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ANALYSIS OF THE P1-APPROXIMATION FOR THE RADIATIVE HEAT TRANSFER IN SKIN TISSUES LOADED WITH NANOPARTICLES

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Abstract. *The effectiveness of the cancer treatment by hyperthermia depends on the heat transfer process, in which the goal is to overheat and kill the cancerous cells, but preserving the surrounding healthy tissues. Computational simulation can be employed for better comprehending the heat transfer process in biological tissues, including tumorous and healthy cells. Here the one-dimensional coupled radiation, conduction and blood perfusion in hyperthermia therapy is solved using two different techniques for the radiation modeling: P1-approximation and Discrete Ordinate method. Using the P1-approximation for the radiation model in complex coupled simulations of the heat transfer in hyperthermia therapy is highly desirable, since the method is simple and provides considerable CPU time reduction. The results obtained in this paper demonstrate that the P1-approximation can be accurately employed under conditions which can occur in hyperthermia applications for skin cancer.*

Keywords: *cancer hyperthermia therapy; coupled radiation-conduction-blood perfusion simulations; Discrete Ordinate method ; P1-approximation.*

1. INTRODUCTION

Hyperthermia is a cancer therapy in which the temperature of a tissue region is increased with the goal of killing the cancerous cells. Laser light is largely employed as the heat source in this type of therapy, which use has been preconized in the sixties [Hairong and Diakides (2003)] and more recently has becoming of increasing interest. Nanoparticles can be employed in order to enhance the laser energy absorption in the cancerous region. Tijahjono and Bayazitoglu (2008) simulated the radiation absorption of nanoparticles embedded in a host transparent media. Vera and Bayazitoglu (2009) analyzed the effect of the concentration of gold nanoshells in tissues using the P1-approximation. Maksimova et al. (2007) demonstrated by simulations and experiments that the temperature of tissue regions embedded with gold nano-shells can be substantially larger than the temperature of the surrounding regions without nanoparticles.

The effectiveness of the treatment depends on the control of the heat transfer process, since the goal is to overheat and kill the cancerous cells, but preserving the surrounding healthy tissues. Determining the temperature field in a hyperthermia laser therapy requires the coupled solution of the transient heat transfer due to radiation, conduction and blood perfusion. Computer simulation is a powerful tool in solving heat transfer problems. Therefore, its application to simulate and better comprehend the heat transfer process in hyperthermia laser therapy has been of increasing interest when nanoparticles are employed or for cases in which no nanoparticles are used, as in the works by Kim and Guo (2004), Kim and Guo (2007), Jaunich et al. (2008), among many others.

Accurate solution of the heat transfer problem requires accurate data about physical properties of the different healthy and cancerous tissues, such as: absorption and scattering coefficients, density, specific heat, thermal conductivity, blood perfusion parameters and others. All these parameters need to be determined. This is often accomplished using inverse procedures, such as those presented by Kapio and Somersalo (2004).

Although useful information can be inferred from modeling one-dimensional cases, the geometry of real tumors is three-dimensional and usually complex. In addition, anisotropic scattering can occur. Considering three-dimensional tumor geometries and anisotropic scattering make the solution of the coupled physics involved in the heat transfer processes occurring in hyperthermia therapy considerably more expensive. The computer time consumption can become a major problem, especially when statistical Monte Carlo techniques are employed for solving inverse problems in order to infer the thermophysical properties and quantify the uncertainty associated to them, or for the cases of numerical simulation under uncertainty. Thus, simple and computationally cheap techniques are highly desirable for the

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radiation heat transfer solution. The P1-approximation is one of the less time consuming methods for solving the radiative transfer equation (RTE). However, the method can lead to considerably inaccuracy for cases in which the absorption coefficient is much larger than the scattering coefficient, as presented by Modest (1993) and Howell et al. (2011).

This paper aims at investigating the application of the P1-approximation for solving the RTE in hyperthermia therapy simulations. Results were obtained with the P1-approximation and compared with other results computed using the Discrete Ordinate method.

2. MATHEMATICAL FORMULATION

Aiming at obtaining a model for simulating the three-dimensional heat transfer problem occurring in hyperthermia therapy, the P1-approximation formulation was implemented in the COMSOL commercial code. In addition to three-dimensional geometry, this code is able to consider deal with important coupled physical phenomena occurring in the heat transfer process, which are frequently neglected. For instance, the anisotropy of the radiation scattering process can be important, as demonstrated in the results presented in figure 3 of the following section.

Although able to consider a series of phenomena, the P1-approximation implemented in the COMSOL code can fail for optically thick media and loses accuracy, when an optically thin medium acts as a barrier between hot and cold surfaces, in the presence of collimated irradiation [Modest (2003)]. In addition, the P1-approximation is applicable when the absorption coefficient is lower than the scattering coefficient [Welch and van Gemert (2011)]. Thus, for investigating how the the P1-approximation behaves when applied to simulate the heat transfer occurring in hyperthermia therapies. The P1-approximation results were obtained and compared with results obtained using the Discrete Ordinate method, expected to be more accurate.

Accordingly Modest (2003), the Discrete Ordinate method was first proposed by Chandrasekhar (1960), for applications to stellar atmosphere radiation and primarily extended to general radiative heat transfer problems by Fiveland (1982, 1984, 1987, 1988), Hyde and Truelove (1977) and Truelove (1987, 1988). Unlike the P1-approximation, which is limited to the first-order approximation, the Discrete Ordinate method may be carried out to any order of accuracy. In addition the method application is not conditioned to certain ranges of the absorption and scattering coefficients.

For obtaining the Discrete Ordinate results, a one-dimensional Finite Volume code was developed. This code is able to solve one-dimensional transient coupled radiation, conduction and perfusion heat transfer which occur in hyperthermia therapy. This code was used to obtain results for two test cases, which are described in the following section.

The transient heat transfer problem through an inhomogeneous media composed of different tissues that considered here is formulated in terms of energy equation with position-dependent physical properties, given by:

$$\rho(\mathbf{r},t)c(\mathbf{r},t)\frac{\partial T(\mathbf{r},t)}{\partial t} = \nabla\cdot\mathbf{q}_c(\mathbf{r},t) + \nabla\cdot\mathbf{q}_r(\mathbf{r},t) + Q(\mathbf{r},t) \tag{1}$$

where $\mathbf{r}$ is the position vector, T is the tissue temperature, ρ is the density, c is the specific heat, Q is the heat source that takes into account the metabolic heat generation and the heat removal due to arterial blood perfusion, $\mathbf{q}_c$ is the conduction heat flux vector and $\mathbf{q}_r$ is the radiation heat flux vector. The heat conduction flux can be obtained by Fourier's law

$$\mathbf{q}_c(\mathbf{r},t) = -k_c(\mathbf{r},t)\nabla T(\mathbf{r},t) \tag{2}$$

where k_c is the thermal conductivity.

The volumetric heat source, $Q(\mathbf{r},t)$, takes into account the arterial blood perfusion by using the approximation proposed by Penne (1948), as well as the metabolic heat generation, $Q_m(\mathbf{r},t)$. It is written as

$$Q(\mathbf{r},t) = \rho_b(\mathbf{r},t)c_b(\mathbf{r},t)v_b(\mathbf{r},t)\left[T(\mathbf{r},t) - T_b(\mathbf{r},t)\right] + Q_m(\mathbf{r},t) \tag{3}$$

where the subscript b denotes the blood and $v_b(\mathbf{r},t)$ is the blood perfusion rate .

The radiative heat flux $\mathbf{q}_r(\mathbf{r},t)$ is computed from the solution of the Radiative Transfer Equation (RTE) which is given by:

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$$\frac{dI}{ds}(\mathbf{r},\mathbf{s},t) = kI - \sigma_s I + \frac{\sigma_s}{4\pi}\int_{\omega_i} I(\mathbf{s}_i)\,\Phi(\mathbf{s}_i,\mathbf{s})\,d\omega_i + S_c \tag{4}$$

where I is the radiation intensity, s is the path of the propagation of a radiation ray with intensity I, ω_i is the solid angle related to the incident rays, $\mathbf{s}$ is the direction of propagation, $\mathbf{s}_i$ is the direction of incidence of the radiation rays scattered in the $\mathbf{s}$ direction, k is the absorption coefficient, σ_s is the scattering coefficient and ϕ is the scattering phase function. The quantity S_c is a source term related to collimated laser radiation and will be better described next, along with the boundary conditions for the radiation problem. In equation (4), the term which accounts for radiation emission is not included, since the temperature of the tissue is low and therefore emission from the tissue is insignificant compared to the laser radiation. The same assumption was adopted by Dombrovsky (2011) and others. As the radiation from the laser is monochromatic, the equation is not spectrally dependent.

Specific boundary conditions are considered for the solution of the radiation heat transfer in hyperthermia therapy, which is considered here in one Cartesian dimension. The left surface of the medium is irradiated by collimated laser radiation. For this surface, it is assumed that the radiation imposed by the laser crosses the boundary without any absorption or reflection, that is, the absorptivity and reflectivity of the external surface of the tissue is zero to the laser radiation. On the other hand, the internal reflectivity of this surface is equal to one, that is, the surface totally reflects internal radiation. Differently, the internal reflectivity of the right boundary surface lies between zero and one, so that part of the radiation incident on this surface escapes the tissue.

The collimated laser radiation which penetrates the tissue can be either absorbed or scattered in all directions. The quantity S_c, in equation (4), accounts for the fraction of the collimated laser radiation which is scattered away of the direction of propagation of the collimated laser radiation can be computed, according Modest (2003), as

$$S_c(\mathbf{r},\mathbf{s}) = \frac{1}{4\pi}\,\mathbf{q_o}\,e^{-\tau}\Phi(\mathbf{s}_c,\mathbf{s}) \tag{5}$$

where $\mathbf{q_o}$ is the laser emissive power, the subscribed c in the vector $\mathbf{s}_c$ refers to the laser collimated radiation direction end the optical thickness is given by

$$\tau = \int_0^s (k+\sigma_s)\,ds' \tag{6}$$

where s = $|\mathbf{r}-\mathbf{r_w}|$ and $\mathbf{r_w}$ is the coordinate at the surface irradiated by the laser.

After obtaining the radiation intensity field by solving equation (4), the radiative heat flux can be obtained by integrating the radiation intensity over the total spherical solid angle interval:

$$\mathbf{q_r}(\mathbf{r},\mathbf{s}) = \int_{\omega=4\pi} I(\mathbf{s})\,d\omega \tag{7}$$

The RTE (in the form of equation (4)), is solved using the Discrete Ordinate method and with the P1-approximation. The Discrete Ordinate method was implemented following the procedures described by Howell et al. (2011) and Modest (2003). For the code verification, results were obtained and compared with a series of exact solutions presented by Howell et al. (2011) for isothermal gray isotropic scattering media confined between two infinite parallel black walls. The solution was verified by obtaining results using different numbers discrete ordinates, namely, 2 to 8. No significant difference was verified for the one-dimensional considered cases. Further details of the Discrete Ordinate Method are omitted here for the sake of brevity, but can be readily found in et al. (2011) and Modest (2003).

The P1-approximation is a particular case of the more general diffusion approximation of the radiative transfer equation. The diffusion approximation holds in media within which scattering events dominate absorption. Although the diffusion approximation can be derived from energy conservation law, its rigorous derivation is obtained as an approximation of the RTE [Welch and van Gemert (2011)]. A basic assumption made in the derivation of the diffusion approximation is that, the radiance is slightly anisotropic and thus may be represented by few terms of its Legendre series expansion [Welch and van Gemert (2011)]. The P1-approximation is obtained by cutting off the Legendre series expansion of the radiance after the second term. This expansion in terms of Legendre series introduces a new quantity, namely the fluence rate or photon density. So the P1-approximation yields to the following equations, where, again the radiation intensity has been separated in a diffuse and in a collimated component.

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The collimated component of the fluence rate follows Beer's law and is given by [Welch and van Gemert (2011)]:

$$\Phi_c(\mathbf{r}) = \mathbf{q_o}\, e^{-\tau} \tag{8}$$

The diffuse part of the fluence rate is given as [Welch and van Gemert (2011)]:

$$\nabla\left[-D(r)\nabla\Phi_d(\mathbf{r}) + \frac{\sigma_s(r)g(r)}{\mu_{tr}(r)}\Phi_c(\mathbf{r})\right] + k(r)\Phi_d(\mathbf{r}) = \sigma_s(r)\Phi_c(\mathbf{r}) \tag{9}$$

Where,

$$\mu_{tr} = k + \sigma_s(1-g) \tag{10a}$$

and

$$D = \frac{1}{3\mu_{tr}} \tag{10b}$$

The quantity D is the diffusion coefficient, μ_{tr} is the transport coefficient, and g is the anisotropy coefficient, which describes the anisotropy of the scattering. Note that for isotropic scattering g is zero and for a peaked forward scattering medium its value tends to one.

Scattering in biological tissues is highly forward directed, thus, diffusion theory is not always a good approximation near boundaries or sources [Star (1989), Star (1995)]. An improvement is possible by introducing a Dirac delta function in the scattering phase function, and derived a similar equation with transformed coefficients. This is known as the Delta-Eddington approximation. Basically, the Delta-Eddington approximation tries to approximate the Henyey-Greenstein, which one is considered as a good model phase function for scattering in biological tissue [Welch and van Gemert (2011)]. The P1-approximation together with the Delta-Eddington approximation yields the following equations for the diffuse fluence rate and the collimated fluence rate [Welch and van Gemert (2011)]:

$$\nabla\left[-D^*(r)\nabla\Phi_d(\mathbf{r}) + \frac{\sigma_s^*(r)g^*(r)}{\mu_{tr}(r)}\Phi_c(\mathbf{r})\right] + k(r)\Phi_d(\mathbf{r}) = \sigma_s^*(r)\Phi_c(\mathbf{r}) \tag{11}$$

where

$$\Phi_c(\mathbf{r}) = \mathbf{q_o}\, e^{-\tau^*} \tag{12}$$

Where for the second order Delta-Eddington approximation, the coefficients σ_s^*, μ_{tr}^*, g^*, and τ^* are given as [Welch and van Gemert (2011)]:

$$\sigma_s^* = \sigma_s\left(1-g^2\right) \tag{13a}$$

$$\mu_{tr}^* = k + \sigma_s^*\left(1-g^*\right) \tag{13b}$$

$$g^* = \frac{g}{1+g} \tag{13c}$$

$$\tau^* = \int_0^s \left(k + \sigma_s^*\right) ds' \tag{13d}$$

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The total fluence rate $\Phi_t(\mathbf{r})$ is obtained by summation of the diffuse fluence rate $\Phi_d(\mathbf{r})$ and the collimated fluence rate $\Phi_c(\mathbf{r})$. Laser heat deposition is due to solely photon absorption, so the volumetric heat source is obtained as the product of the absorption coefficient and the total fluence rate and is given as [Welch and van Gemert (2011)]:

$$\mathbf{q_r}(\mathbf{r}) = \mathrm{k}(\mathbf{r})\Phi_\mathrm{t}(\mathbf{r}) \tag{14}$$

The most common boundaries conditions used for the diffusion equation are the partial boundary current, the extrapolated boundary condition and the zeroth order boundary condition [Schweiger et al., (1995)].When there is a refractive index mismatch at the boundary, as it is the case for an interface air-tissue, the partial boundary current and the extrapolated boundary condition give better results than the zeroth order boundary condition [Schweiger et al., (1995)]. The partial boundary current is used in the present work. It is derived from energy conservation law, and corresponds to a Robin type boundary condition and is given by [Schweiger et al., (1995), Welch and van Gemert (2011)]

$$\mathbf{n}(D(\mathbf{r})\nabla\, \Phi_d(\mathbf{r})) + \frac{1}{2\,A}\,\Phi_d(\mathbf{r}) = 0 \tag{15}$$

for the isotropic case, that is g =0. In the case of anisotropic scattering within the Delta-Eddington approximation, the partial boundary current condition is given by [Schweiger et al., (1995), Welch and van Gemert (2011)]:

$$\mathbf{n}\left(D(\mathbf{r})\nabla\, \Phi_d(\mathbf{r}) + \frac{\sigma_\mathrm{s}^*(\mathrm{r})\mathrm{g}^*(\mathrm{r})}{\mu_\mathrm{tr}(\mathrm{r})} \right) + \frac{1}{2\,A}\,\Phi_d(\mathbf{r}) = 0 \tag{16}$$

Where A is a parameter related to the Fresnel coefficient r_{id} and is defined by [Schweiger et al., (1995)]:

$$A = \frac{1 - r_{id}}{1 + r_{id}} \tag{17}$$

The P1-approximation together with the bioheat equation were implemented with a commercial FEM package (COMSOL Multyphysics 4.3.a, Comsol Inc., Burlington, MA), by using its partial differential interface, which allows user defined equations, and a built-in interface for the bioheat equation.

3. RESULTS AND DISCUSSION

Three test cases are considered in this work, all based on the work by Dombrosvky et al. (2011). They consist of the transient one-dimensional heat transfer in a multi-layer slab of a medium that simulates human tissues. The slab is composed of five layers, as illustrated by figure 1. Each layer presents different thermophysical properties to simulate different types of tissues, which are presented in table 1. The left-hand side surface of the slab is irradiated by collimated laser radiation with wavelength of 0.6328 μm. The emissive power of the laser is 20 kW which is imposed on the tissue in the form of the periodic step function presented in figure 2. For the conduction problem, the temperature of the left boundary is prescribed, while a convective boundary condition is imposed for the surface on the right to simulate heat transfer to other inward tissues. The convective heat transfer coefficient is taken as h = 50 W/(m^2 K). The initial temperature, the temperature at surface on the left and the temperature of the surrounding medium exchanging heat with the surface on the right are all equal to 37°C. The radiation from the laser incident on the tissue crosses the surface, totally penetrating the tissue. On the other hand, the internal reflectivity of the left surface is ρ = 1, therefore all radiation which reaches this surface after being scattered inside the media is reflected back and does not escape the tissue. The internal reflectivity of the right surface is ρ = 0.49, thus about half of the radiation incident on this surface escapes the tissue.

Three test-cases are examined in this paper. The two first test-cases were considered with the goal of analyzing and comparing the P1-approximation and the Discrete Ordinate method. The difference between them relies on the tissue properties, which vary with the concentration of nanoparticles. For the first test-case, the tissues are assumed to be free of nanoparticles, while for the second test-case two tissue layers are assumed to contain nanoparticles. The third test case aims at demonstrating the effects of the scattering phase function. It shows the differences between the results for isotropic and anisotropic scattering for a case where two tissue layers contain nanoparticles.

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For the second and the third test-cases examined below the first and second tissue layers were considered embedded with nanoparticles (gold nanoshells). The absorption and scattering coefficients for these two layers were then computed according the procedure presented by Dombrovsky et al. (2011). The nanoparticles are gold nanoshells and the coefficients are computed as a function of the volume fraction of the nanoparticles in the base tissue. For the second test case, the volume fraction that the nanoparticles occupy in the host tissue is of $2x10^{-6}$. For the third test-case, the volume fraction of the nanoparticles in the tissue is considered as 10^{-5}. The corresponding absorption and scattering coefficients for the tissues loaded with gold nanoshells are presented in table 2, for test-cases 2 and 3. The test case 2 corresponds to a situation in which, the diffusion approximation holds. That is, the scattering coefficient is higher than the absorption coefficient.

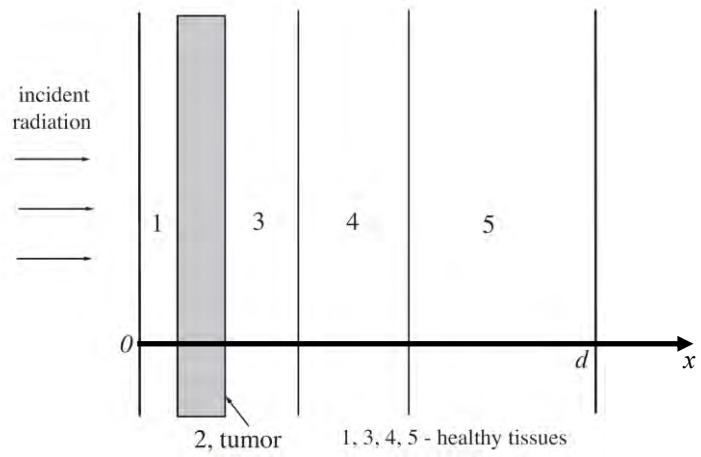

Figure 1 – Representation of the one-dimensional model problem with five layers of tissues [from Dombrovsky et al. (2011)].

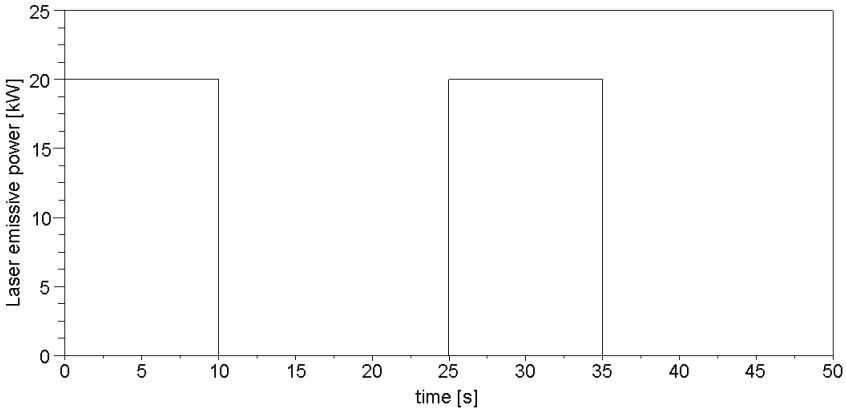

Figure 2 – Variation of the laser emissive power in periodic heating.

Table 1. Physical properties of the tissue layers [Dombrovsky et al. (2011)].

Layer number	1	2	3	4	5
Tissue type	Epidermis	Tumor	Papillary dermis	Reticular dermis	Fat
Layer thickness [mm]	0.1	0.3	0.4	0.8	2.0
ρ [Kg/m^3]	1200	1030	1200	1200	1000
c [J/(Kg K)]	3589	3582	3300	3300	3674
k_c [W/(m K)]	0.235	0.558	0.445	0.445	0.185
Q_m [W/m^3]	0	3680	368.1	368.1	358.3
w_b [1/s]	0	$63 \cdot 10^{-4}$	$2 \cdot 10^{-4}$	$13 \cdot 10^{-4}$	10^{-4}
k [m^{-1}]	180	50	20	20	10
σ_s [m^{-1}]	2360	600	200	200	400

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Table 2. Absorption and Emission coefficients of the first and second tissue layers relative to the test cases 1 and 3

Test Case	Property	Layer 1	Layer 2
2	σ_a [m^{-1}]	3115.5	637.1
	σ_s [m^{-1}]	2777.75	683.55
3	σ_a [m^{-1}]	3115.5	767.1
	σ_s [m^{-1}]	2777.75	2443.55

Test Case 1

In this test case the results obtained with the P1-approximation and with the solution of the Radiative Transfer Equation with the Discrete Ordinates Method are compared. For the present test-case, all tissues are assumed to be free of nanoparticle and their properties are given by table 1. Figure 3 presents a comparison of the temperature profiles obtained with these two methods of solution of the radiation problem. It is interesting not notice in his figure that the maximum temperature resulting from the laser heating at the surface is not in the tumor region. The maximum temperatures are observed at the depth of 1.4 to 1.6 mm within the health tissues of the reticular dermis. Hence, in this case the laser heating is more likely to kill healthy cells than the cells in the tumor region, that is, the hyperthermia treatment in this case would be harmful for the patient.

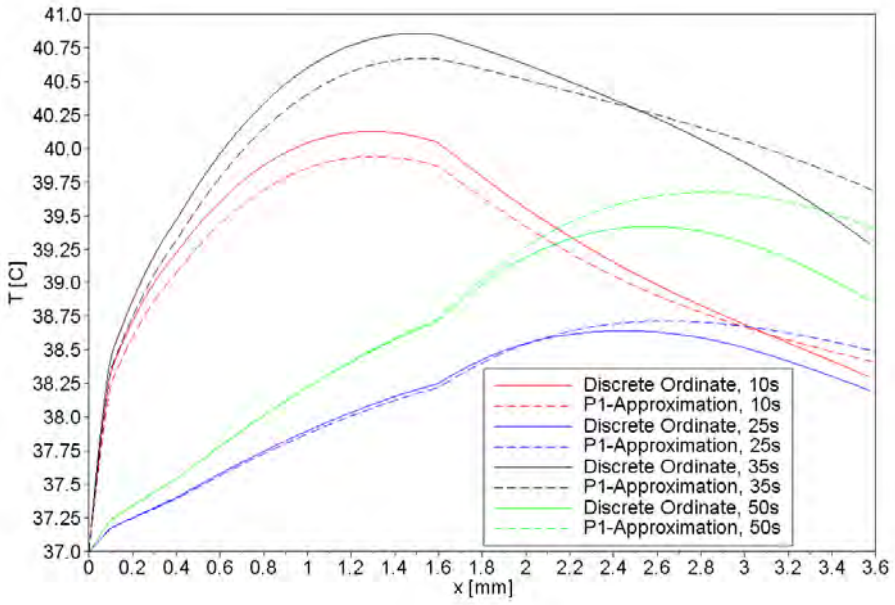

Figure 4 – Comparison of the temperature distribution at computed with the Discrete Ordinate method and the P1-approximation for the tissue without nanoparticles.

The results presented in figure 3 exhibit discrepancies between the temperature profiles computed with the Discrete Ordinate Method and with the P1-approximation. However, as expected, these discrepancies are very small, since the absorption coefficient is smaller than the scattering coefficient and the conditions for the P1 approximation are appropriate. The largest discrepancy between the two solutions is about 0.5°C at the right boundary, for the time of 35s. The difference is lower than 0.25◦C for the maximum peak of temperature, that is, about 0.6%. These results exemplify the important conclusion that the computationally faster P1-approximation is reliable for solving the heat transfer in tissues without nanoparticles, irradiated by laser radiation with wavelength of 0.6328 μm.

Test Case 2

The difference of this test case relative to the previous one is that now nanoparticles are embedded in the first and second layers of the tissue. The volume fraction of the nanoparticles in these tissues is 10^{-5}, which results in the absorption and scattering coefficients presented in table 2. As can be seen in this table, the value of the absorption coefficient is much higher than the value of the scattering coefficient. Thus the P1-approximation is expected to be less accurate than for the case presented above. The temperature profiles within the medium at different times, obtained with

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the P1 approximation and with the Discrete Ordinate Method, are compared in figure 5. This figure shows that the differences between the P1-approximation and Discrete Ordinate results are not considerably high for the present test-case, despite the fact that the nanoparticles increase the absorption coefficients as compared to the scattering coefficients, in the first two layers of the slab. Although the differences are higher close to the boundary, this region is not the target for the hyperthermia therapy. Furthermore, the temperatures predicted with both solution techniques in the neighborhood of the non-heated boundary are smaller than those associated with the killing of cells. In the region of the tumor, where the peak temperature does occur, the discrepancies between the two solution techniques are smaller than 0.7%. This demonstrate that the P1-approximation is able to provide satisfactory results for conditions similar to those encountered in the human tissue during hyperthermia therapy, even when the relation between scattering and absorption coefficient is expected to not provide accurate results.

Nevertheless, a remark should be made about the thickness of the tissue layers. The two layers which contain the nanoparticles are very thin for the cases examined. Thus, although their absorption and scattering coefficients are high, the optical thicknesses associated with them are relatively low. Although the validity of the P1-approximation for accurately simulating the heat transfer in human tissue embedded with nanoparticles has been tested above for the condition of low optical thicknesses, thin layers like those examined here can occur in several applications of hyperthermia for treating skin cancer.

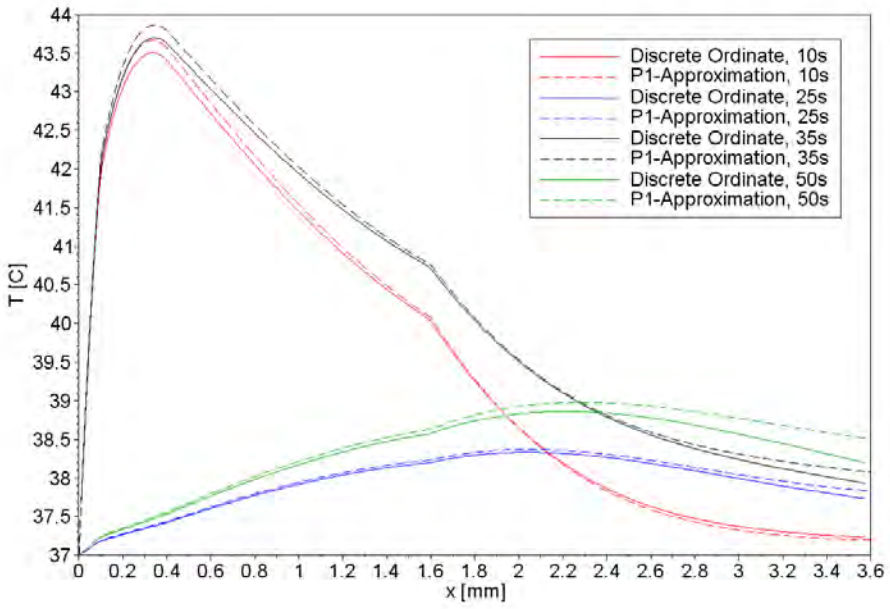

Figure 3 – Comparison of the temperature distribution computed with the Discrete Ordinate method and the P1-approximation for the tissue with nanoparticles.

Test Case 3

Figure 5 presents the temperature profiles within the multilayer slab composed of the tissues for different times, where the first and second layers had their properties modified by assuming that they contain gold nanoshells with a volumetric concentration of $2x10^{-6}$. This figure presents the temperature profiles obtained with isotropic and anisotropic scattering. For the anisotropic scattering, the anisotropy factor is taken as 0.9 for all tissue layers. This value is representative of biological tissues, according to Cheong et al. (1990). In order to account for scattering anisotropy, the P1-approximation together with the second order Delta-Eddington approximation of the Henyey-Greenstein phase function was used.

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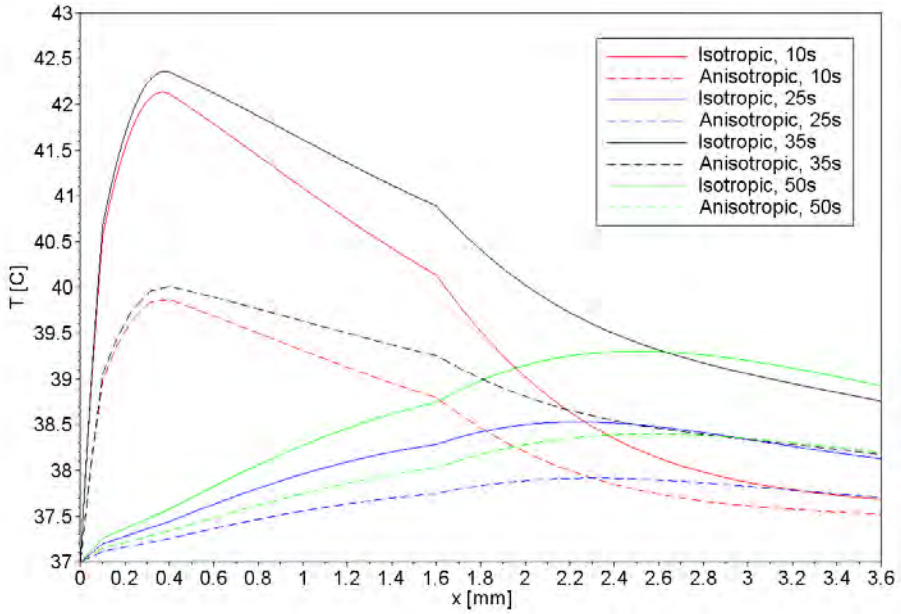

Figure 5 – Comparison of the temperature distribution at different times of exposition to the periodic laser heating for cases of isotropic and anisotropic scattering.

Figure 5 shows that, for both isotropic and anisotropic scattering, the largest temperatures are reached within the tumor, as a result of the large absorption coefficients resulting from the presence of nanoparticles in this tissue layer. Figure 5 also shows that the temperature profiles are significantly different for the case in which the scattering is anisotropic, as compared to the case of isotropic scattering. The difference in the peak temperature is about 6% higher (2◦C) for the isotropic scattering case. As reported in the review presented by Lima et al. (2006), this temperature difference can result in the killing or not killing of the tissue cells. We note that, for the peak of 40◦C in the anisotropic case, the cells are alike to be alive, while for the peak of the isotropic case (above 42◦C) the cells would be killed.

4. CONCLUSIONS

Simulating the transient heat transfer in hyperthermia therapy can be a complex task, involving coupled radiation, conduction and blood perfusion, complex geometry of tumors, anisotropic scattering of radiation, among other phenomena. As a result, computational simulations are generally very time-consuming. In addition, the solution of inverse problems for accurate determination of the thermophysical properties of human tissues are essential. The solution of the inverse problem can become a prohibitive task, when all complex phenomena relative to the heat transfer model are considered in detail. The radiation model is the most time consuming in the solution of the heat transfer problem. For this reason, the computationally inexpensive P1-approximation solution was analyzed in this paper for a case involving the heating of skin tissues by a laser, for cases involving tissues with and without nanoparticles, as well as for isotropic and anisotropic scattering.

The comparison of results obtained with the P1-approximation and with the Discrete Ordinate Method solution of the Radiative Transfer Equation indicated that, as expected, the P1-approximation can be used for simulating the heat transfer in tissues without nanoparticles during hyperthermia therapy. Such was also the case for the cases of tissues embedded with nanoparticles examined in this paper, because their associated optical thicknesses were small. Tissues with thin layers can occur in several applications of hyperthermia therapy dealing with skin cancer.

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