



PARTICLE CLUSTERING IN DECAYING ISOTROPIC TURBULENCE COMPARING ONE-WAY AND TWO-WAY COUPLING

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Abstract. Clustering of small solid particles in decaying isotropic turbulence is studied numerically. Direct numerical simulation (DNS) method is used to solve the three-dimensional, time-dependent velocity field of a homogeneous, non-stationary turbulence combined with a Lagrangian point-particle model for the particulate phase (Eulerian-Lagrangian approach). Numerical grids containing 64^3 , 128^3 and 256^3 points were used to resolve the turbulent motion at the Kolmogorov lengthscale for a range of microscale Reynolds numbers starting from $R\lambda = 27$ and decaying to $R\lambda = 13$. The clustering characteristics of two different solid particles sizes (Light and Intermediate) injected randomly in the flow, were obtained integrating the equation of particle motion along the instantaneous trajectories of 1 million particles for each particle type. The simulation results are presented for both one-way coupling, and two-way coupling. The results also include the time development of the mean-square relative velocity of the particles, the particle density function and the concentration of particles on the fluid. The time development of the Lagrangian velocity frequency spectra of the particles and their surrounding fluid are also presented. The objective of this study is to analyze the physical mechanism associated with particle clustering in a simple turbulent flow by examining the simulation results described above. This work has been concerned with analyzing various aspects of direct numerical simulation of particle laden flows. A survey of the literature indicates a lack of consensus as to the number of particles that must be considered, and questions are raised about the typical method used to determine the statistically significant number of particles to track. Further, the accepted practice for determining the virtual release time for decaying turbulence has been assessed. Results indicate that basing the virtual release time on the time at which the maximum of the mean relative velocities occurs may not be long enough to ensure that particle trajectories are independent of their initial conditions.

Keywords: particle clustering, decaying isotropic turbulence, direct numerical simulation, one-way coupling, two-way coupling.

1. INTRODUCTION

In this study, direct numerical simulation (DNS) of the compressible Navier-Stokes equations (Sesterhenn, 2001) was used to simulate turbulence in a cube of unit volume with periodic boundary conditions. A code that can solve compressible flow was used to allow the possibility that in the future, simulations of interaction between turbulence and shock waves with particles can be made. A very low Mach number was used to allow the simulation of incompressible flows. The non-dimensionalized time-dependent, three dimensional continuity, momentum and energy equations were discretized using a 5th order upwind compact difference scheme. The explicit 4th order Runge-Kutta method was used for the time stepping. Interpolation of fluid velocities between nodes is required in order to calculate the fluid drag acting on the particle. In this case was used Lagrange polynomial interpolation scheme. The initial velocity field for the fluid, was generated by using the spectral method of (Rogallo and Moin, 1984). This method generates an isotropic, incompressible, divergence-free velocity field that is periodic in all three spatial directions.

The emphasis in this work is on the resolution characteristics of the difference approximations rather than their formal accuracy (i.e., truncation error). By resolution characteristics we mean the accuracy with which the difference approximation represents the exact result over the full range of length scales that can be realized on a given mesh (Lele, 1992).

For grids containing 128^3 and 256^3 points, 1 million particles were initialized randomly with initial instantaneous velocity having the same velocity of the fluid. Were used two different diameters for the particles $DP1 = 35 \mu\text{m}$ and $DP2 = 46.5 \mu\text{m}$. Figure 1 show the particle field at $T = 0.14 \text{ s}$, the corners of the cube display the dispersion of the particles, while in the center of the field, particles are concentrated. In Fig. 2 the vorticity modulus is shown.

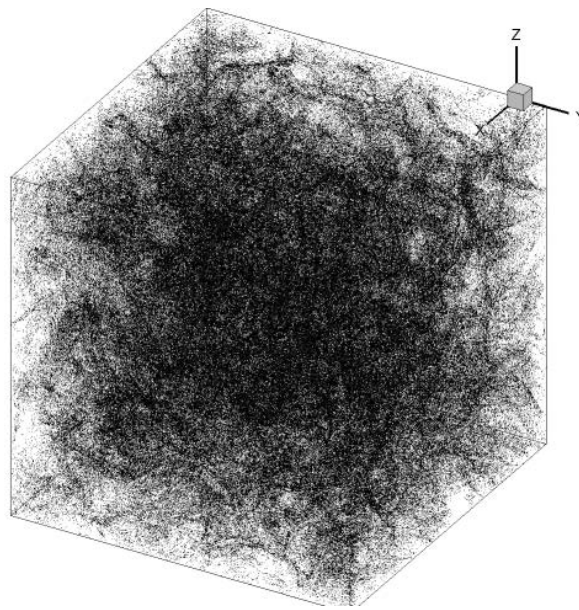


Figure 1. Particle field with 1 million particles inside unitary domain at $T = 0.14$ s (Present work).

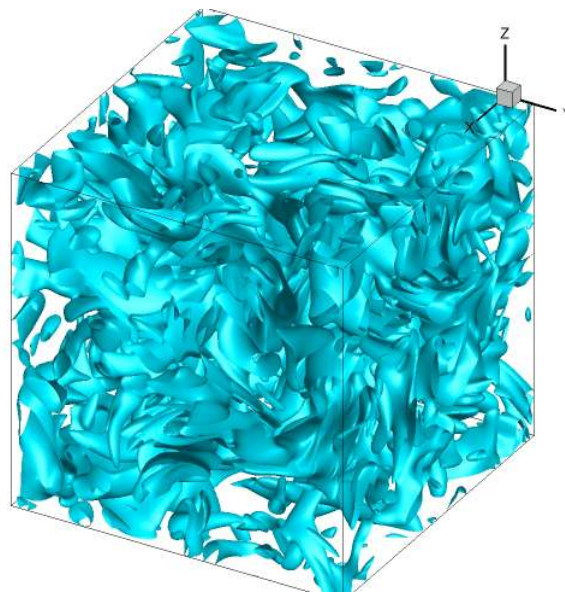


Figure 2. Vorticity module for homogeneous and isotropic turbulence at $T = 0.14$ s (Present work).

2. PARTICLE-LADEN TURBULENT FLOWS

Particle-laden flows are abundant in nature and can be found in various industrial systems. Pneumatic conveying systems in chemical, food and pharmaceutical industries, and the transport of pollutants in the atmosphere are typical examples. Energy production systems such as droplets in internal combustion engines, fluidized bed combustions and gasification for coal and biomass involve efforts to find a combustion process conducive to controlling pollutant emissions and efficient energy generation (Xu, 2008).

It is helpful, to classify the regimes of particle-laden flows from the perspective of the interaction between particles and fluid phase motions. To describe this, the Fig. 3 proposed by (Elghobashi, 1993) shows numerically how the dimensions of the particles and the dimensions of the fluid field are connected to the type of interaction between the particles and turbulence.

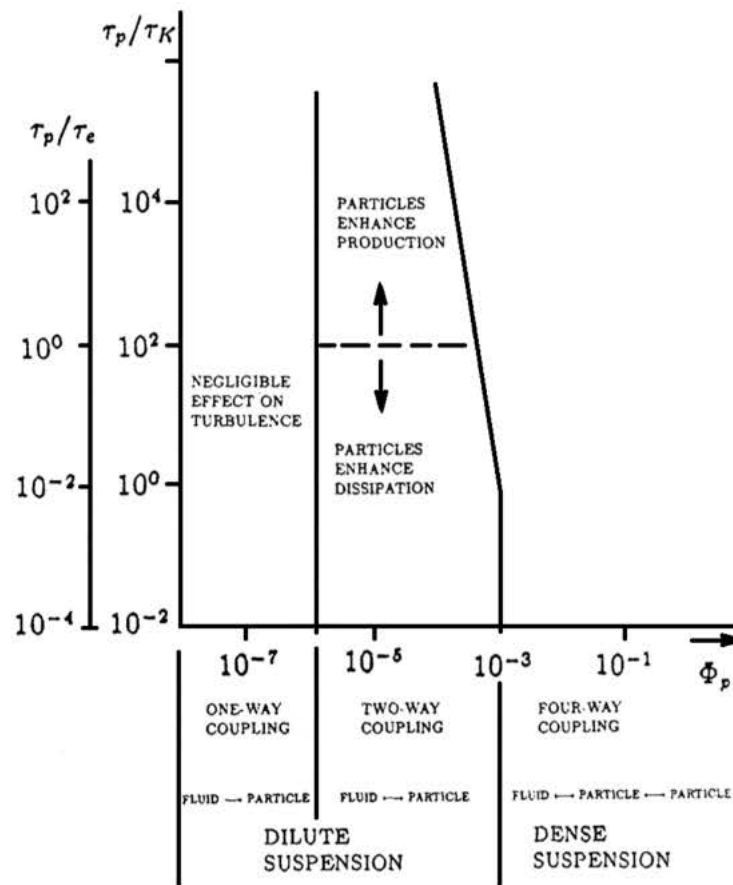


Figure 3: Map of regimes of interaction between particles and turbulence (Elghobashi, 1993).

The quantities on the dimensionless coordinates are defined below:

- Φ_p : volume fraction of the particle, $\Phi_p = N_p V_p / V$
- τ_p : particle response time, $\tau_p = \rho_p d^2 / (18 \rho_f \nu)$ for Stokes flow
- τ_η : Kolmogorov time scale, $\tau_\eta = (\nu / \varepsilon)^{1/2}$
- τ_e : turnover time of large eddy, $\tau_e = l / u$

For very low values of $\Phi_p \leq 10^{-6}$ the particles have negligible effect on turbulence, and the interaction between the particles and turbulence is termed as one-way coupling. This means that particle dispersion, in this regime, depends on the state of turbulence but due to the negligible concentration of the particles the momentum transfer from the particles to the turbulence has an insignificant effect on the flow. In the second regime, $10^{-6} < \Phi_p \leq 10^{-3}$, the momentum transfer from the particles is large enough to alter the turbulence structure. This interaction is called two-way coupling. Now, in this regime and for a given value of Φ_p , lowering τ_p (e.g. smaller d for the same particle material and fluid viscosity) increases the surface area of the particulate phase, hence the increased dissipation rate of turbulence energy. On the other hand, as τ_p increases for the same Φ_p , the particle Reynolds number, R_p , increases and at values of $R_p \geq 400$ vortex shedding takes place resulting in enhanced production of turbulence energy. The vertical coordinate τ_p/τ_e is related to the other coordinate τ_p/τ_η via the turbulence Reynolds number ($R_l = u_l/\nu$) since $(\tau_p/\tau_\eta) = R_l/2$. Thus the coordinates shown are for $R_l = 104$, 1 which is typical in practical flows. Flows in the two regimes discussed above are often referred to as dilute suspensions. In the third regime, because of the increased particle loading, $\Phi_p > 10^{-3}$, flows are referred to as dense suspensions. Here, in addition to the two-way coupling between the particles and turbulence, particle/particle collision takes place, hence the term four-way coupling. As Φ_p approaches 1, we obtain a granular flow in which there is no fluid, and obviously that flow is beyond the scope of this text (Elghobashi, 1993).

As we can see in Figure 3, the line separating the two-way and four-way coupling regimes is inclined. This is intended to indicate the tendency of particle-particle collision to take place at higher values of τ_p/τ_e , thus transforming the two-way to four-way coupling regime.

The behavior of particles in turbulent flows with one-way coupling is reasonably understood, at least in unconfined homogeneous flows. The limitation to this understanding, as mentioned earlier, stems mainly from the incomplete understanding of turbulence itself even in particle-free flows. On the other hand, flows in the two-way or four-way

coupling regimes are still at the infancy stage of understanding due to the highly nonlinear nature of the interactions in these flows. The mathematical approach to be reviewed in this work will take an important care about the two-way coupling formulation.

3. HOMOGENEOUS ISOTROPIC TURBULENCE

Turbulence was recognized as a distinct fluid behavior by Leonardo da Vinci more than 500 years ago. It is Leonardo who termed such motions "turbolente", and hence the origin of our modern word for this type of fluid flow. But it wasn't until the beginning of last century that researchers were able to develop a rigorous mathematical treatment of turbulence. The first major step was taken by G. I. Taylor during the 1930's. Taylor introduced formal statistical methods involving correlations, Fourier transforms and power spectra into the turbulence literature. In a paper published in 1935 in the Proceedings of the Royal Society of London, he very explicitly presents the assumption that turbulence is a random phenomenon and then proceeds to introduce statistical tools for the analysis of homogeneous, isotropic turbulence. In 1941 the Russian statistician A. N. Kolmogorov published three papers that provide some of the most important and most-often quoted results of turbulence theory. These results, which will be discussed in some detail later, comprise what is now referred to as the K41 theory⁵, and represent a major success of the statistical theories of turbulence. This theory provides a prediction for the energy spectrum of a 3D isotropic homogeneous turbulent flow. Kolmogorov proved that even though the velocity of an isotropic homogeneous turbulent flow fluctuates in an unpredictable fashion, the energy spectrum (how much kinetic energy is present on average at a particular scale) is predictable (Glenn, 2010).

The spectral theory of Kolmogorov had a profound impact on the field and it still represents the foundation of many theories of turbulence. It should however be kept in mind that 3D isotropic homogeneous turbulence is an idealization and its almost impossible to encounter in nature. The challenge is then to understand what aspects of these theories apply to natural flows and what are pathological.

A turbulent flow is said to be *isotropic* if,

- There is no variance of the statistical properties of a flow in relation to the rotation coordinate system.

Rotation and buoyancy forces tend to suppress vertical motions and create an anisotropy between the vertical and the horizontal directions. The presence of a mean flow with a particular orientation can also introduce anisotropies in the turbulent velocity and pressure fields.

A flow is said to be *homogeneous* if,

- There is no variance of the statistical properties of the flow when promoted translation of the coordinate system.

It is understood, therefore, that isotropy implies homogeneity. The reciprocal is not true (Aristeu, 1994). An example of 3D isotropic homogeneous flow is shown in Fig. 4.



Figure 4: Instantaneous visualization of the enstrophy distribution in a compressible, homogeneous isotropic turbulent field. Data courtesy of Eric Johnsen, Michigan Institute for Plasma Science and Engineering.

4. TRANSPORT MECHANISM

Particles can be entrained on the fluid motion, or they can roll along the ground. According to the transportation of small solid particles, literature divided into three different mechanisms: suspension, saltation and reptation. This classical division is described on observations of sand movement by (Bagnold, 1941) and shown in Figure 5. There are no provided information on the prevailing rates. However, since it is wind-induced movements, speeds should be within a moderate range.

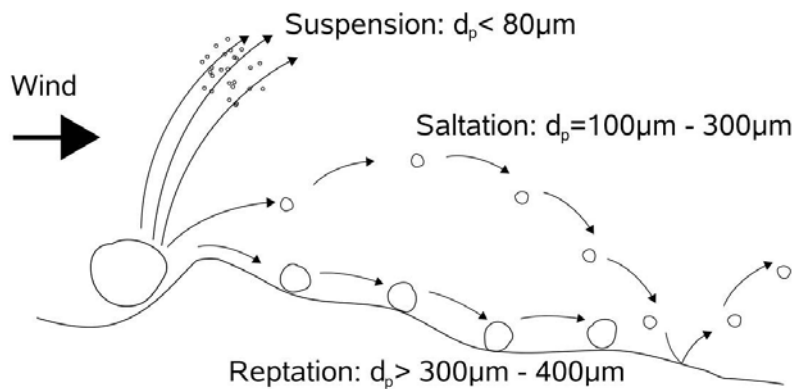


Figure 5: Division of the transport mechanisms of particles indicated by diameter range.

Suspensions indicate a heterogeneous mixture of substances, in which solids are distributed in the fluid. At specific cases, suspensions are adopted for grain sizes smaller than $80 \mu\text{m}$. The saltation is erratic transport of particles in a fluid, with particles in diameter range between $100 \mu\text{m}$ and $300 \mu\text{m}$. There are particles that moves without losing contact with the ground, this process is called reptation. It occurs for particles whose diameter is larger than $300 \mu\text{m}$ and smaller than $400 \mu\text{m}$. This does not mean that "big" objects lie forever, for example, can be entrained ballast stones on high-speed lines also produced by the brindle. These stones do not fall within the described classification (Heinrich, 2011). The next two sections provide a brief overview of the suspension and saltation processes. Reptation is not considered further, because it is not relevant for this work. Below on this text is often spoken of particles, for this work we will consider particles or beads as a round body with a diameter smaller than 1 mm . Its important to have all this concepts in mind in order to associate them with the study case.

5. CONCLUSIONS

There is a considerable evidence for a high correlation between, regions of high strain and low vorticity and regions of high concentration in heavy particles. Figure 6 shows a slice along the $x = 0.5$ of particles positions with respective vorticity field at $t = 0.14 \text{ s}$ and Fig. 7 shows particle concentration for the same position and time. The close interaction between the coherent vorticity structures and particles clustering are observed in Fig. 6. In the review by (Eaton and Fessler, 1994) structured flows are considered in which it is easily seen that heavy particles are surrounding mixing layer or wakes vortices. The basic underlying mechanism is the following: due to their inertia, particles denser than the fluid tend to be ejected from vortical structures while they are easily trapped by convergent regions of the flow. The opposite behavior is expected for particles less dense than the carrier fluid. If these behaviors are easily seen and understood on simple steady flows, there is a wide gap to fill to directly apply them to homogeneous isotropic turbulent flows where, even if structures do exist, they do not necessarily live for long times.

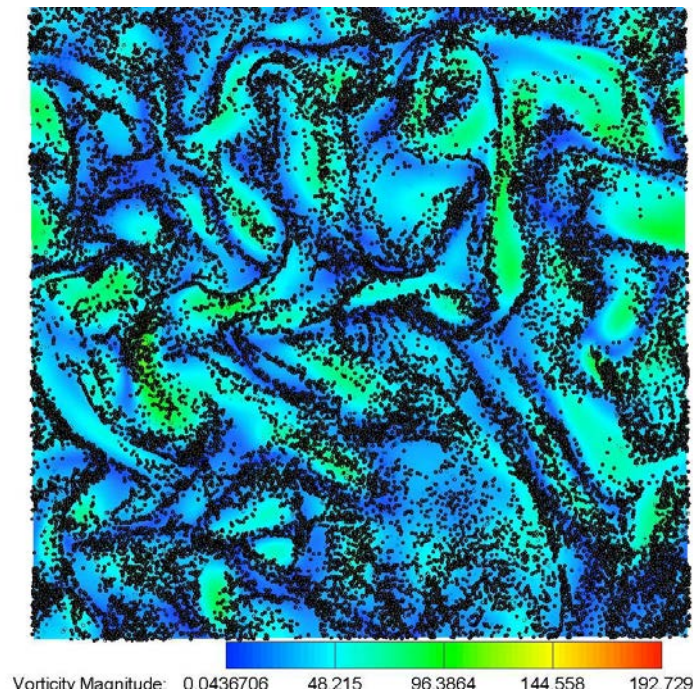


Figure 6. Slice showing the close interactions between coherent vorticity structures and particle clustering at $t = 0.14s$ (Present work).

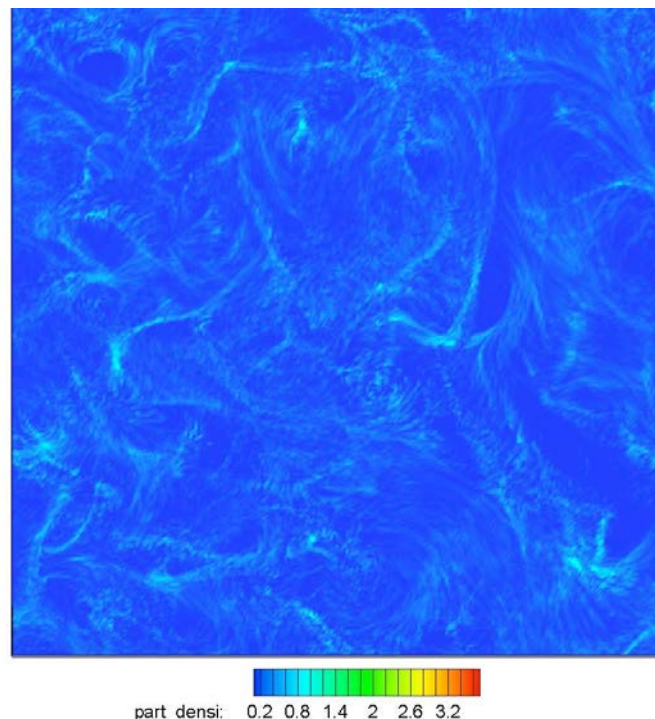


Figure 7. Slice of particle density computed with particles positions and transposed to fluid nodes at $t = 0.14 s$ (Present work).

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6. ACKNOWLEDGEMENTS

The authors would like to gratefully acknowledge the support of the Institute of Fluid Mechanics and Engineering Acoustics (ISTA) and the Laboratory of Fluid Mechanics (MFLab) Fellowship. In addition, this work was made possible by the facilities of the North-German Supercomputing Alliance (HLRN: www.hlrn.de). This optional section must be placed before the list of references.

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