ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

AUXILIARY ALGORITHM FOR ANALYSIS AND AID DESIGN OF THE GEOMETRY OF STEERING WHEEL AND SUSPENSION SYSTEMS

Leonardo Baptista Gitahy
José Stockler Canabrava Filho
Flávio de Marco Filho
Federal University of Rio de Janeiro - Dp$^{\underline{to}}$ de Eng$^{\underline{a}}$ Mecânica - Bl.G/204 - CT Cidade Universitária -21941-909 - Ilha do Fundão-RJ
flavio@mecanica.ufrj.br

Abstract. *This work present an auxiliary algorithm for the design, analysis and selection of the geometry of steering wheel and suspension systems. The method is based in the reduction of the variation of the wheel steering angle during the vehicle curve. That variation is minimized and, simultaneously, is evaluated the geometry and the steering tire angle error. The algorithm was applied in a steering wheel and suspension systems of mini-baja type vehicle and the comparison results are presented. This work aims to present an algorithm to aid the design of the car steering and suspension geometry systems. The method permit to reduce the undesirable variation of the steering angle of the wheels cause by the movement of the front suspension and, simultaneously, to specify a steering mechanism geometry that minimizes the errors of the angles of the wheels during the turns of the cars. The presented algorithm is an easy, fast, safe and powerful tool that provides the most suitable geometry for steering wheel and suspension systems, allowing to make several analysis to enable the appropriate choice of the best set of parameters for these mechanisms. This mathematical algorithm was applied to the design of steering systems and suspension of a off-road vehicle and the results of software simulations will be presented.*

Keywords: *Steering Wheel systems, Auxiliary Systems Design, Automotive Vehicles, Machine Design*

1. INTRODUCTION

The steering system is responsible for changing the direction of the vehicle. It directly interacts with other systems such as the suspension and is commonly made up of a set of mechanisms whose main purpose is to transform the rotational motion of the steering wheel operated by the driver in translational movement bars, which cause the wheels to rotate around an axis close to the vertical direction. The suspension system is responsible for ensuring that the tire be in permanent contact with the ground, providing stability to the steering system. It is also responsible for the uptake and impact of the irregularities of the ground by means of its components, providing comfort to the occupants of the vehicle. This is a very important interaction since the stability and dynamics of the vehicle are dependent on their synchronism. Thus, further study on the interaction between these systems it is necessary. This paper presents a set of mathematical algorithms developed for the design of specific mechanisms used in steering systems and suspension automotive.

Among the various parameters to be determined for the correct functioning of the steering and suspension are the determination of the appropriate dimensions for the components of these mechanisms. The correct choice of these dimensions influences the driving characteristics, tire wear and cornering stability.

The method presented here consists in reduces the unwanted variation of the steering angle of tires on a supposed action of the front suspension and, simultaneously, specify a geometry that minimizes the errors in these angles during a turn. The algorithm, then, asks for some basic design data such as the wheelbase, the gauge of the vehicle and the working angle of the suspension, providing the user the essential dimensions of the mechanisms of steering and suspension systems.

The algorithm was applied to the design of steering systems and suspension of a *BAJA* vehicle and through software simulations the results of your application could be compared.

2. FRONT WHEEL GEOMETRY

According to [1], a factor of great influence on the performance of the direction system is the axis of rotation of the steering wheel, called the *Kingpin* and in some cases is defined by the lower and upper joint articulation or bearing in the shock towers, as shown in Figure 1.

In most applications this axis has an inclination converging toward the center of the vehicle, called kingpin inclination. Typical values are 0 to 5° to trucks and 10 to 15° for cars. The intersection of the kingpin axis and the ground is called *kingpin off-set*. It is positive when it is inside the center of the intersection of the wheel and the ground.

The kingpin off-set variation results in sensibility driver change and reducing the steering straining due to the effect of the tire's rolling replacing the drag effect resulting in increased efforts. The effects of this angle are related to the vehicle tendency to remain straight, the reduction in steering effort and reduce tire wear.

ISSN 2176-5480

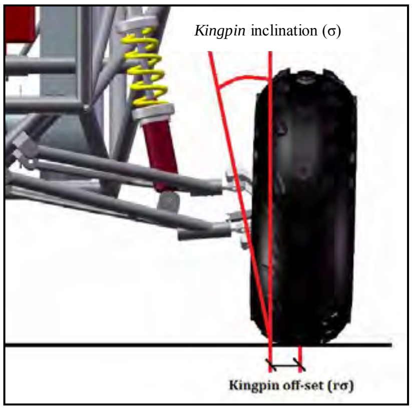


Figure 1 - The *Kingpin* inclination (σ) and *Kingpin off-set* (r_σ).

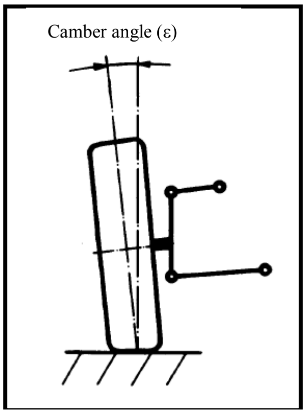


Figure 2 - The camber angle (ε).

The camber angle is the angle between the central plane of the wheel and a plane perpendicular to the track , front view of the vehicle, as shown in Figure 2. Negative values are taken when the wheels point to the vehicle structure and positive ones when pointing out. The force that the tire can develop in a curve is highly dependent on its angle relative to the road surface (camber) and therefore the camber angle has a major effect on the stability (grip) of a vehicle during a turn. When the camber is adjusted correctly allows the entire surface of the tire stick to the track, maximizing the use of the contact area of the tire when making turns. Camber adjustments are used to maintain the highest possible grip of the tire surface with the curves.

The addition of the camber angle with the kingpin angle is important since it determines the point of intersection of the center wheel and kingpin axis. This setting determines the suspension work trending regarding the convergence open or closed ("toe in" or "toe out"). The divergence is determined when the wheels are directed out and closed convergence is the opposite configuration. Vehicles having open or closed convergence tend to have faster wear tires. The suspension system design is made considering that tires are always operating with a certain camber. However, it is not an easy task, since the vehicle when enters the curve the suspension flex slightly vertically.

If we consider that the wheel is connected to the chassis by several links that must move to permit all the movements, the wheel is subject to change position whenever the suspension moves up and down (Figure 3).

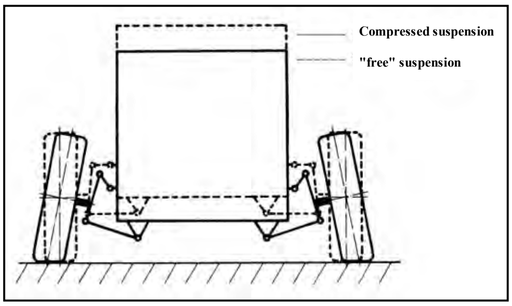


Figure 3 - Camber angle variation during suspension work.

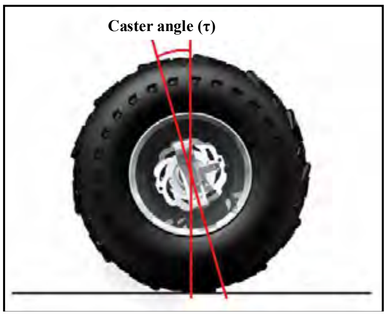


Figure 4 - Caster angle (τ)

According to [1], when inclined in the longitudinal plane, the steering axis is called the *caster*. Is considered positive when its intersection with the ground determines a point ahead the contact center of the front tire. Typically there are caster angles of 0 to 5° which may vary with the suspension deflection. It is important to note that through the caster angle of the kingpin axis can be positioned in front of or behind the vertical axis, see Figure 4. As the inclination of the kingpin, the caster angle causes the wheels to have different angles of camber according to the steering angle.

The effect of the positive caster is to increase the camber of the curve inner wheel and decrease the camber of the wheel outside the turn, or together with positive inclination of the kingpin, offsets the increase in camber of the wheel outside the turn, and causes the inner wheel camber curve assumes an even greater value.

The caster angle positive improves directional stability since the center line of the master pin to pass through the road surface ahead of the center line of the wheel. Another important effect is that the positive caster tends convergence of the front wheels inside, also called convergence closed or "toe-in". This effect occurs due to the pivots of the wheels being positioned internally or when the vehicle tends to go down with his own weight, see Figure 5. So a combination between the angles of inclination of the caster and Kingpin positives increases steering strain, since both generate bending moments with the tendency to keep the vehicle straight trajectory.

ISSN 2176-5480

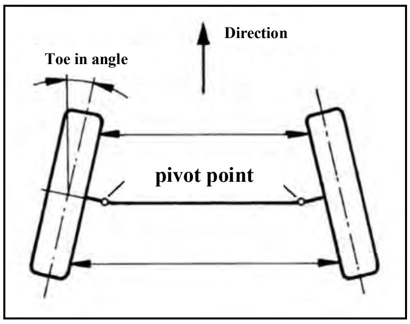


Figure 5 - Top view of the front suspension

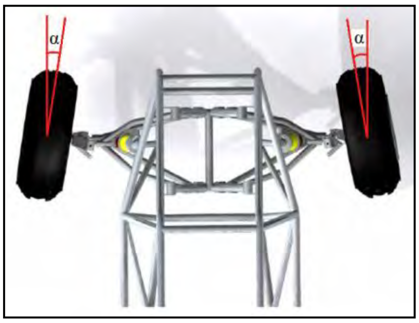


Figure 6 - Convergence angles

Heavy trucks have a tendency to use negative caster angle, in order to reduce this effect of increased effort. Thus, the turn and return of the steering system is determined only from the angle of the kingpin.

The static convergence angle is the angle between the central plane in the longitudinal direction of the vehicle and the line that intersects the central plane of a wheel and the plane of the track. It is positive if the front wheel is facing "inside" of the vehicle and is negative otherwise.

To minimize tire wear and power loss (caused by rolling resistance) wheels should point straight ahead when the car is running straight. Convergence (toe-in) or divergence (toe-out) cause excessive tire "slip" whenever they are turned over (Figure 6). Excessive toe-in causes accelerated wear on the outer edges of the tires. Excessive toe-out causes wear on the inside edges.

3. THE ACKERMAN GEOMETRY

According to [1], the lateral translation transmitted by the steering mechanisms through the bus bar, to the left and right wheels have an important geometric feature. The kinematic geometry of the rods system is not a parallelogram that produces the same steering angles for both wheels, but a trapezoid closest to the Ackerman geometry where the inner wheel has a steering angle larger than the outside wheel. The need for such geometry can also be explained by the fact that the inner wheel describes a radius of curvature smaller than the outer wheel, as shown in Figure 7.

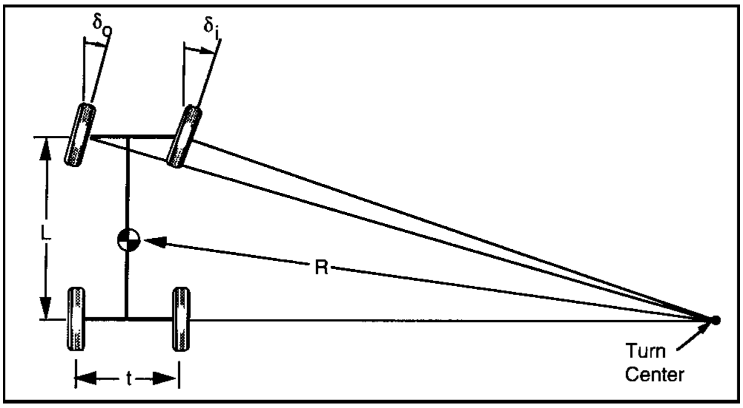


Figure 7 - Ackerman geometry configuration

According to the same bibliographic reference, the computation of the internal and external angles of the wheels regarding Ackerman geometry can be approximated as the following equations:

$$\delta_{\text{interno}} = \tan^{-1}\left[\frac{L}{R-\frac{T}{2}}\right] \cong \frac{L}{R-\frac{T}{2}} \qquad \delta_{\text{externo}} = \tan^{-1}\left[\frac{L}{R+\frac{T}{2}}\right] \cong \frac{L}{R+\frac{T}{2}}$$

where δ_{interno} e δ_{externo} represents the angles of the inner and outer wheel along the curve.

The approach described above can be considered for small angles, which are the most commonly found. Hence the tangent arcs of these angles are approximately equal to their own angles. The perfect Ackerman geometry is hardly satisfied with the design of suspension geometry, but is approached through the concept of the trapezoid, as shown in Figure 8. Therefore, the desired effect of greater steering angle of the inner wheel in relation to the external is generated by trapezoidal geometry.

ISSN 2176-5480

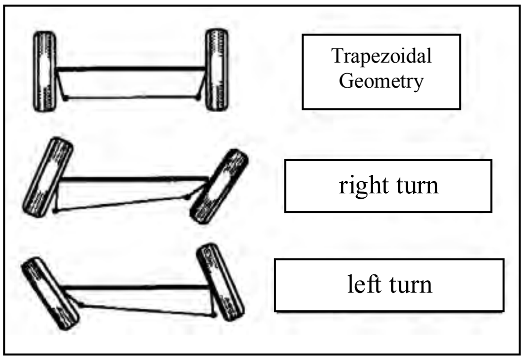


Figure 8 - Trapezoidal geometry

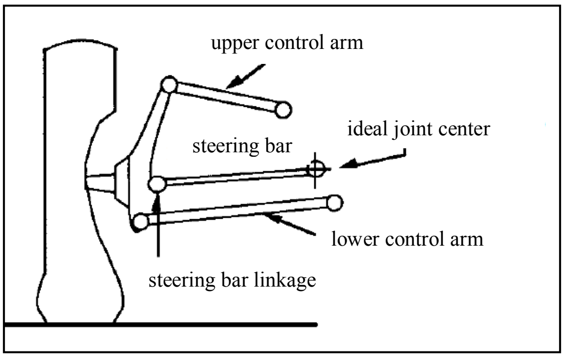


Figure 9 – Ideal steering geometry for independent suspensions

The degree of accuracy of Ackerman geometry of the vehicle has little influence on the directional behavior for high speed, but have influence on self centering maneuvers at low speeds. It may also be noted progressive resistance torque depending on the steering angle. It may also be noted progressive resistance torque depending on the steering angle, however, from a given angle, torque tends to diminish or even be negative for large steering angles.

4. WHEEL GEOMETRY ERRORS

Gillespie, 1992, states that the rods articulated steering system function is to transmit the movement of the steering mechanism for the vehicle's wheels. However the suspension position variation, the geometry of the steering system alters causing errors in the steering geometry. The ideal steering system is formed by a system of articulated bars, where the arc described by the suspension when undergoes deflection is perfectly described the same bars. It is worth noting that there are no steering angles toward this ideal condition. The reasons for this are: the space assemble limitations of the components, nonlinearities in the movements of the suspension and the geometry changes when the system is out of its central position. Consequently suspension deflections causes variation in the convergence of the wheels which can generate steering angles of both wheels.

The center of the sphere of the bar opposite the point where it connects to the axle is the position that determines the errors of the steering system. The ideal condition is illustrated in Figure 9. CAD systems or other geometric methods such as circle inflection can be used to determine this ideal condition.

4.1. Convergence Errors

Figure 10 illustrates the center of the steering remaining at the ideal centre. However its length is less than the ideal length, and may be superior in condition studied in this work.

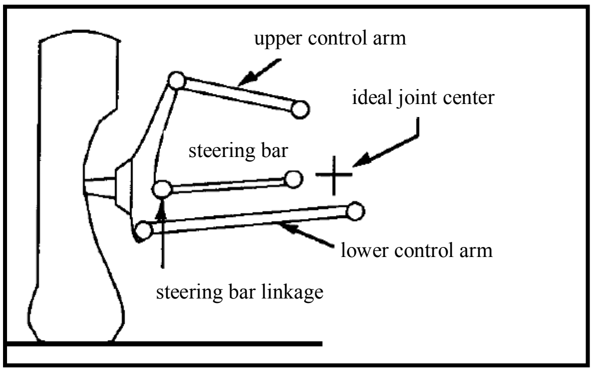


Figure 10 – Convergence error in steering geometry

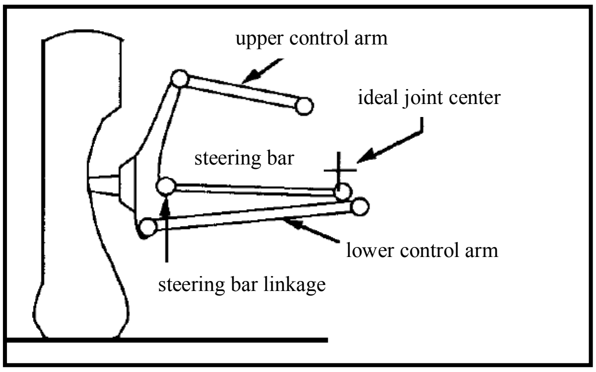


Figure 11 - Error by scrolling effect on steering geometry

In this case, when the left wheel rises or falls there is a difference of arches, where the steering bar arc is less than that described by the suspension which is composed of upper and lower arms, generating the displacement of the steering bar to the right which results in an steering wheel angle of the left wheel to the left, considering the steering bar behind the wheel center. Similarly, the same effect occurs on the right wheel with a steering angle to the right. Therefore convergence errors will occur as far as the load is imposed on the front axle during the work of the

ISSN 2176-5480

suspension system, because they cause unwanted steering angles in the steering system and should be interpreted as steering geometry errors.

4.2. Scrolling Effect Errors

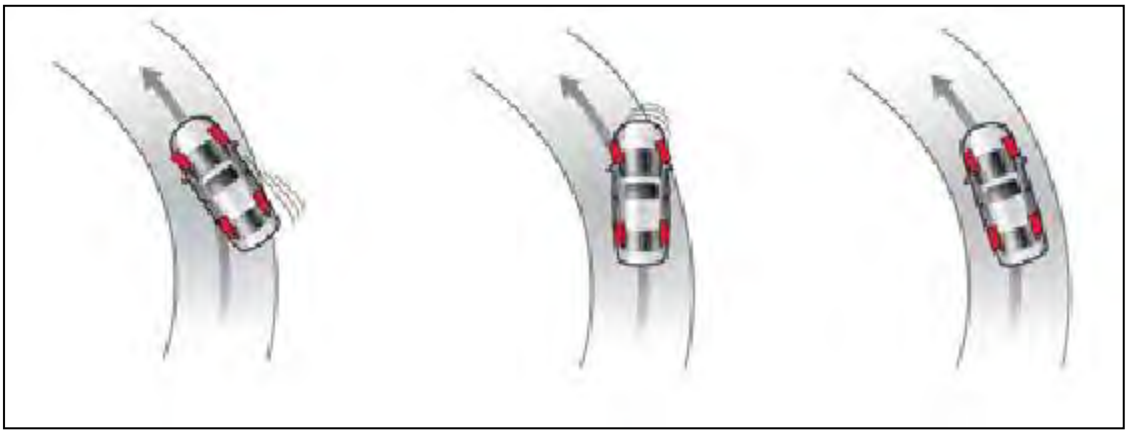
As illustrated in Figure 11, where there is a rear view of the left wheel, the pivot center of the steering bar is below the ideal center. The result is that due to the symmetry both wheels will steer in the same direction when the vehicle body rolls, i.e. the suspensions of the front and rear side are closed and the opposite are open. Besides the scrolling effect we can observe another one: the effect of under-steering angle the directional response of the vehicle, occurring the opposite, i.e., effect over-steering angle, if the center of the steering knuckle is located above the ideal center.

5. VEHICLE DYNAMICS

5.1. Over-Steering Angle and Under-Steering Angle

When a front slip angle is imposed lateral forces appear on the front wheel of the vehicle, which consequently acquires an angular velocity around its axis and develops an "*attitude*" angle, i.e., an angle between its longitudinal central axis and direction of displacement. Due to this angle, also develops a slippage angle at the rear wheel.

If the vehicle is performing a curve of constant radius at constant speed the angular velocity around its own axis is also constant. Therefore, the moments on the center of mass, created by lateral forces on the axles must to cancel. Thus, a vehicle that has its center of mass equidistant from the front and rear axles requires the same slip angle on its wheels to make a turn. This is considered a neutral behavior. Thus, a vehicle with its center of mass closer to the front axle to the rear requires a slip angle at its front wheels greater than the slip angle of the rear wheels. Such behavior is called under-steering and implies an "*attitude*" angle smaller than the set to a neutral vehicle. In this case, the vehicle behavior is the opposite of the vehicle concerned in the previous case, which means that its center of mass is found nearer the rear axle and it is necessary that the slip angle of the rear wheels is greater than the front wheel. Therefore, the attitude angle should be larger than the set to a neutral vehicle. Such behavior is called over-steering.



(a) over-steering (b) under-steering (c) neutral

Figure 12 - Illustration of tendency to under and over-steering

Consider a vehicle in a constant speed and the pilot increases gradually the steering angle, is possible, then, to evaluate the results for different situations. If the vehicle under-steers by definition, then the slip angle of maximum centripetal force will be reached before the front wheels. Therefore, the generated centripetal force on the front axle will be unable to increase the "attitude" angle beyond this point. As the slip angle is increased, the centripetal force will decrease and the vehicle will tend to move straight.

Consider now that the vehicle over-steers; by definition, the slip angle of maximum centripetal force will be reached before by the rear wheels. Thus, the centrifugal force generated by the rear axle will be unable to keep the angle of attitude beyond this point. This behavior in a vehicle is considered unstable because if the front slip angle continue to be increased the car will acquire an angular acceleration around his shaft that will spin out of control unless the driver quickly decreases the steering angle (which in some cases may have to be negative).

For passenger cars, the final behavior considered ideal is under-steering because it guarantees that the driver has some control over the vehicle. The over-steering is not easily controlled by drivers and can be the cause of accidents.

The over and under-steering depend not only on the position of the center of mass of the car, but several other factors that affect the car dynamics.

5.2. Pitch Center and Anti-Dive/Anti-Squat Mechanisms

When a vehicle accelerates or brakes, the force applied by the runway on the tires generates a bending moment around the center of mass of the vehicle. Therefore, when a car brakes, this effect causes the lowering of the car front and the rear be lifted up. Acceleration case the effect is the opposite. The vehicle aerodynamics, the height of the mass center, the tires load distribution and the general comfort of the passengers, are all affects by this effect.
The suspension mechanisms geometry designed to combat these effects are Anti-Dive (for braking) and Anti-Squat (for acceleration) as shown in Figure 5**.**

The way that should be set the suspension geometry to provide these mechanisms, in braking, depends on whether the brake is located: inside the wheel or inside the transmission. If the brake is located inside the transmission the forces generated by the friction between pad and brake disc will be resisted directly by the vehicle structure. In the event that the brake is positioned inside the wheel, these forces will be transmitted from the suspension to the vehicle structure.

Considering only the longitudinal force applied to the tires then the pitch center given by the suspension geometry a moment will be generated resulting in a tendency for compression or extension of the suspension.

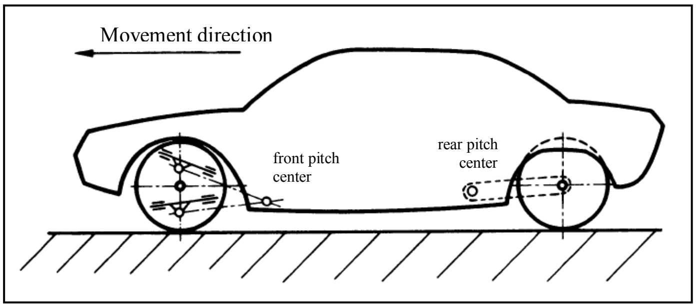


Figure 13 - Vehicle front and rear axis pitch center

The higher the percentage of utilization of these mechanisms greater the fraction of longitudinal forces transmitted by the suspension arms.

5.3. The Steering System and The Suspension System

The steering wheel is connected to the steering mechanism via the system steering column which is basically composed of shafts, bearings and universal joints. Through this connection, the steering mechanism is activated and transforms the rotary motion of the steering wheel steering wheel angle. Usually there are two types of mechanisms used to perform this work: rack and re-circulating ball bearings.

The main functions of the suspension system are maximize the friction between the tires and the ground, to provide stability to the steering system and, through its components, absorb all irregularities of the ground, providing comfort to the passengers. There are different types of suspension. They can be classified into two groups: independent suspension and rigid axle suspensions. These terms refer to the ability of two opposite wheels to move independently.

6. DESIGN ALGORITHM OF THE FRONT SUSPENSION GEOMETRY AND WHEEL GEOMETRY

After establishment of the "ideal" solution, the next step was the development of a mathematical algorithm to assist the design of the vehicle front suspension geometry. It is based on the previously presented theoretical introduction. The comparison factors are: (1) the mechanism stability, (2) geometric variation (changing the mechanism dimensions), (3) the complex mechanism and (4) weight/cost ratio.

The algorithm analyzes the suspension geometry behavior from the mechanism dimensional parameters, allowing the system to find a geometry that minimizes the influence of the dynamics of the suspension in the steering system, reducing unwanted variation of the tires steering angle on a front suspension supposed action.
The main algorithm function is to determine the vehicle wheels behavior (relation between the steering angles of the wheels) from the dimensional parameters of the mechanism, allowing the designer to find the geometry that best suits the Ackerman geometry, minimizing errors in the steering angles of the wheels while performing of a curve.

7. ALGORITHM DEVELOPMENT

7.1. Mechanisms Selection For Suspension and Steering Systems

For the selection of possible mechanisms to be determined by the algorithm some conditions were considered:

ISSN 2176-5480

stability, possibilities for adjustments to the parameters and geometric simplicity. Based on these conditions we found that the best mechanism to be used in the front suspension system is a mechanism "Double-A", since this mechanism allows a wide adjustment range of the suspension geometric parameters, providing a better performance and therefore a good stability, i.e., little variation in the mechanism geometric parameters during the suspension deflection.

The suspension mechanism type Double-A basically consists of two arms, usually triangular shaped, assembled in overlapping planes. The geometry is based on a four-bar mechanism, since each suspension arm contains two pivot points: on the vehicle structure and on the stub axle.

After the selection of the mechanism to be used in the front suspension, the next step is the development of a mathematical algorithm that allows the analysis of the suspension geometry behavior from the dimensional parameters of the mechanism, to find a geometry that minimizes the influence of the dynamic behavior of the suspension steering system. To develop this algorithm, a mathematical model was created from a real model of suspension Double-A (Figure 14).

7.2. Determination of Motion Equations of the Suspension Mechanism

To determine the equations of motion of the suspension mechanism is necessary to determine the equations corresponding to the points where are the spherical joints of the mechanism, as shown in Figure 15.

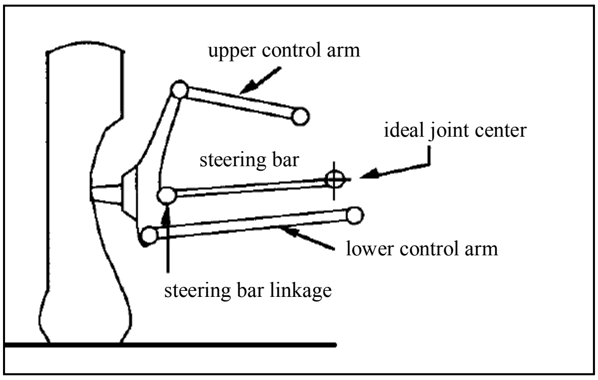


Figure 14 - Real model of suspension Double

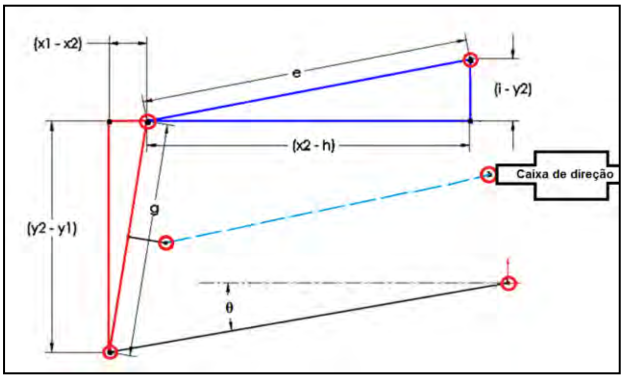


Figure 15 - Trigonometric relations to achieve the equation x_2 and y_2

Considering the coordinates of the points A, B and I as input parameters to the algorithm, is necessary to determine the motion equations of points x_1, x_2, x_3, y_1, y_2 e y_3.

$$x_1 = a \cdot cos(\theta) \quad [1]$$

$$y_1 = a \cdot sin(\theta) \quad [2]$$

To determine the motion equations of points x_2 e y_2 é is necessary to consider the trigonometric relations which can be seen in Figure 16(a) and (b).

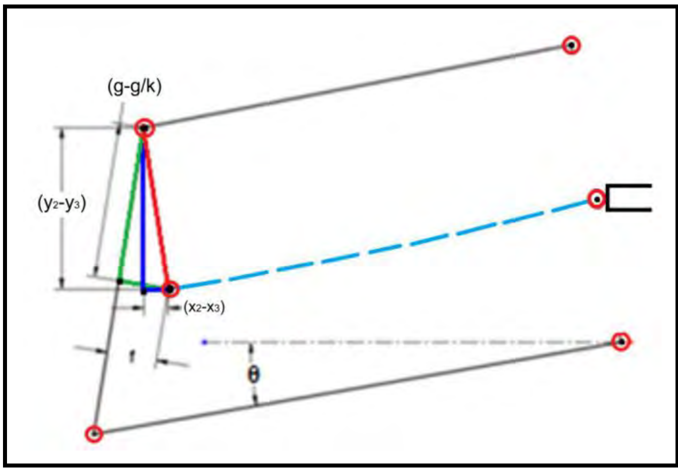


Figure 16(a) - Trigonometric relations to achieve the equation y_3.

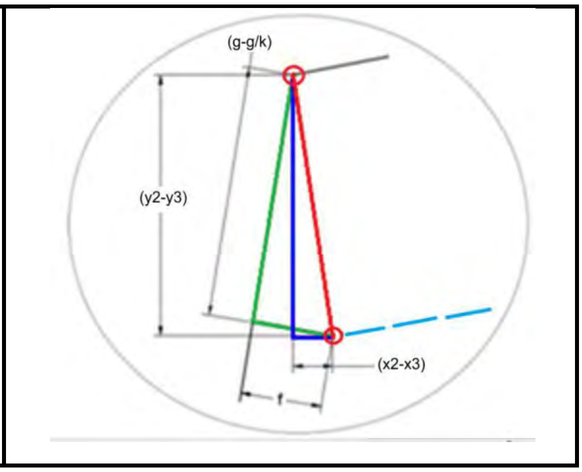


Figure 16(b) - Detail of the interest region.

$$(x_2 - h)^2 + (i - y_2)^2 = e^2 \quad [3]$$

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 = g^2 \quad [4]$$

$$x_2 = h + \sqrt{e^2 - i^2 + 2\, i\, y_2 - y_2^2} \quad [5]$$

Substituting the expression found in equation [4] and solving as a function of y2 we obtain the motion equation at y2.

$$y_2 = \frac{1}{2\,(4\,h^2 + 4\,i^2 - 8\,h\,x_1 + 4\,x_1^2 - 8\,i\,y_1 + 4\,y_1^2)} \Big(-4\,e^2\,i + 4\,g^2\,i + 4\,h^2\,i + 4\,i^3 - 8\,h\,i\,x_1 + 4\,i\,x_1^2 + 4\,e^2\,y_1 - 4\,g^2\,y_1 + 4\,h^2\,y_1 - 4\,i^2\,y_1 - 8\,h\,x_1\,y_1 + 4\,x_1^2\,y_1 - 4\,i\,y_1^2 + 4\,y_1^3 - \sqrt{\big((4\,e^2\,i - 4\,g^2\,i - 4\,h^2\,i - 4\,i^3 + 8\,h\,i\,x_1 - 4\,i\,x_1^2 - 4\,e^2\,y_1 + 4\,g^2\,y_1 - 4\,h^2\,y_1 + 4\,i^2\,y_1 + 8\,h\,x_1\,y_1 - 4\,x_1^2\,y_1 + 4\,i\,y_1^2 - 4\,y_1^3)^2 - 4\,(4\,h^2 + 4\,i^2 - 8\,h\,x_1 + 4\,x_1^2 - 8\,i\,y_1 + 4\,y_1^2)\,(e^4 - 2\,e^2\,g^2 + g^4 - 2\,e^2\,h^2 - 2\,g^2\,h^2 + h^4 - 2\,e^2\,i^2 + 2\,g^2\,i^2 + 2\,h^2\,i^2 + i^4 + 4\,e^2\,h\,x_1 + 4\,g^2\,h\,x_1 - 4\,h^3\,x_1 - 4\,h\,i^2\,x_1 - 2\,e^2\,x_1^2 - 2\,g^2\,x_1^2 + 6\,h^2\,x_1^2 + 2\,i^2\,x_1^2 - 4\,h\,x_1^3 + x_1^4 + 2\,e^2\,y_1^2 - 2\,g^2\,y_1^2 + 2\,h^2\,y_1^2 - 2\,i^2\,y_1^2 - 4\,h\,x_1\,y_1^2 + 2\,x_1^2\,y_1^2 + y_1^4)\big)}\Big) \quad [6]$$

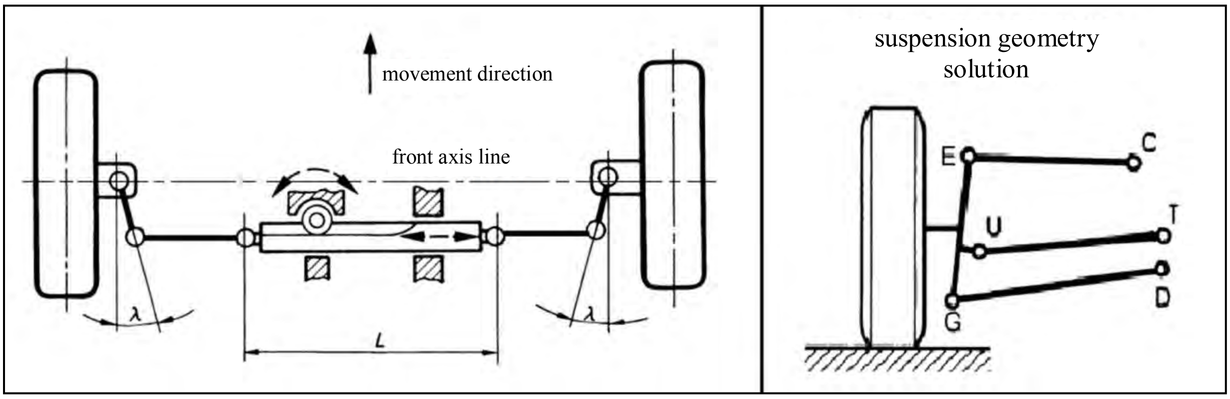


Figure 17 - Suspension geometry is valid when the steering arm is placed behind the axis line.

This variation of the steering mechanism displays only the pivot point $G(x_3,y_3)$ in a different position in relation to the mechanism discussed above. Thus, for this new configuration the equations of motion of the new point need to be determined, according to the trigonometric relations shown in Figures 18 e 19.

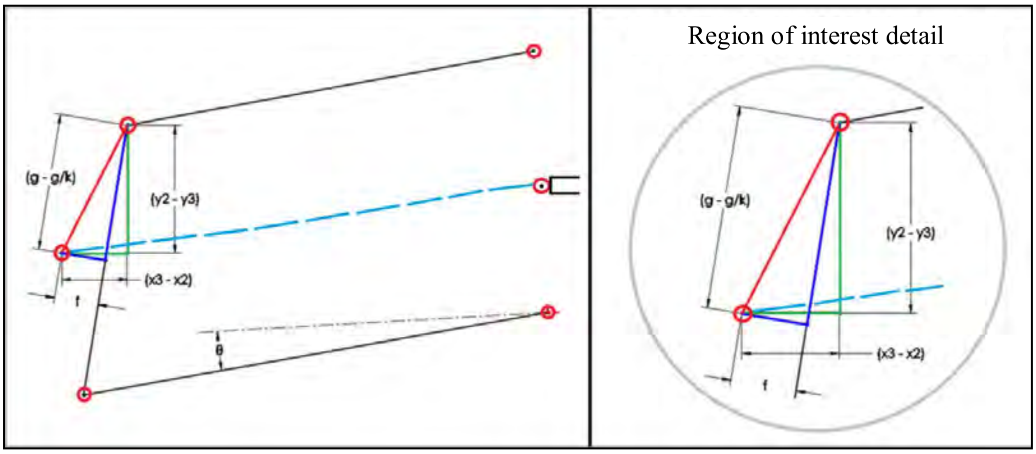


Figure 18 - Trigonometric relations to obtain the equation at y_3'.

ISSN 2176-5480

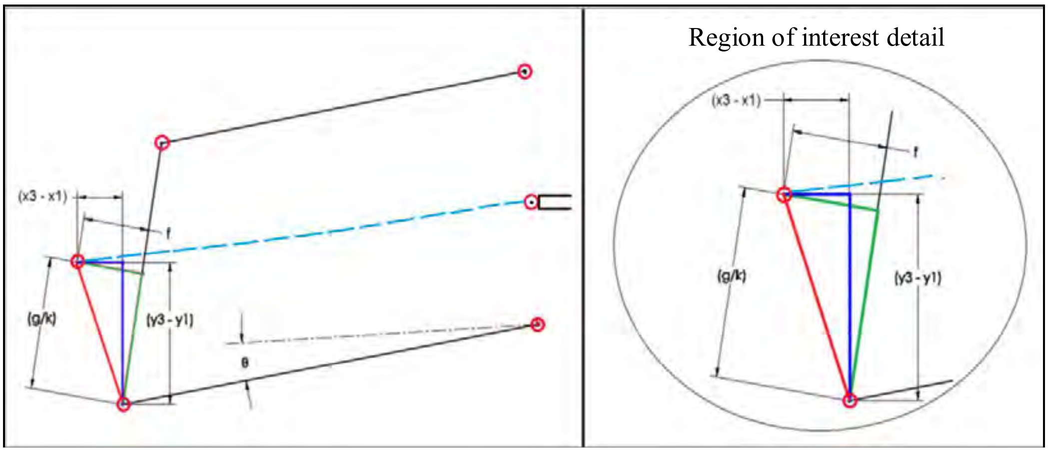


Figure 19 - Trigonometric relations to obtain the equation at x_3'.

Where:

$$(g - {}^{g}/_{k})^2 + f^2 = (x_3' - x_2)^2 + (y_2 - y_3)^2 \qquad [7]$$

$$({}^{g}/_{k})^2 + f^2 = (x_3' - x_1)^2 + (y_3' - y_1)^2 \qquad [8]$$

Solving equation [14] as a function of y_3', we get the motion equation to point y_3':

$$y_3 = \frac{\sqrt{(g^2 k^2 + f^2 k^4 - k^4 x_1^2 + 2 k^4 x_1 x_3 - k^4 x_3^2)}\ (k^2 y_1)}{k^2} \qquad [9]$$

Substituting the expression found for y_3' in equation [7] and solving as a function of x_3': motion equation point x_3':

$$\begin{aligned}
x_3' = {} & \frac{1}{2 k^2 (x_1^2 - 2 x_1 x_2 + x_2^2 + (y_1 - y_2)^2)} \\
& (-2 g^2 k x_1 + g^2 k^2 x_1 + k^2 x_1^3 + 2 g^2 k x_2 - g^2 k^2 x_2 - k^2 x_1^2 x_2 - k^2 x_1 x_2^2 + k^2 x_2^3 + \\
& k^2 x_1 y_1^2 + k^2 x_2 y_1^2 - 2 k^2 x_1 y_1 y_2 - 2 k^2 x_2 y_1 y_2 + k^2 x_1 y_2^2 + k^2 x_2 y_2^2 + \\
& \sqrt{}(-k^2 (y_1 - y_2)^2 (4 g^4 - 4 g^4 k + g^4 k^2 + k^2 x_1^4 - 4 k^2 x_1^3 x_2 + k^2 x_2^4 - 4 g^2 y_1^2 + \\
& 4 g^2 k y_1^2 - 4 f^2 k^2 y_1^2 - 2 g^2 k^2 y_1^2 + k^2 y_1^4 + 8 g^2 y_1 y_2 - 8 g^2 k y_1 y_2 + \\
& 8 f^2 k^2 y_1 y_2 + 4 g^2 k^2 y_1 y_2 - 4 k^2 y_1^3 y_2 - 4 g^2 y_2^2 + 4 g^2 k y_2^2 - \\
& 4 f^2 k^2 y_2^2 - 2 g^2 k^2 y_2^2 + 6 k^2 y_1^2 y_2^2 - 4 k^2 y_1 y_2^3 + k^2 y_2^4 - \\
& 2 x_2^2 (2 g^2 - 2 g^2 k + 2 f^2 k^2 + g^2 k^2 - k^2 y_1^2 + 2 k^2 y_1 y_2 - k^2 y_2^2) - \\
& 4 x_1 x_2 (-2 g^2 + 2 g^2 k - 2 f^2 k^2 - g^2 k^2 + k^2 x_2^2 + k^2 y_1^2 - 2 k^2 y_1 y_2 + k^2 y_2^2) + \\
& x_1^2 (-4 g^2 + 4 g^2 k - 4 f^2 k^2 - 2 g^2 k^2 + 6 k^2 x_2^2 + 2 k^2 y_1^2 - 4 k^2 y_1 y_2 + 2 k^2 y_2^2))))
\end{aligned} \qquad [10]$$

The value of the theoretical length of the steering arm for this mechanism variation can be found in the same way, applying the same trigonometric relation shown in Figure 49.

$$(x_3 - u)^2 + (y_3 - v)^2 = m^2 \qquad [11]$$

$$m' = \sqrt{(x_3' - u)^2 + (y_3' - v)^2} \qquad [12]$$

The equations found for the theoretical length of the steering arm are a function of the working angle and the dimensional parameters of the suspension mechanism. Thus, as the mechanism has constant dimensions, the only variable is the suspension working angle value, is possible then to plot a graph of the steering arm theoretical length as a function of the working angle, thus allowing, an interference analysis of the suspension system in the steering system.

From the trigonometric relations of the steering mechanism shown in Figures 21 and 22 and again using as a reference parameter the distance between the pivot axes of the wheels – T (fixed parameter) is possible to determine the following motion equations for this specific case:

Inner wheel during curve:

$$\frac{T}{2} = -d\ \mathrm{sen}[\emptyset + \delta_{int}] + c\ \cos\left[\mathrm{arcsen}\left[\frac{b - d\cos[\emptyset + \delta_{int}]}{c}\right]\right] + \frac{L}{2} + \Delta L \qquad [13]$$

Outside wheel during curve – when $\phi \geq \delta_{ext}$:

$$\frac{T}{2} = d\ \mathrm{sen}[\emptyset - \delta_{ext}] + c\ \cos\left[\mathrm{arcsen}\left[\frac{b - d\cos[\emptyset - \delta_{ext}]}{c}\right]\right] + \frac{L}{2} - \Delta L \qquad [14]$$

Outside wheel during curve – when $\phi < \delta_{ext}$:

$$\frac{T}{2} = -d\ \mathrm{sen}[\emptyset - \delta_{ext}] + c\ \cos\left[\mathrm{arcsen}\left[\frac{b - d\cos[\emptyset - \delta_{ext}]}{c}\right]\right] + \frac{L}{2} - \Delta L \qquad [15]$$

Equation for arm length direction in the mechanism (top view):

$$c_t = \sqrt{[b - d\cos[\emptyset]]^2 + \left[\frac{T}{2} - \frac{L}{2} + d\ \mathrm{sen}[\emptyset]\right]^2} \qquad [16]$$

Developing the equations [13], [14] e [15] as a function of camber angle of the wheel and for the constants previously presented, we found the new motion equations of the steering mechanism for the situation where the steering arm is in front of the front axle line. The motion equations found for the wheels are a function of the displacement of the steering box and of the dimensional parameters of the steering mechanism. So, as the mechanism dimensions are constants, the only variable is the value of displacement of the steering box, is then possible to plot a graph of the behavior of the inner wheel to the curve due to the behavior of the outer wheel to the curve, allowing a comparison of the actual steering geometry with ideal geometry described by Ackerman.

After determination of the motion equations and other relations, was possible to perform studies of the suspension and steering systems. These studies allow the designer to find one suspension geometry that has the lowest influence in steering system and a steering geometry that suits the Ackerman trapezoidal geometry.

For validation of the algorithm was performed a case study on a vehicle Baja SAE. The results were compared with the simulation results of a 2-D vehicle models developed in the Solidworks 2009.

7.3. The Baja SAE Vehicle

The prototype Baja SAE 2009 (Figure 20) is a tubular chassis single-seat off road vehicle with an independent Double-A front suspension mechanism, which operates together with an spring-damper dual-stage system.

Figure 20 - 3-D Baja SAE vehicle models

Table 1 - Dimensional parameters of the vehicle's suspension geometry

Parameter	Dimension [mm]	Description
T	1120	distance between the pivot points of the steering
L	1340	wheelbase of the vehicle
l	420	steering box length (considering extenders)
d	90	arm length “d”
b	-60	distance between the steering box axis and front axle
Δl	0 to 70	working range of steering box
Steering arm position		positioned in front of the front axle line

The steering system of the vehicle comprises a steering box, which communicates directly with the steering wheel by means of a steering bar. The dimensions of the front suspension mechanism of the vehicle are presented in Table 7 and Figure 21.

ISSN 2176-5480

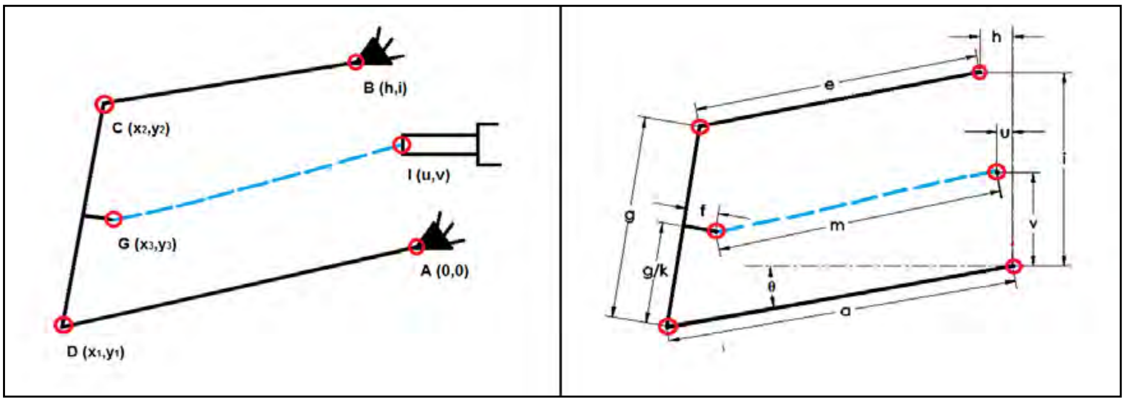


Figure 21 - Dimensional parameters of the Baja SAE 2009 suspension mechanism.

An analysis of dimensional parameters of the vehicle suspension mechanism was performed on the mathematical algorithm developed. Presented below are the results (Figure 22) of the variation of the theoretical length of the steering arm as a function of the working angle of the suspension, found in the mathematical simulation.

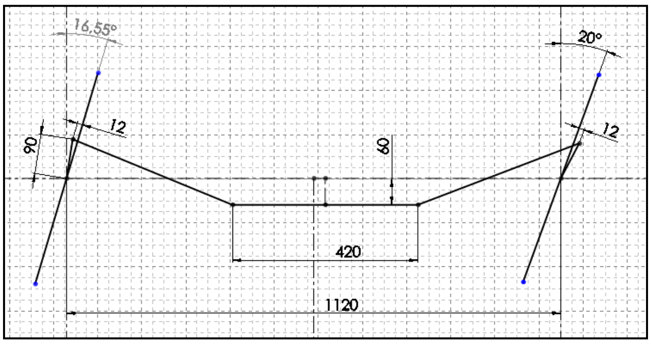


Figura 22 - Imagem do modelo 2D do mecanismo de direção desenvolvido no software Solidworks 2009.

Presented below is the graph of the relation between the angles of steering wheel and shift steering box found by the model (Figure 23). Next a graph was plotted comparing the results for the two cases (Figure 24).

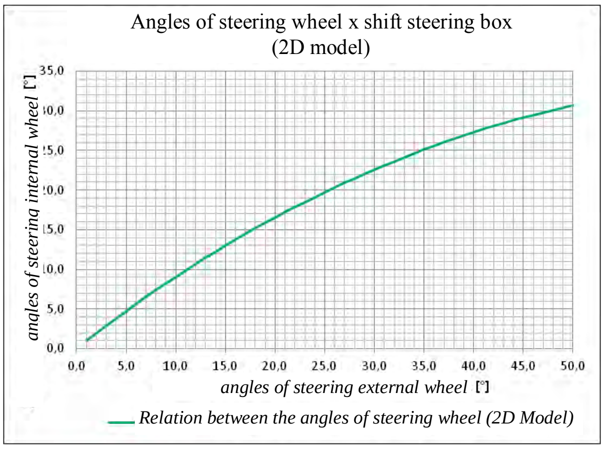


Figure 23 - Relation between the angles of steering wheel and shift steering box model.

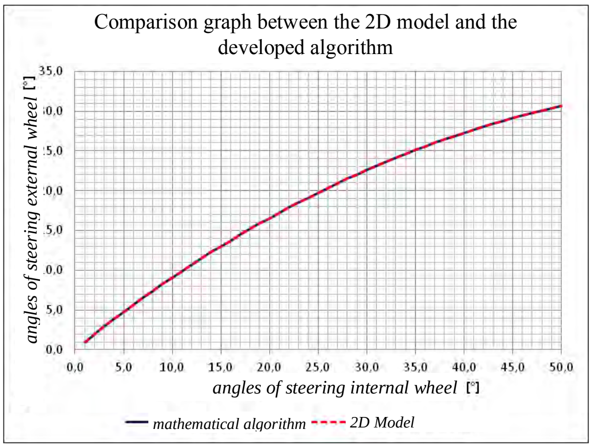


Figure 24- Comparison of the results for the two cases.

Based on the results, we found that the two methods have a good compatibility (more than enough for this type of project, where the manufacturing tolerances of the parts of the mechanism are wider.

After validation of the steering system model, is possible to perform an analysis of the actual mechanism behavior, comparing the relative angles of steering wheel and the steering box shift of the actual mechanism (analyzed by mathematical algorithm) and the trapezoidal mechanism (Ackerman ideal geometry). Below (Figure 25) is presented a comparison between the wheels steering angles behavior of the two mechanisms.

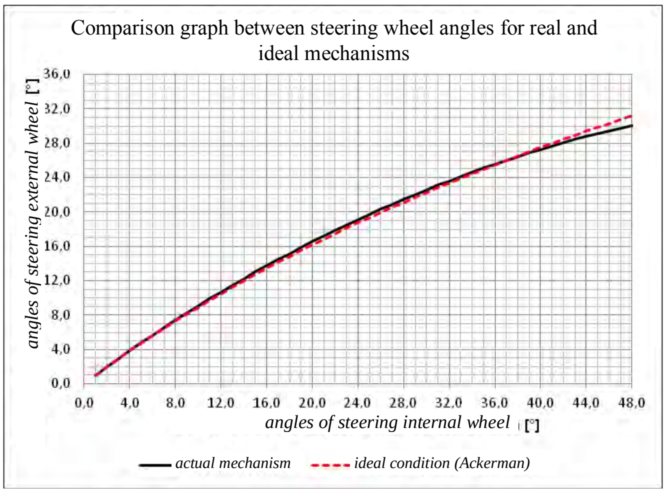


Figure 25 - Comparison of the results for the two mechanisms

The results show that the Baja SAE vehicle steering mechanism (actual mechanism) presents a good match with the geometric condition described by Ackerman (ideal condition), displaying errors approximately 2%. The associated error is very small (less than 1 degree) and can be ignored.

8. CONCLUSION

This study aimed the development of an auxiliary algorithm for the design of steering and geometries automotive suspension. The method used by the algorithm consists basically in reduce unwanted variation of the tires steering angle in a supposed front suspension action and simultaneously, specifying an adequate geometry that minimizes the errors in these angles during the execution of a curve.

After the development of the algorithm, a validation was carried out with 2-D models of steering mechanisms and suspension, using the software *Solidworks* 2009. The results showed that the algorithm has excellent compatibility with the models. The implementation of this algorithm significantly reduces the design time of steering and suspension geometries.

This reduction is justified due the fact that the designer estimates the best possible configuration for the steering and suspension geometries disregarding tire x ground contact (ideal geometry where the friction between tire and ground is considered ideal and no slippage).

From this ideal geometry, is possible to perform a full dynamic simulation using any expert software such as ADAMS CAR, which performs a complex vehicle simulation, including tire x ground contact. The simulation initial results, will not necessarily be ideal for the two conditions, however, allows a good approximation of the real situation of operation of steering mechanisms and suspension.

9. REFERENCES

Gillespie, Thomas D. "Fundamentals of Vehicle Dynamics." Society of Automotive Engineers, 1992.
Reimpell, J. at al. 2001,"The Automotive Chassis: Engineering Principles." Great Britain. 2th edition.
Milliken, William F. & Douglas L., 1995 "Race Car Vehicle Dynamics.", Society of Automotive Engineers,.
Staniforth, Allan. 1999, "Competition Car Suspension: Design, Construction and Tuning." Haynes Publishing, 3th edition.
Crouse, William H. "Automotive Mechanics." New York: McGraw-Hill, 4th edition.
Mabie, H. H. & Ocvirk, 1980, F. W. "Mecanismos." Livro Técnico e Científico S.A.
Shigley, J. E. 1970, "Cinemática dos Mecanismos." Editora Edgar Blücher Ltda.
Tilley, Alvin R., 2002, "The Measure of Man: Human Factors in Design.", Wiley, Revised Edition.
The Car Bible. at: <http://www.carbibles.com/>. Accessed in 21/05/2010.
How Car Steering Works. at: http://auto.howstuffworks.com/steering. Accessed in 21/05/2010.

10. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.