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CHAOS CONTROL OF THE CARDIOVASCULAR DYNAMICS

Bianca Borem Ferreira
Universidade Gama Filho – Department of Mathematics
20.740.900 – Piedade – Rio de Janeiro – RJ – Brazil
biaborem@gmail.com

Marcelo Amorim Savi
Universidade Federal do Rio de Janeiro/COPPE – Department of Mechanical Engineering
21.941.972 – Rio de Janeiro – RJ, P.O. Box 68.503 – Brazil
savi@mecanica.ufrj.br

Aline Souza de Paula
Universidade de Brasília - Department of Mechanical Engineering
70.910.900 – Asa Norte – Brasília – DF - Brazil
alinedepaula@unb.br

Abstract. *Some serious cardiac arrhythmias can be characterized on the basis of the nonlinear dynamics approach. In this regard, it becomes possible to develop new strategies of analysis and treatments, different from those employed in traditional approaches. In this paper we use a mathematical model composed of three modified Van der Pol oscillators connected by time delay couplings to reproduce the ECG signals and to analyze dynamics of heartbeats. A continuous chaos control method is then applied to pathological ECGs, especially the ventricular fibrillation. The idea is to investigate the effectiveness of this technique to control, eliminate or minimize the effects of this pathology.*

Keywords: *Heart, Nonlinear dynamics, Chaos control, Van der Pol oscillators, delayed feedback method.*

1. INTRODUCTION

The human body consists of several interconnected systems and many of them exhibit nonlinear characteristics and chaotic behavior. The heart plays a fundamental aspect in the physiology of living beings and the existence of chaotic behavior of heart rhythms are the objective of several research efforts (Christini *et al.*, 2001; Ferreira *et al.*, 2011; Garfinkel *et al.*, 1992, 1995; Glass *et al.*, 1983, 1987; Gois & Savi, 2009; Kaplan & Cohen, 1990; Savi, 2005).

The cardiac conduction system can be treated as a network of self-excitatory elements composed by the sinoatrial node (SA), atrioventricular node (AV) and the His-Purkinje system (HP) (Gois & Savi, 2009; Grudzinski & Zebrowski, 2004). The electric excitation is primarily generated at the SA node, known as natural pacemaker, located at the right atrium. It initiates the electrical impulse that spreads as a wave, stimulating both atria. The impulse reaches the AV node, which is the electrical connection between the atria and the ventricles. Afterward, electrical impulse goes to the His-Purkinje system, which transmit the electrical impulse to myocardial cells, producing simultaneous contraction of the ventricles.

The electrocardiogram (ECG) is the most widely used mechanism to analyze the heart functioning. The ECG signal records the electrical impulses related to heart function in the form of waves. The dynamics of the heartbeat has been analyzed through both mathematical models and time series analysis (Van der Pol & Van der Mark, 1928; Grudzinski & Zebrowski, 2004; Santos et al. 2004). Gois & Savi (2009) reproduced ECGs through a mathematical model consisting of three modified Van der Pol oscillators, which represent the SA node, AV node and the His-Purkinje system, connected by time delayed couplings. This model is able to reproduce normal and pathological ECGs. Ventricular fibrillation was associated with chaotic behavior, as addressed by Stein *et al.* (1999).

The control of chaotic heartbeats is a key issue in cardiology. Chaos control is based on the richness of chaotic behavior and the most important characteristic is the stabilization of unstable periodic orbits (UPO) embedded in chaotic attractor by employing small perturbations (De Paula & Savi, 2009a, b; 2012). Garfinkel *et al.* (1992; 1995) presented a pioneer work related to the application of chaos control method in cardiac rhythms. They employed a perturbation feedback chaos control strategy, based on OGY approach, to stabilize cardiac arrhythmias induced by a drug called ouabain in rabbit ventricle. Ferreira *et al.* (2011) employed the ETDF chaos control method to the natural pacemaker modeled by the modified Van der Pol equation proposed by Grudzinski & Zebrowski (2004). The main objective was to control or to suppress chaotic responses, avoiding critical pathologies.

In this paper, the ETDF chaos control method is employed to eliminate chaotic cardiac responses associated with ventricular fibrillation. The three-coupled oscillator model proposed by Gois & Savi (2009) is employed to describe the heartbeat dynamics. The idea is to monitor ECG signals generated by the proposed model, treating two distinct signals: normal and ventricular fibrillation. Results show that the ETDF method is able to generate less complex behaviors of the ECG.

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2. EXTENDED TIME-DELAYED FEEDBACK CONTROL METHOD

Chaos control methods can be classified as continuous and discrete approaches. Among the continuous control methods, the ones that stand out are the time delayed feedback (TDF) (Pyragas, 1992) and extended time delayed feedback (ETDF) (Socolar *et al.*, 1994). Among the discrete methods, it is important to highlight the pioneer OGY method (Ott *et al.*, 1990).

The chaos control technique may be understood as a two-stage procedure. The learning stage is the first one where UPOs embedded in the system attractor are identified and controller parameters are estimated. The second stage is the control stage and consists in the use of control law to impose the perturbation needed to stabilize the desired UPO.

ETDF is a control strategy applied to systems modeled as follows (Pyragas, 1992; Socolar *et al.*, 1994):

$$\begin{aligned} \dot{x} &= Q(x,y) \\ \dot{y} &= P(x,y) + C(t,y) \end{aligned} \tag{1}$$

where x and y are state variables, $Q(x,y)$ and $P(x,y)$ defines the system dynamics, while $C(t,y)$ is associated with the control action. In the ETDF method, the control perturbation is based on feedback from the difference between the present state and the delayed states of the system, being given by:

$$\begin{aligned} C(t,y) &= K[(1-R)S_\tau - y] \\ S_\tau &= \sum_{m=1}^{N_\tau} R^{m-1} y_{m\tau} \end{aligned} , \tag{2}$$

where $y = y(t)$, $y_{m\tau} = y(t - m\tau)$, τ is the time delay, $0 \le R < 1$ and K are the controller parameters. In general, N_τ is infinite, but can be set as a function of the dynamical system. Note that, for any value of K and R, the perturbation of the Eq. (2) is zero when the trajectory of the system is on an UPO since $y(t - m\tau) = y(t)$ for all m if $\tau = T_i$, where T_i is the periodicity of the ith UPO. According to the correct choice of the values K and R becomes possible to stabilize the system in one of its UPOs. The TDF is a particular case of the ETDF when $R = 0$.

Note that the dynamical system together with the control law is governed by differential difference equations (DDE). The solution of this type of equation can be carried out by considering an initial function $y_0 = y_0(t)$ over the interval $[-N_\tau\tau, 0]$. In this work, this function is estimated by a Taylor series expansion as proposed by Cunningham (1954) and shown below:

$$y_{m\tau} = y - m\tau\dot{y}. \tag{3}$$

Numerical procedure considers the fourth-order Runge-Kutta method with linear interpolation on the delayed variables (Mensour & Longtin, 1997). Besides, it is assumed three delayed states, $N_\tau = 3$.

During the learning stage, the UPO identification is carried out using the close-return method (Auerbach *et al.*, 1987). Moreover, controller parameters, K and R, are estimated from Lyapunov exponents of each desired UPO (De Paula *et al.*, 2012; Ferreira *et al.*, 2011). The calculation of these exponents is carried out by considering a finite number of elements (Farmer, 1982). Therefore, the initial function, $y_i(t)$, is approximated by N samples. Under this assumption, the system is represented by $n(N+1)$ ODEs, instead of DDEs with n state variables, and the classical algorithm proposed by Wolf *et al.* (1985) is employed.

3. MATHEMATICAL MODEL

Several studies have been developed to model the dynamics of cardiac rhythms. Basically, connected nonlinear oscillators may model the heart functioning. Each oscillator represents the cardiac systems associated with: SA node, the natural pacemaker; AV node; and His-Purkinje system. The combination of waves coming from the SA node, AV node and His-Purkinje system is responsible for ECG aspect.

In this work, we use three-coupled oscillators to represent the ECG signal following the same idea of Gois & Savi (2009). The conceptual model of the cardiac system is presented in Figure 1Figure where general couplings and external forcing are incorporated in order to represent different kinds of behavior.

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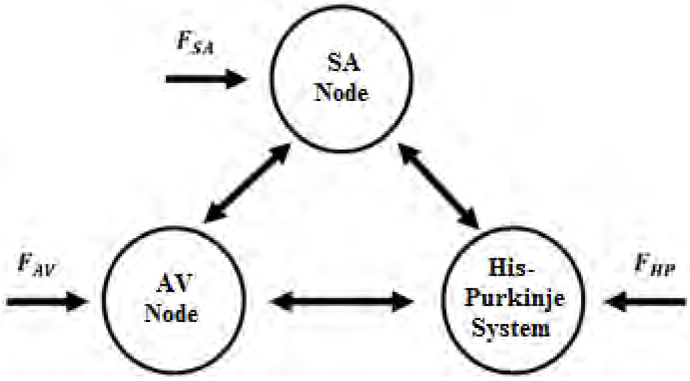

Figure 1. General conceptual model for the cardiac system.

A modified Van der Pol equation proposed by Grudzinski & Zebrowski (2004) is employed to mathematically represent the cardiac system. Therefore, the system is governed by the following equations.

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= F_{SA}(t) - \alpha_{SA} x_2 (x_1 - v_{SA_1})(x_1 - v_{SA_2}) - \frac{x_1(x_1+d_{SA})(x_1+e_{SA})}{d_{SA} e_{SA}} - k_{AV-SA}(x_1 - x_3^{\tau_{AV-SA}}) - k_{HP-SA}(x_1 - x_5^{\tau_{HP-SA}}) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= F_{AV}(t) - \alpha_{AV} x_4 (x_3 - v_{AV_1})(x_3 - v_{AV_2}) - \frac{x_3(x_3+d_{AV})(x_3+e_{AV})}{d_{AV} e_{AV}} - k_{SA-AV}(x_3 - x_1^{\tau_{SA-AV}}) - k_{HP-AV}(x_3 - x_5^{\tau_{HP-AV}}) \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= F_{HP}(t) - \alpha_{HP} x_6 (x_5 - v_{HP_1})(x_5 - v_{HP_2}) - \frac{x_5(x_5+d_{HP})(x_5+e_{HP})}{d_{HP} e_{HP}} - k_{SA-HP}(x_5 - x_1^{\tau_{SA-HP}}) - k_{AV-HP}(x_5 - x_3^{\tau_{AV-HP}}),
\end{aligned}
\tag{4}
$$

where $F_{SA}(t)$, $F_{AV}(t)$ and $F_{HP}(t)$ are harmonic external forces of the type $F(t) = \rho \sin(\omega t)$; k_{AV-SA}, k_{HP-SA}, k_{SA-AV}, k_{HP-AV}, k_{SA-HP} and k_{AV-HP} are coupling constants; $x_i^\tau = x_i(t-\tau)$, with τ representing the delay; $i = 1, \dots, n$, where n is the dimension of the system. Note that the coupling terms have time delay, x_i^τ, which represents the time necessary for the transmission of signals between different regions of the heart.

Gois & Savi (2009) suggested that the ECG signal is formed by the composition of individual signals of the oscillators, and its representation can be done by a linear combination of each oscillator signal as follows:

$$X = ECG = \beta_0 + \beta_1 x_1 + \beta_2 x_3 + \beta_3 x_5. \tag{5}$$

And, similarly, is defined:

$$\dot{X} = \frac{d(ECG)}{dt} = \beta_1 x_2 + \beta_2 x_4 + \beta_3 x_6. \tag{6}$$

3.1 ECG signals

This section deals with numerical simulations of the proposed model showing its capacity to describe some typical ECG signals. Our main goal is to show a qualitative agreement with experimental ECG signals, especially the normal and some pathological signal related to chaotic behavior.

In all simulations time steps are defined as $\Delta t = {}^{2\pi}/_{N_M \omega}$, with $N_M \geq 150$ and $N_P = 30000$. Moreover, the following initial conditions are adopted: $[\,x_1(0) \quad x_2(0) \quad x_3(0) \quad x_4(0) \quad x_5(0) \quad x_6(0)]$ = [-0.1 0.025 -0.6 0.1 -3.3 10/15].

3.1.1 Normal ECG

Normal ECG is now in focus by considering the following parameters: $\alpha_{SA} = 3$, $v_{SA1} = 1$, $v_{SA2} = -1.9$, $d_{SA} = 1.9$, $e_{SA} = 0.55$, $\alpha_{AV} = 3$, $v_{AV1} = 0.5$, $v_{AV2} = -0.5$, $d_{AV} = 4$, $e_{AV} = 0.67$, $\alpha_{HP} = 7$, $v_{HP1} = 1.65$, $v_{HP2} = -2$, $d_{HP} = 7$, $e_{HP} = 0.67$, $\beta_0 = 1\,mV$, $\beta_1 = 0.06\,mV$, $\beta_2 = 0.1\,mV$, $\beta_3 = 0.3\,mV$, $k_{SA-AV} = 3$, $k_{AV-HP} = 55$, $\tau_{SA-AV} = 0.8$, $\tau_{AV-HP} = 0.1$. Note that this is related to a conceptual model with unidirectional couplings as presented in Figure 2.

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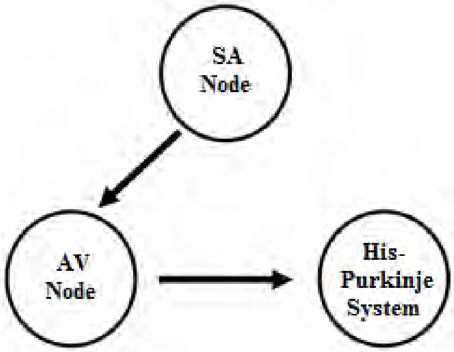

Figure 2. Conceptual model of the normal ECG.

Figure 3 shows the comparison between numerical simulations and an experimental normal ECG obtained from the database "*Physionet*" (www.physionet.org/physiobank/database/#ecg). It is noticeable that numerical ECG captures the general behavior of normal ECG, showing good agreement with real data. Furthermore, analyzing the detail of a cardiac cycle, shown also in Figure 3Figure , it is observed that numerical ECG presents the three basic waves: P wave, QRS complex and T wave.

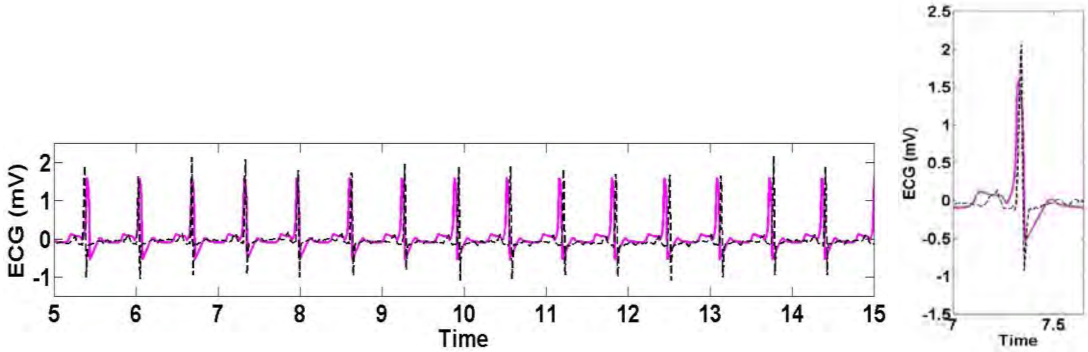

Figure 3. Comparison between numerical and experimental data of the normal ECG.

Figure 4 presents two-dimensional projections of Poincaré sections of normal ECG. The regular behavior is observed pointing to a quasi-periodic response due to the closed curve aspect.

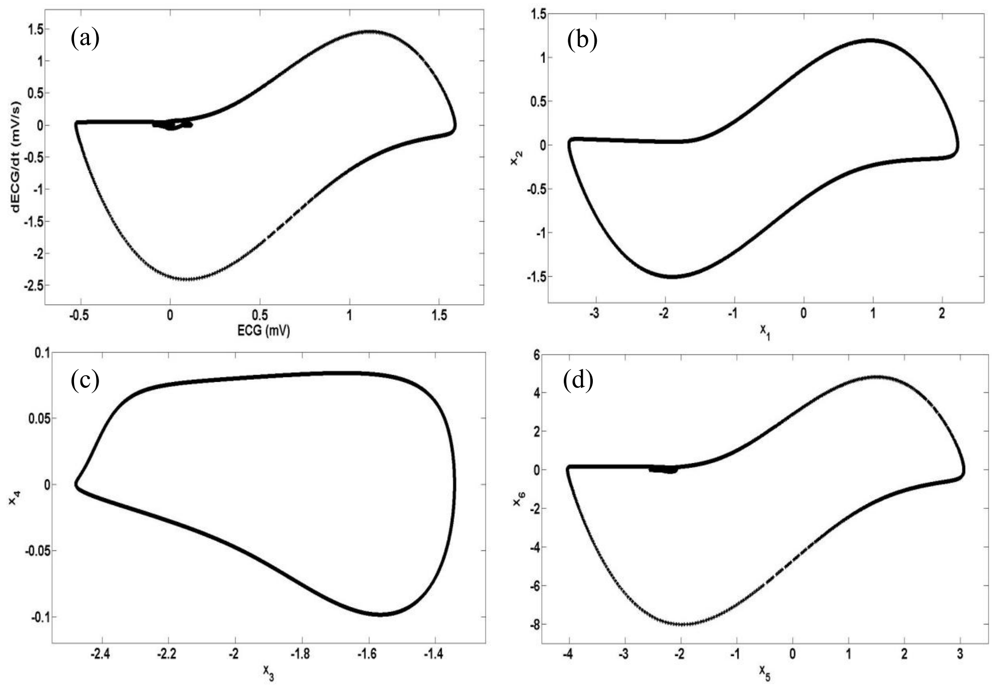

Figure 4. Poincaré section projections related to normal ECG: (a) ECG, (b) SA node, (c) AV node and (d) His-Purkinje system.

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The estimation of Lyapunov exponents furnishes the following final values: $\lambda = (0 \quad 0 \quad -0.2 \quad -0.2 \quad -4.5 \quad -4.6)$. This result confirms the quasi-periodic response observed in the Poincaré section.

3.1.2 ECG with Ventricular Fibrillation

Ventricular fibrillation is a severe cardiac arrhythmia usually associated with chaotic and irregular ventricular contraction (Dubin, 1996). This behavior causes a lack of synchronization necessary for the proper functioning of the heart. This pathological behavior of the heart is critical being responsible to death. ECG related to ventricular fibrillation has irregular aspect as shown by experimental data in Figure 5. It is noticeable the irregular pattern that can vary from different measures.

(www.geicpe-tripod-com)

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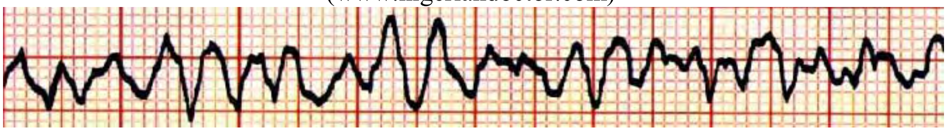

(www.ecg.med.br)

Figure 5. Experimental ECGs of patients with ventricular fibrillation.

In order to reproduce the ECG with ventricular fibrillation, it is considered the conceptual model shown in Figure 6. The model parameters are similar to the one used for normal ECG, except for the parameters related to the SA node (Ferreira *et al.*, 2011): $\alpha_{SA} = 0.5$, $v_{SA1} = 0.97$, $v_{SA2} = -1$, $d_{SA} = 3$ and $e_{SA} = 6$. Besides, external excitation is considered by assuming the following parameters: $\rho_{AV} = 2.5$, $\omega_{AV} = 1.9$; $\rho_{HP} = 5$, $\omega_{HP} = 1.9$; $\rho_{SA} = 2.5$, $\omega_{SA} = 1$.

Figure 7 shows numerical simulations related to the ventricular fibrillation ECG. Note the irregular pattern that properly represents the qualitative behavior of the experimental ECGs. Phase space and the Poincaré sections are presented in Figures 8 and 9. It is clear the chaotic-like structure of results showing an irregular behavior and Poincaré sections with fractal-like structure.

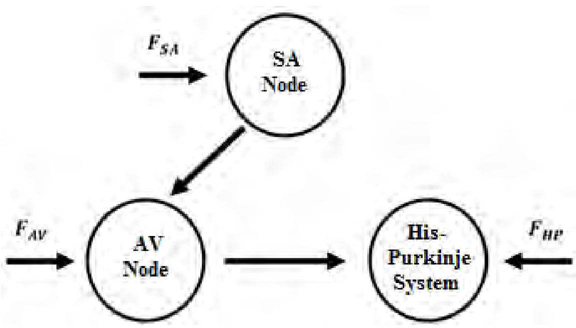

Figure 6. Conceptual model of the ECG with ventricular fibrillation.

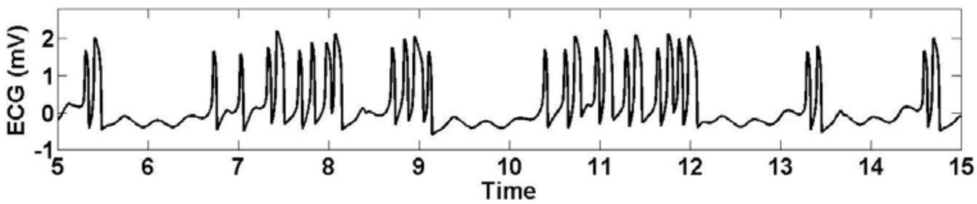

Figure 7. Numerical simulation of the ECG related to ventricular fibrillation.

The estimation of Lyapunov exponents is an important tool to assure the chaotic behavior of the system: $\lambda = (+0.2 \quad +0.2 \quad -0.4 \quad -0.4 \quad -2.1 \quad -2.1)$. Note that there are two positive values that confirm the chaotic nature of this pathology.

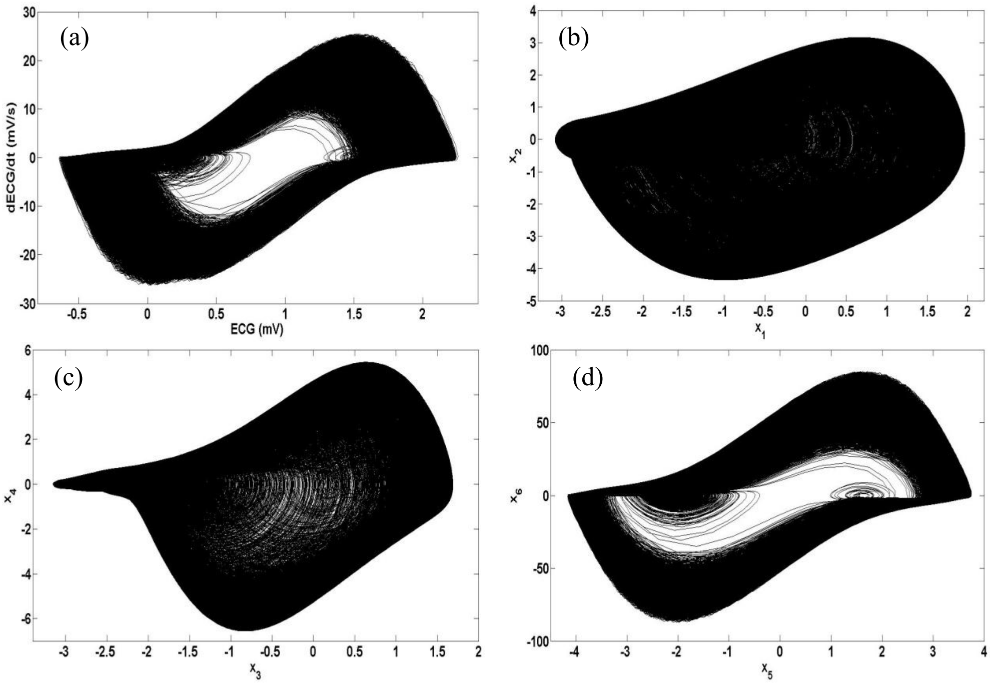

Figure 8. Phase space projections related to ECG with ventricular fibrillation: (a) ECG, (b) SA node, (c) AV node and (d) His-Purkinje system.

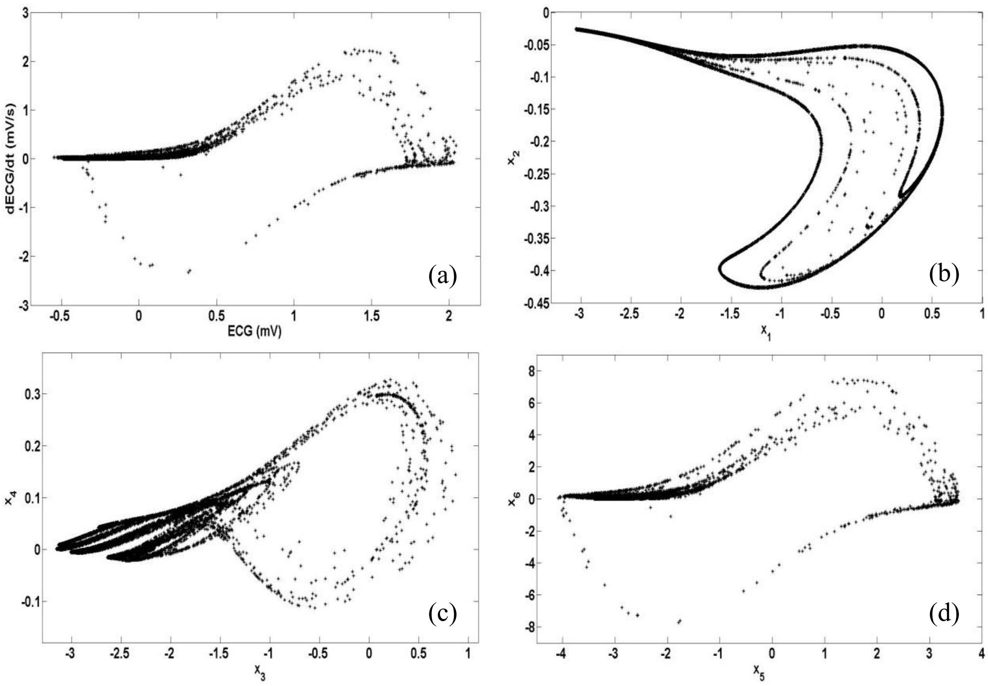

Figure 9. Poincaré section projections related to ECG with ventricular fibrillation: ECG (a), (b) SA node, (c) AV node and (d) His-Purkinje system.

4. CHAOS CONTROL APPLIED TO CARDIAC SYSTEM

The ETDF approach is now applied to the heart rhythms. The goal is to control the ventricular fibrillation ECG. Under this assumption, we consider system perturbations that avoid this pathology. The control action is included in the natural pacemaker, the SA node. The most interesting idea would be to stabilize an UPO related to the normal ECG. Nevertheless, we choose a period-2 UPO in order to observe the general behavior of the controlled system, trying to avoid critical pathological behavior of the cardiac system.

The control action, represented by $C(t, x_2)$, is applied at the SA node (natural pacemaker) and the system dynamics is governed by the following equations.

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$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = F_{SA}(t) - \alpha_{SA} x_2 (x_1 - v_{SA_1})(x_1 - v_{SA_2}) - \frac{x_1(x_1 + d_{SA})(x_1 + e_{SA})}{d_{SA} e_{SA}} - k_{AV-SA}(x_1 - x_3^{\tau_{AV-SA}}) - k_{HP-SA}(x_1 - x_5^{\tau_{HP-SA}}) + C(t, x_2)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = F_{AV}(t) - \alpha_{AV} x_4 (x_3 - v_{AV_1})(x_3 - v_{AV_2}) - \frac{x_3(x_3 + d_{AV})(x_3 + e_{AV})}{d_{AV} e_{AV}} - k_{SA-AV}(x_3 - x_1^{\tau_{SA-AV}}) - k_{HP-AV}(x_3 - x_5^{\tau_{HP-AV}})$$
$$\dot{x}_5 = x_6$$
$$\dot{x}_6 = F_{HP}(t) - \alpha_{HP} x_6 (x_5 - v_{HP_1})(x_5 - v_{HP_2}) - \frac{x_5(x_5 + d_{HP})(x_5 + e_{HP})}{d_{HP} e_{HP}} - k_{SA-HP}(x_5 - x_1^{\tau_{SA-HP}}) - k_{AV-HP}(x_5 - x_3^{\tau_{AV-HP}}) \quad (7)$$

The controller adopted a wait time approach to start its actuation. This means that the actuation only starts when the system visits the neighborhood of the desired UPO. This is a standard procedure for discrete chaos control methods (De Paula & Savi, 2012) and presents good results in cardiac systems (Ferreira *et al.*, 2009).

After UPO identification, it is necessary to define controller parameters, K and R, which is done by the calculation of maximum Lyapunov exponents for each desired UPO, in this case a period-2 UPO. Figure 10 shows the identified orbit through the two-dimensional projections of the phase space. Figure 11 presents the maximum Lyapunov exponents, considering $\tau = 2\,(2\pi/\omega)$, corresponding to the periodicity 2. Note that there are regions associated with negative values of Lyapunov exponents, where it is possible to stabilize UPOs with proper choices of parameters R and K.

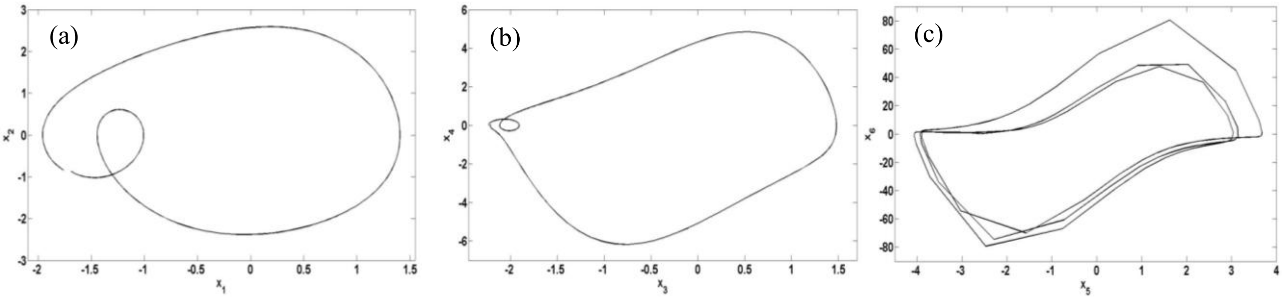

Figure 10. Identified period-2 UPO: (a) SA node, (b) AV node and (c) His-Purkinje system.

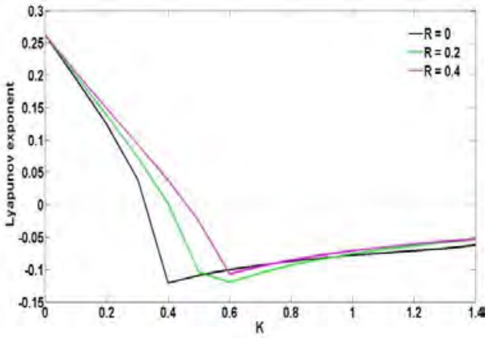

Figure 11. Period-2 UPO: maximum Lyapunov exponents.

The controller performs the stabilization of the period-2 UPO adopting $R = 0$ and $K = 0.4$. Figure 12 shows the uncontrolled (ventricular fibrillation - dashed black line) and the controlled (pink line) ECGs. Figure 13 shows the same behavior by observing phase space projections. Figure 14 presents the control action imposed to the system. It is observed that it is possible to minimize the effects of ventricular fibrillation in chaotic cardiac system using small perturbations.

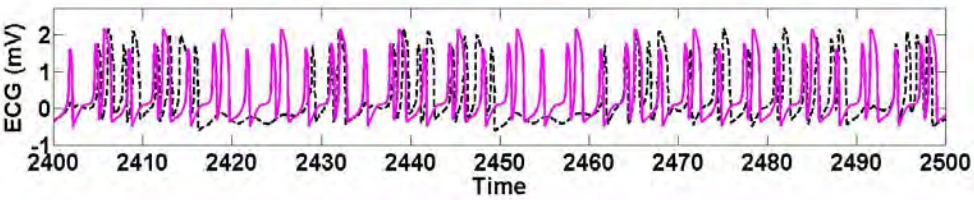

Figure 12. ECG related to ventricular fibrillation and the stabilization of a period-2 UPO: uncontrolled (dashed black line) and controlled (pink line) responses using $\mathbf{R = 0}$ and $\mathbf{K = 0.4}$.

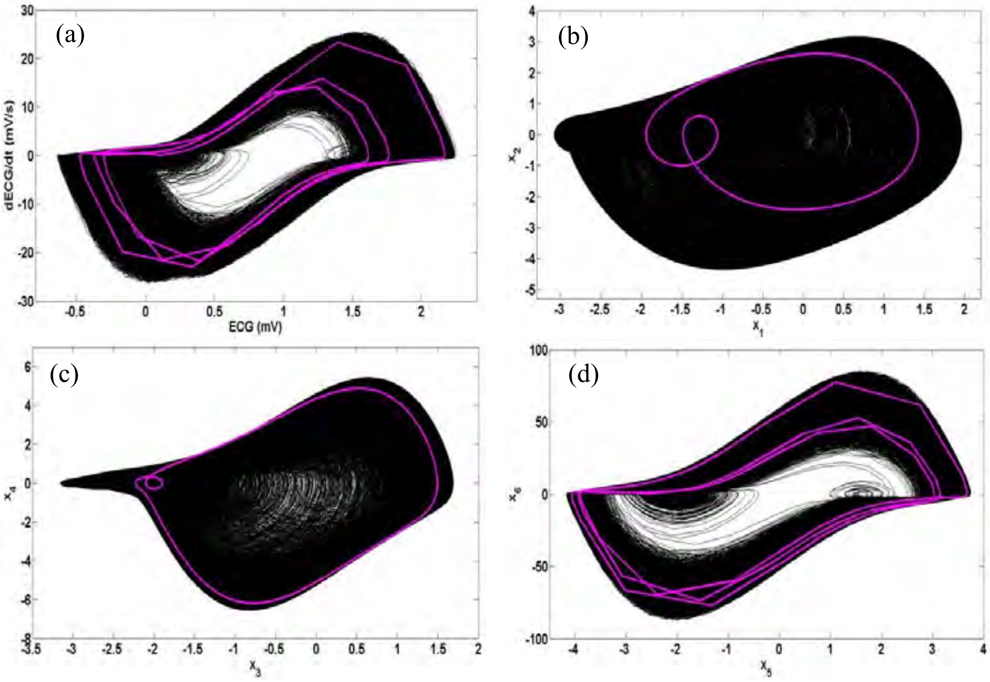

Figure 13. Phase space projection of the ECG related to ventricular fibrillation and the stabilization of a period-2 UPO: uncontrolled (black line) and controlled (pink line) responses using $\mathbf{R = 0}$ and $\mathbf{K = 0.4}$. (a) ECG, (b) SA node, (c) AV node and (d) His-Purkinje system.

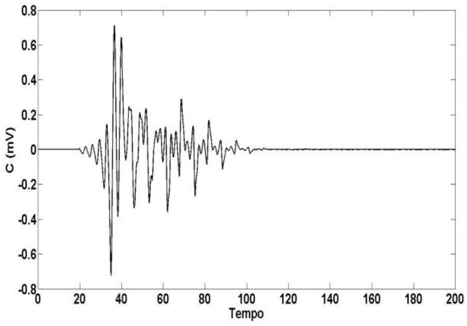

Figure 14. Control action for the stabilization of a period-2 UPO.

4.1 Chaos suppression

The stabilization of an UPO embedded in chaotic attractor is very convenient since this orbit belongs to system dynamics and, therefore, its stabilization requires less controller effort. Nevertheless, there are some situations where this is not possible. In these cases, chaos suppression is an interesting alternative in order to avoid critical pathological behavior of the cardiac system. The major difference between both cases is that chaos suppression is associated with larger control efforts. This procedure evades the central idea of chaos control that uses small perturbations but it is useful for health issues.

Let us employ some controller parameters that can promote control without think in terms of UPOs. Under this assumption, the following parameters are employed: $R = 0.8$ and $K = 2$. Figure 15 shows the ECG record of the uncontrolled (ventricular fibrillation - in black) and controlled (in pink) responses. Figure 16 shows the phase space projections related to both responses while Figure 17 presents the control action.

Obtained results show that controller is able to suppress the chaotic behavior of the ventricular fibrillation. However, it should be highlighted that the stabilized orbits are not related to natural orbits and, as a consequence, controller efforts are greater than situations where orbits that belong to system dynamics are stabilized.

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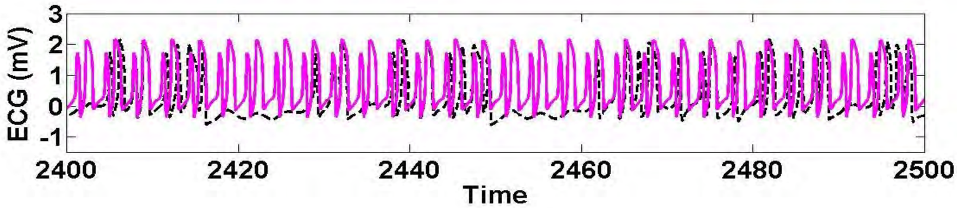

Figure 15. ECG related to ventricular fibrillation and the chaos suppression: uncontrolled (dashed black line) and controlled (pink line) responses using $\mathbf{R = 0.8}$ and $\mathbf{K = 2}$.

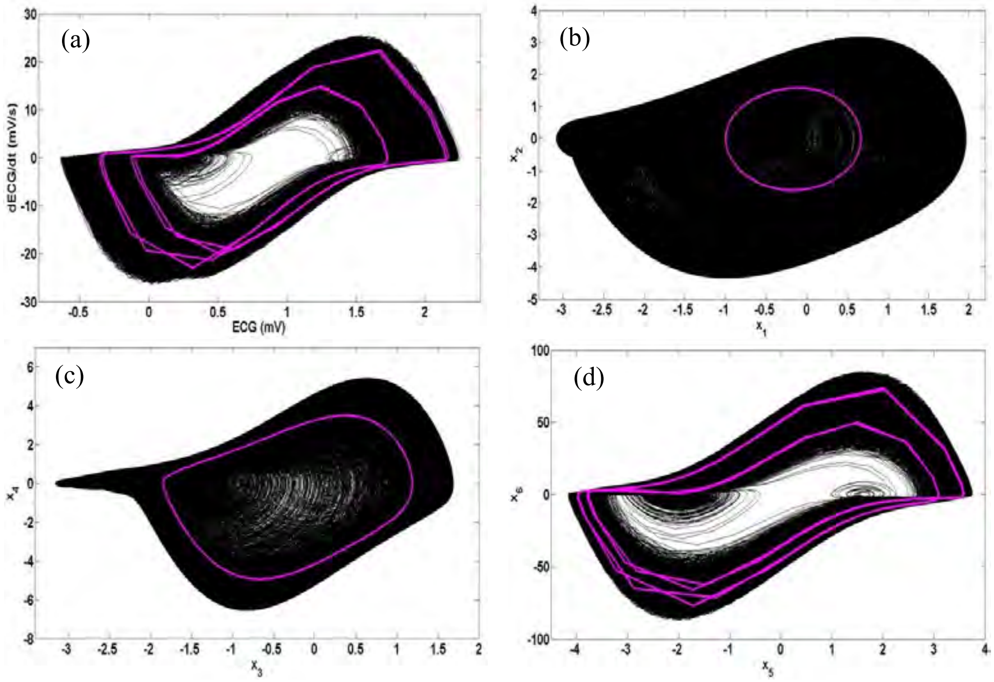

Figure 16. Phase space projections related to ventricular fibrillation and the chaos suppression: uncontrolled (black line) and controlled (pink line) responses using $\mathbf{R = 0.8}$ and $\mathbf{K = 2}$. (a) ECG, (b) SA node, (c) AV node and (d) His-Purkinje system.

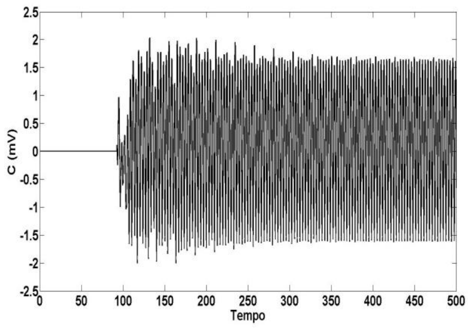

Figure 17. Control action for the chaos suppression using $\mathbf{R = 0.8}$ and $\mathbf{K = 2}$.

5. CONCLUSION

This work deals with the application of the extended time delayed feedback chaos control method to cardiac system modeled as a three-coupled oscillator, connected with time delay couplings. Cardiac behavior is analyzed by considering ECG signals and two different situations are treated: normal and ventricular fibrillation, which is associated with chaos being a critical pathological behavior of the heart. The basic idea is to employ ETDF method to avoid this critical behavior. We stabilize some UPOs embedded in the chaotic attractor, analyzing the resulting ECG. In general, it is possible to say that the ETDF is successful applied generating less critical behaviors of the heart with small control efforts. An alternative approach is also investigated in order to suppress chaos with higher control efforts, by stabilizing orbits that do not belong to system dynamics. Thus, the application of ETDF method is an interesting approach to avoid critical behaviors as ventricular fibrillation with small control efforts. Situations where it does not succeed can be easily solved by increasing the control effort, which leads to the suppression of chaotic response.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Auerbach, D., Predrag, C., Eckmann, J. P., Gunaratne, G. & Procaccia, I., 1987, "Exploring Chaotic Motion Through Periodic Orbits", *Physical Review Letters*, v. 58, n. 23, pp. 2387-2389.

Christini, D. J., Stein, K. M. Markowitz, S. M., Mittal, S., Slotwiner, D. J., Scheiner, M. A., Iwai, S. & Lerman, B. B., 2001, "Nonlinear-Dynamical Arrhythmia Control in Humans", *Medical Sciences*, v. 98, n. 10, pp. 5827-5832.

Cunningham, W. J., 1954, "A Nonlinear Differential-Difference Equation of Growth", *Mathematics*, v. 40, pp. 708-713.

De Paula, A. S. & Savi, M. A., 2009a, "Controlling Chaos in a Nonlinear Pendulum Using an Extended Time-Delayed Feedback Method", *Chaos, Solitons and Fractals*, v. 42, pp. 2981-2988.

De Paula, A. S. & Savi, M. A., 2009b, "A Multiparameter Chaos Control Method Based on OGY Approach", *Chaos, Solitons and Fractals*, v.40, n.3, pp.1376-1390.

De Paula, A. S., Savi, M. A., Wiercigroch, M., Pavlovskaia, E., 2012, "Bifurcation Control of a Parametric Pendulum", *International Journal of Bifurcation and Chaos in Applied Sciences and Engineering*, v.22, n.5, pp.1250111-1250111.

Dubin, D., 1996, "*Interpretação Rápida do ECG...Um Curso Programado*", Rio de Janeiro: Ed. de Publicações Científicas, v.1, 295 p.

Farmer, J.D., 1982, "Chaotic Attractors of an Infinite-Dimensional Dynamical System", *Physica 4D*, pp. 366-393.

Ferreira, B. B., De Paula, A. S. & Savi, M. A., 2011, "Chaos Control Applied to Heart Rhythm Dynamics", *Chaos, Solitions and Fractals*, v. 44, pp. 587-599.

Garfinkel, A., Spano, M. L., Ditto, W. L. & Weiss, J. N., 1992, "Controlling Cardiac Chaos", *Science*, v. 257, pp. 1230-1235.

Garfinkel, A., Weiss, J. N., Ditto, W. L. & Spano, M. L., 1995, "Chaos Control of Cardiac Arrhythmias", *Trends in Cardiovascular Medicine*, v. 5, n. 2, pp. 76-80.

Glass, L., Guevara, M. R., Shrier, A. & Perez, R., 1983, "Bifurcation and Chaos in a Periodically Stimulated Cardiac Oscillator", *Physica 7D*, pp. 89-101.

Glass, L., Goldberger, A. L., Courtemanche, M. & Shrier, A., 1987, "Nonlinear Dynamics, Chaos and Complex Cardiac Arrhythmias", *Proceedings of the Royal Society of London*, v. 413, n. 1844, pp. 9-26.

Gois, S. R. S. M. & Savi, M. A., 2009, "An Analysis of Heart Rhythm Dynamics Using a Three-Coupled Oscillator Model", *Chaos, Solitons and Fractals*, v. 41, pp. 2553-2565.

Grudzinski, K. & Zebrowski, J. J., 2004, "Modeling Cardiac Pacemakers with Relaxation Oscillators", *Physica A*, v. 336, pp. 153-162.

Kaplan, D. T. & Cohen, R. J., 1990, "Is Fibrillation Chaos?", *Circulation Research*, v. 67, pp. 886-892.

Mensour, B. & Longtin, A., 1997, "Power Spectra and Dynamical Invariants for Delay-Differential and Difference Equations", *Physica D*, v. 113, pp. 1-25.

Ott, E., Grebogi, C. & Yorke, J. A., 1990, "Controlling Chaos", *Physical Review Letters*, v. 64, n. 11, pp. 1196-1199.

Pyragas, K., 1992, "Continuous Control of Chaos by Self-Controlling Feedback", *Physics Letters A*, v. 170, pp. 421-428.

Savi, M. A., 2005, "Chaos and Order in Biomedical Rhythms", *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, v. 27, n. 2, pp. 157-169.

Santos, A. M., Lopes, S. R. & Viana, R. L., 2004, "Rhythm Synchronization and Chaotic Modulation of Coupled Van der Pol Oscillators in a Model for the Heartbeat", *Physica A*, v. 338, pp. 335-355.

Socolar, J. E. S, Sukow, D. W. & Gauthier, D. J., 1994, "Stabilizing Unstable Periodic Orbits in Fast Dynamical Systems", *Physical Review E*, v. 50, n. 4, pp. 3245-3248.

Stein, K. M., Walden, J., Lippman, N. & Lerman, B. B., 1999, "Ventricular Response in Atrial Fibrillation: Random or Deterministic?", *American Journal of Physiology - Heart and Circulatory Physiology*, v. 277, pp. H452-H458.

Van der Pol, B. & Van der Mark, J., 1928, "The Heartbeat Considered as a Relaxation Oscillator and an Electrical Model of the Heart", *Philosophical Magazine*, v.6, pp.763-775.

Wolf, A., Swift, J.B., Swinney, H.L. & Vastano, J.A., 1985, "Determining Lyapunov Exponents from a Time Series", *Physical D*, v. 16, pp. 285-317.

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