ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

EVALUATION OF GENETIC ALGORITHM AND DIFFERENTIAL EVOLUTION APPLIED TO PARAMETRIC OPTIMIZATION OF A COMPONENT USED IN REACTIVE ACOUSTIC FILTERS

Key Fonseca de Lima
Nilson Barbieri
Renato Barbieri
Pontifícia Universidade Católica do Paraná. Rua Imaculada Conceição, 1155. Curitiba-PR. Cep.: 80215-901.
e-mails: key.lima@pucpr.br, keyflima@gmail.com, nilson.barbieri@pucpr.br, renato.barbieri@pucpr.br.

Abstract. *This work evaluate the velocity of convergence of the solutions obtained with the parametric optimization of the cylindrical surface tube used in acoustic filters with two methods: Genetic Algorithms (GA) abs Differential Evolution (DE). In this application, the Finite Element Method (FEM) plus the Improved Four-Pole Parameter Method are employed for calculating the value of the objective function. The Objective function is defined from the sound transmission loss (TL), which is the one of the parameters used for evaluating the acoustic filter efficiency in frequency domain. Some results illustrated in this work indicate that the versions implemented for GA e DE are very appealing methods for acoustic problems for situations where the number of design variables is relatively large.*

Keywords: *acoustic filter, GA, DE, optimization.*

1. INTRODUCTION

The Genetic Algorithms (GA) is based on Darwin's concept of the selection and natural evolution applied to mathematical programming. It was originally proposed by Holland in 1975 and it has been widely used in the last years for obtaining optimum solutions for many engineering efforts. The optimization performance through the GA involves the task of making the population with the best fitness evolve and survive through many generations. During optimization, a set of solutions is chosen so it "evolves" towards the optimum solution for the problem. The GA has the ability of searching for a global optimum in discontinuous and multimodal spaces without requiring a proper initial point (Goldberg, 1989). GA efficiency has been widely verified in many fields, and besides that, it has been greatly used for acoustic problems (Chang *et al*., 2005; Balauc *et al*., 2008; and Chiu, 2010).

The Differential Evolution (DE) is a direct search method that was developed for minimizing the problems represented by non-linear and non-differentiable continuous spatial functions. This method was created in 1995 by Storm and Price when attempting to solve a polynomial fitting problem by Chebychev associated with electronic filter designs. In addition to that, the method is fairly simple and presents fast convergence features for problems that involve a small number of particles (individuals) and a large number of design variables. The main idea of the differential evolution is to create vectors composed by the design variables by adding the weighted difference between two randomly chosen vectors of the population and a third particle. This method was widely tested by Storm and Price in 1997 when searching for a minimal global in the study of the functions: De Jong Functions, Corana's parabola, Griewangk's function, Zimmermann's problem, Hyper-Ellipsoid function, Katsuura's Function, Rastrigin's Function and Ackley's function, being very efficient when compared to the results obtained with other search methods: Adaptive Simulated Annealing (ASA), the Annealed Nelder and Mead approach (ANM), the Breeder Genetic Algorithm (BGA), the EASY Evolution Strategy and the method of Stochastic Differential Equations (SDE). Since then, the DE has been used in the solution of problems in several engineering fields: identification of hysteretic systems (Kyprianou *et al*. 2001; non-linear modeling problems of laser diodes under microwave intensity (Akdagli and Yuksel, 2006); flow shop scheduling (Onwubolu and Davendra, 2006) and hydro generation scheduling (Yuan *et al*., 2008),

In this work, the parametric optimization of the surface form of a cylindrical tube used in acoustic filters is carried out using two direct search methods: Genetic Algorithms (GA) and Differential Evolution (DE). The tube acoustic efficiency in relation to the Transmission loss (TL) is evaluated in the optimizations. The transmission loss calculation will be carried out throughout the finite element method in the frequency domain, along with the Improved Four-Pole Parameter Method (Kim and Soedel, 1989-90).

2. THEORY

The GA allows to reach the maximum of a function f(x(i,j)) subject to the following design constraints:

$$x_L(i) \le x_i \le x_U(i) \Rightarrow i = 1, 2, \ldots, m \tag{1}$$

ISSN 2176-5480

where x_i is the particles or individuals set of design variables $x(i,j)$ which lower limit is $x_L(i)$ and the upper limit is $x_U(i)$ and m is the number of design variables to be defined.

The GA represents the design variables as sets of binary numbers of nb bits that are called chromosomes. This way, the viable interval for each variable x_i is divided into N intervals:

$$N = 2^{nb} - 1 \tag{2}$$

So, each variable x_i can be represented by any discreet representation, for e.g., through a binary number: 011011. This number can be decoded as:

$$x_i = x_L(i) + (0 \cdot 2^5 + 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0) \cdot s_i = x_L(i) + 27 s_i \tag{3}$$

where s_i is the variable interval x_i defined by:

$$s_i = (x_U(i) - x_L(i))/27\,. \tag{4}$$

This procedure defines the codification and decodification processes of the GA variables. The first step in the development of the GA is the creation of an initial population. Each particle or individual i of the population is a set with $m \times nb$ bits. The set of variables, x_i, in the binary form, creates the genetic code for each individual. An initial population with a z size is created by a random process. The second step is to decode the genetic code of each individual using the Eqn. (3) and verifying the value correspondent to its respective fitness, i.e., of its objective functions, given by f_1, f_2, …, f_z. The most fit individuals, with higher f values are considered most "optimal". Their fitness value must reflect the design and the restrictions imposed on the problem being studied, Tab. 1.

Table 1. Set of individuals and their genetic codes.

i Individual	x_1	x_2	$\cdots$	x_m	f
1	101101	010001	···	101101	f_1
2	101010	100001	···	111001	f_2
3	110011	001101	···	111011	f_3
$\downarrow$	$\downarrow$	$\downarrow$		$\downarrow$	$\downarrow$
z	101001	111110	···	101001	f_z

The third step includes the selection and crossover phases. Here, the weaker individuals are replaced by the stronger ones. The selection enables the best individuals to survive and serve as parents for the next project generation, through the fitness calculation. In this phase, the individuals are shuffled and have their order changed. After shuffling, two individuals are chosen. These are called parents and the one with the best fitness will originate from a child and will have some of its chromosomes randomly changed, Tab. 2. This procedure is carried out z times, generating the number of z children. The crossover transference makes parts of the parents' chromosomes to create the next generation of the project by combining features in such a way as to create better individuals in average, but not always.

Table 2. Selection and crossover.

i Individual	x_1	x_2	$\cdots$	x_m	f
1	101101	010001	···	101101	f_1
3	110011	001101	···	111011	f_3
		if $f_3 > f_1$			
3	1 1 0 0 1 1	0 0 1 1 0 1	···	1 1 1 0 1 1	f_3
	$\downarrow$	$\downarrow$ $\downarrow$	···	$\downarrow$ $\downarrow$	
Child 1	1 1 0 0 0 1	1 0 0 1 0 1	···	1 0 1 0 1 0	

The fourth step may switch a chromosome bit of an individual to its opposite value (e.g..: 0 to 1). This step is called mutation and aims to introduce a new beneficial feature that does not exist in the current population. The mutation makes the fitness value of a child to suffer a change. This procedure is performed to avoid local optimum in search spaces induced by the random fitness change of the individual. The probability of a member bit to suffer mutation must vary between 0.005 and 0.1, which demonstrates that this will rarely occur in the nature (Carrol, 1996). If the feature

ISSN 2176-5480

introduced is not beneficial to the individual that suffered such mutation, probably it will not survive a future transformation step.

The fifth and last step verifies if the genetic code of the individual with the best fitness was replicated in any of the created children. If this does not happen, a child is randomly removed and the individual with the best fitness is added. This step improves the optimization accuracy. This operation is called elitist reproduction. In order to conclude this generation, the highest f value and its respective set of chromosomes x_i are stored. If the required number of generations in this project is reached, the evolution will be concluded; otherwise, a new decodification step will begin, and continue successively. All the described process transforms an initial randomly chosen population into a population that is more adapted to its environment, making them more optimal.

2.2 Differential Evolution (DE)

The Differential Evolution method consists basically of three steps: (i) create a population with z individuals in a multidimensional space, randomly distributed over all the objective function domain, and consequently, quantify its respective fitness, f; (ii) substitute the current population for one with better fitness; (iii) repeat this substitution until a population with best fitness, that meets the pre-established convergence criteria, is found.

The vector for the initial population $x(i,j)_g$ with j particles is randomly created and must be contained inside the solution space for the problem.

$$x(i,j)_g \quad \text{with} \quad i = 1, 2, \dots m; \quad j=1, 2,\dots z \quad \text{e} \quad g = 1, 2, \dots, G \tag{5}$$

where i is the number of design variables and g is the number of generations to be evaluated.

The DE creates a new parameters vector by adding the weighted difference between two particles of the population into a third. This operation is called mutation. The individual that comes from the mutation is mixed with the parameters from another predetermined particle called target vector, producing the trial vector. The parameters mixture is mentioned in the literature as crossover and will be explained in more details below. If the trial vector produces a upper value for the objective function f than the target vector, then the trial vector will replace the target vector in the following generation. This last operation is called selection. Each population vector has to serve a target vector once, so that z competitions occur in a generation. The mutation and crossover strategies for the DE are described below.

A mutant vector $v(i,j)_{g+1}$ is created for each target vector $x(i,j)_g$ randomly chosen within the population z according to:

$$v(i,j)_{g+1} = x_{r1,g} + F(x_{r2,g} - x_{r3,g}) \tag{6}$$

In Eqn. (6), $x_{r1,g}$ is the target vector. The indexes $r1$, $r2$, $r3 \in \{1, 2, \dots z\}$ are integral and different among themselves. The integral ones $r1$, $r2$, $r3$ randomly chosen are different from the index j in such a way that z must be larger or equal to 4 in order to allow for this condition. Factor F that controls the amplification of the differential variation ($x_{r2,g}$-$x_{r3,g}$) is real and constant $\in$ [0,2], (Storn and Price, 1995).

In order to increase the diversity of the particles, the crossover is introduced. For this purpose, the trial vector is:

$$u(i,j)_{g+1} = [u(1,j)_{g+1}, u(2,j)_{g+1}, \dots, u(m,j)_{g+1}] \tag{7}$$

where

$$u(i,j)_{g+1} = \begin{cases} v(i,j)_{g+1} \Rightarrow \text{if } (rand1(i) \le CR) \text{ or } i = rand2(j) \\ x(i,j)_g \Rightarrow \text{if } (rand1(i) > CR) \text{ and } i \ne rand2(j) \end{cases} \tag{8}$$

In Eqn. (8) $rand1(i)$ is a real number chosen randomly $\in$ [0,1]. CR is a crossover constant $\in$ [0, 1]. $rand2$ (i) is a randomly chosen index $\in$ [1, 2, ..., m], which guarantees that $u(i,j)_{g+1}$ receives at least a parameter of $v(i,j)_{g+1}$, see Refs. (Storn and Price, 1995-97). The selection, the last step of the process, decides if the trial vector $u(i,j)_{g+1}$ should or not become a member of the generation $g+1$. In this step $u(i,j)_{g+1}$ is compared to the target vector $x(i,j)_g$ using the best fitness criterion, i.e.:

$$x(i,j)_{g+1} = \begin{cases} u(i,j)_{g+1} \Rightarrow \text{if } f(u(i,j)_{g+1}) > f(x(i,j)_g) \\ x(i,j)_g \Rightarrow \text{if } f(u(i,j)_{g+1}) \le f(x(i,j)_g) \end{cases} \tag{9}$$

ISSN 2176-5480

3. NUMERICAL ANALYSES – OPTIMIZATION PROBLEM

The main objective of this paper is evaluating the quality of the solutions obtained with the parametric optimization of the surface of a cylindrical straight tube used in acoustic filters. The evaluated optimization methods are: GA and DE.

The straight cylindrical tube is 300 mm long and has a radius of 17.3 mm. The optimization region is defined as the central part of the tube with 100 mm in length. In the optimization region, for each particle, the boundary radius, (r_i) in relation to the center of the tube, is defined by the subtraction of the design variables x_i from the tube radius value, i.e.:

$$r_i = 17.3 - x_i \tag{10}$$

where the design variable restrictions x_i of the particle $x(i, j)$ are given by the Eqn. (1) and for this study they are: $0 \leq x_i \leq 5$ mm with i =1, 2, ... m design variables.

The objective function value is calculated for each change in the tube surface boundary. All the results shown in this article are obtained by employing the objective functions written as a function of the average TL in the frequency range between 1000 and 3500 Hz. The average TL has already been used by Lima, *et al*., (2011) for the optimization of acoustic silencers and its mathematical expression is:

$$f_i(x(i,j)) = \overline{TL}(\Delta\omega_i) = \frac{1}{\omega_i - \omega_f}\int_{\omega_i}^{\omega_f} TL(\omega)\, d\omega \tag{11}$$

where ω_i = 1000 hz – the beginning of the frequency interval in study and ω_f = 3500 hz – the end of the frequency interval in study.

The value for the objective function defined in Eqn. (11) is determined using the Finite Element Method (FEM) in addition to the Improved Four-Pole Parameters Method. The numerical analyses comprised in this article use the air at 20°C as a work fluid, the specific mass being 1.21 kg/m^3 and sound speed c equal to 343 m/s.

In the design region, for the highest excitation frequency, f is equal to 3500 hz, the finite element mesh has a characteristic length of 1.087 mm. This provides for a sufficient number of elements by wavelength (λ) in order to reduce amplitude and phase errors in the evaluation of the objective function using the FEM (Ihlenbur et al., 1997). In this article, the wavelength is defined by:

$$\lambda = c/f = 0.098 \text{ m} \tag{12}$$

With λ = 0.098 m we assure that all the numerical analyses will be carried out with at least 90 elements by wavelength. The finite element analyses were carried out by employing the linear isoparametric element with axi-symmetric formulation. The design region boundary is defined by the known Hermite polynomials for each particle. This approximation aims to locally and smoothly approximate the boundary (continuity C^1) and to reduce the number of design variables. This way, we only have the need to obtain 22 points equally distant from the optimization algorithm, i. e., m = 22. The points between two design variables are obtained by using the Hermite polynomials as shown in Fig. 1.

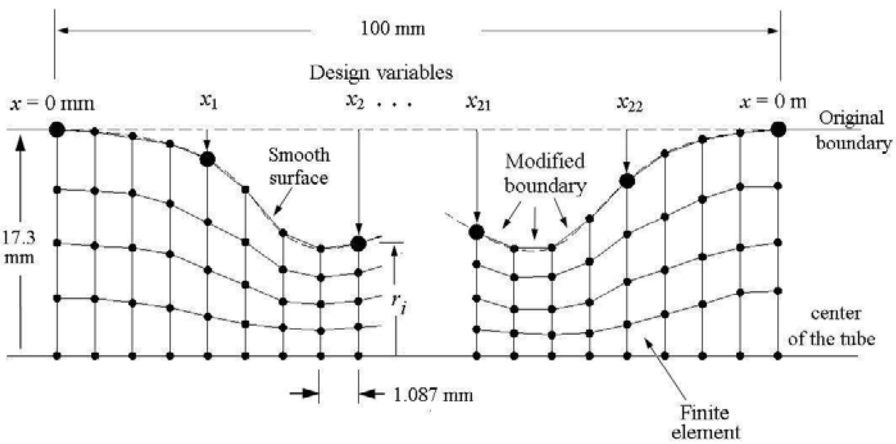

Figure 1. Definition of the optimized tube profile and the design variables

ISSN 2176-5480

This work will be conducted in three 3 steps:

I. Definition of the DE parameters;
II. Study of the convergence speed for the two methods;
III. Analysis of the tube acoustic efficiency with the obtainment of the optimized profile.

3.1 Step I – Determination of the DE parameters

GA effectiveness in optimization analyses of acoustic filters have been widely studied by several authors (Chang *et al*., 2005; Chiu *et al.*, 2008, Chiu, 2010 and Airaksinen *et. al.*, 2011). However, there are not many works describing the efficiency of the DE. The following parameters were used in the analyses with the GA: crossover probability = 50% and mutation probability = 2%. The DE is a relatively simple method that represents only 2 parameters: *CR* and *F*. In order to determine the best set of parameters for the problem in study, 16 analyses are carried out for the DE, Tab. 3. Several parameter configurations are shown in these analyses. Besides that, the number of evaluations necessary to reach the maximum value for the objective function is shown in the tables. As a stopping criterion, 40,000 evaluations of the objective function were done for each analysis. In Tab. 3, it can be easily noticed, with the exception of the analysis 4, that the objective function reaches the same value for all the analyses carried out, i.e., 3.73 dB. However, for the next step of this work, the analysis parameters are chosen where the convergence of the objective function was faster: *CR* =5 and *F* = 0.5 (Analysis 3).

Table 3. DE parameters.

Analysis	Population	*CR*	*F*	Number of evaluations	Maximum objective function (dB)
1	20	0.9	0.5	19680	3.73
2	20	0.7	0.5	19080	3.73
3	20	0.5	0.5	18400	3.73
4	20	0.3	0.5	35040	3.72
5	20	0.9	0.7	21160	3.73
6	20	0.7	0.7	20960	3.73
7	20	0.5	0.7	25200	3.73
8	20	0.3	0.7	23600	3.73
9	20	0.9	0.9	26300	3.73
10	20	0.7	0.9	23160	3.73
11	20	0.5	0.9	20360	3.73
12	20	0.3	0.9	21520	3.73
13	20	0.9	1.3	39840	3.73
14	20	0.7	1.3	29400	3.73
15	20	0.5	1.3	28000	3.73
16	20	0.3	1.3	37840	3.73

3.2 Step II - Study of the convergence speed of the GA and DE

To verify which is the most efficient optimization method in relation to the convergence speed, four evaluations per method are carried out, i.e., with the populations of 10, 20 and 40 particles. The parameters defined in Sec. 3.1 are used in this step. The results for the convergence behavior of the optimizations are shown in Figs. 2 and 3. Figure 4 presents a comparative amongst the best results for the two methods. According to Figs. 2 and 3, it can be clearly noticed that the increase in the number of particles has only increased the convergence speed for the optimization with the GA. In Fig. 4, it can be noticed that the convergence is obtained faster through the GA after 5,900 evaluations of the objective function (see continuous vertical line). However, the maximum objective function value is only reached after 29,600 (GA) and 10,820 (DE). The maximum values reached for the objective function through the two methods evaluated are: 3.70 (GA) and 3.73 (DE), Tab. 4. Table 5 shows the design variables values for the two methods when they all reached 5,900 evaluations and when their objective functions reached the maximum values. In addition, GA and the DE results are similar. In this case, the surface that was found in the tube optimization is shown in Fig. 5. The GA and DE presented results without significant differences when their objective functions reached the maximum values.

ISSN 2176-5480

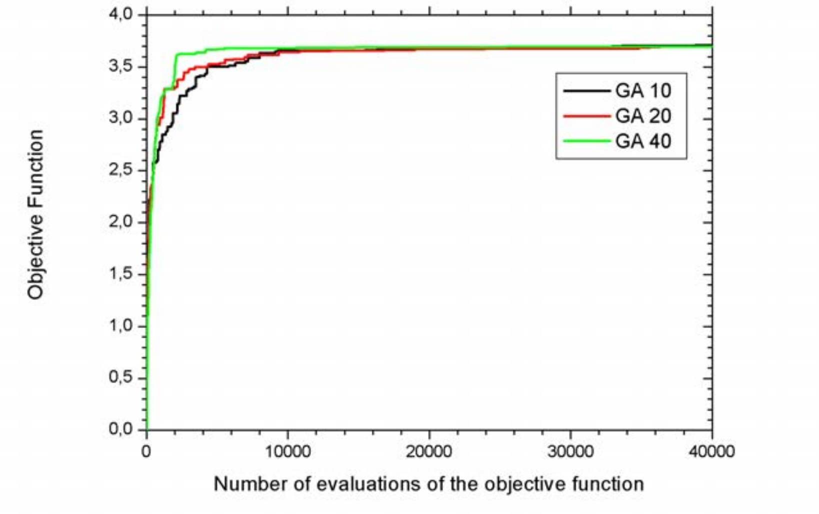

Figure 2. GA convergence behavior.

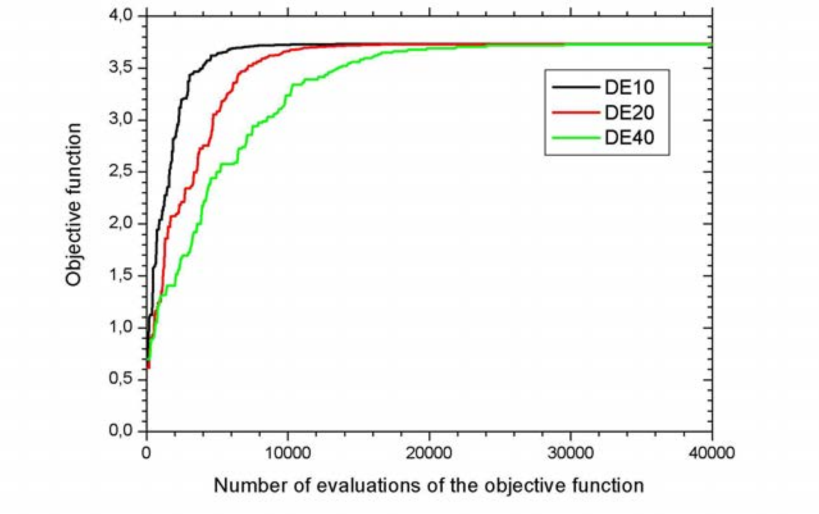

Figure 3. DE convergence behavior.

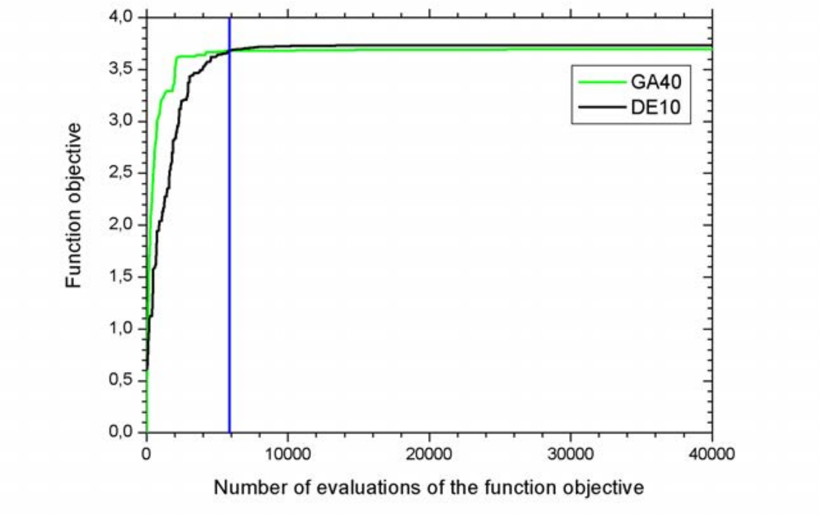

Figure 4. The best results of the convergence behavior between the GA 40 and DE 10.

ISSN 2176-5480

Table 4. Values for the objective function obtained through the optimization methods.

Evaluation number	Optimization method	Number of particles	Objective function value (dB)
5,900	DE	10	3.69
	GA	40	3.68
10,820	DE	10	3.73
29,600	GA	40	3.70

Table 6. Values of the design variables obtained through the optimization methods.

Evaluation number	Optimization method	Values of design variables in (mm).										
5,900	DE	4.894	4.990	4.915	4.979	4.980	4.994	4.987	0.012	0.023	0.000	0.009
		0.006	0.008	0.001	0.001	4.891	4.994	4.988	4.997	4.995	4.968	4.984
	GA	4.971	4.990	4.990	5.000	4.990	4.976	4.980	0.020	0.063	0.049	0.024
		0.044	0.010	0.034	0.010	4.961	4.956	4.985	4.946	4.995	5.000	4.995
10,820	DE	5.000	4.999	5.000	4.999	5.000	4.999	4.998	0.000	0.007	0.000	0.000
		0.001	0.000	0.001	0.006	4.999	5.000	5.000	5.000	5.000	5.000	4.995
29,600	GA	5.000	4.990	4.990	5.000	4.985	4.980	4.980	0.029	0.000	0.049	0.020
		0.005	0.000	0.015	0.005	4.961	4.956	4.995	4.995	4.969	4.995	4.985

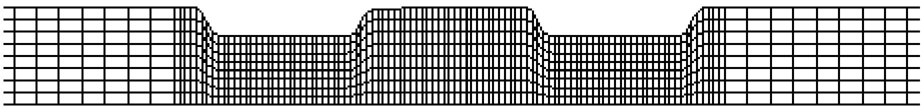

Figure 7. Profile of the optimized tube after 5,900 evaluations of the objective function for the GA and DE.

3.3 Analysis of the tube acoustic efficiency with the obtainment of the optimized profile

The acoustic efficiency of the optimized tube is measured through the determination of the sound transmission loss in the frequency domain, TL(ω). Fig. 8 shows the TL(ω) results for the tube of Fig. 7 in a black line. For just one optimized tube, the average TL reaches the average value of approximately 6 dB for a frequency range between 2100 and 3400 Hz. In order to show the optimized tube efficiency, the result of the two optimized tubes assembled in series is showed red line in the Fig. 8 with red line. The TL(ω) results reach higher values in the same frequency range for this configuration.

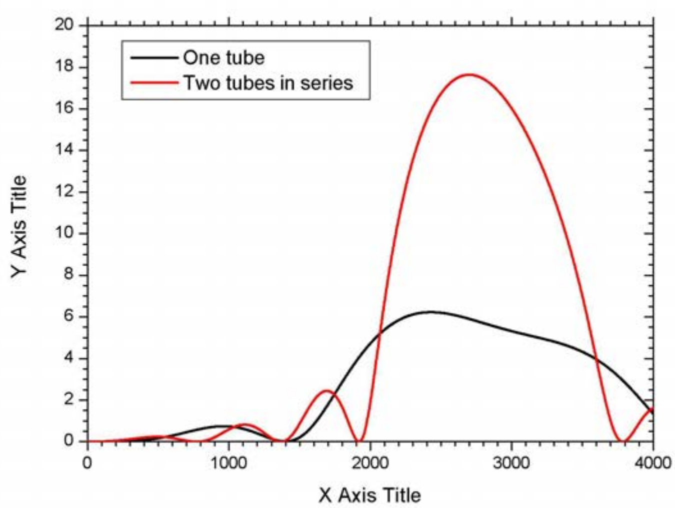

Figure 8. TL(ω) behavior for one and two optimized tubes.

4. CONCLUSIONS

The main observations and conclusions obtained in this work are:
a) The optimization with the GA and the DE present are fast.
b) The optimization with the DE has proven to be more efficient for this type of problem;
c) The increase in the number of particles in a population is not always beneficial to the convergence of the solution, see Figs. 2, 3 and 4;
d) The use of tubes with an optimized profile has proven to be effective for attenuating the noise in high frequencies for acoustic filter.

5. REFERENCES

Airaksinen, T. and Heikkola, E., 2011. “Multiobjective muffler shape optimization with hybrid acoustics modeling”. *Journal Acoustical Society of America*, Vol. 130, pp. 1359-1369.

Akdagli, A. and Yuksel, M. E., 2006. “Application of differential evolution algorithm to the modeling of laser diode nonlinearity in a radio-over-fiber network”. *Microwave and Optical Technology Letters*, Vol. 48, pp. 1130–1133.

Baulac, M., Defrance, J. and Jean, P., 2008. “Optimization with genetic algorithm of the acoustic performance of T-shaped noise barriers with a reactive top surface”. *Applied Acoustic*, Vol. 69, pp. 332–342.

Carrol, L. D., 1996. “Genetic algorithms and optimizing chemical oxygen-iodine lasers”. *Development in Theoretical and Applied Mechanics*, Vol. 18, pp. 411–424.

Chang, Y. C., Yeh, L. J. and Chiu, M.C., 2005. “Shape optimization on double-chamber mufflers using a genetic algorithm”. *Journal Mechanical Engineering Sciences*, Vol. 219, pp. 31–43.

Chiu, M. C., 2010. “Shape optimization of multi-chamber mufflers with plug-inlet tube on a venting process by genetic algorithms”. *Applied Acoustics*, Vol. 71 (6), pp. 495–505.

Chiu, M. C. and Chang. Y. C., 2008. “Numerical studies on venting system with multi-chamber perforated mufflers by GA optimization”. *Applied Acoustics*, Vol. 69, pp. 1017–1037.

Goldberg, D. E., 1989. *Genetic Algorithms in Search, Optimization, and Machine Lerning*. Addison&Willey, New York.

Holland, J., 1975, *Adaptation in Natural and Artificial Systems*. University of Michigan Press, Ann Arbor.

Ihlenbur, F., Babuska, I. and Sauter, S., 1997. “Reliability of finite element methods for numerical computation waves”, *Advances in Engineering Software*, Vol. 28, pp. 417–424.

Kim, J. and Soedel,W., 1989. “Analysis of gas pulsation in multiply connected three dimensional acoustic cavity with special attention to nature mode or wave cancellation effects”, *Journal of Sound and Vibration*, Vol. 131, pp. 103–114.

Kim, J. and Soedel, W., 1990. “Development of general procedure to formulate four pole parameters by modal expansion and its application to three dimensional cavities”. *ASME Transaction, Journal of Vibration and Acoustic*, Vol. 112, pp. 452–459.

Kyprianou, A. And Worden, K. and Panet, M., 2001. “Identification of hysteretic systems using the differential evolution algorithm”. *Journal of Sound And Vibration*, Vol. 248, pp. 289–314.

Lima, K., F., Lenzi, A. and Barbieri, R., 2011. “The study of reactive silencers by shape and parametric optimization techniques”. *Applied Acoustics*, Vol. 72 , pp. 142–150.

Michalski, K. A., 2001. “Electromagnetic imaging of elliptical–cylindrical conductors and tunnels using a differential evolution algorithm”. *Microwave and Optical Technology Letters*, Vol. 28, pp. 164–169.

Onwubolu, G. and Davendra, D., 2006. “Scheduling flow shops using differential evolution algorithm”, European *Journal of Operational Research*, Vol. 171 , pp. 674–692.

Storn, R. and Price, K., 1995. “Differential Evolution: a simple and efficient adaptive scheme for global optimization over continuous spaces”, *International Computer Science Institute – Technical Report*, TR-95-012, Berkeley.

Storn, R. and Price, K., 1997. “Differential evolution: a simple and efficient heuristic for global optimization over continuous spaces”. *Journal of global Optimization*, Vol. 11, pp. 341–359.

Yuan, X., Zhang, Y., Wang. L. And Yuan, Y., 2008. “An enhanced differential evolution algorithm for daily optimal hydro generation scheduling”. *Computers & Mathematics with Applications*, Vol. 55, pp. 2458–2468.

6. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.