ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

A CELL-CENTERED FINITE VOLUME METHOD FOR THE SIMULATION OF TWO-PHASE FLOWS IN HETEROGENEOUS AND ANISOTROPIC OIL RESERVOIRS

Contreras, F. R. L.
ferlicapac@gmail.com

Lyra, P.R. M.
prmlyra@padmec.org

Carvalho, D. K. E.
dkarlo@uol.com.br

Universidade Federal de Pernambuco

Abstract. *The simulation of the incompressible displacement of oil by water in anisotropic and heterogeneous petroleum reservoirs represents a major challenge from a numerical point view. The mathematical model that describes the fluid flow in the oil reservoir includes an elliptic equation for the pressure field and a quasi-linear hyperbolic equation for the saturation field, which are solved via an Implicit Pressure Explicit Saturation (IMPES) procedure. In this method, given an initial saturation field, the pressure equation is solved implicitly, then the velocity field is computed and used in the saturation equation which is solved explicitly. In the present work, the pressure equation is discretized by a Multipoint Flux Approximation Method with a Diamond type stencil (MPFA-D) that is very robust and capable of reproducing piecewise linear solutions exactly by means of a linear preserving interpolation with explicit weights that avoid the solution of locally defined systems of equations which is a common feature of other MPFA methods. For the solution of the saturation equation, we use a traditional First Order Upwind Method (FOUM). In order to validate our formulation, we solve some problems involving heterogeneous and anisotropic reservoirs. Our results compare very well with others found in literature.*

Keywords: *Finite volume formulation; Two-phase flows; Non-homogeneous reservoir oil*

1. INTRODUCTION

The main objective of the simulation of two-phase flows in porous media is to depict accordingly the complex physical and chemical fluid flow interactions in order to predict correctly the fluid flow displacement in heterogeneous reservoir rocks. The numerical simulation of reservoirs is basically the use of computational tools to solve the differential equations that govern the transport phenomena of fluids through highly heterogeneous and anisotropic porous media. The methodology that we use to discretize the equations governing is a Cell-Centered Finite Volume (CCFV) which is locally and globally conservative. The CCFV method is quite popular in engineering and computational physics due its easy implementation and robustness in handling of unstructured grids and complex geometries. One of the most popular methodologies used to solve the governing equation of immiscible displacement of oil-water in oil reservoir is the Implicit Pressure-Explicit Saturation (IMPES) strategy (Carvalho, et al., 2006). In this strategy or procedure, a sequential time stepping procedure is used to split the computation of the pressure field from the saturation field. In the IMPES method, initially, the pressure equation is solved implicitly from an initial saturation distribution, and then, the velocities are computed from this pressure field. In sequence, the velocity field is used as an input for the saturation equation, which is finally solved explicitly. The process is repeated until the end of the analysis. In commercial simulators the pressure equation is solved based on conservative Finite Difference scheme (FD) or Two Point Flux Approximation (TPFA), according (Aavastmark, et al. 1996) this flux approximation is inconsistent because used only two point causing numerical errors that no disappear with the mesh refinement, this occurs particularly in grid not *K*-orthogonal. We present a fully cell-centered finite volume formulation, initially formulated by (GAO,et al., 2010) for heterogeneous and anisotropic diffusion problems. For simulating the immiscible displacement of oil-water, we introduce the term mobility in proposed formulation by (GAO,et al., 2010), then pressure equation is solved by called *MPFA–D (Multi-Point Flux Approximation type Diamond)* scheme, in this scheme, the flux on each cell edge is explicitly expressed by two cell-centered unknowns defined on the cells which are sharing that edge, and two vertex unknowns at the two endpoints. The vertex unknowns are express as certain linear combinations of the neighboring cell-

ISSN 2176-5480

centered unknowns, which reduces the scheme to a completely cell-centered one. The weights of this linear combination are derived in order to make the robust method and able reproducing a piecewise linear solution accurately. MPFA-D is an important generalization in spatial discretization, since it provides a consistent and accurate flux representation for general non-orthogonal grids with full tensor, furthermore, the weights are calculated of form explicit, without need to solve locals matrix, as occurs in others classics schemes, for example, MPFA-O (Aavastmark, et al, 1996). The saturation equation is discretized using the finite volume scheme, where hyperbolic flux is calculated by first order upwind scheme which is robust and produces monotones solutions near shock regions (Hirsch, 1994).

2. MATHEMATICAL FORMULATION

The mathematical formulation consists in describing the governing equations of the two-phase flow of oil and water in the interior of the reservoirs. We assume, without loss of generality, that the fluid and rock are incompressible and that the flow is isothermal, despite the gravitational effects and capillary. We use a segregated formulation in the which the basic equations are obtained from the Conservation of Mass and the Darcy's Law, which can be written for *i*-phase (*i*=*o* (oil), *w* (water)), as:

$$\frac{\partial(\phi\rho_i S_i)}{\partial t} = -\nabla\cdot(\rho_i \boldsymbol{v_i}) + q_i \text{ and } \boldsymbol{v_i} = -\frac{k_{ri}}{\mu_i}\boldsymbol{K}\nabla p_i \tag{1}$$

where $\boldsymbol{v_i}$ and ρ_i, represent, the Darcy velocity and densities for each *i*-phase, ϕ and q_i are the porosity and the source term (or sink). Besides, p_i and S_i is pressure and saturation for each *i*-phase. The mobility of phase in question is denoted by $\lambda_i = k_{ri}/\mu_i$, with k_{ri} and μ_i represent viscosity and relative permeability, respectively. $\boldsymbol{K}$ is absolute permeability tensor of the rock that satisfies the ellipticity condition $\left(\boldsymbol{K}_{xx}\boldsymbol{K}_{yy} \geq \boldsymbol{K}_{xy}^2\right)$. We assume that the rock is fully saturated by oil and water and the following equation holds:

$$S_o + S_w = 1 \tag{2}$$

We assume that $p = (p_w + p_o)/2$, furthermore $\boldsymbol{v} = \boldsymbol{v_o} + \boldsymbol{v_w}$. Using Eqs. (1) to (2) and after algebraic manipulations, we write the pressure equation for two-phase flow, as:

$$\nabla\cdot\boldsymbol{v} = Q \text{ with } \boldsymbol{v} = -\lambda_T\,\boldsymbol{K}\nabla p \tag{3}$$

Where $\lambda_T = \lambda_o + \lambda_w$ and Q=Q_w + Q_o, the total mobility total and specific flow rate, respectively.
Again, using Eqs. (1) - (3) and after some algebraic manipulation we can write the saturation equation, as:

$$\phi\frac{\partial S_w}{\partial t} = -\nabla\cdot F(S_w) + Q_w,\ f_i = \lambda_i/\lambda_T \tag{4}$$

where $F(S_w) = f_w\boldsymbol{v}$ is the flux function which is dependent on the water-phase saturation and f_w, is fractional flow function, more detail over mathematical formulation in (Ewing 1983), (Carvalho et al, 2006) and (Chen et al 2006).

2.1 Initial conditions and boundary conditions

The pressure and saturation equations are completely determined when we use an appropriate set of initial and boundary conditions, as follows:

$$\begin{aligned}
&\boldsymbol{v}\cdot\boldsymbol{n} = 0 \text{ in } \Gamma_N\times[0,t]\ ,\ p(\boldsymbol{x},t) = \bar{p} \text{ in } \Gamma_D\times[0,t]\\
&S_w(\boldsymbol{x},t) = \bar{S}_w \text{ in } \Gamma_I\times[0,t]\\
&S_w(\boldsymbol{x},0) = \bar{S}_w^0 \quad \text{in } \Omega
\end{aligned} \tag{5}$$

ISSN 2176-5480

where Γ_D, Γ_N represent the Dirichlet (prescribed pressures) and Neumann (prescribed fluxes) boundary conditions, respectively, Γ_I represents the injection wells, $\bar{S}_w$ represents the saturation prescribed in a set of injection wells and finally, $\bar{S}_w^0$ is the initial saturation distribution.

3. NUMERICAL FORMULATION

As mentioned at the beginning of section 2, we use the IMPES methodology (Ewing, 1983) to solve governing equations displacement immiscible of two-phase flow oil-water.

3.1 Implicit formulation of equation of pressure by the method MPFA-D

Integrating the pressure equation (Eq. (3)) over the whole domain Ω, we have:

$$\int_{\Omega} \nabla \cdot \boldsymbol{v} \partial\Omega = \int_{\Omega} Q \partial\Omega \tag{6}$$

Applying the Divergence Theorem of Gauss in arbitrary control volume Ω_k, we have:

$$\int_{\Omega_k} \nabla \cdot \boldsymbol{v} \partial\Omega = \int_{\partial\Omega_k} \boldsymbol{v} \cdot \boldsymbol{n}\, dA \tag{7}$$

where $\boldsymbol{n}$ is the unit normal vector to $\partial\Omega_k$. In equation (7) after using Mean Value Theorem, the discretization at each control surface of the control volume is approximated using the Finite Volume Method, thus we have:

$$\boldsymbol{v_{IJ}} \cdot \boldsymbol{N_{IJ}} = \int_{\partial\Omega_k} \boldsymbol{v} \cdot \boldsymbol{n}\, dA \text{ and } \sum_{IJ \in \partial\Omega_k} \boldsymbol{v_{IJ}} \cdot \boldsymbol{N_{IJ}} = Q_k V_k \tag{8}$$

where $\boldsymbol{v_{IJ}}$ is the approximate velocity over the control surface formed by nodes *I* and *J*, and $\boldsymbol{N_{IJ}}$ is the normal area vector, and Q_k is the value specific injection/production in the control volume *"k"*, V_k is the volume of a generic triangular control volume *"k"*. The interpolation of the flow mobility in the control surface is given by:

$$\lambda_{IJ} = \frac{\lambda_I + \lambda_J}{2}, \text{ where } \lambda_i = \sum_{k=1}^{n(i)} \lambda_{T(k)} V_k \Big/ \sum_{k=1}^{n(i)} V_k \,, \; i = I, J \tag{9}$$

where *n(i)* represent the number of cells that concur in the *i*-vertex. The discretization of the Eq. (8) is obtained using the proposed method by (Gao, et al., 2010). This method ensures that the solutions obtained for the pressure field is piecewise linear. Therefore, the estimated gradient is written as:

$$\nabla p \cong \frac{p_J - p_I}{|\boldsymbol{IJ}|} \boldsymbol{IJ} + \frac{\Re \boldsymbol{IJ}}{|\boldsymbol{IJ}|^2} \left[\left(p_I - p_{\hat{L}} \right) ctg \measuredangle IJ\hat{L} + \left(p_J - p_{\hat{L}} \right) ctg \measuredangle \hat{L}IJ \right] \tag{10}$$

where $p_{\hat{L}}$, p_I and p_J are the pressures at the vertices of the triangle $\triangle \hat{L}IJ$, $\boldsymbol{IJ}$ is the vector that belong to control surface and $\Re$ is the rotation matrix in counterclockwise direction. The set of geometric parameters additional used in the present formulation is shown in Figure 1.

From the equation (10) we can approximate the velocities and consequently the flow on the control surfaces, as:

$$\boldsymbol{v} \cdot \boldsymbol{N_{IJ}} \cong -\boldsymbol{K}_{IJ}^{(n)} \left((p_I - p_{\hat{L}}) ctg \measuredangle IJ\hat{L} \; + \; (p_J - p_{\hat{L}}) ctg \; \measuredangle \hat{L}IJ \right) - \boldsymbol{K}_{IJ}^{(t)} (p_J - p_I) \tag{11}$$

Where the geometric and physical constants are given by:

ISSN 2176-5480

$$K_{IJ}^{(t)} = \frac{(\boldsymbol{N_{IJ}})^T \boldsymbol{K}_{\hat{L}}(\boldsymbol{IJ})}{|\boldsymbol{IJ}|^2}, K_{IJ}^{(n)} = \frac{(\boldsymbol{N_{IJ}})^T \boldsymbol{K}_{\hat{L}}(\boldsymbol{N_{IJ}})}{|\boldsymbol{IJ}|^2}, \; ctg\measuredangle IJ\hat{L} = \frac{\boldsymbol{J\hat{L}}\cdot\boldsymbol{JI}}{|\boldsymbol{JI}|h_{IJ}^L} \text{ and } ctg\measuredangle \hat{L}IJ = \frac{\boldsymbol{I\hat{L}}\cdot\boldsymbol{IJ}}{|\boldsymbol{IJ}|h_{IJ}^L} \tag{12}$$

Therefore, integrating Eq. (11) in the control surface *IJ*, and taking into account the triangle $\triangle IJ\hat{L}$, we have:

$$\int_{IJ} \boldsymbol{v}\cdot\boldsymbol{N_{IJ}}dA \cong -\frac{K_{IJ}^{(n)}}{|\boldsymbol{IJ}|}\left((p_I - p_{\hat{L}})\frac{(\boldsymbol{J\hat{L}}\cdot\boldsymbol{JI})}{|\boldsymbol{IJ}|} + (p_J - p_{\hat{L}})\frac{(\boldsymbol{I\hat{L}}\cdot\boldsymbol{IJ})}{|\boldsymbol{IJ}|}\right) - (p_J - p_I)K_{IJ}^{(t)} \tag{13}$$

Analogously, integrating the Eq. (11) in surface *JI*, and taking into account the triangle $\triangle JI\hat{R}$, we have:

$$\int_{JI} \boldsymbol{v}\cdot\boldsymbol{N_{JI}}dA \cong -\frac{K_{JI}^{(n)}}{|\boldsymbol{JI}|}\left((p_J - p_{\hat{R}})\frac{(\boldsymbol{J\hat{R}}\cdot\boldsymbol{JI})}{|\boldsymbol{JI}|} + (p_I - p_{\hat{R}})\frac{(\boldsymbol{I\hat{R}}\cdot\boldsymbol{IJ})}{|\boldsymbol{IJ}|}\right) - (p_I - p_J)K_{JI}^{(t)} \tag{14}$$

Therefore, sum the Eq. (13) and (14), and using the flux continuity in the control surface, and after algebraic manipulations, we can write the approximate flow in the control surface, as:

$$\boldsymbol{v_{IJ}}\cdot\boldsymbol{N_{IJ}} = -k_{IJ}^{(1)}|\boldsymbol{IJ}|\left[p_{\hat{R}} - p_{\hat{L}} - d_{IJ}^{(1)}(p_J - p_I)\right] \tag{15}$$

where $p_{\hat{R}}$ and $p_{\hat{L}}$ are the pressures in the barycenter of the control volume, p_I and p_J are the pressures in the vertex of the control surface $\boldsymbol{IJ}$. The parameters in Eq. (15) are given by:

$$k_{IJ}^{(1)} = \frac{K_{IJ}^{(n)}K_{JI}^{(n)}}{K_{IJ}^{(n)}h_{JI}^R + K_{JI}^{(n)}h_{IJ}^L}, \; d_{IJ}^{(1)} = \frac{\boldsymbol{\hat{L}\hat{R}}\cdot\boldsymbol{IJ}}{|\boldsymbol{IJ}|^2} - \frac{1}{|\boldsymbol{IJ}|}\left(\frac{K_{IJ}^{(t)}}{K_{IJ}^{(n)}}h_{IJ}^L + \frac{K_{JI}^{(t)}}{K_{JI}^{(n)}}h_{JI}^R\right) \tag{16}$$

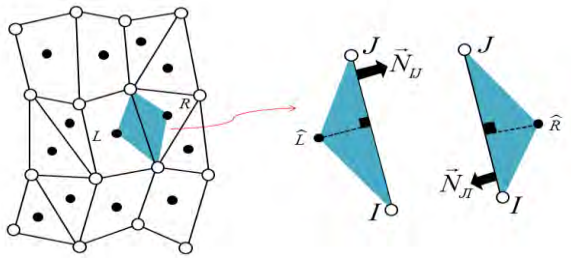


Figure 1. Local diagram of any polygonal mesh, illustrating the *Diamond Path*.

3.2 Treatment of boundary conditions

If $\boldsymbol{IJ}$ is located on the Dirichlet boundary, we obtain directly the approximation of flux from Eq. (13):

$$\boldsymbol{v_{IJ}}\cdot\boldsymbol{N_{IJ}} = \frac{-K_{IJ}^{(n)}}{h_{IJ}^{\hat{L}}|\boldsymbol{IJ}|}\left(\left(\boldsymbol{J\hat{L}}\cdot\boldsymbol{JI}\right)g_D(I) + \left(\boldsymbol{I\hat{L}}\cdot\boldsymbol{IJ}\right)g_D(J) - |\boldsymbol{IJ}|^2 p_{\hat{L}}\right) - \left(g_D(J) - g_D(I)\right)K_{IJ}^{(t)} \tag{17}$$

$$\boldsymbol{v_{IJ}}\cdot\boldsymbol{N_{IJ}} = g_N|\boldsymbol{N_{IJ}}| \tag{18}$$

where $g_D(J)$, $g_D(I)$ are the pressures defined in Dirichlet boundaries and g_N are the fluxes defined in Neumann boundaries.

3.3 Interpolating the pressures at the mesh vertices

The pressures at the vertices can be expressed as linear combinations of cell-centered pressures in order to the MPFA-D method become completely a Cell-Centered method. Therefore, the pressure in arbitrary vertex *"I"* of the primal mesh is given by:

ISSN 2176-5480

$$p_I = \sum_{k=1}^{n(I)} w_k p_k \tag{19}$$

where w_k is the weight associated to cell "k". In this paper, we use two types of explicit weights proposed by (Gao, et. al. 2010). The "explicit" term is used to indicate that the weights are computed explicitly, without the need of solve a local system of equations as in other classic MPFA-O methods (Aavatsmark, et al., 1996). The explicit weight is given in Eq.(20), and derived using the continuity equation in the iteration region (Gao, et. al. 2010), see Figure 2. Other authors have proposed various ways to calculate the interpolation weights (Wu, et al., 2003), (Huang, et al., 1998), (G. X, et al., 2007), but, in general, these interpolations are not very robust whenever treating anisotropic and heterogeneous (possibly discontinuous) media.

$$w_k = \frac{\bar{\psi}_k}{\sum_{k=1}^{n(I)} \bar{\psi}_k}, \quad k = 1,2,...,n(I) \tag{20}$$

and

$$\bar{\psi}_i = \tilde{K}_{i,1}^{(n)} \eta_{i,1} \xi(i) + \xi(i+1) \tilde{K}_{i,2}^{(n)} \eta_{i,2} \tag{21}$$

Where

$$\eta_{k,1} = \left\| \vec{Ik} \right\| \Big/ h_{Ik}^k, \ \eta_{k,2} = \left\| \overrightarrow{Ik+1} \right\| \Big/ h_{Ik+1}^k \tag{22}$$

and

$$\xi(i) = \frac{\bar{K}_{i-1}^{(t)} - \bar{K}_i^{(t)} + \bar{K}_{i-1}^{(n)} \cot \vartheta_{i-1,1} + \bar{K}_i^{(n)} \cot \vartheta_{i,2}}{\tilde{K}_{i-1,2}^{(n)} \cot \theta_{i-1,2} + \tilde{K}_{i,1}^{(n)} \cot \theta_{i,1} - \tilde{K}_{i-1,2}^{(t)} + \tilde{K}_{i,1}^{(t)}} \tag{23}$$

The physical and geometrical parameters are calculated analogously to the terms given in Eq. (12).

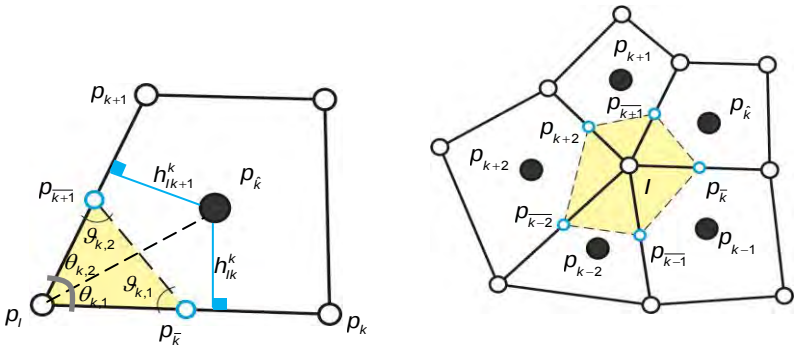


Figure 2. Sketch of the region of iteration involving geometric entities needed to calculate the weights of type 1.

3.4 Explicit saturation equation

Integrating Equation (4) along the domain Ω, we can write:

$$\int_\Omega \phi \frac{\partial S_w}{\partial t} \partial\Omega = -\int_\Omega \nabla \cdot F(S_w) \partial\Omega + \int_\Omega Q \partial\Omega \tag{24}$$

Considering an arbitrary control volume Ω_k, the Eq. (24), can be written by Green-Gauss theorem as

ISSN 2176-5480

$$\int_{\Omega_k} \phi \frac{\partial S_w}{\partial t} \partial\Omega_k = -\int_{\Gamma} F(S_w) \cdot \boldsymbol{n} \partial\Gamma + \int_{\Omega_k} Q \partial\Omega_k \tag{25}$$

Considering the general case of an inhomogeneous porous media, a semi-discrete numerical scheme for the solution of the non-linear hyperbolic saturation equation, can be written as

$$S_w^{n+1} = S_w^n + \frac{\Delta t}{\phi V_k}\left[Q_k V_k - \sum F_{IJ}(S_w) \cdot \boldsymbol{N_{IJ}} \right] \tag{26}$$

In Equation (26), the term spatial is solve using the classical First Order Upwind scheme, which can be computed as

$$F_{IJ} \cdot \boldsymbol{N_{IJ}} = \begin{cases} f_{w,L} \boldsymbol{v_{IJ}} \cdot \boldsymbol{N_{IJ}}, & \text{if } \boldsymbol{v_{IJ}} \cdot \boldsymbol{N_{IJ}} \geq 0 \\ f_{w,R} \boldsymbol{v_{IJ}} \cdot \boldsymbol{N_{IJ}}, & \text{if } \boldsymbol{v_{IJ}} \cdot \boldsymbol{N_{IJ}} < 0 \end{cases} \tag{27}$$

where $f_{w,L}$ and $f_{w,R}$ are the fractional flow of water on the left and right control volumes of face *IJ*, respectively.

4. NUMERICAL RESULTS

In this section, we solve some "benchmark" problems in order to show the potential of the formulation to deal with heterogeneous and anisotropic petroleum reservoirs.

4.1 Study field pressures in heterogeneous and anisotropic domain

The problem analyzed in this example was adapted from (Crumpton, et al, 1994), (Hyman, et al, 1997), (Carvalho, 2005) and (Contreras, 2012). Domain is a square $[-1,1]^2$, with Dirichlet boundary conditions given by analytical solution. The analytical solution and the term sink are given by:

$$p = \begin{cases} \left(2\sin(y)+\cos(y)\right)\alpha x + \sin(y)\ , x \leq 0 \\ e^x \sin(y)\ ,\ x > 0 \end{cases},\ q = \begin{cases} \left(2\sin(y)+\cos(y)\right)\alpha x + \sin(y)\ ,\ x \leq 0 \\ -2e^x \alpha \cos(y)\ ,\ x > 0 \end{cases} \tag{28}$$

The permeability is given as follows:

$$K = \begin{cases} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \text{ if } x < 0 \\ \alpha \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \text{ if } x \geq 0 \end{cases} \tag{29}$$

In order to calculate the convergence rates, we define the following norms of errors:

$$\mathrm{E_{MAX}} = \max|p - \tilde{p}|,\ \mathrm{E_{RMS}} = \left(\sum_{i=1}^{N} \frac{(p-\tilde{p})^2}{N} \right)^{0.5} \text{ and } \mathrm{E_{VEL}} = \left(\sum_{i=1}^{N_A} (v - \tilde{v})^2 V_i \Big/ \sum_{i=1}^{N_A} V_i \right)^{0.5} \tag{30}$$

Where $\mathrm{E_{MAX}}$, $\mathrm{E_{RMS}}$ and $\mathrm{E_{VEL}}$ are the maximum, the root mean square and the velocity-L2 errors norms, respectively, and N is the number of cells, $\tilde{p}$ numerical solution. Being v velocity analytically computed and $\tilde{v}$ velocity numerically computed, N_A is the number of faces which are in mesh and V_i is the sum of the volumes of the cells that share the i-th face. In this example, we used the MPFA-D, with explicit weight on a triangular structured mesh. It can be seen that the rates of convergence for the pressure are of second order and that, for velocity, they are higher than one, presenting very similar results to those of (Crumpton, et al, 1994) and (Hyman, et al, 1997). As higher is the parameter α, as more severely discontinuous is the porous medium, which is characteristic of oil reservoir. The scheme MPFA-D represent properly discontinuous medium, due this, even for very higher alpha, the scheme continues to present rates of convergence of second order. This discretization is suitable for problems in which the permeability tensor is heterogeneous and represented by non-matrix diagonal, with strong discontinuities.

ISSN 2176-5480

Table 1. Results by the method MPFA-D, with α=1.

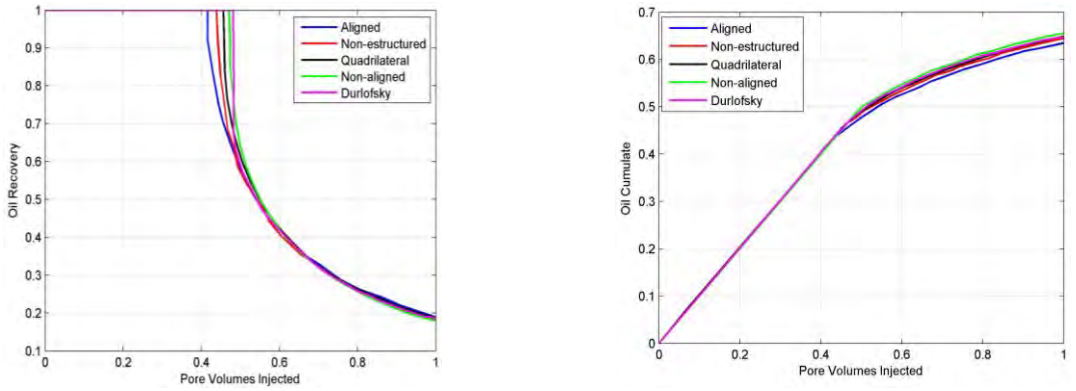
$1/h$	MPFA-D		Hyman, et al., 1997		Crumpton, et al., 1994		Rate of Convergency MPFA-D		
	$E_{MÁX}$	E_{RMS}	$E_{MÁX}$	$R_{MÁX}$	E_{RMS}	R_{RMS}	R_{MAX}	R_{RMS}	R_{Vel}
8	0.0156	0.0062	---------	-------	$3.33\text{x}10^{-3}$	-------	---------	-------	-------
16	0.0046	0.0016	$3.74\text{x}10^{-3}$	-------	$9.37\text{x}10^{-4}$	1.8294	1.7618	1.9542	1.5538
32	0.0013	$3.95\text{x}10^{-4}$	$9.66\text{x}10^{-4}$	1.9529	$2.45\text{x}10^{-4}$	1.9353	1.8231	1.20181	1.6415
64	$3.43\text{x}10^{-4}$	$9.89\text{x}10^{-4}$	$2.45\text{x}10^{-4}$	1.9792	$6.25\text{x}10^{-5}$	1.9709	1.9222	1.9978	1.7681

Table 2. Results by the method MPFA-D, with α=1000

$1/h$	MPFA-D		Crumpton, et al., 1994		Rate of Convergency MPFA-D	
	$E_{MÁX}$	E_{RMS}	E_{RMS}	R_{RMS}	R_{MAX}	R_{RMS}
8	7.7343	3.1410	1.83	-------	---------	-------
16	2.2107	0.7825	$4.79\text{x}10^{-1}$	1.9337	1.8068	2.0051
32	0.6324	0.1945	$1.22\text{x}10^{-1}$	1.9731	1.8056	2.0083
64	0.1729	0.0488	$3.07\text{x}10^{-2}$	1.9906	1.8709	1.9948

4.2 Two-phase flow in a quarter five-spot wells configuration with reason viscosity moderately adverse M=4.0

This problem was adapted from (Durlofsky, 1993), (Carvalho, 2005) and (Contreras, 2012), it is a non-dimensionalized version of the one quarter five-spot problem wells. We consider a homogeneous and isotropic reservoir, where the porosity value is irrelevant, in which it serves only to non-dimensionalize the time step. The residual saturations and viscosities of oil and water phases are given, respectively by: S_{rw}=S_{ro}=0 and the mobility ratio is $M = 4$. The water saturation in the injector well is 1 and the pressure in the production wells are zero. This problem is slightly sensitive to the grid orientation effect (GOE), due the adverse mobility ratio. In this problem we used four meshes: one non-structured, one structured and aligned, one structured and non-aligned and one quadrilateral mesh, see Figure 3. According (Durlofsky,1993), theoretically, the solutions should be the same regardless of the mesh configuration. From the analysis of Figure 5, using as reference the results obtained with the non-structured mesh which, in principle, is free GOE. We note that the results obtained with the aligned mesh shows an anticipation of the breakthrough, while the results from the non-aligned and the quadrilateral meshes delay the arrival of the water saturation front, compared with the results on non-structured mesh. We note that our results are similar with the results obtained by Durlofsky on non-aligned triangular mesh using a combination of Mixed Finite Elements to solve the pressure equation and Finite Volume scheme to solve the saturation equation.



(a) (b)

Figure 3. Two-phase flow in a configuration the quarter five-spot wells with ratio moderately adverse viscosity M=4 (a) recovery oil and (b) accumulate oil.

4.3 Two-phase flow in a configuration a quarter five-spot wells flow through a confined region with four low permeability barrier

This is a variation of the classical one quarter five spot problem, which was adapted from (Edwards, 2006). In this case, it involves a high-permeability domain divided by four non-intersecting low permeability barriers (with drop by

ISSN 2176-5480

six orders of magnitude in permeability) aligned in a cross shape, see Figure 3. The configuration permits fluid flows through the mid-section. The permeability tensor in zones 1 and 3 is an identity tensor and in zones 2 and 4 it is a full permeability tensor, with principal axes at 45º to the domain axes and larger principal value (ratio 9:1) along the orthogonal to the right hand diagonal. The residual saturations of oil and water are S_{ro} =S_{rw} =0 and the mobility ratio is M=4. The boundary conditions at the injection well are given by: S_{inj} =1, Q_{inj} =1 and the pressure in the producing well is p_{prod} =0, see Figure 4 a. Indeed, in Figure 5, we can observe that, as expected, that there is no flow through the low permeability barriers, making it clear that the velocity field was also calculated accurately "respecting" the abrupt variation in the permeability field between the barriers and the rest of the reservoir. A brief analysis of Figures 5, allow us to conclude that in both the coarser mesh as the mesh finer, solutions represent the same behavior, and the representation of the saturation front in the mesh finer is very accuracy presenting lower scattering along the field, that in the mesh coarser. The low-permeability barriers are clearly detected by the first order upwind (see, Figure 5) and the computed fronts conform to the expected quarter five-spot 'cascade'. The test show the fundamental advantages of the scheme (for two-phase flow), the scheme's ability to detect rapid changes in permeability and yield consistent flow behavior.

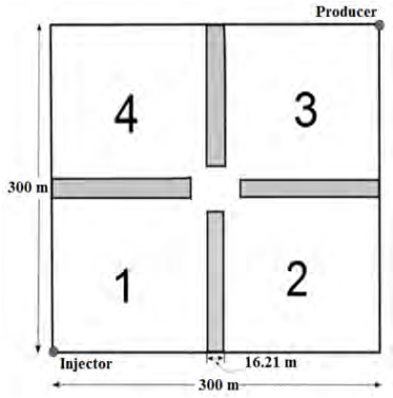
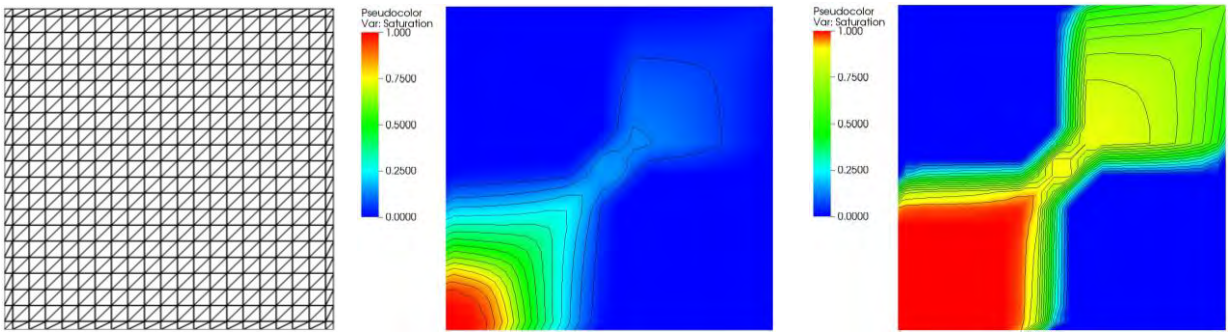
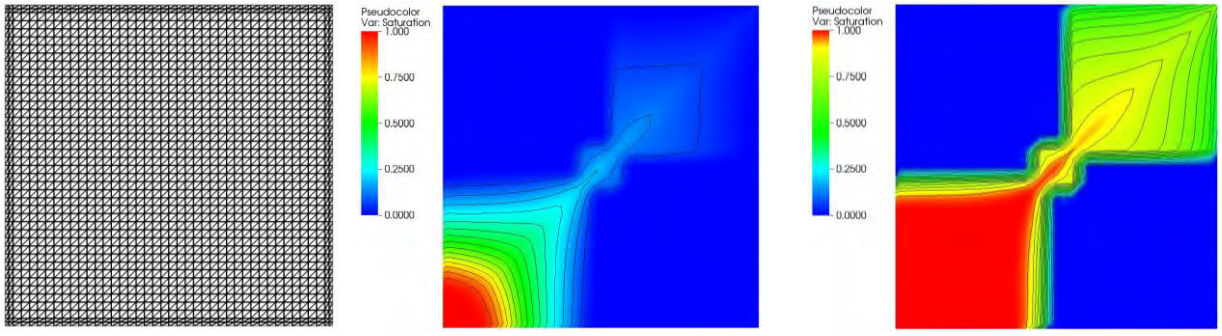


Figure 4. (a) Geometry of the domain with four different sections.



(a) (b) (c)

Figure 5. (a) Coarse mesh, adapted of (Edwards, 2006), (b) profile saturation field with VPI=0.1, and (c) profile saturation field for VPI =0.95.



(a) (b) (c)

Figure 6. (a) Fine mesh, (b) profile saturation for VPI=0.1, and (c) profile saturation for VPI =0.95.

ISSN 2176-5480

5. CONCLUSIONS

In this article, we presented a full cell-centered finite volume (CCFV) method to simulate the immiscible displacement of oil and water in heterogeneous and anisotropic in oil reservoirs. The pressure equation is discretized using a MPFA-D. The first order upwind method is used here to solve the saturation equation. The presented method is able to deal with any polygonal meshes by producing convergent solutions without need to solve local systems which are common in other classical MPFA schemes. This FV formulation is capable of handling highly discontinuous coefficients in an elegant and accurate manner. Some representative model examples were used in order to show the ability of the proposed method to deal with discontinuous permeability fields in which this characteristic vary in orders of magnitude throughout of the reservoir. The results indicate the potential of the method for modeling reservoirs with complex geometries. In near future, we aim to extend this methodology to deal with more realistic problems, including the effects of gravity and capillarity in three dimensions.

6. ACKNOWLEDGEMENTS

The authors would like thank the following Brazilian research agencies: Fundação de Amparo à Ciência e Tecnologia do Estado de Pernambuco (FACEPE), Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), CENPES-PETROBRAS (SIGER - Rede Temática de Simulação e Gerenciamento de Reservatórios).

7. REFERENCES

1. Aavatsmark, I., T. Barkve, Ø. B. & Mannseth, T., 1996. Discretization on Non-Orthogonal, Quadrilateral Grids for Inhomogeneous, Anisotropic Media. *Journal of Computational Physics*, 15 August, Volume 127, pp. 2-14.
2. Carvalho, D. K., Willmersdorf, R. B. & Lyra, P. R. M., 2006. A Node-Centered Finite Volume Formulation for the Solution of Two-Phase Flows in Non-Homogeneous Porous Media. *International Journal for Numerical Methods in Fluids*, 19 March.
3. Carvalho, D. K. E., 2005. *Uma Formulação do Método dos Volumes Finitos com Estrutura de dados por Aresta para a Simulação de Escoamento em Meios Porosos,* Tese Doutorado,Universidade Federal de Pernambuco, Refice.
4. Chen, Z., Huan, G. & Ma, Y., 2006. *Computational Methods for Multiphase Flows in Porous Media.* Philadelphia: SIAM.
5. Contreras, L. R. F., 2012. *Um Método dos Volumes Finitos Centrado na Célula para Simulação de Escoamento Bifásico em Reservatórios de Petróleo Heterogêneos e Anisotrópicos*. Dissertação de Mestrado, UFPE, Recife
6. Durlofsky, L. J. A., 1993. Triangle Based Mixed Finite Element-Finite Volume Technique for Modeling Two Phase Flow Through Porous Media. *Journal of Computational Physics*, 105(252-266).
7. Edwards, M. G., 2006. Higher-Resolution Hyperbolic-Coupled-Elliptic Flux Continuous CVD Schemes on Structured and Unstructured grids in 2-D. *International Journal for Numerical Methods in Fluids*, Volume 51.
8. Ewing, E. R., 1983. The Mathematics of Reservoir Simulation. Philadelphia: SIAM.
9. GX, L., LG, S. & ZJ., S., 2008. Numerical Methods for Energy Flux of Diffusion Equation on Unstructured Grids. *International journal for Numerical Methods in Biomedical Engineering*, Volume 26.
10. Hirsch, C., 1994. Numerical Computation of Internal and External Flows. Bruselas- Belgium: A Wiley-lnterscience Publication.
11. Huang & Kappen, 1998. A Study of Cell-Centered Finite Volume Methods for Diffusion Equations. *Mathematics Research Report*.
12. Lipnikov, K., Shashkov, M., Svyatskiy, D. & Vassilevski, Y., 2007. Monotone Finite Volume Scheme for Diffusion Equations on Unstructured Triangular and Shape-Regular Polygonal Meshes. *Journal of Computational Physics*, Novembro, Volume 227, pp. 492-512.
13. Wu, J., Fu, S. W. & Shen, L. J., 2003. A Difference Scheme with High Resolution for the Numerical Solution of a Nonlinear Diffusion Equation. *Journal on Numerical Methods and Computer Applications*, Volume 24.
14. Gao, Zhiming & Wu, Jiming, 2010. A linearity-preserving cell-centered scheme for the heterogeneous and anisotropic diffusion equations on general meshes. *International Journal for Numerical Methods in Fluids*, 10 December, Volume 67, pp. 2157-2183.

8. RESPONSIBILITY NOTICE

The author(s) is (are) the only responsible for the printed material included in this paper.