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MODELING, CONTROL AND IMPLEMENTATION OF A BALL AND BEAM SYSTEM

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Abstract. *The ball and beam system is a classical mechanical system consisting of a ball that moves over a beam in a planar movement. The beam can rotate around its center of gravity, and an elastic belt attached to the beam extremities (and an electric motor) allows the transmission of control forces to the beam in order to cause the movement. The ball translates and rolls, always maintaining a contact with the beam. The ball's rolling movement can be without or with slipping, and this last kind of rolling movement is more likely to occur in high beam's angles (in relation to a horizontal line) and in higher ball's velocities. The friction model between the ball and the beam (and the beam and its bearing) is also complex, involving possibly dry and viscous friction together. We present the modeling, control and implementation of a closed loop control system for a ball and beam system. Firstly, we present and compare the mathematical model, considering rolling without and with slipping. A closed loop controller is then designed and implemented in the real system in order to do a comparative analysis. Despite of being a didactical system, the ball and beam presents a complex dynamics, with several nonlinearities, with an infinite number of equilibrium points (if we apply a torque in the beam) and a difficult-to-determine friction model. Finally, conclusion for the modeling, simulation and control techniques are drawn, and future research directions are pointed out.*

Keywords: *Ball and Beam, Closed Loop Control, Rolling Without Slipping*

1. INTRODUCTION

The system to be studied in this work is the Ball and Beam system, which is a didactical plant found in many control laboratories. There are many (slightly different) configurations, differing from manufacturer to manufacturer. In this work we use the Amira-Elwe system (Amira-Elwe,1999). In Figure 1, at the left hand side, we present a photo of the complete system, with the mechanical plant (behind), and the power-control module. The ball moves over a trail in the beam, and its position is measured by a CCD camera. The position is calculated via real time image processing techniques (in the control module) and sent as an analog signal to the computer. It is simply the distance from the left end of the bar. The beam's angular position is also measured, but it is not used for the closed loop control in this work. The light bulbs must be turned on for the camera works properly. A current is sent from the power-control module to the DC motor, which is the system actuator (not visible in the photo) and causes the beam to rotate around its center by means of an elastic belt, which is attached at the ends of the beam. The system is then a single-input-single-output (SISO) system. All the signals are voltage signals between zero to 10 volts. In order to control the system, the power module communicates with a PC computer via a PCI acquisition board.

The Ball and Beam system (in its several versions) is a common benchmark for testing linear and nonlinear control techniques. The model complexity (in terms of order an nonlinear terms) does not vary considerably in the references consulted, but some minor effects are considered in some of them. Feedback control is always used, as it is naturally unstable system, but the complexity of the control laws are very different. In (Yu, 2009), proportional + derivative control (PD) is designed and tested experimentally, with the addition of nonlinear compensation in some cases. In (Hauser, 1992), a modified feedback linearization technique is used for the case in which the ball starts at the center for the beam, where the model presents singularities for the standard technique (such singularity is not present in the model above). In (Colón and Teixeira, 2009), simulation results are presented for different control laws (linear and nonlinear) for the model in (Hauser, 1992). In (Andreev, 2000), on the other hand, a geometric matching control strategy is used, which is a complex nonlinear design technique. In (Lemos *et al*, 2002), an adaptive control strategy is used for a similar plant. In all those works, the control strategies manage to reduce significantly the effect of disturbances in simulation (and some of them, experimentally). In the present work, on the other hand, despite the more detailed modeling (that considers the belt elasticity, for example), the effect of non-viscous friction is stronger in the experimental system, and is not properly modeled, as precise models for these effects should use partial differential equations or differential inclusion (Stewart, 2000). The control laws must then be sufficiently robust in order to cope with such uncertainty.

The paper is organized as follows: In section 2, some models obtained by Newton's method are presented, in which the friction forces must be calculated. In section 3, the lagrangean modeling is done in order to obtain a model where the contact forces do not appear explicitly. In section 4, the linearized model of the system is obtained and presented, and also the time discretization is realized. In section 5, we present two control design techniques for the ball and beam system: the pole placement design, which is very simple to obtain, and the robust LQG/LTR control design, which is more involved in the theoretical aspects, but produces a robust controller, which is desirable in a scenario of high model uncertainty (as in the case of complex friction model).

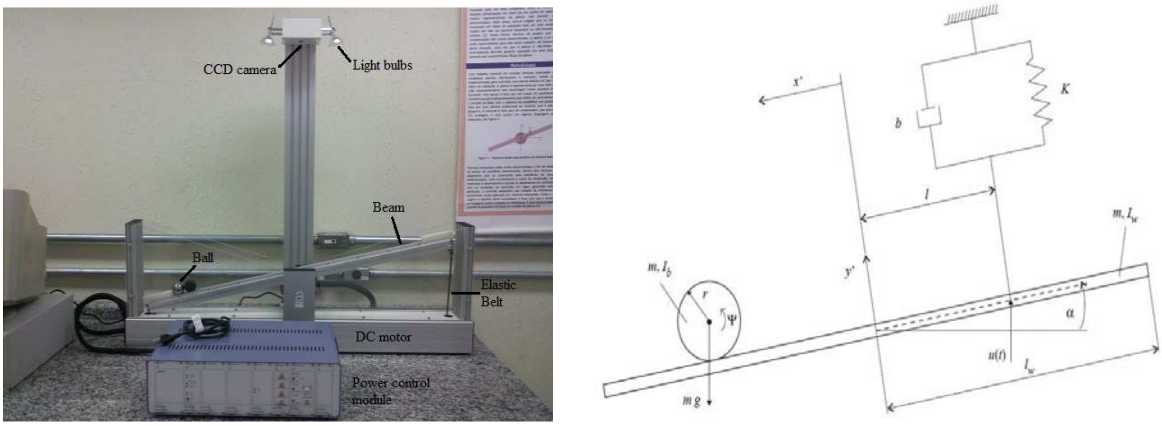

Figure 1. Ball and Beam system.

2. NEWTON MODELING OF THE SYSTEM

The modeling of the system presented in Figure 1, in the right side, is presented in this section. We present the modeling based in the Newton's method. It is also possible that the ball rolls and slip at the same time for large α, but for the most part of the time, the rolling is supposed to be without slipping. In this case, the system can be considered as non-holonomic (Bloch, 2003), which allows a reduction in the number of degrees of freedom from three to two. In the Newton's method, it is necessary to model the ball as developing its movement in a *non-inertial system*, that is fixed in the beam (Craig, 1989). In this case, the contact forces must be calculated, as the friction forces are complex when there is slipping.

2.1 Newton's method

In a non-inertial reference frame, the Newton's Law must be properly modified, by using the concept of *total derivative* (or *covariant derivative*) which accounts for the variations of the frame of reference itself their effect in the total movement. If

$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \tag{1}$$

is the transformation matrix from the non-inertial to the inertial frame of reference, and $\Omega = R^{-1}\dot{R}$ is the matrix representing the variation of the moving frame expressed in the moving frame (that is an anti-symmetric matrix), the covariant derivative can be written as:

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$$D_t = \frac{\partial}{\partial t} + \vec{\Omega}\times = \frac{\partial}{\partial t} + \Omega \tag{2}$$

where $\vec{\Omega}$ is the vector equivalent to *Ω*. The Newton's law can be simply written as:

$$\vec{T} = D_t \vec{H} \tag{3}$$

and the same equation for linear motion is simply:

$$\vec{F} = mD_t D_t \vec{S}' = m\left(\frac{\partial}{\partial t} + \vec{\Omega}\times\right)\left(\frac{\partial}{\partial t} + \vec{\Omega}\times\right)\vec{S}' \tag{4}$$

where *S'* is the ball's center of mass position in the moving reference frame. This is the covariant second Newton's law, that is valid for any reference frame (Sattinger and Weaver, 1986). After some manipulation, the second Newton's law can be written as:

$$m\ddot{S}' = F - m\Omega^2 S' - m\dot{\Omega}S' - 2m\Omega\dot{S}' \tag{5}$$

where *S'* is the ball's center of mass position, *m* is the ball's mass, and *F* is the total force applied in the ball (expressed in the non-inertial frame of reference). Let *R* be the orthogonal matrix transforming the non-inertial reference frame to the inertial frame (fixed in the laboratory). That is:

$$R = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \tag{6}$$

and

$$\Omega = \begin{bmatrix} 0 & -\dot{\alpha} \\ \dot{\alpha} & 0 \end{bmatrix} \tag{7}$$

Note: The matrix $\Omega=\dot{R}R^{-1}$ represents a different velocity, which is the angular velocity of the non-inertial frame expressed in the inertial frame. The change between non-inertial frame to the inertial (or vice-versa) is a similarity transformation, also known as adjunct transformation.

Consider now the decomposition of the system in two independent bodies (in contact by friction and normal forces), as shown in Figure 2.

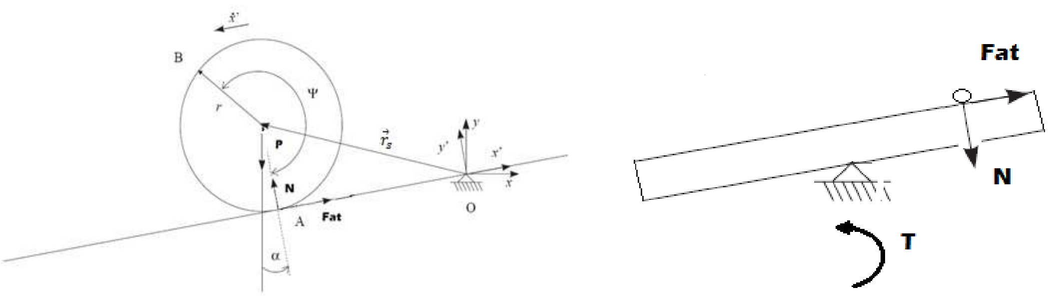

Figure 2. Decomposition of the system.

Applying the second Newton's Law in the ball alone (in the moving reference frame), we have the equation:

$$m\ddot{S}' = \begin{bmatrix} F_{at} - mg\sin\alpha & N - mg\cos\alpha \end{bmatrix}^t - m\begin{bmatrix} 0 & -\dot{\alpha} \\ \dot{\alpha} & 0 \end{bmatrix}\begin{bmatrix} 0 & -\dot{\alpha} \\ \dot{\alpha} & 0 \end{bmatrix}S' - m\begin{bmatrix} 0 & -\ddot{\alpha} \\ \ddot{\alpha} & 0 \end{bmatrix}S' - 2m\begin{bmatrix} 0 & -\dot{\alpha} \\ \dot{\alpha} & 0 \end{bmatrix}\dot{S}' \tag{8}$$

In any situation, the relative position of the ball's center of the mass is always constant and $y' = r$, so that the S' can be written as:

$$S' = [x' \quad r]^t \ , \ \dot{S}' = [\dot{x}' \quad 0]^t \text{ and } \ddot{S}' = [\ddot{x}' \quad 0]^t \tag{9}$$

so that we have:

$$[m\ddot{x}',0]^t = [F_{at} - mg\sin\alpha, N - mg\cos\alpha]^t + [m\dot{\alpha}^2 x', m\dot{\alpha}^2 r]^t - [-m\ddot{\alpha}r, m\ddot{\alpha}x']^t - 2m[0,\dot{\alpha}\dot{x}']^t \tag{10}$$

which implies that the normal force must be:

$$N = m\left(g\cos\alpha - r\dot{\alpha}^2 + \ddot{\alpha}x' + 2\dot{x}'\dot{\alpha}\right) \tag{11}$$

and the movement of the center of mass is

$$m\ddot{x}' = -F_{at} - mg\sin\alpha + m\dot{\alpha}^2 x' + m\ddot{\alpha}r \tag{12}$$

In order to determine the relative rotational movement of the ball, described by the angle Ψ, we have to apply the rotational second Newton Law, in a non-inertial reference frame, which is expressed as:

$$\vec{T} = \frac{d\vec{H}}{dt} = \frac{\partial\vec{H}}{\partial t} + \vec{\Omega}\times\vec{H} \tag{13}$$

where $\vec{H}$ is the angular momentum, $\vec{T}$ is the total torque applied in the system and $\vec{\Omega}$ the angular velocity vector, that is related to the matrix Ω by a Lie Algebra isomorphism (Bloch *et al.*, 2003) and (Bullo *at al.*, 2005). In fact, it is possible to represent all the vector quantities as anti-symmetric matrices, with the vector product substituted by the matrix Lie Bracket. As the vectors $\vec{H}$ and $\vec{\Omega}$ have the same directions, which means that their vector product is null, the equation is reduced to

$$\vec{T} = \frac{\partial\vec{H}}{\partial t} = I_b\ddot{\Psi} = rF_{at} \tag{14}$$

The friction force $\vec{F}_{at}$ depends on the normal force in the ball and a friction coefficient. Differently from the sliding friction coefficient (Coulomb's Law) this friction coefficient (rolling friction coefficient) is very small (for example, for steel contact the value is around 0.0003). There was a great controversy in those kinds of friction models for rigid body dynamics. The most accepted approach would be consider the body's elastic nature, which would imply, in control and simulation, the real time solution of complicated partial differential equations (Stewart, 2000). Considering now the torque applied by the motor in the beam (in Figure 2), and the contact normal force that opposes the movement, we have

$$I_w\ddot{\alpha} = lu(t) - x'N - bl\dot{\alpha} - kl\alpha \tag{15}$$

In which $u(t)$ is the input control and all the other torques are opposing the movement, where k is the elastic coefficient of the rubber belt (that transmit the force to the beam), b is the viscous friction coefficient of the beam and I_w is the beam moment of inertia.

Note: some details about the geometry of the beam are ignored here, and the interested reader should consult (Cazzolato, 2007) for a more thorough modeling.

2.2 State space representation

The state-space representation of the system is: defining the state variables as $x_1 = x'$, $x_2 = \dot{x}'$, $x_3 = \alpha$, $x_4 = \dot{\alpha}$, $x_5 = \Psi$, $x_6 = \dot{\Psi}$ and writing the normal force not depending on accelerations, we have:

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$$N = \frac{m\left(g\cos x_3 - r\dot{x}_4 + 2x_2x_4 - lx_1u(t)/I_w\right)}{1 + m{x_1}^2/I_w} \tag{16}$$

the state space representation is then:

$$\dot{x}_1 = x_2 \tag{17}$$

$$\dot{x}_2 = F_{at} - mg\sin x_3 + mx_1{x_4}^2 + mr\left(\frac{lu(t) - x_1N - blx_4 - klx_3}{I_w}\right) \tag{18}$$

$$\dot{x}_3 = x_4 \tag{19}$$

$$\dot{x}_4 = \frac{lu(t) - x_1N - blx_4 - klx_3}{I_w} \tag{20}$$

$$\dot{x}_5 = x_6 \tag{21}$$

$$\dot{x}_6 = \frac{1}{I_w}F_{at} \tag{22}$$

Determining the real nature of the friction force F_{at} is a very hard task, as it involves a very complex microscopic dynamic (between the molecules of the contact surfaces). A consistent model (that is, without discontinuities) would take into account the elasticity of the "rigid" bodies, which means to deal with partial differentia equations. This would produce a model very difficult to simulate numerically. Simplifications would involve discontinuous models, like the Coulomb friction, that frequently presents numerical difficulties too.

3. LAGRANGE MODELING OF THE SYSTEM

A more successful approach to the modeling of the system would be the lagrangean one, where one does not have to calculate the contact forces. Considering the rolling without slipping of the ball, one has $x' = r\Psi$. The total kinetic energy is the sum of rotational and translational energy of the ball and the translational energy of the beam, that is:

$$T_b = \frac{1}{2}mv_s^2 + \frac{1}{2}I_b\omega_b^2 \tag{23}$$

$$T_w = \frac{1}{2}I_w\dot{\alpha}^2 \tag{24}$$

where T_b is the kinetic energy of the ball and T_w is the kinetic energy of the beam, v_s is the velocity of the center of mass of the ball, m is the mass of the ball, I_b is the moment of inertia of the ball and I_w is the moment of inertial of the beam. Ψ is the angular position of the ball. The total kinetic energy are then:

$$T = T_b + T_w = \frac{1}{2}\left\{I_w\dot{\alpha}^2 + m\left(\dot{x}'^2 + 2\dot{x}'^2\dot{\alpha}r + \dot{\alpha}^2r^2 + \dot{x}'^2\dot{\alpha}^2\right) + I_b\left(\frac{\dot{x}'}{r} + \dot{\alpha}\right)^2\right\} \tag{25}$$

It is important to note that this kinetic energy is calculated in the inertial reference frame, so the angular velocity of the ball is the sum of the two angular velocities (ball and beam). The potential energy of the system is the gravitational potential energy of the ball and the elastic energy in the belt, that is:

$$V_b = -mgx'\sin\alpha \tag{26}$$

$$V_f = \frac{1}{2} k l^2 \alpha^2 \tag{27}$$

so the total potential energy is given by:

$$V = -mgx' \sin\alpha + \frac{1}{2} k l^2 \alpha^2 \tag{28}$$

It is considered that all the dissipation (by friction effects) is concentrated in the beam (rotational movement) and is of linear nature (which is a rough approximation), that is, the friction force is modeled by

$$F_R = -bl\dot{\alpha} \tag{29}$$

so the Rayleigh dissipation function is:

$$Y = \frac{1}{2} b l^2 \dot{\alpha}^2 \tag{30}$$

If we apply the Euler-Lagrange formula and consider the rolling without slipping restriction $x' = r\Psi$, it was chosen two degrees of freedom, that are α e x'. The Euler-Lagrange equations results in

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = u(t) l \cos\alpha - \frac{\partial Y}{\partial \dot{\alpha}} \tag{31}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = -\frac{\partial Y}{\partial \dot{x}} \tag{32}$$

where $L = T - V$ is the lagrangean of the system.

Defining the same state space variables (from one to four, of course), the resulting equations (coming from the Euler-Lagrange equations) will be in number of four. Introducing the coefficients:

$$a_1 = m + \frac{I_b}{r^2},\ a_3 = mg,\ b_1 = I_b + I_w,\ b_2 = 2m,\ b_3 = bl^2,\ b_4 = kl^2,\ b_5 = \left(mr^2 + I_b\right)\frac{1}{r},\ b_6 = mg \tag{33}$$

We have the state-space equations (see (Amira-Elwe,1999)):

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{a_2\left[\left(b_2 x_1 x_2 + b_3\right)x_4 + b_4 x_3 - b_6 x_1 \cos x_3\right]}{a_1\left(m x_1^2 + b_1\right) - a_2 b_5} + \frac{\left(m x_1^2 + b_1\right)\left(a_3 \sin x_3 + m x_1 x_4^2\right)}{a_1\left(m x_1^2 + b_1\right) - a_2 b_5} - \frac{a_2 l \cos x_3 u(t)}{a_1\left(m x_1^2 + b_1\right) - a_2 b_5} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{-\left(b_2 x_1 x_2 + b_3\right)x_4 + b_4 x_3 - b_6 x_1 \cos x_3}{m x_1^2 + b_1} - \frac{b_5\left(a_3 \sin x_3 + m x_1 x_4^2\right)}{a_1\left(m x_1^2 + b_1\right) - a_2 b_5} - \frac{a_2 b_5\left[\left(b_2 x_1 x_2 + b_3\right)x_4 + b_4 x_3 - b_6 x_1 \cos x_3\right]}{\left(m x_1^2 + b_1\right)\left[a_1\left(m x_1^2 + b_1\right) - a_2 b_5\right]} + \\
&\quad \left(1 + \frac{a_2 b_5}{a_1\left(m x_1^2 + b_1\right) - a_2 b_5}\right)\frac{l \cos x_3 u(t)}{m x_1^2 + b_1}
\end{aligned} \tag{34}$$

4. SYSTEM LINEARIZATION

In order to design a linear closed loop controller for this system, one has to linearize it, that is, the equations must be approximated by a linear time invariant system in the vicinity of an equilibrium point. After some calculations, and for the following values of the system parameters: $m = 0.27\,kg$, $M = 1.122\,kg$, $l = 0.49\,m$, $r = 0.018\,m$, $R = 0.02\,m$, $b = 1.0\,Ns/m$, $I_w = 0.5m$, $k = 0.001\,N/m$, $g = 9.8\,m/s^2$, we have the matrices of the state space linear model, that are:

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$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.0342 & 0 & 6.592 & 0.031 \\ 0 & 0 & 0 & 1 \\ 18.898 & 0 & -0.344 & -1.713 \end{bmatrix} ; \quad B = \begin{bmatrix} 0 \\ -0.0633 \\ 0 \\ 3.4960 \end{bmatrix} \tag{35}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}; \quad D = \begin{bmatrix} 0 \end{bmatrix}; \tag{36}$$

In order to implement the controller in a computer system, it must be discretized, and the sampling time is 0,05 seconds. By using standard functions in MATLAB, it is possible to obtain the discretized model, that is given by the matrices:

$$A_D = \begin{bmatrix} 1 & 0.05 & 0.008239 & 0.0001721 \\ 0.001542 & 1 & 0.3295 & 0.009494 \\ 0.02296 & 0.0003854 & 0.9996 & 0.04791 \\ 0.9054 & 0.02296 & -0.01394 & 0.9176 \end{bmatrix} ; \quad B_D = \begin{bmatrix} -7.101e-005 \\ -0.002563 \\ -0.004247 \\ 0.1675 \end{bmatrix} \tag{37}$$

$$C_D = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}; \quad D_D = \begin{bmatrix} 0 \end{bmatrix}; \tag{38}$$

The eigenvalues for this discretized plant are then 1.1601, 0.9654 + 0.1601j, 0.9654 - 0.1601j, 0.8262, which clearly indicates that the system is unstable (one eigenvalue has module greater that one). It is also easy to check that this model is controllable and observable.

5. CONTROL DESIGN AND EXPERIMENTAL RESULTS

In this section, we apply the pole-placement technique in order to control the ball and beam system. The controller must guarantee that reference signals (for the ball's position) will be followed as close as possible. After that, we design a robust LQG/LTR controller in order to guarantee also the robustness of the system. This property of the system is very important, as the friction between the ball and the beam, and the beam and its bearing are very complex, and the controller, and the complete closed-loop system, should be not affected by this uncertainty.

5.1 Pole placement control technique

By using the linearized and discretized model of the system presented in section 4, it is possible to design a linear time-invariant controller (Ogata, 2005). The pole-placement design technique is based on the state feedback control philosophy, which assumes that all the states of the system are measurable (that is, there are sensors for all the states) and those variables are used in feedback loops. For the system in question, on the other hand, we do not have sensors for all the states. In fact, the acquisition system only measures the ball position ($x_1 = x'$) by the camera, and the beam's angle ($x_3 = \alpha$) by the encoder. It seems tempting to simply derive those signals in real-time, in order to obtain the other states, but this process often causes more problems than it solves. In fact, all the sensor signals have noise, and consequently have high amount of power in high frequencies. When we apply time derivative in those signals, the result is more noise, which affects negatively the controller performance. The solution is to construct a *state observer*, which is part of the controller and is, in a certain sense, a plant model running in parallel (constantly corrected by the real measurements of the sensors). The states estimates are the used for feedback purposes. In Control Theory books, like Ogata (2005), it is proved that the whole closed-loop system is stable.

The discrete time closed loop poles were selected to be 0.9512, 0.7788, 0.4724 e 0.4724. The state feedback gains for this case are 27.170, 29.82, 58.78 and 6.39, respectively. In Figure 3, it is shown the experimental result for the closed-loop system just designed. The reference signal is simply a square wave with amplitude 0.1 meters (extreme positions for the ball). It is clear that the performance is unsatisfactory, as the nonlinear effect (mainly due to the friction) is causing in limit-cycle (oscillations when the reference signal stabilizes). The force applied by the CC motor is presented in Figure 4.

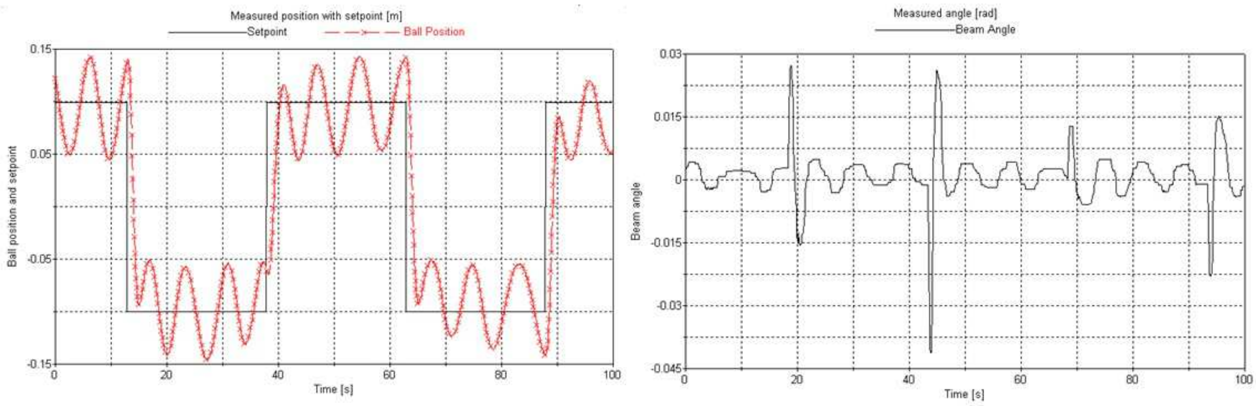

Figure 3. Ball position (real and reference) and beam's angle (in radians).

5.2 Disturbance rejection for the pole-placement controller

In order to evaluate the controller performance for disturbances, it was applied a constant/null reference signal (that is, the ball's desired position is in the center of the beam, and consequently the beam's angle must be zero) and manual disturbances were applied in the system (that is, the ball was slightly deviated from its position). In

Figure **5** and Figure 6, we can see the disturbance generated in about 14 seconds and 32 seconds, and the control system reactions right after the events. It can be observed that the controller can attenuate the disturbances very rapidly, but the oscillatory behavior, almost certainly caused by a limit-cycle, is persistent. The controller manages to attenuate the disturbances, but the oscillations remain.

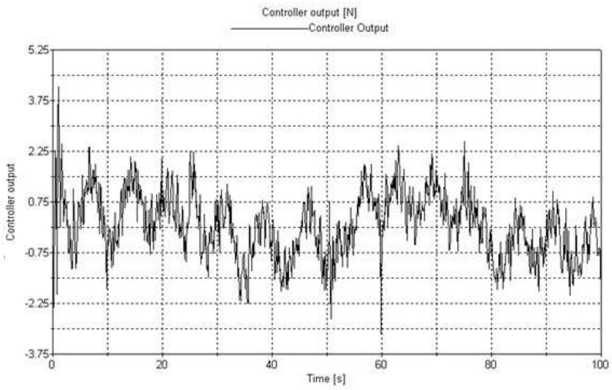

Figure 4. Force applied by the CC motor.

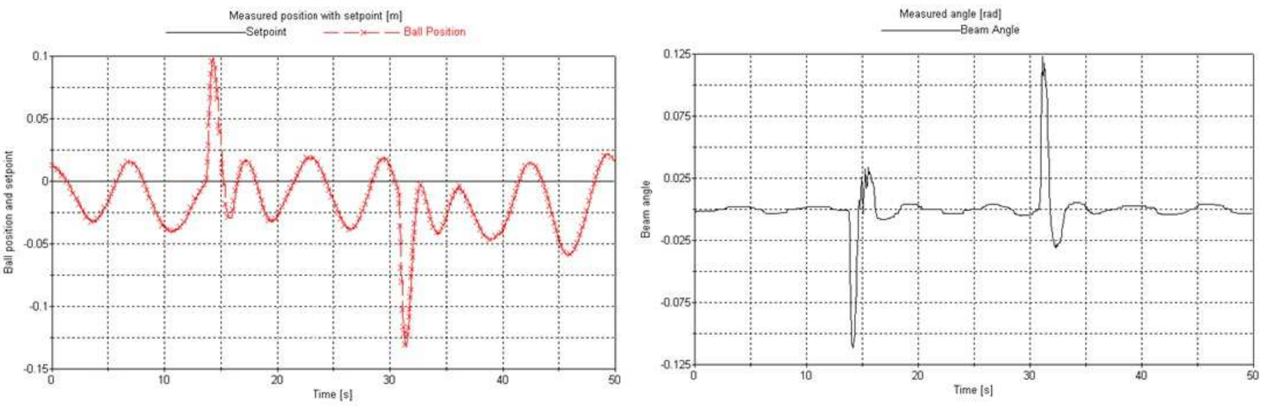

Figure 5. Ball position (real and reference) and beam's angle (in radians) with disturbances

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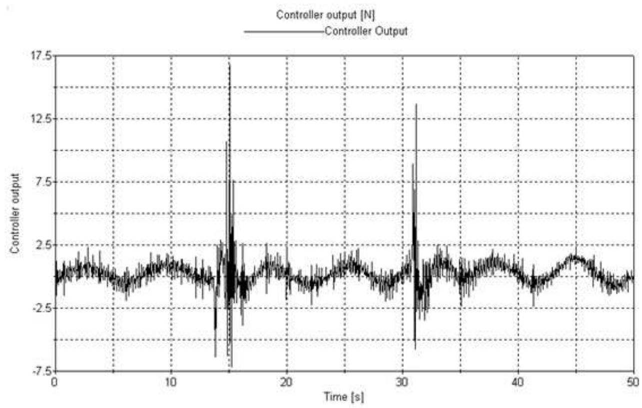

Figure 6. Force applied by the CC motor.

5.3 Robust LQG/LTR control technique

Now we design a robust controller by using the LQG/LTR control technique. The structure of the controller is very similar to the pole-placement, that is, it is a linear time-invariant controller based in the state feedback and state estimate philosophy. The controller gains, on the other hand, are very different, and the system is designed to have robustness properties, that is, to be few sensitive to model uncertainties, which includes nonlinearities (from the friction, for example). Also, the parameters of the state observer (in this case, it is an optimal state estimator called Kalman Filter) are especially designed in order to guarantee robustness (Cruz, 1996).

In Figure 7, we see the reference signal (the same as in the previous case) and the system response. It can be seen that the performance improved due to the fact that the system is less sensitive to plant uncertainties. On the other hand, in some cases, the oscillatory behavior appears, but with less amplitude. In Figure 8, we see the force applied by the motor.

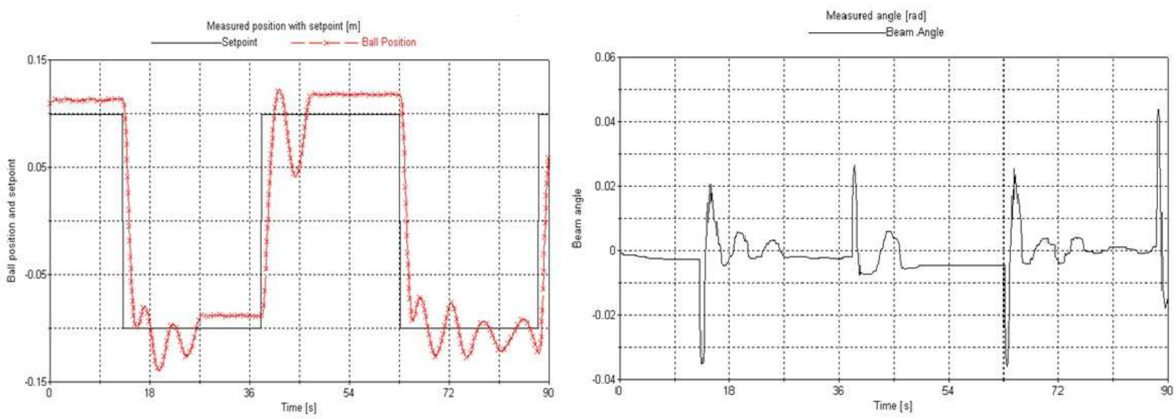

Figure 7. Ball position (real and reference) and beam's angle (in radians).

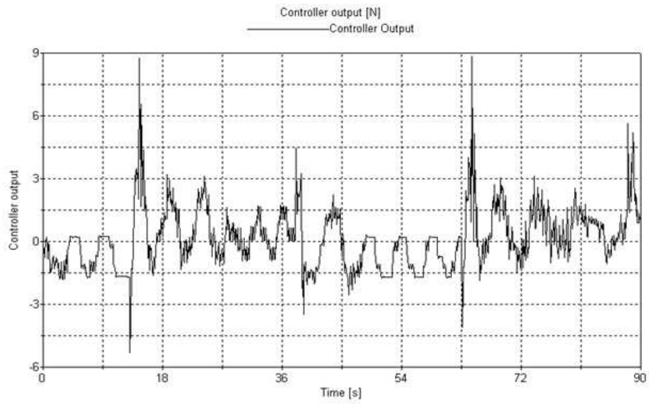

Figure 8. Force applied by the CC motor.

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6. CONCLUSIONS

It was presented two different models for the ball and beam system, one of them obtained by the Newton's method, where the forces of contact, in particular the friction force between the ball and the beam, must be determined. The mathematical formulation of this force is complex and frequently is discontinuous in nature, even in the case of rolling friction. The correct modeling should use elasticity of the "rigid bodies", what increases significantly the complexity of the algorithms (and the computational time). Coulomb friction models frequently present numerical difficulties. The other model presented is based on the lagrangean model, which is based in energy considerations and does not need to calculate the force. But the model in this case does not consider the nonlinear nature of the friction, that does not participate in the linearization process and is not predicted in the control design process.

It was also designed different linear control techniques for the ball and beam system. One of them did not considered robustness properties and presented poor performance (limit cycles) due to the nonlinear behavior of the friction (not considered in the design model). The other controller (LQG/LTR), that is robust, could deal better than the other in reducing the oscillations, but it points to a nonlinear controller as a more effective controller for those matters.

Future works includes a more thorough analysis of the friction model, possibly including the differential inclusion technique (Stewart, 2000), the nonlinear elasticity of the belt, and more advanced nonlinear control techniques like the Feedback Linearization (Hauser, 1992) with a second loop to provide robustness properties and the Sliding Modes technique, that is nonlinear and robust at the same time.

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