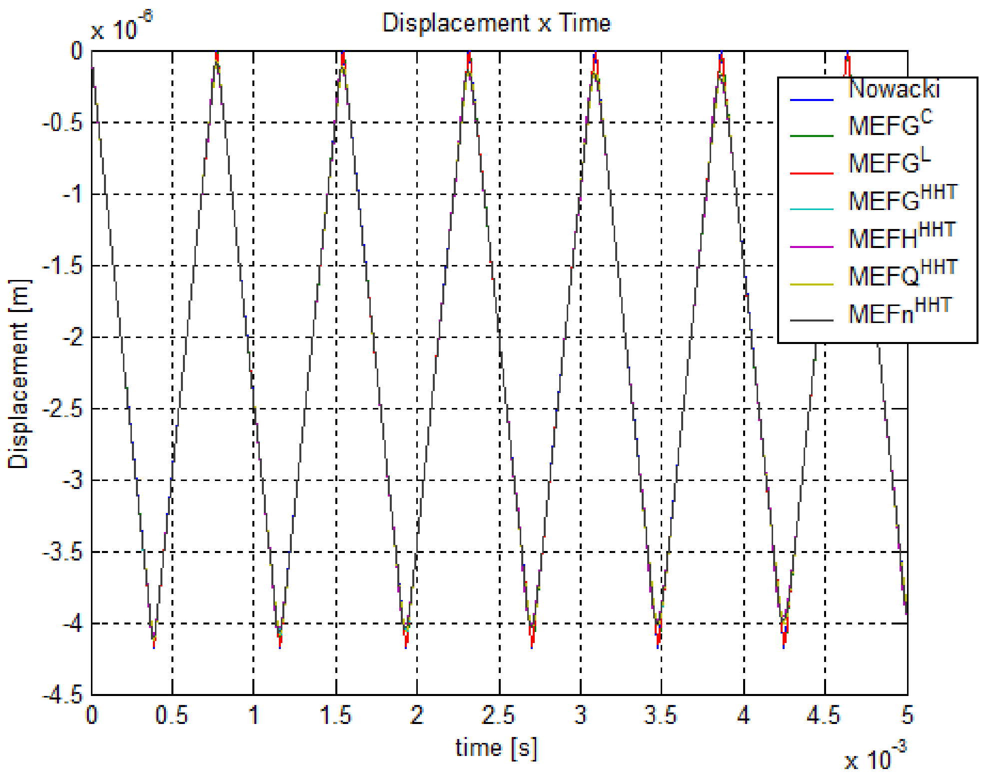

Figure 4.2 Displacement curve of example 1 evaluated by several different methods.

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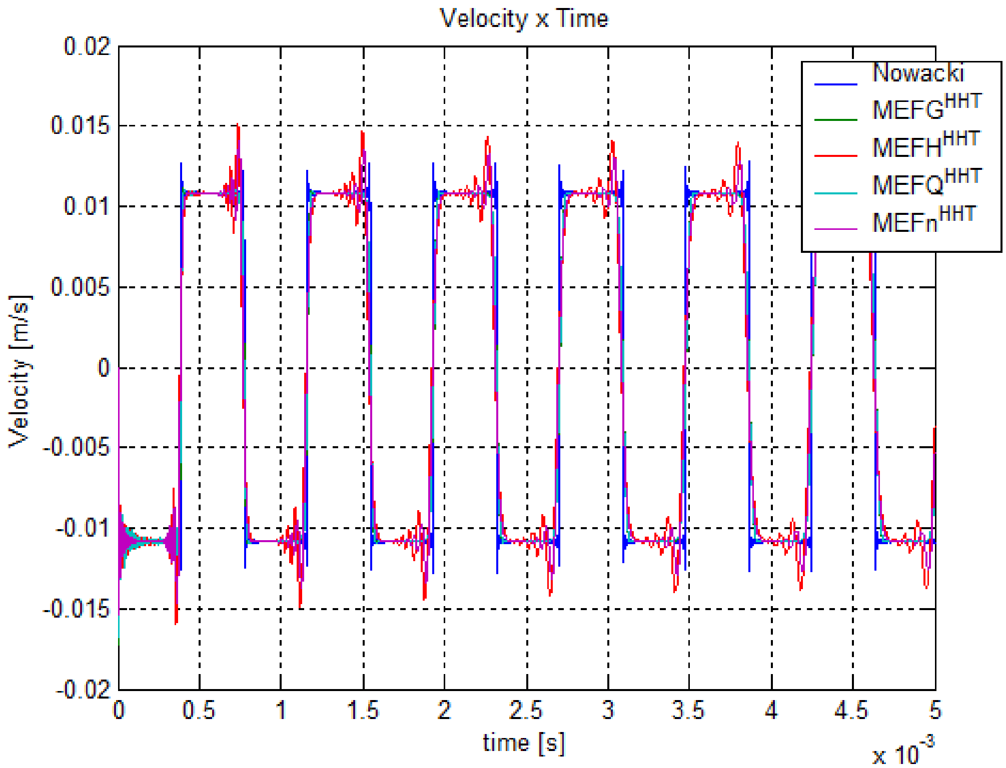


Figure 4.3. Velocity curve of different methods by using HHT.

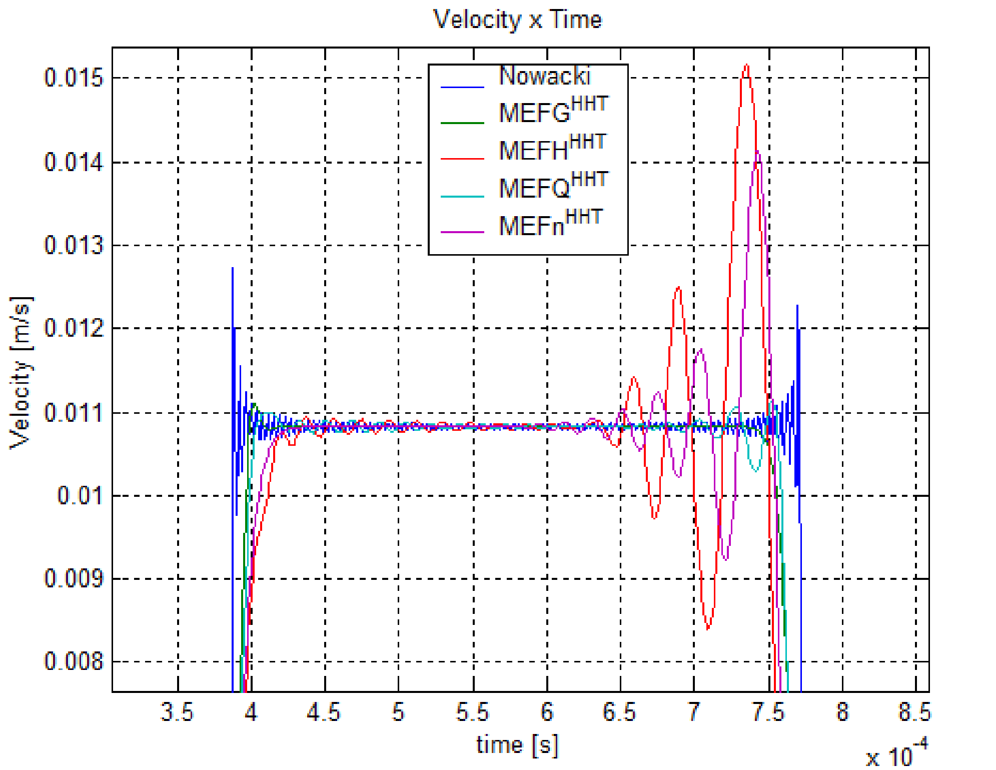


Figure 4.4. A zoom of figure 4.3 at the discontinuity of velocity curve.

ISSN 2176-5480

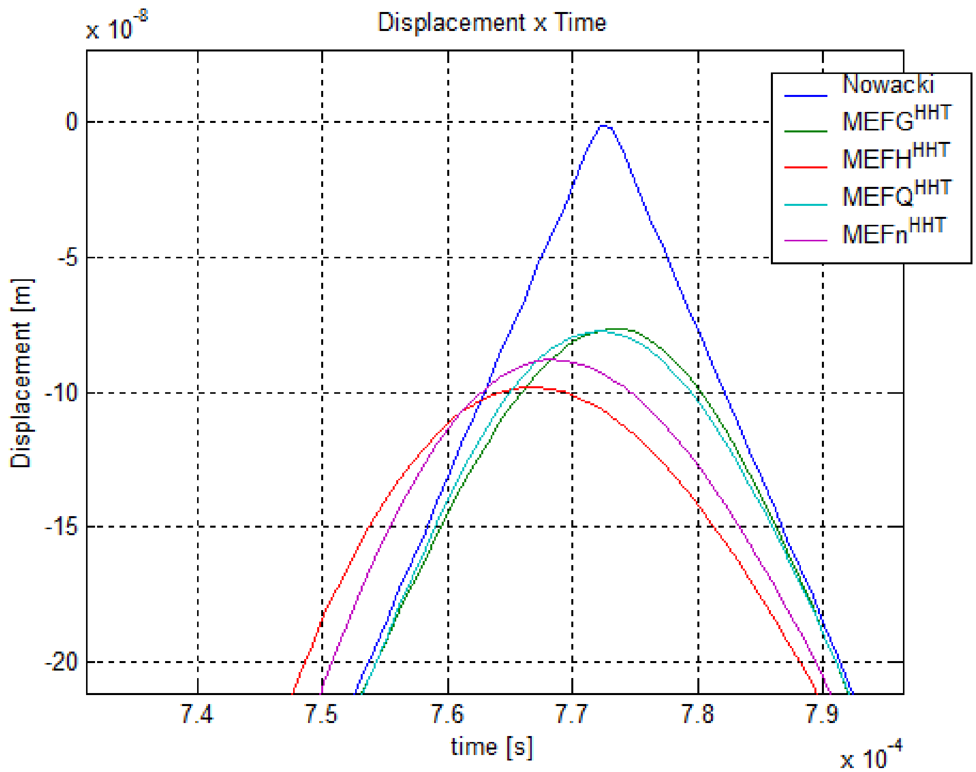


Figure 4.5. A zoom at the peak of displacement curve.

4.2 Example: bar subjected to impact load

This example deals with the bar that has same mechanical and geometrical properties as shown by example 1. In this case, an impulsive loading was applied to the free end with time interval $1x10^{-6}$s. In this example, only HHT method was employed due to the reason presented in the previous example. The figure 4.6 shows the displacement curve as degree function, due to the phenomenon produced by impulsive load applied to the free end. This type of load introduces more numerical instability as it presents singular point along the time. Even with HHT method, several finite element formulations present numerical oscillation at the displacement degree. However, the GFEM shows reasonable accuracy, as shown by figure 4.7. While other element formulation present numerical oscillation once approximation the singular point, but GFEM has more stable behavior. This type of stability is particularly desirable, due to the fact that the strain could be evaluated with more accuracy.

ISSN 2176-5480

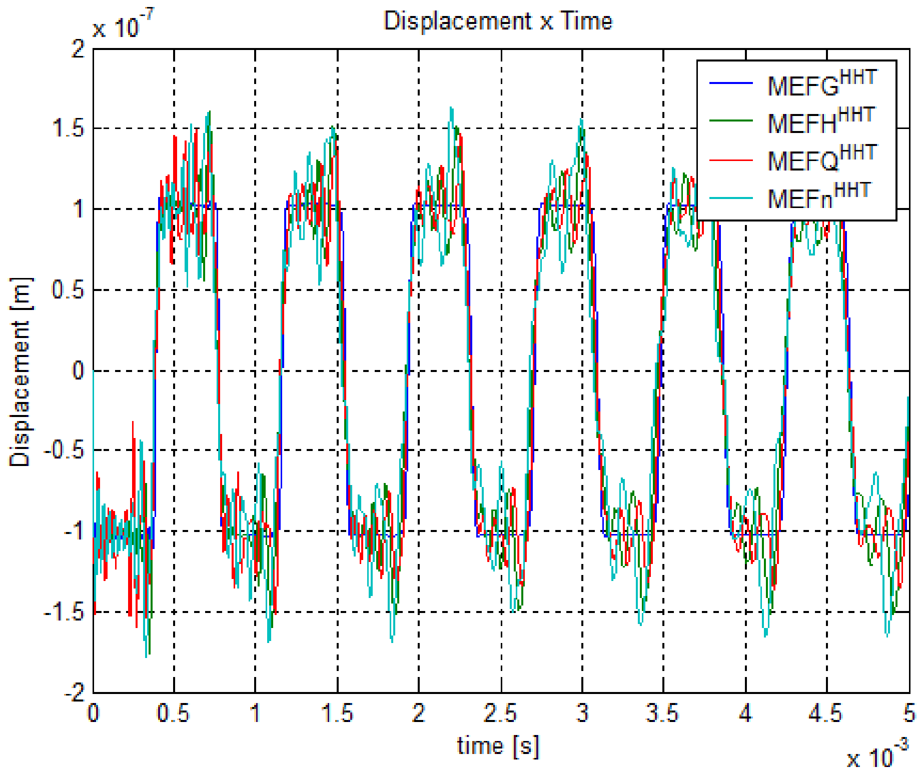


Figure 4.6 Velocity curves of several methods by using HHT method.

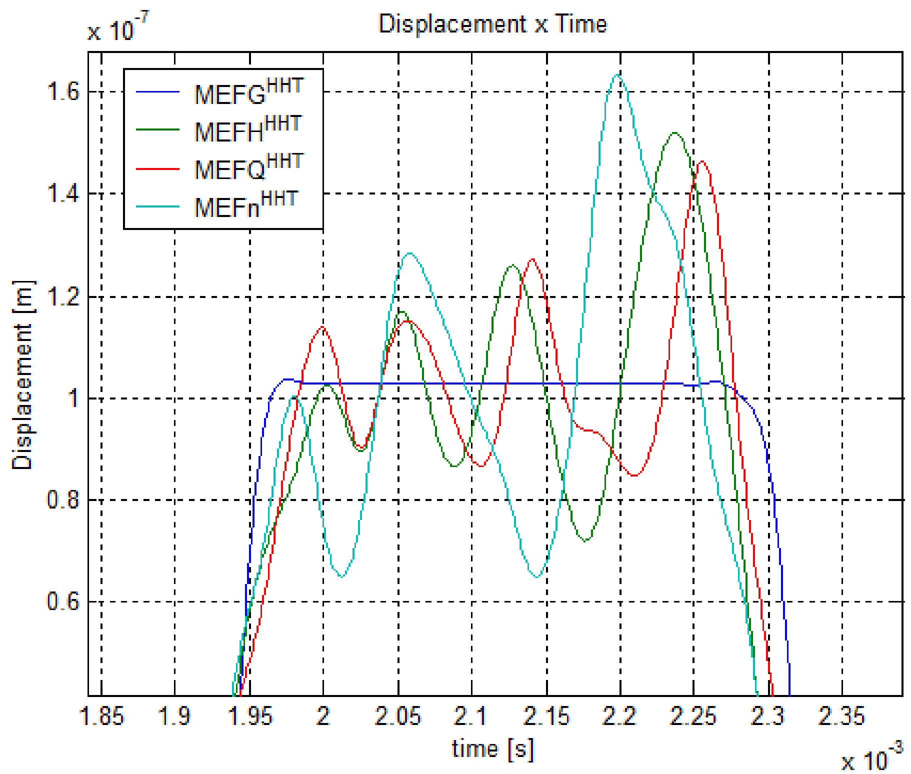


Figure 4.7. Displacement curves of several methods by using HHT method

4.3 Example: beam subjected to initial displacement

ISSN 2176-5480

This example work with an example presented by Torri (2012), with material properties chosen to provide $c = \sqrt{E/\rho} = 1m/s$. The length of bar is 1 m and has both end clamped, as show by figure 4.8. The initial longitudinal displacement is shown below, with maximum value of 0,25 in the middle of bar. The result for comparison of Newmark linear acceleration method and HHT method will be show below.

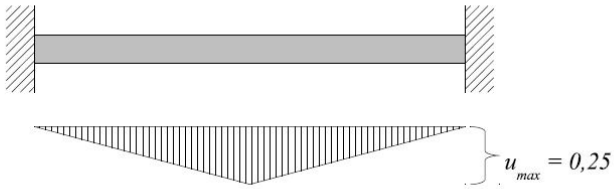
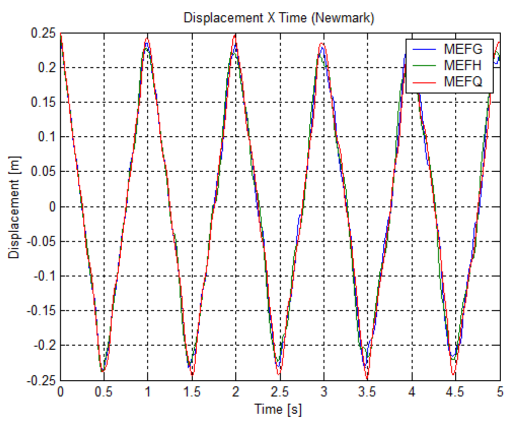
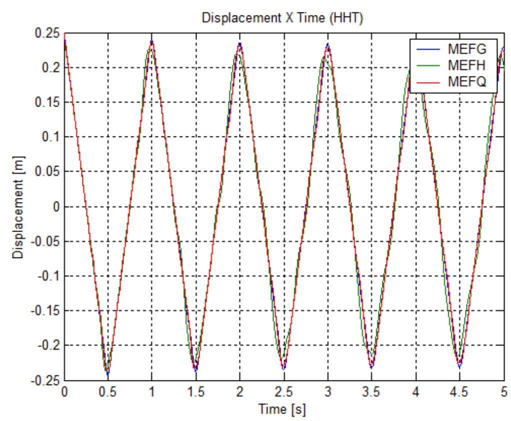


Figure 4.8

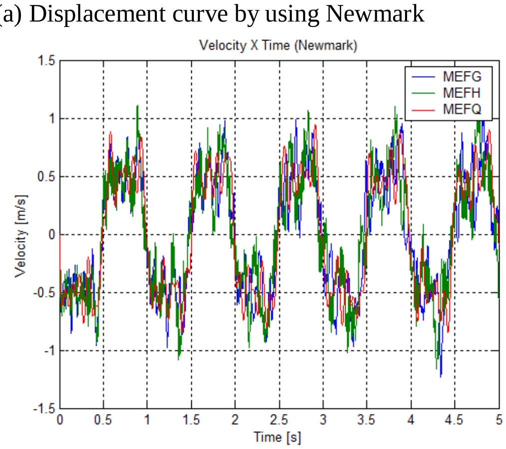
As it was demonstrated in previous example, for continuous function as bar behavior, all element formulation has presented reasonable result and accuracy, as it can be observed in figure 4.9 (a) and (b). By the other hand, for discontinuous function, as velocity, all element formulation has presented numerical oscillation, even with GFEM in this case, as in figure 4.9 (c) and (d). Even with HHT method, the GFEM presents numerical oscillation, especially in the singular point. As it is know, the velocity is derivative of displacement in time, it is reasonable to expect that, even the displacement curve was remarkably attained, but the derivative produce discontinuity in this case, in which an initial displacement was introduced and it is similar to have an impulsive loading at the beginning of analysis.



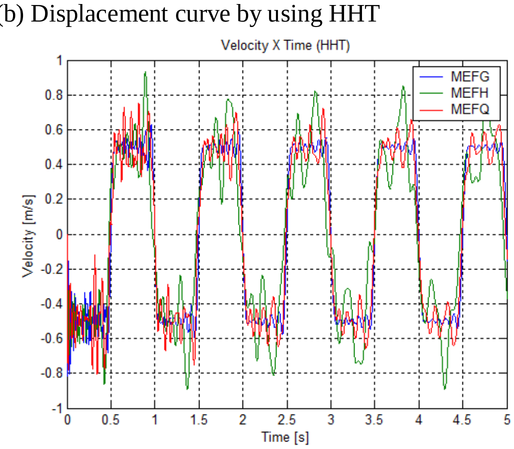
(a) Displacement curve by using Newmark



(b) Displacement curve by using HHT



(c) Velocity curve by using Newmark



(d) Velocity curve by using HHT

Figure 4.9Displacment and velocity results by using different methods of solution.

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5. CONCLUSIONS

By using hierarchical e generalized finite element method, the elastodynamics problem was analyzed and results of several cases were presented. The results show that HFEM and GFEM have remarkable behavior once the HHT was adopted. Especially, once the singularity problem was encountered, such as velocity curve in the case of bar subjected to impact. In the other hand, once the Newmark algorithm was adopted, HFEM and GFEM present numerical oscillation and have behavior such as other conventional finite element method.

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