ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

COMPUTATIONAL ALGORITHM BASED ON ALTINTAS MODEL FOR CUTTING DYNAMICS OF END MILLING PROCESS

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Abstract. *The study of cutting dynamics is important to predict regenerative vibration to avoid tool breakage and bad surface quality. In this study the critical stability condition of an end milling operation is simulated based on Altintas Model for dynamics of metal cutting. This analytical model considers the milling tool and the workpiece as system of two degrees of freedom. The required routine calculates the depth of cut that defines the stable and unstable region for all the possible spindle speed disposable on the machine. Using all the results for the limit depth of cut is possible to build the diagram of stability lobes. The algorithm considers as input parameters: the tool geometry, workpiece material, specific cutting forces and dynamic coefficients of the tool. Using the algorithm it is possible to evaluate the contribution of each parameter to reduce chatter and in this study some parameter contribution are analyzed by comparing the different lobes diagrams. Material removal rate is considered to compare different situations for the exit angle. For the specific parameters described, it was possible to find one value for the exit angle, and consequently the width of cut, that maximizes the removal rate without chatter.*

Keywords: *Metal cutting, End milling, Chatter, Vibration stability*

1. INTRODUCTION

The manufacturing scientific research usually focus topics that involves the analysis of materials, tool wear, geometry, mechanics and dynamics of the process. Several machining operations are executed by a large variety of machine-tools in order to produce all needed shapes and tolerances. Being under the desired tolerances is a target for the machine tools to produce non rejected pieces. Machined pieces presents dimensional or geometry error as a function of the inherent process and also due to vibrations and misplacing of tool and workpiece. Controlling the workpiece is a problem when chatter appears due to low stiffness of tool-machine-tool system or workpiece specially when using high cutting velocities values, as in high speed milling - HSM (Patel *et al.*, 2008).

The chatter, or self-excited vibration, is one of the greater limiters of the metal cutting processes since the increase in the amount of removed material can induce instabilities that may forego the tool to breakage. The presence of this type of vibration in processes like milling may cause bad superficial workpiece quality, low dimensional tolerance, decrease in the life cycle of the tool, or even damage in the bearing of the machine. Thus, the study and modeling of the chatter are essential for the tool optimization and increase of productivity in machining industry (Araujo *et al.*, 2009).

Tobias (1965) and Koenigsberger and Tlusty (1967) were first researchers to publish analytical models explaning the regenerative mechanism of chatter and its prediction for the case of orthogonal cut. Afterwards Altintas and Budak (1995) developed the analytical model prediction of chatter for the two degree of freedom and applied to end milling process. The model considered the average cutting forces and it was able to achieve the prediction of chatter conditions valideted with experiments. The analytical model however did not reach good results for the prediction when radial depth of cut was small compared to the milling tool diameter. The stability of the milling process was investigated by Zhao and Balachandran (2001), and Insperger *et al.* (2003), by considering single and two degree of freedom systems for different experimental conditions and the mathematical model is represented by delay-differential equations (Moon, 1998).

The objective of this paper is to develop an algorithm in MATLAB software to generate the diagram of lobes of the end milling processes using Altintas model. Using this graphic tool, it is possible to analyze the influence of the parameters of the processes in this stability. The control of the stability can be done not only by choosing an optimal situation in the diagram, but also by choosing the correct input parameters which to stabilize the system. In this study it is presented a comparison with different values for these parameters and the results are compared to a reference.

2. ALTINTAS MODEL FOR CHATTER CONSIDERING TWO DEGREES OF FREEDOM

The end milling tool is dynamically similar to a cylindrical beam fixed in one extremity by the tool-holder and forced on the other side by the contact with the workpiece under cutting process. The dynamic of the system composed by the end milling tool, tool-holder, machine-tool and the workpiece can present complex nonlinear behavior associated to extreme conditions, as chatter, which can severely affect the quality of the machined workpiece. Modeling can be very useful for establish optimal operational conditions. Several authors proposed simple models to study tool dynamics during machining using one (Insperger *et al.*, 2003) or two degrees of freedom mass-spring-dashpot mechanical system models (Altintas and Budak, 1995).

When the tool is considered rigid, the tangential and radial forces acting in tooth j can be written as function of the uncut chip thickness $h(t)$, the axial depht of cut a, the tangential specific cutting pressure K_t and the radial cutting constant k_r as can be noted in in Eq 1:

$$F_{tj}(t) = K_t.a.h(t) = K_t.a.f_t.\sin(\phi(t)), \qquad F_{rj}(t) = k_r.F_{tj}, \tag{1}$$

Note that the chip thickness for end milling is normally described by Martellotti equation as a function of f_t the feed per tooth and the spindle angle ϕ. The forces can be decomposed in the x and y directions as:

$$F_{xj}(t) = -F_{tj}(t).cos\phi_j - F_{rj}(t).sin\phi_j, \qquad F_{yj}(t) = +F_{tj}(t).sin\phi_j - F_{rj}(t).cos\phi_j, \tag{2}$$

where ϕ_j is the position angle of tooth j and it vary according to the spindle speed n, as can be seen in Fig. 1. The cutting force is a function of time acting in the end milling tool is the sum of the contribution of each tooth:

$$F_x(t) = \sum_{j=0}^{N-1} F_{xj}(\phi_j, t); \quad F_y(t) = \sum_{j=0}^{N-1} F_{yj}(\phi_j, t). \tag{3}$$

Depending on the volume of material removed, the force can be distributed differently in one revolution. The material removal rate, or MRR, represents the volume of material removed by unit of time. The MMR is an usual parameter of control and optimization of the time of production. For the milling process the material removal rate is described for Cheng (2009) as:

$$MRR = a.b.n.N.f_t, \tag{4}$$

Where b is the radial depth of cut, n the spindle speed and N the number of teeth.

When the tool can no longer be considered rigid, it is necessary to analysis the dynamic of the cut in order to study the stability of the system. Altintas and Budak (1995) modeled the cutting forces as can be seen in Fig. 1. The end milling tool is assumed to have N number of teeth with a zero helix angle. In the figure k_x, k_y, c_x and c_y are the modal parameters of the mass-spring-dashpot system in the x and y direction, and dF_{tj} and dF_{rj} are the tangential and radial cutting force respectively.

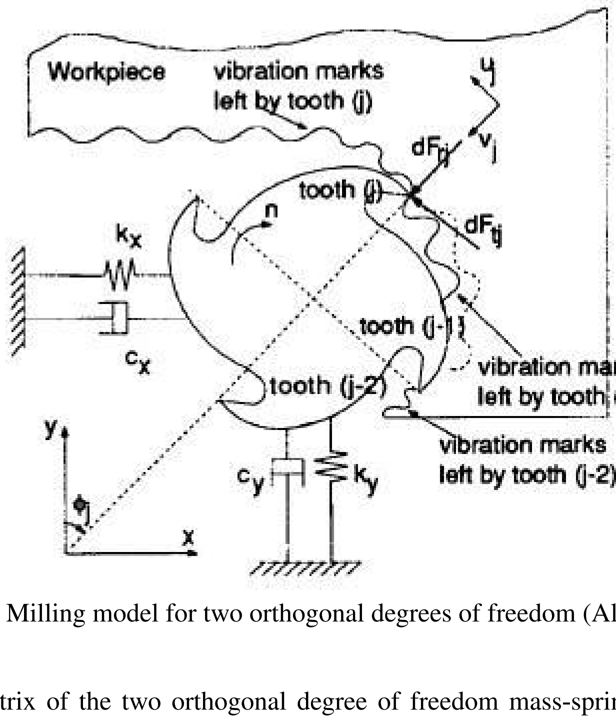

Figure 1. Milling model for two orthogonal degrees of freedom (Altintas, 2000).

The transfer function matrix of the two orthogonal degree of freedom mass-spring-dashpot mechanical system is trivial as it has only the diagonal terms G_{xx} and G_{yy}. The transfer function matrix then is obtained by doing the laplace transform in the caracteristic diferential equation as showed in the folowing steps:

$$m\ddot{x} + c_x\dot{x} + k_x x = F_x, \tag{5}$$

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$$m\ddot{y} + c_y\dot{y} + k_y y = F_y, \tag{6}$$

where m is the mass, x and y are the current dynamic displacement of the cut in the time domain. In the frequence domain, the transfer function $\Phi(s)$ is write as

$$\Phi(s)_{xx} = \frac{X(s)}{F_x(s)} = \frac{1}{(ms^2 + c_x s + k_x)} = \frac{\omega_{nx}^2}{k_x(s^2 + 2\xi_x\omega_{nx}s + \omega_{nx}^2)}, \tag{7}$$

$$\Phi(s)_{yy} = \frac{Y(s)}{F_y(s)} = \frac{1}{(ms^2 + c_y s + k_y)} = \frac{\omega_{ny}^2}{k_y(s^2 + 2\xi_y\omega_{ny}s + \omega_{ny}^2)}, \tag{8}$$

where s is the Laplace variable, $X(s)$ and $Y(s)$ are the current displacement in the frequence domain, ω_{nx} and ω_{ny} are the natural frequence in the x and y direction, ξ_x and ξ_y are the system damping ratio in the x and y direction.

The last step in order to study the stability of the system is to obtain it caracteristic equation by developing a expression for the chip thickness h, that in the non rigid tool model it is a time varying variable. In his work, Altintas and Budak (1995) shows that the chip thickness can be expressed by the static chip thickness h_0, the difference between the previous and present dynamic displacement $\{\Delta x \quad \Delta y\}^T$ left by the tooth $j-1$ and j respectively, and the step funtion $g(\phi_j)$ which determines whether the thooth is in or out of cut as show in Eq.9.

$$g(\phi_j) = \begin{cases} 1, & \text{se} \quad \phi_s < \phi_j < \phi_e, \\ 0, & \text{se} \quad \phi_j < \phi_s \quad ou \quad \phi_j > \phi_e, \end{cases} \tag{9}$$

where ϕ_s is the entry angle of the cut and ϕ_e the exit angle, and they can be seem in Fig. 2.

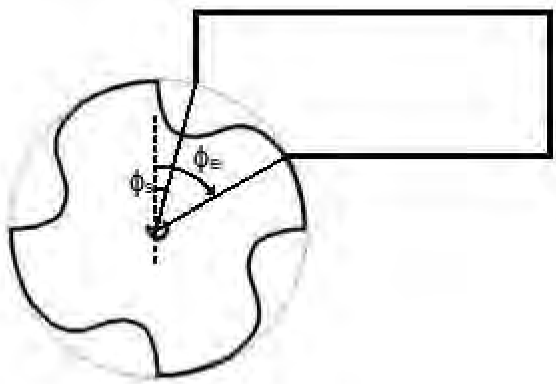

Figure 2. Entry and exit angles of the cut.

To simplify, the static chip thickness h_0 is removed from the expression as it does not influence the regenerative mecanism, so only the dynamic chip thickness $h(\phi_j)$ is considered for chatter analysis

$$h(\phi_j) = [\Delta x sin\phi_j + \Delta y cos\phi_j]g(\phi_j). \tag{10}$$

With the cutting force model, the transfer function matrix and the time varying chip thickness relation, Altintas achieved in his work the caracteristic equation of the chatter regenerative mecanism which is

$$\{F(s)\} = \frac{1}{2}aK_t[1 - e^{-sT}][A_0][\Phi(s)]\{F(s)\}, \tag{11}$$

where T is the tooth passing period and A_0 is the average matrix of directional factors. The caracteristic equation only has a non trivial solution when the determinant is zero.

$$det = \left[[\,I\,] - \frac{1}{2}K_t a(1 - e^{-sT})[A_0][\Phi(s)]\right] = 0. \tag{12}$$

Using this approach and through the study of the stability of the Laplace constant done by Altintas, it is possible to calculate the depth of cut limit a_{lim} for no chatter on a specific spindle rotations speed by using the Eq. 13 and Eq. 14 respectively:

$$a_{lim} = -\frac{2\pi\Lambda_R}{N.K_t}(1 + \kappa^2), \tag{13}$$

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$$T = \frac{1}{\omega_c}(\epsilon + 2k\pi) \longrightarrow n = \frac{60}{NT}. \quad (14)$$

where Λ_R is the real part of the eigenvalue, $\kappa = \Lambda_R/\Lambda_I$ is the phase of the eigenvalue, k is a integer, $\epsilon = \pi - 2\psi$ is the phase shift between present and previous vibration marks and, where $\psi = tan^{-1}\kappa$.

The stability lobes diagram is the graphic that presents a_{lim} in function of the spindle speed rotation n of each lobe k of the Eq. 14 as show in Fig. 3. For values of depht of cut a above the found for the stability limit a_{lim}, the system is instable and chatter occurs. For values below, the system is found in the meter of stability and chatter does not occurs.

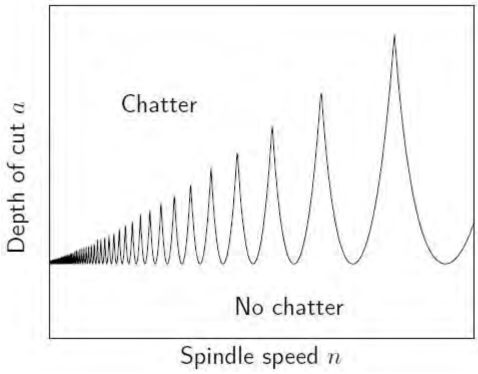

Figure 3. Diagram of stability lobes (Faassen, 2007).

3. NUMERICAL SIMULATIONS

The stability lobe diagram is plotted using the equations presented. For the analytical prediction of chatter, it is necessary a programming routine using known parameters k_y, ω_n and ξ, and the diagram may be plotted following the steps described in Altintas (2000):

- Select a frequency of chatter of the transfer function close to a dominating vibration mode.
- Solve the eigenvalues equation.
- Calculate the depth of cut limit a_{lim} (Eq. 13).
- Calculate rotation of the tool for each stability lobe k. (Eq. 14)
- Repeat the procedure for other chatter frequencies close to the dominating vibration modes of the structure, made evident by the transfer function.

The necessary algorithm to run this routine was developed utilizing MATLAB software.

3.1 Case study

The objective of the case study is to analyze the influence of each parameter of the process on the diagram of lobes built by the algorithm. For this, a group of input data was used in conjunct with the standard data for comparison. This way, altering the values of the input parameters it is possible to compare both diagrams, standard and altered, and analyzing which influence this parameter exercise on the dynamic stability of the system. Parallel to the influences on stability, some other focus of interest also will be considered to determine parameters on input, such as strength of cut and material removal rate (MRR).

The table 1 presents the information of all data of standard input, as well as the changes that will be done for the study case. The altered input parameters will be the number of teeth in the milling tool N, the workpiece material, the stiffness of the set machine-tool, the damping of the set machine-tool, the enter angle ϕ_s and the exit angle ϕ_e.

Using the standard input parameters of the simulation it is obtained a diagram of lobes conform the Fig. 4. In this base model it is important to note that the depth of cut for any spindle speed is found around 1.5 mm and the last two spindle speed where it can be obtained a value of a_{lim} much above the stable depth of cut for all rotations are 5000 RPM and 9000 RPM approximately.

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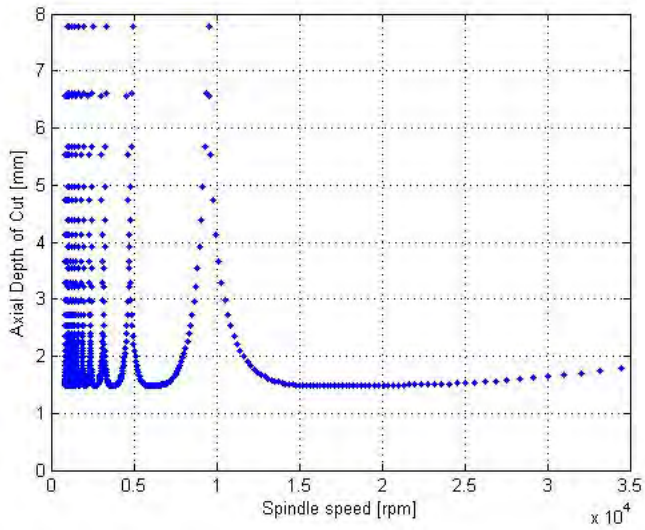

Figure 4. Stability lobes diagram using standard parameters (Reference values of Table 1)

Table 1. Standard input data and variation data 1 and 2.

Input Parameter	Reference Value	Option 1	Option 2
Number of teeth N	4 teeth	8 teeth	2 teeth
Workpiece material	Aluminium	Carbon steel 1095	-
K_r	0.07	0.67	-
K_t	600 MPa	2500 MPa	-
Dynamic Proprieties			
k_x	5.59×10^6 N/m	6.7×10^6 N/m	-
k_y	5.71×10^6 N/m	6.85×10^6 N/m	-
ω_{nx}	593.75 Hz	650 Hz	-
ω_{ny}	675 Hz	740 Hz	-
ξ_x	3.9×10^{-2}	4.68×10^{-2}	-
ξ_y	3.5×10^{-2}	4.2×10^{-2}	-
Entry angle ϕ_s	0°	18°	-
Exit angle ϕ_e	60°	90°	180°

a) Tools with different number of teeth

There is the possibility of work with mills with different number of teeth or inserts available in the market, and the decision of choice of tool to be used in the process may be taken based on the stability of the system. For the comparison were chosen two values of N distinct of the standard.

As can be seen in the Fig. 5, the multiplication of the number of teeth for an arbitrary value has an inversely proportional effect, thereby dividing the critical depth o cut and the equivalent spindle speed by the same factor, the shape of the graphic remain equal and differ by a scale factor. Thus, a tool with two teeth can reach a stable depth of cut for all spindle speed two times higher than that achieved with 4 teeth, while with 8 teeth reaches a depth of cut two times smaller. This effect on stability may be explained by the increased frequency of excitation, which increases the instability while the excitation amplitudes are lower.

Although the decrease in the number of teeth can provide a greater range of stability, as all other parameters were considered constant, stress on the tool in this new scenario is much higher, since more material will be removed once for each tooth, thereby increasing the chip area and hence the cutting force. Therefore, depending on the characteristics of the process and the operating point of the lobe diagram, the use of a greater number of teeth may be a better decision since their use saves life of the tool.

The material removal rate remains constant with the variation of the number of teeth, since the increase of N causes a reduction of f_t and N reduction causes an increase of f_t considering the same feed speed for both cases.

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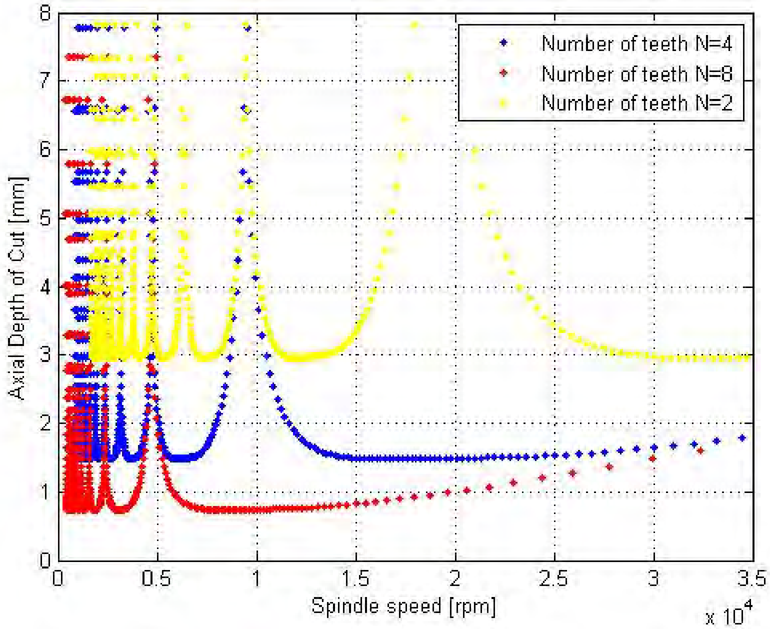

Figure 5. Stability lobes diagram using different number of teeth.

b) Different workpiece materials

Since it is not usually possible to vary the cutting constant K_r without altering the specific cutting pressure K_t, and vice versa, since both are mostly defined by the material of the workpiece and process parameters, their influences on the plot diagram will be evaluated by changing the material and then their combined effects can be predicted.

Changing the aluminum material for carbon steel 1095, the new values of K_t and K_r can be found in Kim and Ehmann (1993). By comparing the two graphs in the Fig. 6 it can be seen that the variation of the material significantly change the stability of the system. For higher strength materials can be observed a notable reduction in a_{lim} for stability at any spindle speed rotation from about 2 times. The spindle speeds where the peaks of a_{lim} occurs were also sensitive to the change of material and were shifted to the left. For higher speeds however, it is possible to obtain a higher value of a_{lim} for the steel than for aluminum.

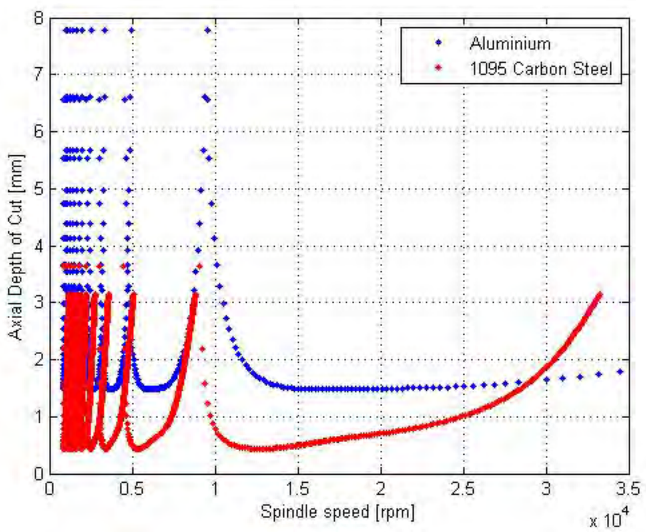

Figure 6. Stability lobes diagram using different material.

Observing the variations in stability no more by the variation of the material, but on the direct variation of the cutting

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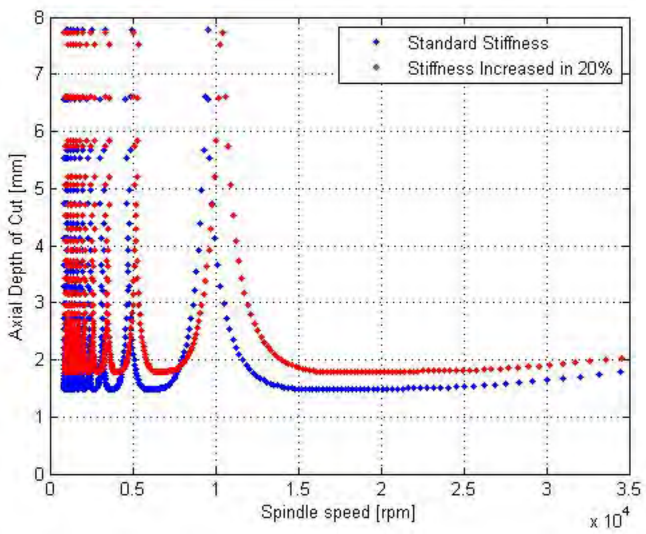
pressure by the process parameters, it is still possible to make an analysis in terms of material removal rate (MRR). According Lima *et al.* (2012) with the increase of the feed per tooth, the value of K_t decreases, which would result in the increase of the stability range, thus enabling an increase in the material removal rate by increasing both the depth of cutting as the feed per tooth since increasing depth of cutting has little influence on the value of K_t, allowing this large gain in MRR. As this gain occurs at the cost of greater effort on the tool and hence a lower life, it is necessary to do an economic analysis of the process to calculate the optimal point between increased MRR and reduced tool life.

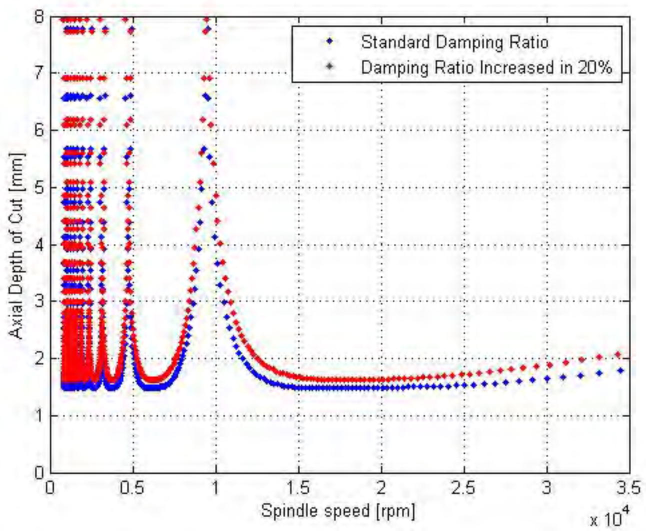
c) Different modal parameters

The change of the modal parameters can be achieved for example by comparing two different machine tools. To analyze the effect of each parameter separately, stiffness and damping, its value was increased by 20 % and then its new natural frequencies were found.

Then plotting the lobe diagram, by the analysis of the Fig. 7 (a) it is possible to note a reduction in the occurrence of chatter with the increase of stiffness. One would expect a greater range of stability greater the rigidity of the system, but it is also important to note that this gain in stability occurs usually at the cost of larger/different machinery, and so stiffness is often a difficult parameter to be changed given availability of tools and machines.



(a) Stability lobes diagram using different stiffness.



(b) Stability lobes diagram using different damping ratio.

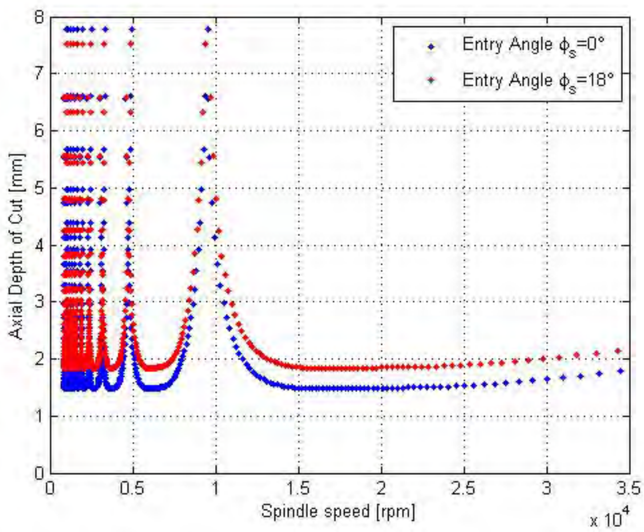
Figure 7. Stability lobes diagram using diferent modal parameters.

The variation of the damping constant was also made increasing its value by 20%. By varying the damping, the natural frequency remained constant.

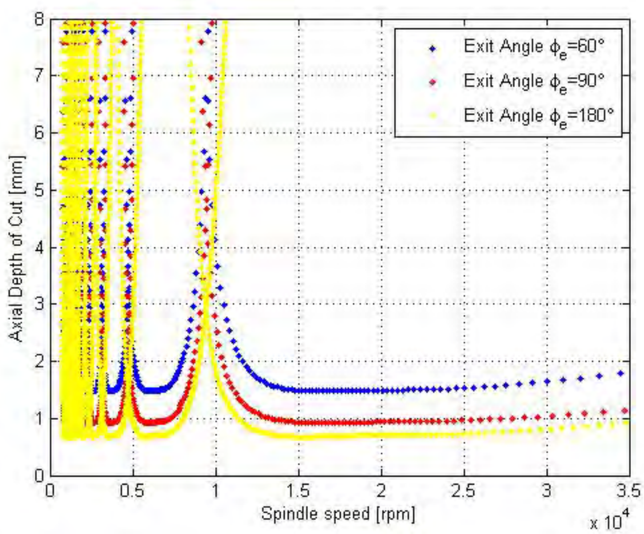
Evaluating the diagram in Fig. 7 (b) the greater the damping that represents the ability to dissipate the energy of the system, the less is the occurrence of chatter and therefore the greater the stability range, although an increase of 20% of the value of ξ has only increased depth of cutting limit at 9.3%. It is also possible to notice that the great speed remain the same for different values of ξ. For the change of ξ, it is also necessary to replace the used tool or machinery.

d) Different cutting geometry

The variation of cutting geometry is done by using differents entry and exit angles. Looking at the diagram in Fig. 8 (a), the variation of ϕ_s increases the value of the depth of cut in which it should operate. However, the increased entry angle usually depends on the geometry of the workpiece often occurs an entry angle greater than zero in up milling only by a short period of time a pass.



(a) Stability lobes diagram using different entry angle.



(b) Stability lobes diagram using different exit angle.

Figure 8. Stability lobes diagram using diferent cutting geometry.

The variation of the exit angle becomes more interesting since its value can be more easily chosen as a parameter of the process. To analyze the variation of the exit angle, were chosen two different values from standard, $90°$ and $180°$.

Analyzing Fig. 8 (b) it is expected that the possible depths of cut are lower for a greater exit angle, because more teeth were in contact at once and so the excitation frequency of the system would be greater, effect similar to which occurs when increasing the number of teeth of the tool. However, the small difference between the diagrams of the exit angles of

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$90°$ and $180°$ proved to be much less trivial, because a reduction of approximately 0.2 mm of the possible depths of cut, the largest exit angle of $180°$ allows the removal of the double material in one pass due to radial depth of cut b be double, being possible a large increase in the volume of chips removed, which will be studied further. In general, the smaller the exit angle, the more sensitive is the stability lobe diagram to the variation of it.

4. MAXIMING THE MATERIAL REMOVAL RATE

To account the increasing in the MRR due to the variation of the exit angle, the MRR was calculated for different values of ϕ_e. The MRR equation (4) can be further recast as the effect of the feed per tooth in the stability is small and can be dropped from the expression. A normalized material removal rate then can be written by the equation 15

$$MRR^* = \frac{MRR}{f_t} = a.b.n.N. \tag{15}$$

To calculate the exit angle ϕ_e which maximizes the material removal rate were calculated the depth of cutting stable for all spindle speeds for different exit angles.

As the values of a used are stable for any spindle speed, rotation will also be drawn from the equation 15 as it not alter the stability of the system. The exit angle that maximizes the MRR in the stability band can be seen in Fig. 9. By observing the figure it is possible to see that the angle that maximizes the MRR is $160°$.

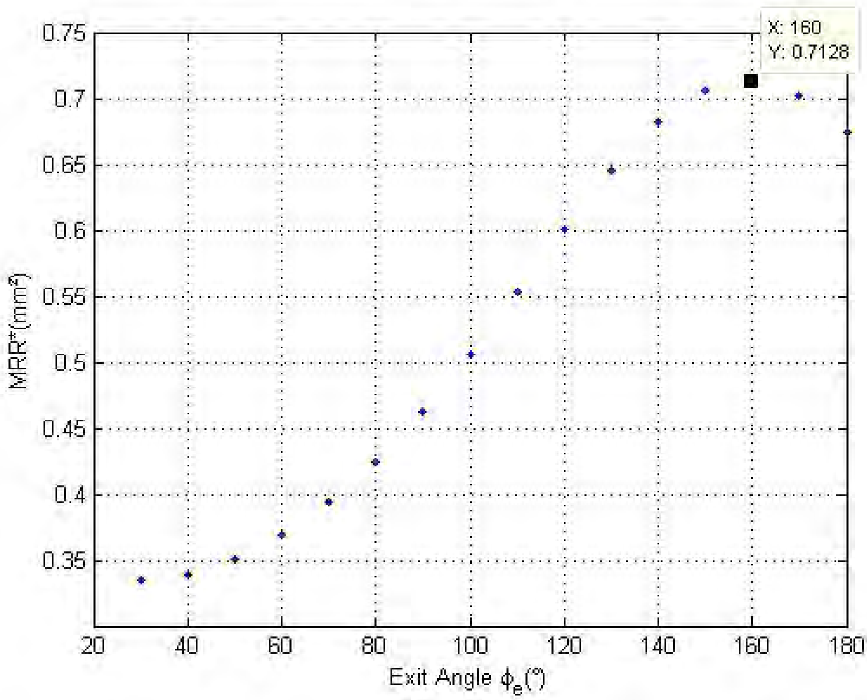

Figure 9. Material removal rate as a function of the exit angle

5. CONCLUSIONS

This article was carried out the development of the algorithm for the prediction of chatter and consequent reduction in production costs prolonging tool life, improving surface finish of the workpiece, and reduced production time by optimizing the material removal rate.

It was conducted a case study analysis of different input parameters, and it was concluded that it is often not possible to increase the range of stability without giving up a greater cutting force or a lower material removal rate. It is necessary to design the machining process analyzing the cost-benefit that offers the variation of each parameter to decide which should be changed to eliminate the chatter vibration. The study of the input parameters shows that:

- The increase of the teeth number, the using of higher strength materials and the increase in the exit angle increased the chatter band of occurrence, while the increase in the system stiffness, the increase in the system damping ratio and the increase in the entry angle decreased the chatter band.

- The using of higher strength materials and the increase in the exit angle increased the cutting force value, while the increase in the teeth number and the increase in the entry angle decreased the cutting force.
- The increase in the teeth number and the increase in the exit angle increased the material removal rate, while the increase in the entry angle decreased the material removal rate.

Besides the influence of each parameters in the stability lobe diagram, it was also possible to achieve an exit angle value of 160 degrees that maximize the material removal rate in a band of stability for all spindle speeds studied, making possible to optimize the manufacturing process by beyond the instability challenge.

6. ACKNOWLEDGEMENTS

The authors thanks to the Mechanical Engineering Department to the academic support and for PR-3 UFRJ to the support on conference expenses.

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