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Surrogate model to quantify uncertainties in the hydraulic fracture process

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Abstract. *Hydraulic fracture is the process by which fracture initiates and propagates due to the fracturing fluid injected in the well with high pressure. This process is used to improve the permeability of reservoir. The fracturing fluid used for this simulation contains water and a wide variety of chemical additives. It is important to take into account the variability of rock reservoir properties to avoid penetration of the fracturing fluid in the freshwater zone. We assess the reliability of the process by considering the presence of uncertainties in the situ-stress, young modulus, permeability and fluid-loss. Hydraulic fracture is the complex non-linear and free boundary problem simulated in this work using the implicit level set algorithm.*
In this study, we employ stochastic analysis to evaluate the probability of environmental accidents take place and analysis the effect of the uncertainties in the hydraulic fracture output. A new stochastic solver, based on surrogates built upon Gaussian processes modeling, is used to resolve the stochastic the nonlinear evolution equations that describe the evolution of hydraulic fractures and might rely on the high stochastic dimension inputs.

Keywords: *Hydraulic fracture, environmental risks, Gaussian process modeling, stochastic analysis*

1. INTRODUCTION

Hydraulic fracture is a process widely used in the petroleum industry to increase the permeability of rock. In most cases, the engineered fracture occurs in very heterogeneous rocks featuring natural fractures, confining stress and elastic modulus variations, irregular interfaces, high permeability zones. These properties of the rocks have a very large influence on the propagation of the hydraulic fracture (see "Fig. 2"). "Fig. 2" shows the evolution of the fracture created by a dye through an interface that divides the rock into two areas with distinct elastic modulus. We observe that the width of the fracture is different in these two areas. This result shows the effect of the modulus contrast in the hydraulic fracture propagation. These picture presents the effect of interfaces on the fracture propagation,we observe that when the fracture through the interface there is a sudden change of fracture growth. (Anderson, 1981) observed the same mechanism in the laboratory experiments.

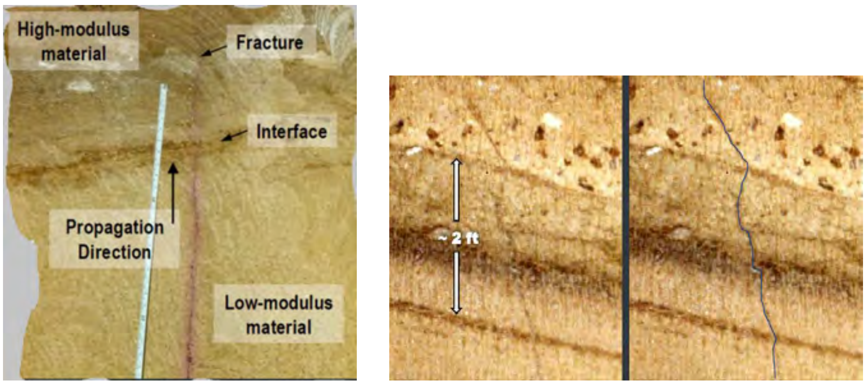

Figure 1. (Warpinski, 2011) Fracture propagating across interface and with modulus contrast

We show the impact of rock properties in the propagation on the hydraulic fracture. Generally, it is expensive or even impossible to have a measurement of reservoir properties without uncertainties. This lack of measurement can lead to economic or environmental risks. So, in this work we access reliability of hydraulic fracture using Uncertainty Quantification (UQ) tools to take into account uncertainties in the process.

Uncertainty Quantification (UQ) can help to cope with this, many time, missing or only partially know of information by solving the hydraulic fracture evolution problem considering the impact of not well know rock properties, heterogeneity, uncertainty in the interface position. In the present approach, UQ stars by considering the missing information about the complexity of the rock formation through the use of random variables (or fields) modeling the key parameters, and, consequently, all other variables. Therefore, we need an effective stochastic solver. Methods such as Monte Carlo,

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polynomial chaos, stochastic collocation showed their difficulties to solve the stochastic equation in a high stochastic dimension. To solve this problem (Bilionis and Zabaras, 2008) proposed the new Bayesian Uncertainty Quantification framework using a novel treed Gaussian process model. This method construct the surrogate by splitting the stochastic space adaptively and consider that the prior information of all output is a gaussian process. This assumption allow the calculation of all the statistics of the outputs and the few call of forward model.

The objective of this work is to quantify uncertainties in the Hydraulic Fracture(HF)process using the surrogate model construct by the use of the Multi-output Gaussian process. In the Multi-output process we consider that all output have the save covariance function and consider the correlation between outputs. This method builds the surrogate locally by splitting the stochastic space adaptively and obtained the global surrogate by the combination of the local surrogate. Before we discuss the simulation results, we will introduce in section 1 the hydraulic fracture process. In section 2 describe the prototype scenario in which the rock (in the fracture propagation area) features three layers. Uncertainty in this introduced by random confining stress and random elasticity modulus. The layer is located at the distance X_1 and X_2 of the well. In the section 3 we present the Multi-output Gaussian process model as method to UQ. In the section 4 we present some numerical results and finally the conclusion.

2. Hydraulic fracture modeling

Hydraulic fractures naturally tend or are designed to propagate in planar regions orthogonal to the direction of the minimal confining stress. Its evolution is driven by fracturing fluid injected at high pressure, breaking the rock formation, and this fluid is partially lost because it might leak off toward the ambient rock. Therefore, the physics of the hydraulic fracture propagation involve the non linear equation at each time in the free boundary condition. Both principal equation and the boundary condition describe this process: the non linear fluid flow equation, the elasticity equation and the boundary condition. The implicit level set algorithm propose by (Peirce and Detourney, 2008) is use to resolve HF process. The model for describing HF addressed here is a one-dimensional version presented by (Peirce and Detourney, 2008) as long as plane strain evolution is assumed. The resulting mathematical model, to be described below, results in a coupled non linear free boundary problem involving integral and partial differential equations.

Before considering, in more detail, the governing equation, we introduce the dimensionless quantities: $x = \ell\chi, t = t_*\tau, \ell(t) = \ell_*\gamma(t), p = p_*\Pi, w = w_*\Omega, \sigma_0 = p_*\Sigma_0\Phi(\chi)$.

$$g_e = \frac{E' w_*}{p_* \ell_*}, g_m = \frac{w_*^2 p_* t_*}{\mu' \ell_*^2}, g_v = \frac{Q_0 t_*}{w_* \ell_*}, g_c = \frac{C' t_*^{1/2}}{w_*}, g_k = \frac{k' \ell_*^{1/2}}{E' w_*}.$$

Where E and v are the rock elasticity modulus and Poisson's; $\mu' = 12\mu$, where μ is the dynamic fluid viscosity; Q_0 is the volumetric injection rate per unit length in the out-of-plane direction; C_l is the cartes's leak-off coefficient; and $K' = 4(\frac{2}{\pi})^{1/2}K_{IC}$, K_{IC} is the modified stress intensity factor.

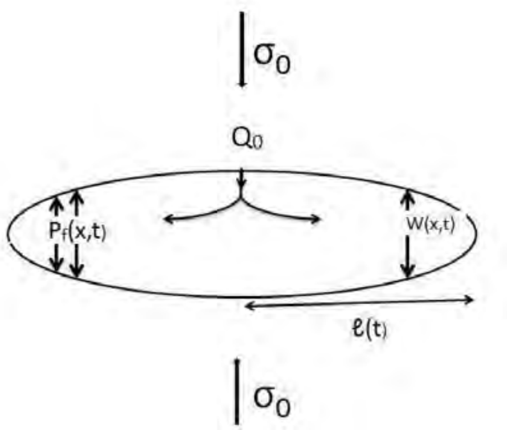

Figure 2. Schematic view of a hydraulic fracture

2.1 Elasticity Equation

$$\Pi = \Pi_f(\chi, \tau, \varpi) - \Sigma_0(\omega)\Phi(\chi, \omega) = \frac{-ge}{4\pi}\int_{-\gamma(\tau,\omega)}^{\gamma(\tau,\omega)} \frac{\Omega(\chi\prime, \tau, \omega)}{(\chi - \chi\prime)}, \quad (1)$$

where Π stands for the net pressure.

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2.2 Fluid Flow Equation

$$\frac{\partial \Omega(\chi,\tau,\omega)}{\partial \tau} + gc\frac{H(\tau-\tau 0(\chi))}{\sqrt{(\tau-\tau_0(\chi))}} = \frac{1}{g_m}\frac{\partial}{\partial \chi}[\Omega(\chi,\tau,\omega)^3 \frac{\partial \Pi_f(\chi,\tau,\omega)}{\partial \chi}] + g_v\Psi(\tau)\delta_0(\chi), \quad (2)$$

$-\gamma(\tau,\omega) < \chi < \gamma(\tau,\omega),$

2.3 Boundary and Propagation Conditions

A zero flux boundary condition is imposed on the fracture boundary $\chi = \gamma(\tau, w); \chi = -\gamma(\tau, w)$

$$\lim \xi \to 0\Omega^3 \frac{\partial \Pi_f}{\partial \xi} = 0. \quad (3)$$

Where $\xi = \gamma - \chi$ is a local coordinate representing the distance from a fracture interior point to the fracture tip. The evolution of the moving boundary is governed by the classical condition $\lim \xi \to 0\frac{\Omega}{\xi^{1/2}} = g_k$ where g_k is the dimensionless rock toughness.

3. Uncertainty Quantification

In order to obtain reliable predictions from numerical simulations, two key ingredients are need: a robust numerical solver and a method to propagate unavoidable uncertainty present in the input data. Here, in order to understand better the impact of such uncertainties on the output of simulations, we employ the implicit level set algorithm (ILSA) developed by (Peirce and Detourney, 2008), which has been exhaustively assessed and proved to be efficient and stable as a deterministic solver. The most celebrated method for uncertainty propagation Monte Carlo(MC) method. This method gives complete statistics of the solution, while it becomes inefficient in high stochastic dimension. Another approach to uncertainty quantification such as polynomial chaos(gPC)(Ghanem and Spanos, 1991),multi-element polynomial chaos (ME-gPC), and stochastic collocation method as been developed (Ma and Zabaras, 2008). This method has been applied to resolve the HF in low dimension by (Zio and Rochinha, 2012). When high stochastic dimension and complex deterministic solver which present strong non linearity and discontinuities such as HF are to be considered, those methods are no longer adequate. To improve those difficulties (Bilionis and Zabaras, 2008) propose an alternative stochastic solver the Multi-output local Gaussian Process Regression (MGPs). The idea of this method is to decompose adaptively the stochastic space in the stochastic element and for each element we construct the surrogate using the multi-output gaussian process(MGPs). This method can provide the semi-analytic calculation of the statistics and the error bar of their prediction. The error bar or uncertainties of the statistics prediction is used to select the new input of the stochastic element using the Active learning Mackay(ALM)(MacKay, 1991). MGPs can construct the local surrogate by an adaptive sampling procedure that automatically selects optimal sample point, identify discontinuity region that can present the abrupt change of the solution. These advantage of MGPs motivate the use of the method to resolve the hydraulic fracture propagate in the complex rock medium present in next section.

4. Multi-output Gaussian process regression

We consider a deterministic simulator returning M outputs $y \in \mathbf{R}^M$ from inputs x lying in input space $X \subset \mathbf{R}^K$ $(K \geq 1)$, $X = x_{k=1}^K[a_k b_k], -\infty \leq a_k < b_k \leq \infty$. Assume that $p(x)$ is a probability density of all $x \in X$ such that $p(x) = \Pi_{k=1}^K P_k(x_k)$, where $P_k(x_k)$ is the probability density pertaining to the k-th dimension. The simulator is essentially a function $f : X \longrightarrow \mathbf{R}^M$, $f(.)$ takes values $f(x)$ for $x \in X$. We may or may not have an explicit expression for $f(.)$. From a bayesian perspective, we regard $f(.)$ as unknown function (S. Conti, 1992). In this paper, we consider that the prior information about $f(.)$ is a Gaussian process

$$f(.) \sim \mathbf{GP(m, C)},$$

i.e that $f(.)$ is specified by its mean m and covariance C. We assumed that The function $f(x)$ is differ from the observation y by additive noise which follows the independent Gaussian distribution with zero mean and variance σ_n^2,

$$y(x^n) = f(x^n) + \epsilon,$$

where $\epsilon = \mathbf{N(0, \sigma_n^2)}$. We assume that we have observed a fixed number $N \geq 1$ and a training set of n observations $D = \{x^n, y^n\}$, where $y^n = f(x^n)$ is the result of computer program with input x^n.

The Gaussian process(GP) on N data point involve the computation of the Cholesky decomposition of an $N \times N$ symmetric positive definite matrix. If the M independent output is modeled the training cost would be $0(M \times N^3)$. In the Multi-output Gaussian process regression (MPGs) (Bilionis and Zabaras, 2008) we consider that all output have the same covariance function. This assumption reduce the computational time to $0(N^3)$.

The mean of observed is:

$$\mu_{obs,r} = \frac{1}{N}\sum_{n=1}^{N} y_r^n, \tag{4}$$

and the observed variance:

$$\sigma^2_{obs,r} = \frac{1}{N}\sum_{n=1}^{N}(y_r - \mu_{obs,r})^2, \tag{5}$$

for $r = 1, .., M$ of the data D. The scaled response functions $g_r : X^i \longrightarrow \mathbf{R}$, defined by:

$$g_r(x) = \frac{f_r(x) - \mu_{obs,r}}{\sigma_{obs,r}}, \tag{6}$$

is necessary for putting all outputs in the same signal strengths. We assume that g_r is the Gaussian Process with zero mean and covariance function $c(x, x\prime; \theta)$:

$$g_r \sim \mathbf{GP}(\mathbf{0}, \mathbf{c}(\mathbf{x}, \mathbf{x}\prime, \theta)), r = 1, ..M,$$

where $\theta \in \Theta \subset \mathbf{R}$ are the $S \geq 1$, unknown hyper-parameters of the covariance function. In this work, we choose the Square Exponential(SE) as the covariance function with the unknown hyper-parameters $\theta = \{sf > 0, \ell_k > 0; k = 1, ..K\}$, the Square Exponential(SE):

$$c_{SE} = s_f^2 \exp(-\frac{1}{2}\sum_{k=1}^{K}\frac{(x_k - x'_k)^2}{\ell_k^2}), \tag{7}$$

where s_f can be interpreted as the *signal strength* and ℓ_k as the *length scale* of each stochastic input. The hyper-parameters are obtain by maximizing the logarithm of the marginal likelihood. In the Multi-output GP case, we consider that the logarithm of the joint marginal likelihood is the sum of the marginal likelihoods of each output, i.e.,

$$\begin{aligned}
\ell(\theta) &= \log p(z_1, .., z_2|X, \theta) \\
&= \sum_{r=1}^{M}\log p(z_r \mid X, \theta) \\
&= -\frac{1}{2}\sum_{r=1}^{M} z_r^T C^{-1} z_r - \frac{M}{2}\log|C| - \frac{NM}{2}\log 2\pi.
\end{aligned}$$

Thus θ is obtain by maximize $\ell(\theta)$: $\theta^* = arg\max \ell(\theta)$ using a Conjugate Gradient method (Fletcher - Reeves algorithm) Dunlavy *et al.* (2010), where $\theta_1 = \log s_f, \theta_{k+1} = \log \ell_k$. The initial value $\theta_0 = \{\theta_{1,0}, .., \theta_{k+1,0}\}$ is used to start the optimization, where $\theta_{1,0} = 0$ for the signal parameter and $\theta_{k+1,0} = \log(\frac{1}{3}L_k)$, for the length scale parameters, where $L_k = b_k - a_k$. Knowing the hyper-parameter θ^*, we can easily calculate the predictive distribution, mean and variance at any test point $x \in X$:

predictive distribution

$$f_r(x)|D, \theta^* \sim \mathrm{N}(\mu_{f_r}(x; \theta^*), \sigma^2_{f,r}(x, \theta^*)), \tag{8}$$

predictive mean:

$$\mu_{f,r}(x; \theta^*) = \sigma_{obs,r} c^T C^{-1} z_r + \mu_{obs,r}, \tag{9}$$

and predictive variance

$$\sigma^2_{f,r}(x; \theta^*) = \sigma^2_{obs,r}(c(x, x; \theta^*) - c^T C^{-1} c), \tag{10}$$

where $c = (c(x, x^1; \theta^*), .., c(x, x^{(N)}; \theta^*))$ and the covariance matrix C evaluated at θ^*

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4.1 Calculation of the local and global statistics

In the MGPs we decompose adaptively the stochastic space X in $X^i, i = 1,..,I$ stochastic element. In all local element we calculate the surrogate and using this, we can calculate the local statistics. The global statistics is obtained by the combination of the statistics over each stochastic element. Using Eq 9 and 10 in X^i, we can calculate all moments $m^q = (m_q^1,..,m_M^q), q \geq 1$, where $m_r^q = \int_{X^i} f_r^q(x)p^i(x)dx$.
$p^i : X \longrightarrow \mathbf{R}$ is the conditional probability density related to X^i:

$p^i(x) = \frac{p(x)}{P(X^i)} 1_{X^i}(X)$, where $P(X^i)$ is the probability of an input point residing in the stochastic element X^i, i.e, $P(X^i) = \int_{x^i} p(x)dx$.

Local prediction:
the predictive distribution:

$$m_r^q | D, \theta^q \sim \mathbf{N}(\mu_{m_r^q}; \sigma^2_{m_r^q}), \tag{11}$$

predictive mean of m_r^q is:

$$\mu_{m_r^q} = \int_{x^i} \mu^q_{f_r}(x;\theta^q)p^i(x)dx, \tag{12}$$

predictive variance:

$$\sigma^2_{m_r^q} = \int_{x^i} \sigma^2_{f_r^q}(x;\theta^q)p^i(x)dx. \tag{13}$$

The integrals involved can be calculated using the Monte Carlo method. When $p^i(x)$ is the uniform distribution we can obtain the semi-analytical expression see *Appendix*.
Global prediction Eq 11 - 13 provide a predictive distribution $m_r^{q,i}$, mean $\mu_{m_r^{q,i}}$ and variance $\sigma^2_{m_r^{q,i}}$ for each element $X^i, i = 1,..,I$. We combine the statistics over each stochastic element to obtain the global statistics:
the predictive distribution:

$$m_r^q | D, \theta^q \sim \mathbf{N}(\mu_{m_r^q}; \sigma^2_{m_r^q}), \tag{14}$$

the predictive mean is:

$$\mu_{m_r^q} = \sum_{i=1}^{I} \mu_{m_r^{q,i}} P(X^i), \tag{15}$$

and the predictive variance:

$$\sigma^2_{m_r^q} = \sum_{i=1}^{I} \sigma^2_{m_r^{q,i}} P(X^i). \tag{16}$$

The predictive moments of the variance are obtained via the calculation of the hyper-parameters θ^1, θ^2. The predictive distribution of the variance ν_r is:

$$\nu_r = \mathbf{N}(\mu_{\nu_r}, \sigma^2_{\nu_r}), \tag{17}$$

predictive mean of ν_r is given by:

$$\mu_{\nu_r} = E[m_r^2 | D, \theta^1, \theta^2] - E[(m_r^1)^2 | D, \theta^1, \theta^2]$$

$$\mu_{\nu_r} = \mu_{m_r^2} - \mu^2_{m_r^1} - \sigma^2_{m_r^1}, \tag{18}$$

the predictive variance of ν_r is:

$$\sigma^2_{\nu_r} = \sigma^2_{m_r^2} + 4\mu^2_{m_r^1}\sigma^2_{\mu_r^1} + 2\sigma^4_{\mu_r^1} \tag{19}$$

4.2 Adaptivity

In this section, we present the procedure to decompose adaptively the stochastic space X in the rectangular element X^i i.e $X^i = [a^i, b^i] \times ... \times [a_k^i, b_k^i]$. The procedure begin by the calculation of the uncertainty

$$\sigma^2_{f^q,p} = \sum_{i=1}^{I} \sigma^2_{f^q,p^i} P(X^i) \tag{20}$$

where

$$\sigma^2_{f^q,p^i} = \frac{1}{M}\sum_{r=1}^{M}\sigma^2_{m_r^{q,i}}$$

of the single element X. If this uncertainty is greater than a certain threshold $\delta > 0$, the element X is split in two element $X^{i,1}$ and $X^{i,2}$ with $X^i = X^{i,1} \cup X^{i,2}$ and $X^i = X^{i,1} \cap X^{i,2} = \oslash$. The set of observation D is splitting according to the input $X^{i,1}, X^{i,2}$ i.e,

$$D^{i,l} = \{(x,y) \in D^i : x \in X^{i,l}\}, l = 1, 2,$$

where $D^i = \{(x^{i,(n)}, y^{i,(n)})\}_{n=1}^{N^i}$.

A new input is added in all elements $X^{i,1}$ or $X^{i,2}$ according to the joint uncertainty of all outputs:

$$x^{new,n+1} = \arg\max \sigma^2_f(x;\theta^*, D^{i,1..n})p(x), \tag{21}$$

where $\sigma^2_f(x;\theta^*, D^{i,1..n}) = \frac{1}{M}\Sigma_{r=1}^{M}\sigma^2_f(x;\theta^*; D^{i,1..n})$.

5. Numerical Results

We study here a challenging problem in which the fracture propagates within a three-layered medium, as described schematically in Fig. 3. In this complex medium, the fracture growth tends to experiment a complex non-symmetric evolution growth due to abrupt changes in the confining stresses which assumes different values at each layer (σ_1 for the layer 1,σ_2 layer 2 and σ_3 for layer 3). The layers are separated by interfaces χ_1 and χ_2, that the fracture will pass through at an unknowns injection times τ. In such a scenario, numerical simulation of the fracture propagation might help the operation in the field, trying to reducing the risks or improving the stimulation. However, non accurate descriptions of the input parameters obtained by indirect measurements (Ask (2003), 2003) might hamper the ability of the numerical model to deliver trustworthy predictions. So in this work, we present the numerical modelling of HF which also takes into account uncertainties inherited from imprecise measurement of rock properties by combining the robust numerical method developed in (Peirce and Detourney, 2008) , that takes care of the deterministic aspects of the problem, with the MGPs described before that will play a role of a stochastic solver. Those uncertainties are represented in the model by considering some of the input parameters as random variables. We elected the Elasticity Modulus and the confining stresses on each layer as those uncertain parameters. Indeed, those choices entails a reasonable first scenario to be analyzed, as those parameters have an important impact in the fracturing, and their values in the technical literature show a substantial dispersion that could be represented with the help of a probabilistic model. The random Elasticity Modulus is considered homogenous along the medium and described as $E = \overline{E} + \widetilde{E}\ \varepsilon$, where $\overline{E}$ is the expected value (mean) and $\widetilde{E}$ the standard deviation. Moreover, ε is a uniform random variable with zero mean and unity variance. Each layer features its own random confining stress $\sigma_i = \overline{\sigma_i} + \widetilde{\sigma_i}\ \xi_i\ (i = 1, .., 3)$ where $\overline{\sigma_i}$ and $\widetilde{\sigma_i}$ represent, respectively, the mean and standard deviation, and ξ_i are independent uniform random variables. We are using simple stochastic models for the input parameters and will not discuss the sensitivity of the final results with regard to them, but in (Zio and Rochinha, 2012) we present a study assessing the impact of using different models,for the same raw data, in the predictions drawn from simulations.

We investigate the fracture behavior by considering as quantities of interest directly computed from the simulations: fracture length in both directions $\ell_r(t)$, $\ell_d(t)$, the aperture at the interfaces position the fracture aperture at the end of injection $\Omega(\chi, \tau_{final})$. After the convergence study of the deterministic solver, we fix the value of the time and space size dτ=3 and $d\chi$=0.44 with the maximum time step=50 and space=61. Using these value we construct the surrogate with 261 outputs values $\{\ell_r(t), \ell_d(t), \Omega(\chi_1,\tau), \Omega(\chi_2,\tau), \Omega(\chi,\tau_{final})\}$. The interfaces are locate at the position $\chi_1 = -2.66$ and $\chi_2 = 4$, we consider that $\overline{\sigma_2} = \overline{\sigma_3} = 0.6$ and $\overline{\sigma_1} = 0.2$ with their standard deviation $\widetilde{\sigma_1} = 0.31$ and $\widetilde{\sigma_2} = \widetilde{\sigma_3} = 0.44$. The modulus of elasticity have for mean $\overline{E} = 1$ and $\widetilde{E} = 0.54$.

Fig. 4- 9 present the deterministic solution of the problem calculate using the mean value of the input $\overline{E}, \overline{\sigma_1}, \overline{\sigma_2}, \overline{\sigma_3}$. In Fig 4 we observe that, the fracture reaches a greater length in the left side of rock than the right. The two length values begin to be different at the injection time τ=60. Fig 4 show the evolution of the opening of the fracture into the rock, we observed that when the fracture through the interface χ_1 at time τ=31 we have an abrupt change of the aperture value. The first interface χ_2 is achieved only at time τ=73, but it does not change influence the aperture value.

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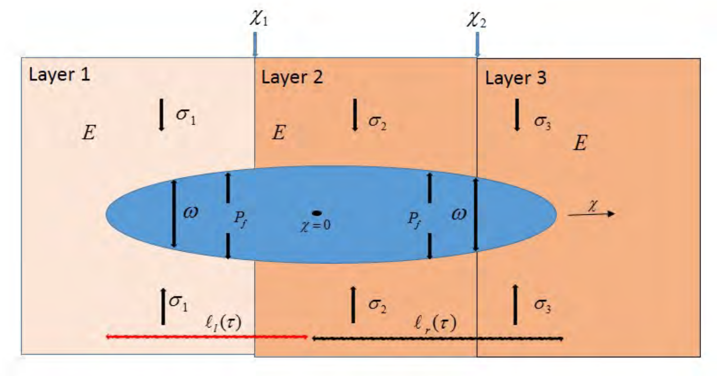

Figure 3. Schematic view of the layered medium

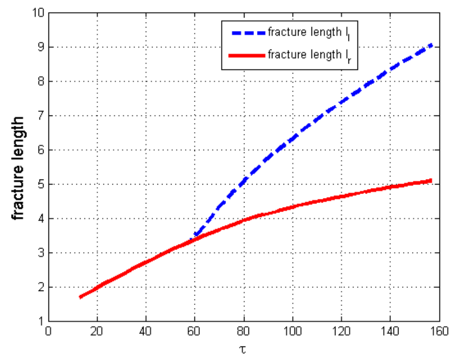

Figure 4. Left and write non-symmetric fracture growth along time

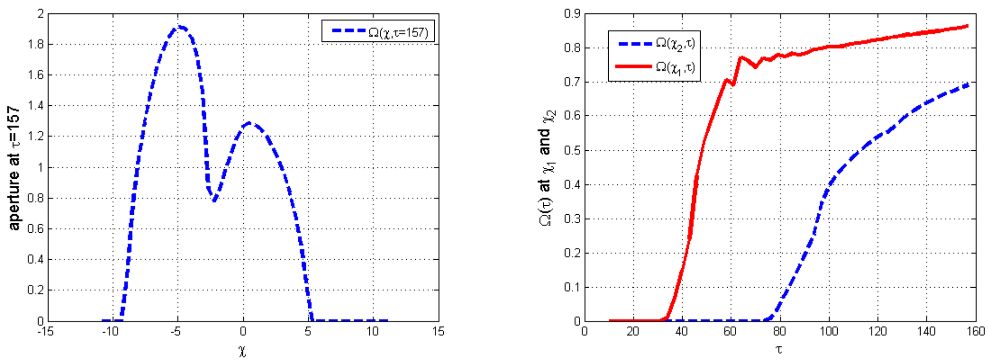

Figure 5. Fracture opening along the domain $\Omega(\chi, \tau = 157)$. Fracture openings at the interfaces $\Omega(\tau, \chi_1)$ and $\Omega(\tau, \chi_2)$

In this section, we present the result of the stochastic HF across interfaces. The global surrogate is obtained by the combination of 15 local surrogate each one built setting $\delta = 10^{-4}$. The deterministic solver has been called 96 time to construct the global surrogate. The afterword statistics of the quantities of interested are computed through sampling the global surrogate and a maximum number of samples $N_{max} = 10000$ was stablished. and use the statistic(mean and variance) calculate at N_{max} to evaluate the convergence of the relative error of mean(mre) and variance(vre) using $N < N_{max}$. We stop the calculation of the relative error when the successive error bar of mean(mre) and variance(vre) is close to the (IS=10^{-3}). In Fig 6 we observe that at N=8000 samples the convergence of mre and vre are close to the IS value. So the surrogate obtain with the error= 1e-4 is evaluate at this sample to calculate the statistic solution of the problem. Fig 7- 8 present the prediction mean and uncertainties of the engineering quantities. The uncertainties of the fracture length and aperture increase with the injection time. Also, the uncertainties in the interface position χ_1 is great compare to other position. These statistics information can give the confidence interval of the engineering quantities. Fig 9 present the PDF of the fracture aperture at both interfaces position.

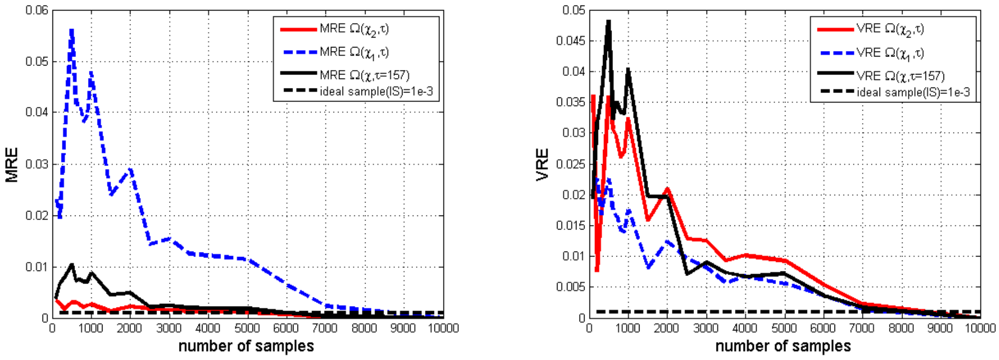

Figure 6. mean and variance relative error(mre and vre)

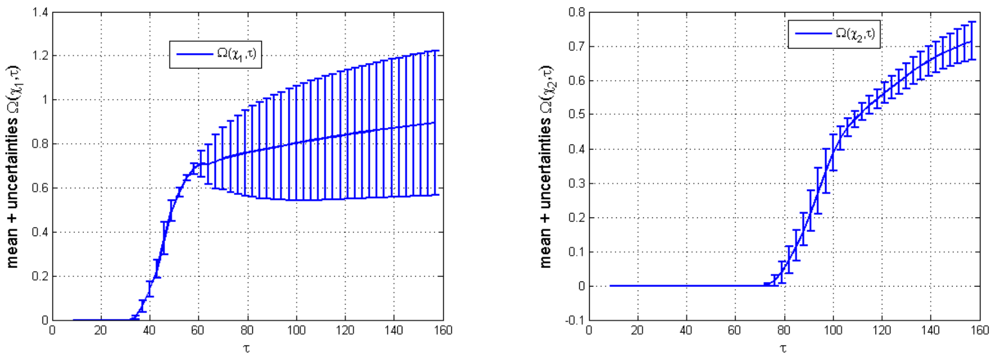

Figure 7. predictive mean and uncertainties

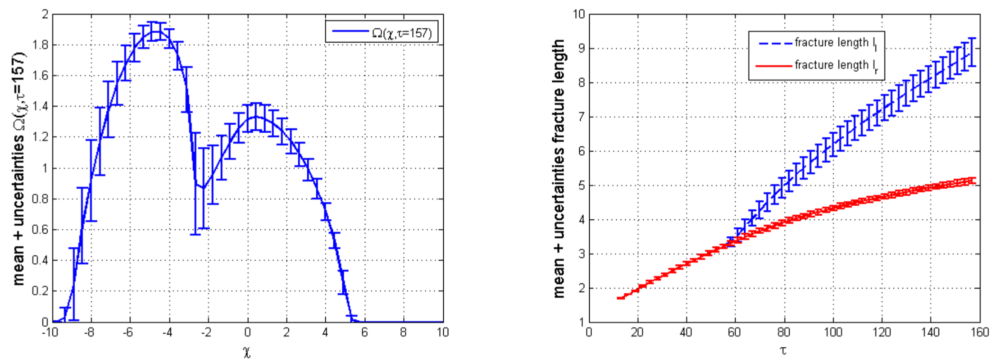

Figure 8. predictive mean and uncertainties of fracture length and PDF of the aperture at $\Omega(\chi_2\ \tau = 157)$

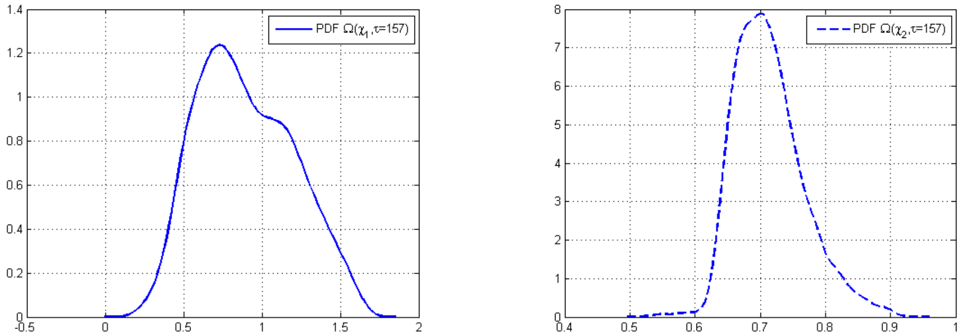

Figure 9. PDF $\Omega(\chi_1, \tau = 157), \Omega(\chi_2, \tau = 157)$

6. Final Remarks

In the present work we investigate the performance of the computational surrogate introduced in Bilionis and Zabaras (2012) in the context of UQ involving hydraulic fracturing numerical simulation. A particular challenging situation modeling the growth of the fracture within a three-layered elastic medium was studied and the preliminarily results were presented here. Difficulties were expected due to the stress barriers promoted by the layered medium. In a future work, we intend to extend this analysis to more complex situations, with special emphasis on stochastic input spaces with larger dimension.

7. ACKNOWLEDGEMENTS

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