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THE ROTARY INVERTED PENDULUM: MODELING, SIMULATION AND CONTROL AIDED BY COMPUTER: CAD-CAE

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Abstract: *This study aims to perform the modeling, simulation and control of the system known as the rotary inverted pendulum. This system was first developed at the Tokyo Technology Institute by Katsuhisa Furuta. Two mathematical models for the same system were obtained: a mathematical model based on Lagrangian formulation, and a computational model based on CAE (Computer Aided Engineering). The control law was designed with a state observer that allowed the allocation of the system closed loop poles. The controller was also tested on a computational nonlinear prototype and its validity was verified nearby the equilibrium point.*

Keywords: *computer-aided engineering, computational modeling, rotary inverted pendulum, Furuta pendulum.*

1. INTRODUCTION

Computational resources are increasingly been used in several areas of engineering due to hardware and software evolution, allowing the development of powerful tools of design and manufacturing. One of these tools is known as Computer Aided Engineering (CAE), a set of computational resources for system validation, simulation and optimization. When compared to classical prototyping techniques, CAE models have greater versatility by providing a significant reduction in developing time and costs.

The system modeling process starts on a Computer Aided Design (CAD), where the geometrical design normally starts. The information obtained in the CAD system can subsequently be exported in a standard digital format (e.g. IGES, STL, VDA, STEP) for a CAE system, allowing numerical simulations of models (Nakamura, et al., 2003). To complete the process, the geometric data can be acquired by a measure coordination machine and the comparison is performed by a CAI system (Rozenfeld, 1996). Figure 1 shows the support software integration, revealing CAD as it´s center, base of the whole process.

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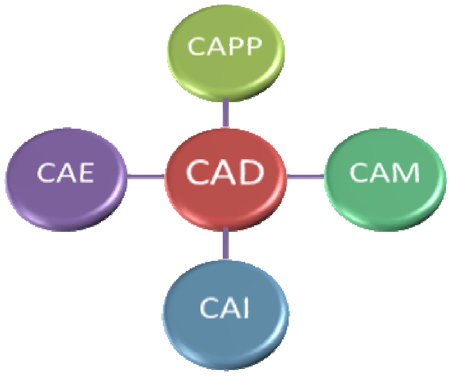

CAD – Computer Aided Design.
CAPP – Computer Aided Process Planning.
CAE- Computer Aided Engineering.
CAI- Computer Aided Inspection.
CAM- Computer Aided Manufacturing.

Figure 1 – Computer Aided Software Integration.

Automatic control has a prominent role in the advancement of engineering and science due to the major importance in systems like space vehicles, missile guidance and robotic, becoming an integral part of modern industrial processes and production. For example, the automatic control is essential in computer numerical control (CNC) machine in manufacturing industries, autopilot systems design in the aerospace industry and cars and trucks design in the automotive industry. It is also essential in industrial operation, such as pressure, temperature, moisture, viscosity and flow control in industrial process (Ogata, 2003).

Nowadays, most engineers and scientists must have good knowledge in this area due to the automatic theoretical and practical control advances applied to optimize the dynamic system performances, improve productivity and reduce the hard work of several repetitive manual routines (Ogata, 2003).

2. COMPUTER AIDED TOOLS

In recent years, there was a significantly reduction in the product development time due to the improvement of computational tools. These systems, called Computer Aided “x” (CA “x”) are integrated to each other in a way that a particular procedure adopted in one phase can improve the time saving in the others (Foggiatto, et al., 2007). Of particular interest for this work are the Computer Aided Engineering (CAE) tools, where one can obtain a mathematical/computational model for the system. CAE is the analysis and evaluation of engineering design using computational techniques to calculate products operation and functionality as well manufacturing parameters too complex for classical methods (Rehg, 1994). In other terms, CAE is the use of computational resources for simulation of mechanical system, encompassing it´s validation, simulation and optimization. By using CAE is possible to simulate the model behavior in several conditions to verify the viability of the physical system construction.

The breakthrough of CAE technology revolutionized the way designers evaluate the performance of their projects, saving time and money since in many situations it is eliminated the need for building prototypes. CAE software’s perform different types of analysis such as structural, thermal, dynamic and fluid and with the use of this tool a large amount of computation is achieved in less time, resulting in data from stress, strain, force and speed.

The use of CAE software allows the designer to focus for longer in technical and creative steps of the process and detect possible errors early in the project, improving the quality and reducing the time of delivery.

2.1 ADAMS

The CAE system used in this study is the MSC ADAMS® for mechanical systems and it enables users to produce virtual prototypes, simulating the behavior of complex mechanical systems movements and analyze design variations until the ideal system is achieved. In this work, simulations are solutions used to movement equations to describe a mechanical system while animations are graphic playbacks simulations previously completed, i.e. animations are interactive models obtained in the simulations (ADAMS / FULL, 2005). ADAMS/Controls is a plug-in that allows users to add sophisticated control methods to the established system. This tool enables the inclusion of ADAMS model into block diagrams developed in control tools as MATLAB software or EASY5 (ADAMS/CONTROLS, 2005).

ADAMS/Controls offers options to simulate the mechanical system and the entirely controller within the control tool, to simulate the complete system in ADAMS or solving the control equations within the control tool and the mechanical equations in ADAMS. This plug-in also allows the user to view the simulation results interactively and the use of the same model for two designs (mechanical and control).

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3. MATHEMATICAL MODELING

A model is a representation of the knowledge that one has about a system and the main tool for studying the behavior of complex systems. Modeling is the first step in the system analysis of any kind and in any respect. If the model is a faithful representation, significant information can be obtained about its dynamics and performance (Trivelato, 2003). On the other hand, modeling is a complex process that involves deduction and inference ability, and frequently is a trial and error process, which is simpler if the laws governing the system are known. Therefore it is "a physics, mathematics, computational or logic representation for any system, process, phenomenon or entity" (Trivelato, 2003). According to their nature models are classified into physical, mathematical, logical, and recently, computational.

The rotary inverted pendulum, also known as "Furuta Pendulum", is an inverted pendulum supported by a rotating base. The system input variable is the torque applied to the base, and the most common output variables are the arm position and the pendulum angle. The rotary pendulum was first developed at the Institute of Technology in Tokyo by Katsuhisa Furuta (Cazzolato & Prime, 2011). Since then, a great amount of articles and thesis have used this system to demonstrate linear and nonlinear control laws.

In Figure 2 one has a schematic representation of the Furuta Pendulum. The torque τ_1 is applied in the base of the arm 1 and the connection between the arms 1 and 2 are free for rotate. The two arms have a length L_1 and L_2 respectively and their masses m_1 and m_2 are represented as concentrated in their center of masses, which are located in the points l_1 and l_2 respectively. Those points are the respective distances between the pivot (rotation) points of the arms and their centers of mass.

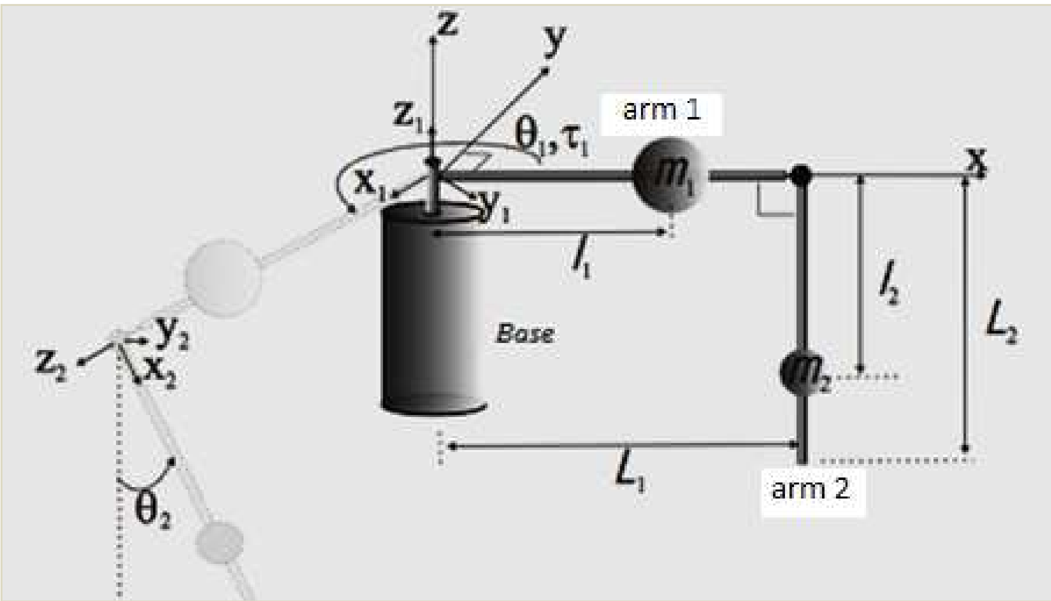

Figure 2 - Rotational inverted pendulum modeling

The angular rotation of the arm 1, that is θ_1, is measured on the horizontal plane where a counterclockwise direction (when viewed from above) is positive. The angular rotation of the arm 2, that is θ_2, is measured in a vertical plane where a counterclockwise direction (when viewed from the front) is positive. When arm 2 is hanging in its equilibrium position ($\theta_2 = 0$), the torque applied to the arm 1 is positive in the counterclockwise sense (when viewed from above). The following assumptions were made:

- The base and arm 1 were considered tightly coupled and infinitely rigid.
- The arm 2 was assumed to be infinitely rigid.
- The axes of the arms 1 and 2 are considered the major axes such that the inertia tensors are diagonal.

In order to write down the Lagrangian equations, the energies expressions are:

- arm 1 potential energy: $E_{p1} = 0$,

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- arm 1 kinetics energy: $E_{p1} = \frac{1}{2}\dot{\theta}_1^2\left(m_1 l_1^2 + J_{1zz}\right)$

and

- arm 2 potential energy: $E_{p2} = g m_2 l_2 (1 - \cos\theta_2)$,

- arm 2 kinetics energy:

$$E_{k2} = \frac{1}{2}\dot{\theta}_1^2\left(m_2 L_2^2 + \left(m_2 l_2^2 + J_{2yy}\right)\sin^2\theta_2 + J_{2xx}\cos^2\theta_2\right) + \frac{1}{2}\dot{\theta}_2^2\left(J_{2zz} + m_2 l_2^2\right) + m_2 L_1 l_2 \cos\theta_2 \dot{\theta}_1 \dot{\theta}_2$$

so, total energy (potential and kinetics) are given by: $E_p = E_{p1} + E_{p2}$ and $E_k = E_{k1} + E_{k2}$,

The Lagrangian is given by the difference between the kinetic energy and potential energy, that is:

$$L = E_k - E_p$$

and the dynamic equations are given by the Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q$$

where $q = [\theta_1 \quad \theta_2]$ is the generalized coordinate and $Q = [\tau_1 \quad 0]$ is the generalized momentum.

By writing down the Euler-Lagrange equation, one has

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \ddot{\theta}_1\left(J_{1zz} + m_1 l_1^2 + m_2 L_1^2 + \left(m_2 l_2^2 + J_{2yy}\right)\sin^2\theta_2 + J_{2xx}\cos^2\theta_2\right) + m_2 L_1 l_2 \cos^2\theta_2 \ddot{\theta}_2$$
$$- m_2 L_1 l_2 \sin\theta_2 \dot{\theta}_2^2 + \dot{\theta}_1 \dot{\theta}_2 \sin(2\theta_2)\left(m_2 l_2^2 + J_{2yy} - J_{2xx}\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = \ddot{\theta}_1 m_2 L_1 l_2 \cos\theta_2 + \ddot{\theta}_2\left(J_{2zz} + m_2 l_2^2\right) - \dot{\theta}_1 \dot{\theta}_2 m_2 L_1 l_2 \sin\theta_2 ,$$

$$-\frac{\partial L}{\partial \theta_1} = 0$$

$$-\frac{\partial L}{\partial \theta_2} = -\frac{1}{2}\dot{\theta}_2^2 \sin(2\theta_2)\left(m_2 l_2^2 + J_{2yy} - J_{2xx}\right) + \dot{\theta}_1 \dot{\theta}_2 m_2 L_1 l_2 \sin\theta_2 + g m_2 l_2 \sin\theta_2 ,$$

which is clearly a fourth order model, that can be put in a state space form.

In order to design a linear controller for the system, it must be linearized around an equilibrium point, in which case is the vertical position of arm 2. The linearized state equations for the vertical position are obtained using a Jacobian linearization with matrices

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ A_{31} & A_{31} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ B_{31} \\ B_{41} \end{bmatrix} \tau_1$$

where,

$$A_{31} = 0\ ;\ A_{32} = g m_2^2 l_2^2 L_1 / (\tilde{J}_0 \tilde{J}_2 - m_2^2 L_1^2 l_2^2)\ ;\ A_{33} = 0\ ;\ A_{34} = 0$$

$$A_{41} = 0\ ;\ A_{42} = g m_2 l_2 \tilde{J}_0 / (\tilde{J}_0 \tilde{J}_2 - m_2^2 L_1^2 l_2^2)\ ;\ A_{43} = 0\ ;\ A_{44} = 0$$

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$$B_{31} = \tilde{J}_2 /(\tilde{J}_0 \tilde{J}_2 - m_2^2 L_1^2 l_2^2)\,;\; B_{41} = \tilde{J}_2 /(\tilde{J}_0 \tilde{J}_2 - m_2^2 L_1^2 l_2^3)$$

where

$$\tilde{J}_1 = J_1 + m_{\cdot 1} l_1^2$$

$$\tilde{J}_2 = J_2 + m_{\cdot 2} l_2^2$$

$$\tilde{J}_0 = \tilde{J}_1 + m_{\cdot 2} L_1^2$$

The following values for the model parameters were utilized (all units in the International System):

$$\begin{array}{ll} m_1 = 1.9606 & m_2 = 1.10284 \\ L_1 = 0.2000 & L_2 = 0.2000 \\ l_1 = 0.1000 & l_2 = 0.1000 \\ J_1 = 0.00673 & J_2 = 0.00373 \end{array}$$

By applying the formulas above, we have the state space model:

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 43.38 & 0 & 0 \\ 0 & 138.56 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 26.81 \\ 40.09 \end{bmatrix} \tau_1$$

3. COMPUTATIONAL MODELING

The computational model was constructed in MSC ADAMS® by connecting three basic parts: base, arm1 and arm2, that are the same elements of the mathematical model. It was used two different types of joints (*fixed joint* and *gasket revolution*) to simulate the interaction between the parts of the computational model. As fixed joints have the function of fixing the motion of a part, they can be defined in two ways: one can fix a part so that it does not perform any movement or it can fix a part to another reference, so that the fixed portion moves according to the reference part. The joints of revolution have the function to fix all the movements of a part except the rotational motion around a reference axis. Revolution joints were used to prevent translational movement and set the movement direction of the base and arm 2 rotation as shown in Figure 3.

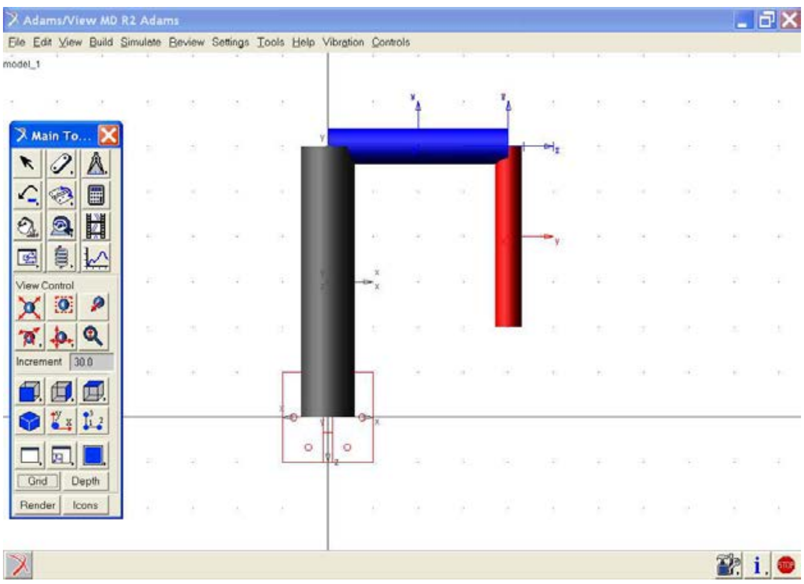

Figure 3 – Base and axis revolution joints.

With the pendulum at the equilibrium position and no force acting on it, we used the export function of the plugin Adams/Controls to export the computational model in the of state space form for MATLAB/Simulink®, as shown in Figure 4.

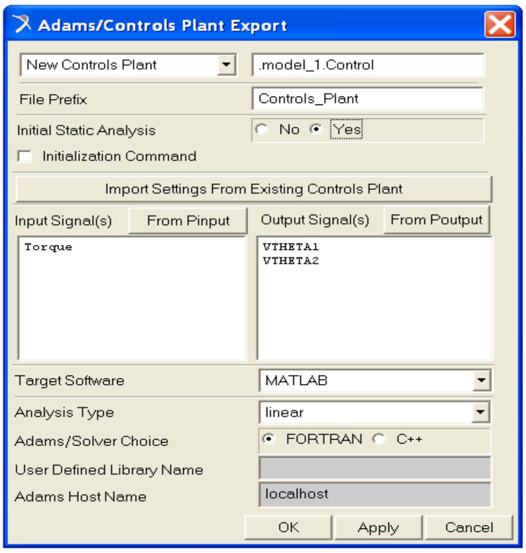

Figure 4- Plant export.

The computational model was imported into MATLAB using *ADAMS_sys*, that generatea the linear model represented by state space matrices *ADAMS_a*, *ADAMS_b*, *ADAMS_c* and *ADAMS_d*, shown in Figure 5.

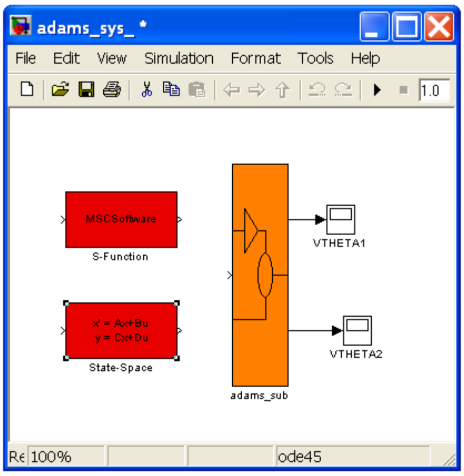

Figure 5 – Subsystem generated by *ADAMS_sys*.

This computational model is, in state space form:

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 39.78 & 0 & 0 \\ 0 & 132.66 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24.62 \\ 36.78 \end{bmatrix} \tau_1 \; ;$$

Which is slightly different from the mathematical model, but is less than 10%. Evidently, this difference comes from the different linearization processes (numerical in the case of ADAMS).

4. CONTROL DESIGN

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Considering the state space representation of the system, modern linear control design techniques uses this kind of representation in order to derive the control laws. The block diagram is shown in Figure 6. The feedback gain vetor $K = [K_1 \quad \cdots \quad K_n]$ must be selected in order to stabilize the system in closed loop, as well as provide robustness and good performance in signal tracking and disturbance rejection. If the state variables are not measurable (or its measurement is not convenient), and state observer must be designed and implemented in software, as part of the controller (Ogata, 2003).

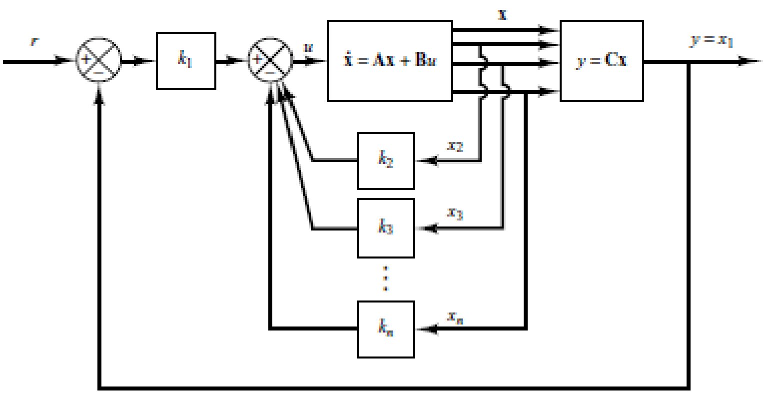

Figure 6 – State Feedback Control Law.

Based in the mathematical model, the state feedback gains were designed based in a Linear Quadratic Regulator (LQR) with weight matrices $Q = CC^T$ and R = 1. The gain vector is then

$$K = [-1 \quad 11.2951 \quad -0.5640 \quad 1.1053]$$

A similar design methodology was used to design the state observer by using the measurements of both angles. The same weight matrices were used (and the algebraic Riccati equation was solved). The observer gain matrix obtained is:

$$K_e = \begin{bmatrix} 2.845 & 6.2546 \\ 6.2546 & 21.76 \\ 23.1730 & 80.0934 \\ 73.8071 & 255.8263 \end{bmatrix}$$

Both the linear mathematical model and the computational model (imported from ADAMS) were compared as shown in Figure 7.

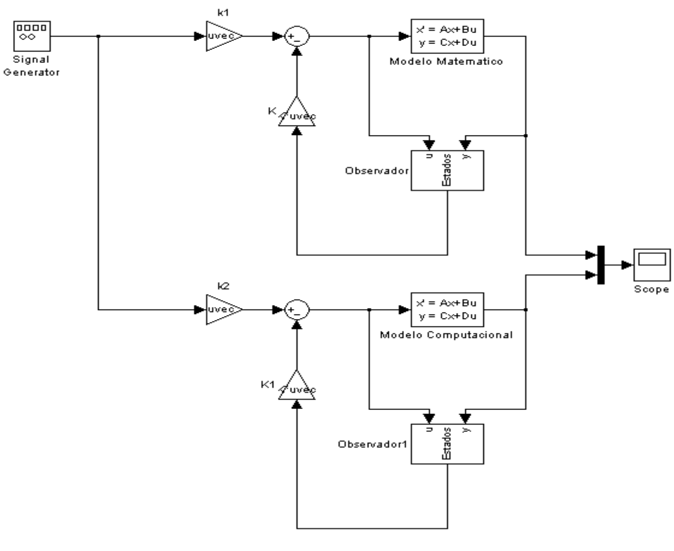

Figure 7 – Closed Loop Control of the Linear Models.

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In Figure 8 it is shown the evolution (in time) of the angles of the arms in closed loop (calculated above). It is clear that the control law (with observer) manages to stabilize the pendulum for both models, despite the fact that the performance is different due to the difference in the models (the computational model is more oscillatory). The unit step response was used, as it is a standard test signal.

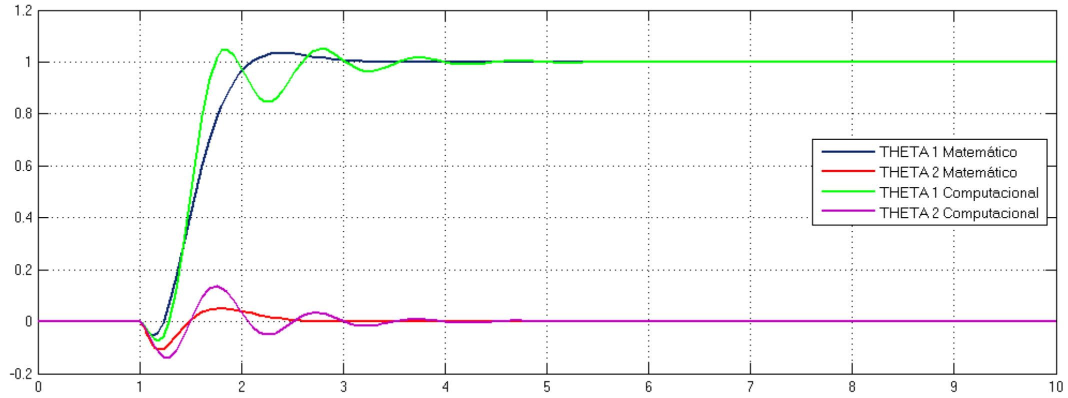

Figure 8 – Step Response for Both Models (Same Control Law).

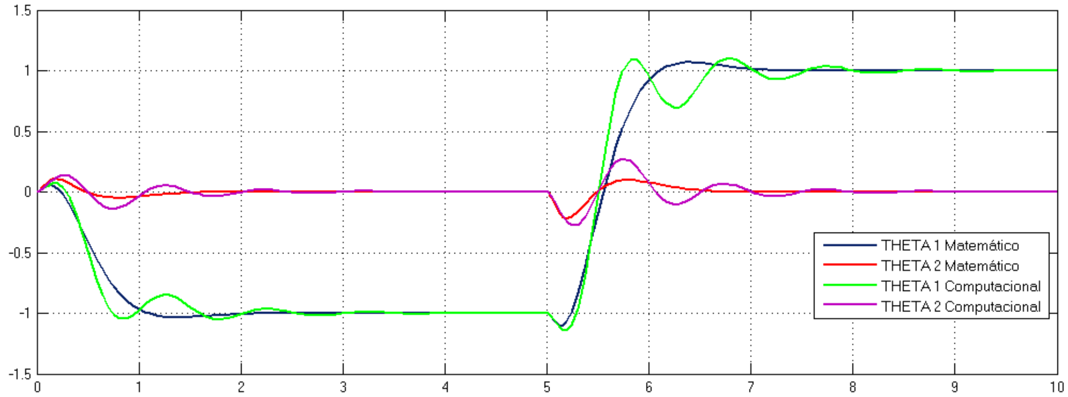

Figure 9 – Square Wave Response for Both Models (Same Control Law).

In order to have a more precise idea of the performance of the control law in the real system, the same control law (with the same observer) was used to control the nonlinear state space model directly in ADAMS, that is, MATLAB and ADAMS interacted in real time in this case. The first one controlled the system in the second one. In Figure 10 it is shown the MATLAB block diagram for this process.

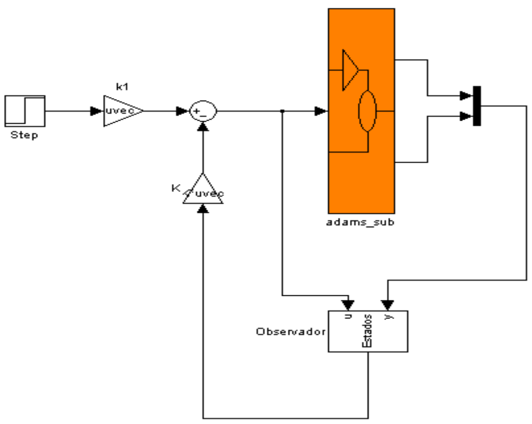

Figure 10 – MATLAB Diagram for the Control of the Nonlinear Plant.

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The result is shown in Figure 11, where the control law clearly stabilizes the Furuta Pendulum. It is important to point out that for large angles in θ_2 (or even in θ_1), there is no guarantee that the control law will stabilize the system. In this case, on the other hand, applying a unit step only in θ_1 (that is a large angle in radians) and giving an initial θ_2 angle as zero (that is, arm 2 in the unstable equilibrium position) the controller succeeded in its objectives. For large initial angles in θ_2, the controller fails.

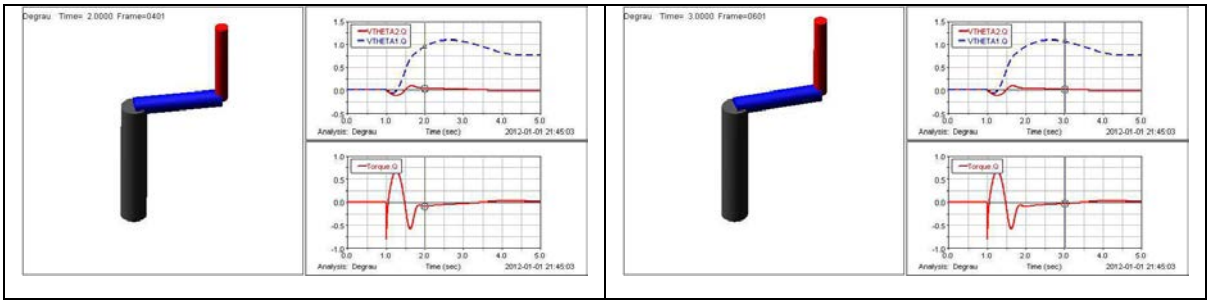

Figure 11 – Closed Loop Step Response for the Nonlinear Plant.

5. CONCLUSIONS AND FUTURE WORK

It was shown that the computational model obtained by ADAMS methods produces a model very similar to the one obtained analytically. This similarity is confirmed by the little difference between the respective linear models. Also, the control law obtained by using one of the linear models (in this case, the analytically obtained) manages to control the other linear model and the ADAMS nonlinear model. The computational model is then an effective tool for design and simulation of modern control laws. In particular, CAD/CAE tools are very effective in saving time avoiding tedious and time-consuming mathematical deductions of analytical models.

Future work includes the development of control laws for more complex systems, including more degrees of freedom, elasticity and fluid phenomena in ADAMS, as well as the design of more advanced control laws, like the nonlinear and switched ones.

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