EMISSION OF VORTICES IN CIRCULAR CYLINDERS WITH DIFFERENT RUGOSITIES

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Abstract. This paper shows an experimental study done in the Current Channel located in Universidade Católica de Petrópolis – UCP with support of Petrobras. Using images, it was possible to measure the distance between vortices in both in-line and cross directions for one fixed cylinder. This cylinder has an intermediate rugosity, choose to be the more significant cylinder. Some results were in accordance with periodic solution following the Strouhal frequencies. But some of the results were different from the literature. The values of in-line direction followed the excitation frequencies which are proportional of the velocity of the current. But in cross-flow direction the values do not match as expected. In both directions the distances were measured from the von Karman wake.

With the information of the source of the vortices, the next step wasto test three different cylinders with different rugosities: glass, polyetilene and a cylinder smooth with sandpaper. The experimental study measured the amplitudes A/D in both in-line and cross-flow directions for these three cylinders an the curve A/D versus Reduced Velocity (Vr) were built. A comparison of the results showed that it is important to take in consideration the presence of the external rugosity of the production risers.

Keywords: Vortex, Vortex Induced Vibrations, Fluid Mechanics, Fluid Dynamics Computation

1. INTRODUCTION

When a fluid with low viscosity passes through an obstacle forms a boundary layer next to it, where its influence is confined to a small range in which the velocity varies rapidly from zero at the wall of the obstacle until the characteristic flow within the fluid. [1] This variation of the magnitude of the transverse velocity in the flow direction represents a rotational flow within the boundary layer, thereby forming the vortex [2], defined as a flow streamlines which have a circular pattern, which moves around a center of rotation [3].

The boundary layer concept was first introduced by Ludwig Prandtl [4] and is directly related to the interaction between the fluid and the surface of the obstacle. The friction between the fluid and the structure determines the response conditions of the structure, being directly proportional to the roughness of the outer surface. The material has a rough surface area in contact with the fluid and thus it is expected to have a greater loss of energy in the kinematic boundary layer and the flow will behave as if it is at high Reynolds number [3].

In a flow where an obstacle is free to move in the direction transverse to the flow, the pressure field is no longer constant. The loss of kinetic energy by friction due to the viscosity of the fluid does not have enough momentum to overcome the adverse pressure gradient [4], and the system tends to balance, migrating from the region of higher pressure to lower pressure, having a movement in opposite direction to the flow, causing the shedding of the boundary layer at a point called separation [5].

Cylindrical structures immersed in water and subjected to a flow, are subject to this vortex formation around them. The result of the shedding of these vortices is an hydrodynamic forces applied on the cylinder causing the Vortex Induced Vibrations (VIV). These forces cause oscillations that arise from the separation of vortices, due to the presence of a force called Lift Force, shown in Figure 1, which in a time t, the force is in a direction and immediately after , at time t + Dt, the force reverses its direction.

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Emission of Vortex Induced Vibration in Circular Cylinders with Different Rugosities
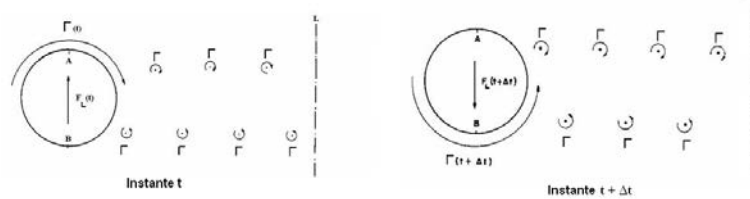

Figure 1 - lift force variation in time

The vortices are shedding periodically and alternately for Reynolds number greater than 40, the frequency which the vortices shed is known as the Strouhal frequency [5]. According to Bernard (1908), the periodicity of the wake is caused by the fluid over a cylinder and associated with the formation of vortices. Von Karman (1912) showed that the pattern of alternating vortices would be stable if the ratio between the longitudinal and the lateral distance between the center of a vortex and another was equal to 0.286 [6], and the wake vortex formation is related to the Reynolds number. This dimensionless number represents the relationship between inertial force and viscous force [7].

For example, steel catenary riser (SCR's) used in production platforms, besides having the external roughness of the steel tube are subject to marine fouling, which makes the tube even rougher. With the presence of these animals, the SCR's has variations of external roughness over time. Besides the anti-fouling coatings, anti-thermal coatings are widely used and have approximately the same external roughness.

The phenomenon of vortex induced vibrations is just one of the known methods of fluid-structure interaction that make up the field of fluid dynamics. The VIV correspond to the nature of self-oscillations, so the magnitude of oscillation is approximately one diameter in cylindrical structures, and can lead the structure to a collapse by structural fatigue [4].

The main consequence of the phenomenon of vortex shedding is the oscillatory forces, and the most important being the cross-flow applied to the cylinder. Although its magnitude is much smaller than the drag force applied in the direction of the flow, the oscillatory movements has two serious structural consequences: structural response can significantly amplify if the frequency of oscillation approaches the natural frequency of the structure, and the tension variation in a large number of cycles may result in fatigue failure which may cause operational and maintenance problems [4].

Therefore, the frequencies that act on a cylindrical structure, subject to ocean currents, such as risers, are: the natural frequency, the Strouhal frequency and frequency response. Usually the response frequencies are close to the natural frequencies. The Strouhal frequency depends on three parameters: the Strouhal number (St), a non-dimensional number, normally between 0.18 and 0.20; flow velocity (U) and the hydrodinamic diameter of the cylinder (D). The relationship between Strouhal number and the Reynolds number is shown in Figure 2. [5]
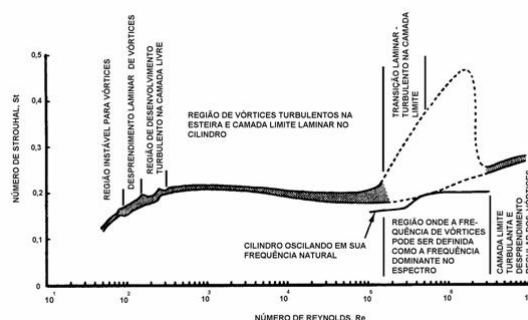

Figure 2 - Reynolds number x Strouhal number

There are two types of responses that the cylinders can present due to vortex shedding, in the direction of the current (in-line), and in the direction perpendicular to the current direction (cross flow). The frequency of the first response is twice that of the second response, but the amplitude in the direction perpendicular to the flow is of the order of 5 to 10 times the response in the flow direction. [5]

The interaction fluid-structure is one of the main objectives in engineering and it has several applications. The study of the phenomenon of vortex induced vibrations may help in developing strategies to mitigate this problem, avoiding the interruption of oil and gas and environmental disaster with irreversible damage, and financial losses.

This article summarizes a study conducted at Universidade Católica de Petrópolis - UCP, with objectives of demonstrate using images, the formation of vortices and measure the distance between them in the direction of flow and perpendicular to it, with Reynolds number between 500 and 620 and compare the results with published literature.

Furthermore, we examined the responses of three cylinders with different roughness, such as glass cylinders, polyethylene (PVC) and polyethylene coated with sandpaper number 40, and the results are presented in curves A / D versus Reduced Velocity for each of them and compare the results with one another.

2. MATERIALS AND METHODS

The experimental work was performed in the current channel located in the hydraulics laboratory of UCP, which has 4 m long, 15 cm wide and 40 cm deep. The channel is fixed and flow is by gravity due to an inclination given in the channel. However, there was a fluctuation in the value of the flow velocity of approximately 0.002 m/s, though the walls and bottom of the channel do not have any protrusions that can create a flow variable. In order to avoid interference of the side wall of the channel of the vortex shedding, we adopted a distance of three times the diameter of the tube, between the channel wall and the cylinder. Thus, the value of the diameter considered in the tests was 0.02 m.

The test apparatus includes a rigid cylinder fixed with 45 cm length, 2 cm in diameter and 40 cm submerged fixed at both ends and 213 images were captured using 4 recordings from high-definition video cameras. To make measurements of the horizontal and vertical distances between the vortices, the water was stained with potassium permanganate injected through a syringe (50 ml) parallel to the cylinder. A millimeter ruler was affixed to the upper edge of the channel (as shown in Figure 3). The Reynolds number used for the tests was between 500 and 620, measured using a Pitot tube. With this measure the path of the vortex is turbulent, as shown in Figure 4, however, the boundary layer is still laminar flow [5].
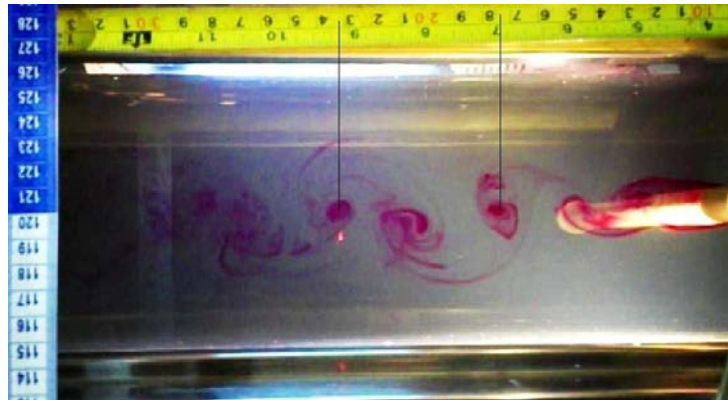

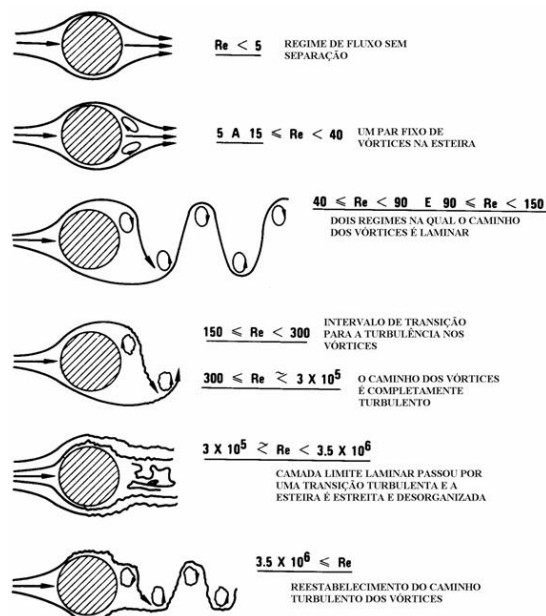Figure 3 - Distance between horizontal vortices

Figure 4 - Standard of vortices as a function of Reynolds number

Each video was shot on the computer and paused for better visualization of the vortex. The videos generated 140 images to measure the horizontal distance and 73 pictures to measure the vertical distance. In each image, two straight lines were drawn perpendicular to the center of the vortex to the millimeter ruler to measure the distance between the vortices, as described above. This process was the same for the distance between the vertical and horizontal vortices. The collected data were analyzed using SPSS statistical program.

For the tests of roughness rigid cylinders were used with 303.4 g weight, 0.025 m diameter, 1.10 m long, submerged 0,225 m, pinned at the upper end and free at the lower end as a pendulum showed in Figure 5. The submerged part of the cylinders were buffered in its their ends, for not constitute a cause of possible variations in the results.

Figure 5 - Channel Currents and support for the cylinder

The tests were divided into three steps: First the experiment was conducted using a roughened PVC cylinder with 0.020 m outside diameter (OD) coated with an emery paper waterproof number 80 until it reaches the diameter of 0.025 m, with the goal of making the three tubes as similar as possible. The final mass was 303.4 g. For the second cylinder, PVC was used with 0.025 OD. Sand was added inside the pipe to achieve the final mass of 303.4 g.

The technique used to equalize the masses of the cylinders using sand, generates a concentrated mass, and this fact is relevant if the equation of the pendulum frequency is used, but the difference of the frequencies in the three tubes was small. The equation for natural frequency of a pendulum, Fn, made with chains and mass at the end is given by:

$$\text{Fn} = \frac{1}{2\pi} \sqrt{\frac{g}{L} \cdot \frac{M + \dfrac{M_c}{2}}{M + \dfrac{M_c}{3}}} \tag{1}$$

In this study, M_C is the linear mass of the cylinder and M is the mass of the submerged pipe including the inner mass and added mass.

The third experiment was performed using a glass tube, with a mass of 296.3 g, 0.025 m in diameter. It was also necessary to fill the tube with sand to stayed with mass of 303.4 g. Figure 6 shows the three samples of tested tubes.

The values of roughness of the three cylinders were obtained in the literature since the equipment available was not sensitive enough to read the values of the sandpaper (too big) and glass (too small).

To identify the optimal speed range of flow during the experiments, it was necessary to determine the natural frequency of the system through the decay test. Due to the rigid cylinder and the shallow depth of the channel is only possible to determine the frequency in the first mode (F1). After performing the decay test, it was concluded that it was not necessary to add springs in the system because the responses of the cylinders were at the expected frequency response.
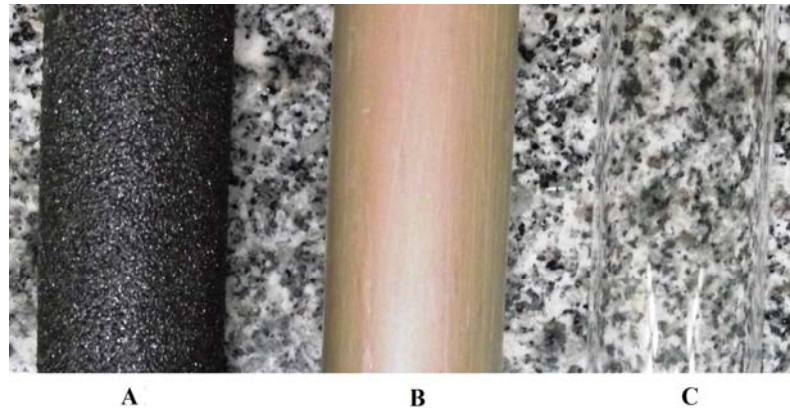
Figure 6 - rough pipe (A), PVC (B) and the glass pipe (C)

The band of velocities was calculated by the formula: Vr = U / D.Fn where Vr is the Reduced Velocity, U is the flow velocity, D is the diameter of the cylinder and Fn is the natural frequency of the corresponding cylinder. The Reduced Velocity range was from 2 to 14, with is possible to determine the optimal speed range from 0.0100 m/s to 0.100 m/s. The measurement of current flow was verified by a Pitot tube.

Amplitudes of vibration in cross and in-line directions were obtained, using a caliper positioned close to the water surface and supported on the edge of the channel. This apparatus allowed the measurement with a greater number of significant digits. The error was considered a tenth of a millimeter, due to error in reading of the displacements. To ensure that the caliper was perpendicular to the channel a graph paper stuck was used on the edge. At the moment the tube reached its maximum displacement the instrument was locked and the vibration amplitude of the cylinder was read. Measurements of displacements in-line and cross were checked five times to find the average value and the procedure was performed 195 times at different speeds for each cylinder. The apparatus described is shown in Figure 7.
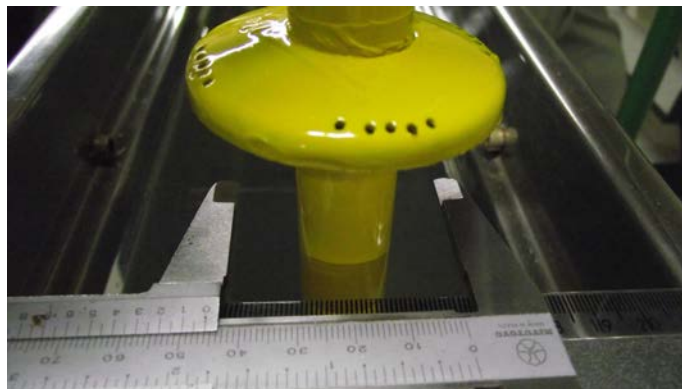

Figure 7 - Reading by maximum values of amplitudes

3. TEST RESULTS OF EMISSIONS VORTICES

Table 1 shows the statistical results of the emission of the vortices measured in the Laboratory. The calculated distance shows that the mean and median differ slightly, indicating that there is a central tendency of the response, where the observations of the distances between the vortices in the lower half of the sample almost equaled the upper half.

The distribution curve is practically a normal curve since the mode and the median has almost the same value in one order of magnitude for and spread of the distribution is mesocúrtica. The mean value of the distances between the horizontal vortices is 6.84 cm (3.14 D) with a standard deviation of 0.645 for 140 images. Figure 8 shows the distribution of the distances between the vortices in the flow direction. [8]

Table 1 - Statistical measurements of the distances between the vortices

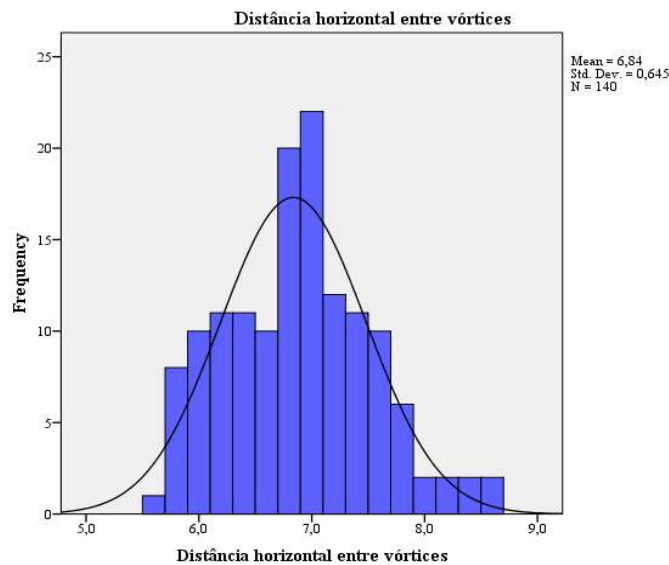|	Horizontal distance (cm)	Vertical distance (cm)
Number of samples	140	73
Mean	6,836	1,719
Median	6,800	1,700
Mode	6,8	1,6
Satndard Desviation	0,6454	0,2952

Figure 8 - Distribution of horizontal distances of the vortices

For the vertical distances, the distribution curve is also nearly a normal where the mode, mean and median have practically the same value and the scattering distribution is also mesocúrtica. The mean value of the vertical distance of the vortices is 1.72 cm (0.86 D) with a standard deviation equal to 0295 using 73 pictures. Figure 9 shows the distribution of the distances between the vortices in the direction perpendicular to the flow [8].
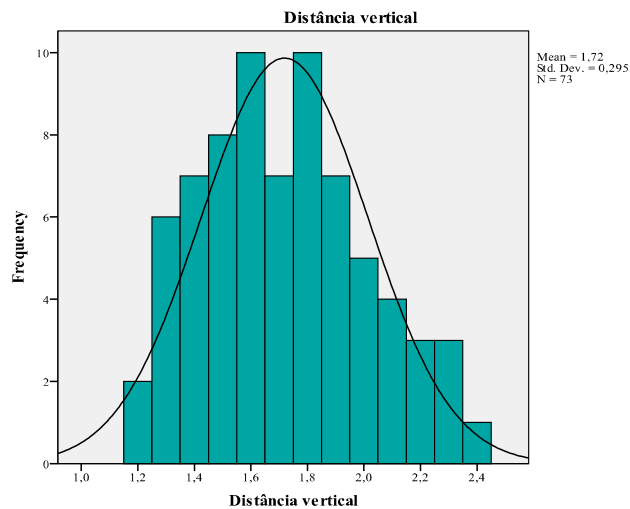

Figure 9 - Distribution of the vertical distances of the vortices

For Reynolds Numbers greater than 40 the von Karman wake shows vortices alternating in both sides of the cylinder and regularly spaced between them. Von Karman (1914) determined in his experiments that the distance between the vortices follow a pattern proportional to the diameter of the cylinder [5]. So, the value of the distance between the vortices in this experiment is 8.6 cm in the horizontal distance and 2.2 cm in the vertical distance to, as shown in Figure 10.
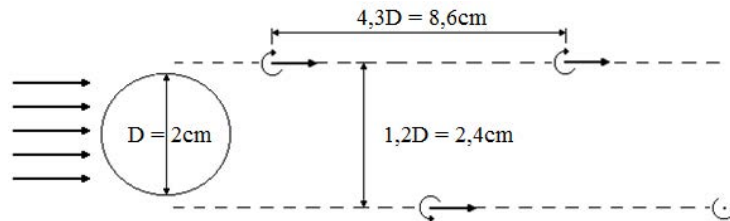

Figure 10 - von Karman wake

Only two values found in the experiments (vertical and horizontal distance) corresponded to the values described by von Karman. That is, a measurement value reached 8.6 cm (horizontal distance) and another had a distance of 2.2 cm (vertical distance). These measurements were verified with the Reynolds Number equal to 600, but for other values of Reynolds, these distances were not confirmed.

According von Karman, the importance of the wake is evident when examining the dynamics of the problem of VIV is analyzed. He modeled the wake as two parallel lines of vortices with opposite signs (see Figure 6) and studied under which conditions these vortex lines would be stable (Acheson, 1990):

$$\cosh = \frac{h\pi}{w} = \sqrt{2} \qquad (2)$$

where h is the vertical distance between the vortices and w is the horizontal distance. Using the measured data in the channel, we obtain for $\cosh = \frac{h\pi}{w}$ the values of 1.33 to 1.32 and the average values for the median values, 1.29 for mode and 1.41 for the theoretical value, giving a difference of 6% compared to the mean, indicating good approximation in the results, considering that the measurements are not very accurate.

The relation between the variations of Reynolds Number is proportional to the variation of the distance between horizontal vortices. There was a difference in proportionality in relation to the vertical spacing of the vortices. These proportions are shown in Table 2 below.

Tabela 2 – Proportions of the Reynolds Number and the distance between vortices

	Variation of vertical distance	Variation of horizontal distance	Variation of Reynolds Number
Porcentage	55%	21%	20%

4. RESULTS OF AMPLITUDES OF VIBRATION WITH DIFFERENT ROUGHNESS

For the experiment 1 with rough cylinder, the maximum amplitude of the in-line direction occurred in a range of Reduced Velocity between 5 and 6, while the maximum amplitude in the cross direction was in a range between 6 and 7 as shown in Figure 11 and Table 3.
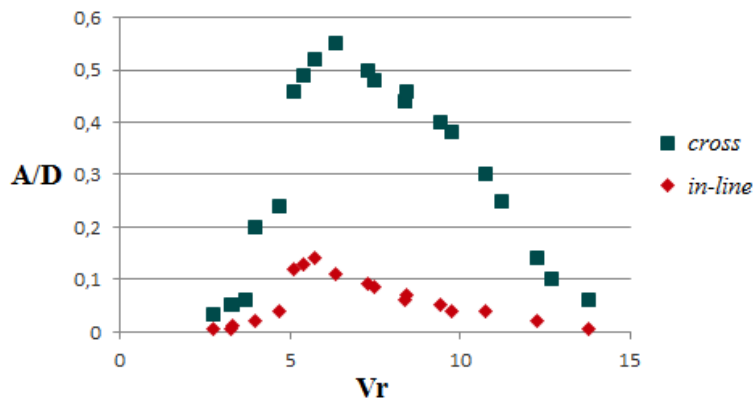
Figure 11 - A/D versus Vr for the rough cylinder

Figure 11 shows that between 5 and 5.2 was not possible to measure the amplitude of vibration due to a large increase of them in a short values of Vr. The maximum amplitudes in-line and cross occurred in different Reduced Velocities. The points of high magnitude of A/D are between 0.4 and 0.6. After reaching Vr values above 10, the curve tends to stabilize A/D equal to 0.05. The curve of the in-line amplitude tends to stabilize after Vr values above 9.

Table 3: **Relationship between the maximum amplitude and velocity for rough cylinder**

Direction	Maximum Amplitude (cm)	Maximum A/D	U (m/s)	Vr
In-line	0,32	0,13	0,040	5,36
Cross	1,38	0,55	0,047	6,33

For experiment 2 with PVC cylinder, it was found that the maximum amplitude of the in-line direction occurred for a range of Reduced Velocity between 5 and 6, while the maximum amplitude in the cross direction appeared for a range of 6 to 7, as shown in Figure 12 and Table 4.
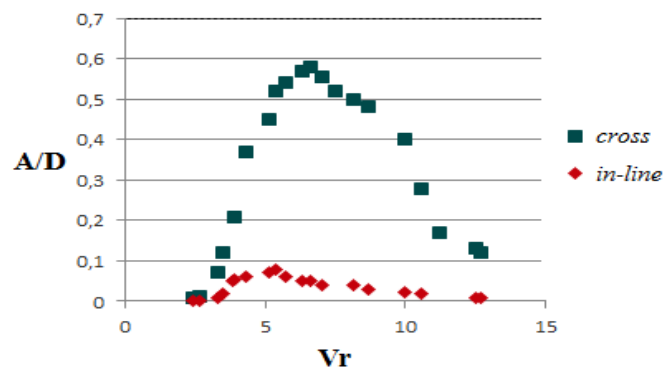

Figure 12 - A/D versus Vr for the PVC cylinder

Figure 12 shows the maximum amplitude in-line and cross low for different Reduced Velocities. In this case it was possible to measure the amplitudes in the entire length of the curve. The points of maximum amplitude represent mostly an A/D in the range of 0.5 to 0.6. After reaching Vr above 10, the curve tends to stabilize in A/D equal to 0.1. The curve of the in-line amplitude tends to stabilize for Reduced Velocity above 7.

Table 4: **Relationship between the maximum amplitude and its respective velocity for PVC cylinder**

Direction	Maximum amplitude (cm)	Maximum A/D	U (m/s)	Vr
In-line	0,2	0,08	0,040	5,36
Cross	1,47	0,59	0,049	6,60

In the experiment 3, with a glass cylinder, it was found that the maximum amplitude of the in-line direction occurred for a range of Reduced Velocity between 4.5 and 5.5 while the maximum amplitude in the cross direction has a range of 6 to 7, as illustrated in Figure 13 and Table 5.
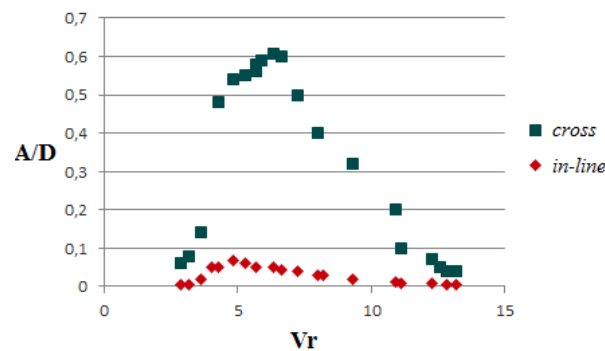

Figure 13 - A/D versus Vr for the the glass cylinder

Figure 13 shows the maximum amplitude in-line and cross flow for different Reduced Velocities. Similar to Figure 11, it was not possible to measure the amplitudes along the entire length of the curve due to the increase of the amplitude in a small speed variation. The points of maximum amplitude represent mostly an A/D in the range of 0.5 to 0.6 concentrated in a small range of Reduced Velocity. After reaching a Vr above 10, the curve tends to stabilize in A/D equal to 0.04. The curve of the in-line amplitude tends to stabilize with Reduced Velocity greater than 7.

Table 5: **Relation between the maximum amplitude and its velocity for the glass cylinder**

Direction	Maximum amplitude (cm)	Maximum A/D	U (m/s)	Vr
In-line	0,2	0,08	0,037	4,86
Cross	1,52	0,61	0,051	6,45

Tsahalis in 1984 concluded that the vortex shedding frequency approaches of the natural frequency of the system from values of Vr greater than 5 and remains close until less than 8, approximately. While the vibration of in-line observed for cylinders with two degrees of freedom occurs at Reduced Velocity greater than 4.5 and smaller than 8 [3].

Table 6 shows a comparison of the values of A/D and their respective Reduced Velocity for in-line and cross directions. The amplitude in-line for the rough cylinder is 186% higher than the amplitude in-line PVC cylinder and glass, which were equal. The range for the cross direction for the glass cylinder is 4% greater than the PVC cylinder and 10% and greater than the amplitude of the rough cylinder in cross direction. The maximum amplitudes and Reduced Velocities their speeds are different for each kind of roughness coefficients. It is explained by the difference in the fluid-structure interaction, which undergoes change according to the material of the cylinder.

Table 6: **Experimental results of maximum amplitude and its associated Reduced Velocity for each type of roughness**

Tubo	A/D máxima cross	Vr cross	A/D máxima in-line	Vr in-line
Rough	0,55	6,33	0,13	5,36
PVC	0,59	6,60	0,08	5,36
Glass	0,61	6,45	0,08	4,86

It was observed that there was greater tilt cylinder in accordance with the increase of A/D. Despite the drag coefficient of the glass tube is small its dynamic vibrations were high, but for the rough cylinder which has high drag coefficient the results showed low dynamic amplitudes compared to the glass tube.

Due to the error in reading the amplitudes, estimated at one-tenth of a millimeter, it was not possible to conclude that the in-line amplitudes of the glass tube and PVC pipe were equal. Similarly, there is a possibility that the value of the maximum cross amplitude of the tube glass and PVC were equal. The amplitudes are in the same order of the error.

Table 7 presents the relation of the roughness of each cylinder with the vibration amplitude in each direction. The roughness of the glass cylinder is lower than 6.6 times the roughness of the PVC cylinder and 66.6 times less than the roughness of sandpaper cylinder.

Table 7: Relation between the maximum amplitudes and the roughness of the cylinders

Pipe	Roughness (mm)	Maximum A/D cross	Maximum A/D in-line
Rough	0,1	0,55	0,13
PVC	0,01	0,59	0,08
Glass	0,0015	0,61	0,08

5. DISCUSSIONS

The configuration of the wake was not confirmed in this work because the results of the distances between vortices have not configured in a standard spacing between them.

The fluctuation in the value of the flow velocity of approximately 0.002 m/s resulted in an alternating emission frequency of the vortices because the behavior of the vortex shedding character showed a hysteresis case, that is, varies if the Reynolds Number is increasing or decreasing.

The values of horizontal distance function of the Reynolds Number showed that the values are close comparing to the literature, but the value of vertical distance varies, for a reason yet unknown in the present study.

The measurements were made using images, projecting perpendicular lines from the center of the vortices to measure in a calibrated ruler, which generates an error in the results due to low image distinctness when enlarged. It is possible to estimate this error inherent in the measurement process in 1.0mm, or an error of 5.81% for the variation of the vertical distances and 1.47% for the variation of the horizontal distances.

The effect of roughness on the emission of the vortices is related to the boundary layer. The speed is considered null next to the wall of the cylinder, with little loss of energy by friction. However, if the irregularities on the walls are large enough to overcome the boundary layer region, there is now a greater energy loss through friction [6]. Thus, the timing of emission of vortices decreases reducing the amplitudes in cross direction. Moreover, the effect of roughness increases the in-line amplitudes due to increased drag force on the structure, moving the graph A/D versus Vr to the left or to the right.

The roughness can modify the flow regime in the vicinity of the wall of the cylinder wall. If this regime is laminar, the shedding of the vortices will occur at an angle α about 82° and for turbulent regimes the angle β will increase to about 120° [1] according to Figure 14.
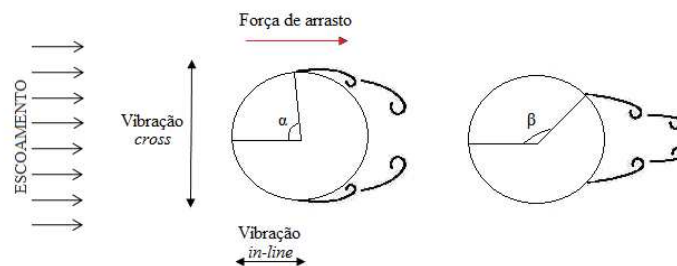
Figure 14 - Angles of shedding of the vortices

Vandiver [4] developed an equation which calculates the amplification of the drag coefficient due to vibrations of a cylinder. This equation, shown below, indicates that the amplification increases when A/D increases.

$$C_{D,amp}(i) = 1.0 + 1.043 \left(2\frac{y_{rms}(i)}{D} \right)^{0,65} \qquad (5)$$

where $y_{rms}(i)$ is the RMS value of the structural displacement at node i, which can be calculated by a dedicated program of structural analysis of VIV using modal superposition, called SHEAR7 developed by MIT (Massachusset Institute of Technology), from USA. More tests can be done in the same channel current of UCP to evaluate the accuracy of this equation. As a first approximation, to the PVC pipe, which is close to the roughness of the steel pipes used in risers by offshore companies, if A/D in the cross direction is 0.59, the amplitude of the drag coefficient can reach to 2.16, which implies duplicating the hydrodynamic forces acting on a cylinder subject to ocean currents.

6. CONCLUSIONS

For the values of Reynolds Number tested the wake of the vortices behavior is in subcritical state, where the transition occurs in the shear layer near the separation, resulting in a change in the flow regime in the boundary layer from laminar for turbulent. This phenomenon causes significant discontinuities in the drag coefficient and the frequency of vortex shedding, consequently influences the distance between the vortices. Thus, the horizontal and vertical distances not set the standard defined by von Karman.

Despite the limitations of the lack of precision of the captured images, the fluctuation of the value of the flow velocity and the error inherent in the measurement process, the results were considered satisfactory, because the results were close to the literature. For the results of measurements of in-line and cross amplitudes be more accurate would be necessary to use a set of accelerometers and improvement in capturing images with sophisticated equipment, which will be done at a later stage.

Regarding the roughness of the cylinders, the study confirmed that it is relevant for the analysis of vortex induced vibrations. The effect of roughness on the cylinder when the vortices are shedding is related to their effect on the boundary layer. There were differences in the amplitude of vibration in-line and cross due to the change in the surface roughness of the tube. Thus, increasing the roughness of the cylinder is directly proportional to amplitude of vibration in-line and inversely proportional to cross amplitude of vibration.

7. ACKNOWLEDGEMENTS

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