# ATTITUDE AND ORBIT MODELLING, SIMULATION AND CONTROL FOR THE BEPICOLOMBO MISSION 

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Abstract. This paper presents the formulation and analysis of gravitational capture process around the Planet Mercury motivated by the BepiColombo Mission, to be launch in 2015 by European Space Agency (ESA). It presents too the Mercury Planet Orbiter (MPO) attitude control after the cruise from Earth to Mercury, when the modules that composes the Mission are considered inside Mercury's sphere of influence. The controller must act in the MPO in a way that it points to Mercury in all instant of time, during its navigation. The vibration of the solar panel, composed by three segments, is analyzed during the attitude maneuvers.

Keywords: BepiColombo Mission, attitude control, gravitational capture.

## 1. INTRODUCTION

The BepiColombo Mission will be the first space mission to orbit Mercury and is considered a cornerstone to the aerospace engineering (ESA, 2000). Another missions, just like Mariner 10 and Messenger, already realized fly-bys around the planet. Mariner 10 was the first mission that realized a passage near Mercury in 1974 and registered a local magnetic field and high radiation taxes (NASA, 2011).

The Mission is composed by two main modules, the Mercury Planetary Orbiter (MPO), the Mercury Magnetospheric Orbiter (MMO) (ESA, 2000). The MPO and MMO are illustrated in Fig. 1. The modules are coupled in the cruise, when will be subject to fly-bys with Venus, Earth and the Sun.

After entry in the Mercury's sphere of influence the modules are captured gravitationally, in a process in that the total energy of the orbit initially positive turns in a negative value, configuring in this way an elliptical orbit around Mercury. This temporary orbit is not stable because of the intense Sun perturbation as a third-body. In that case the modules must improve an orbital maneuver inside the sphere of influence to reduce again the energy of the orbit.

Based in the three-body problem, is possible to find the conditions (operational margins) of the entry parameters in the sphere of influence, just like position and velocity for that the capture be possible.

A second problem to be analyzed is the attitude control of the modules, after the gravitational capture. In this moment the perturbation of the Sun is not considered and the just the MPO attitude are considered because this module demand a most rigorous control (in three axis, pointing to Nadir). The simulations are made considering reaction wheels and gas jets individually and finally coupled. Finally the effects of the control of attitude in the MPO solar panels vibration are present.

## 2. GRAVITATIONAL CAPTURE OF BEPICOLOMBO MODULES

The gravitational capture is a physical phenomenon that reduces the energy of an arbitrary orbit without use of a propulsion system. Yamakawa (1992) presented the concept of temporary gravitational capture in that the process of capture reduces the energy, but the vehicle stay inside the sphere of influence during a limited time. Belbruno (2000) realized calculus for the weak stability boundary and proposed a simple method for ideal insertion velocity calculation. Vieira Neto (1999) formalized a model based in three body problem restricted given by Eq. (1) in that the radio vectors $\mathrm{r}_{1}$ and $\mathrm{r}_{2}$ are given by Eq. (2).

The gravitational capture has been studied for many natural systems just like Earth - Moon (Prado, 2005), Netuno Triton (Agnor and Hamilton, 2006; Solorzano, 2006). The last author presented relations between the temporary capture time and parameters just like the entry angle in the sphere of influence, the Jacobi constant and the orbit energy. A more sophisticated model methodology for optimal design of space mission based in the gravitational capture can be found in Jerg. Et Al. (2009).

(a)
(b)

Figure 1. BepiColombo modules illustration. (a) MPO, (b) MMO. Font: (ESA, 2000)

$$
\begin{align*}
& \ddot{x}-2 n \dot{y}-n^{2} x=-\left[\mu_{1} \frac{x+\mu_{2}}{r_{1}^{3}}+\mu_{2} \frac{x-\mu_{1}}{r_{2}^{3}}\right] \\
& \ddot{y}-2 n \dot{x}-n^{2} y=-\left[\frac{\mu_{1}}{r_{1}^{3}}+\frac{\mu_{2}}{r_{2}^{3}}\right] y  \tag{1}\\
& \ddot{z}=-\left[\frac{\mu_{1}}{r_{1}^{3}}\right] z \\
& r_{1}^{2}=\mu_{2}^{2}+y^{2}+z^{2} \\
& r_{2}^{2}=\left(-\mu_{1}\right)^{2}+y^{2}+z^{2} \tag{2}
\end{align*}
$$

The set of equations given by Eq. (1) represents the law of universal gravitational extended for the case in that there are two primaries (Sun and Mercury) in a system that rotates around the barycenter of both primaries. The distance between the modules and the barycenter is indicated by $r_{1}$, just like its distance of Sun $r_{13}$, and Mercury $r_{2}$. The distance between the Sun and the barycenter is $a$ and $b$ is the distance between it and Mercury. The x-y plane rotates around the inertial plane $\eta-\xi$. The coordinates system for three-body problem is illustrated in Fig. 2.a.

(b)

Figure 2. (a) Coordinates system for three-body problem. (b) Gravitational capture parameters

The set of equations given by Eq. (1) does not present an analytical solution, being necessary resolve then numerically. A interesting information, that can be extracted from the model, is the space region where the movement of the modules is possible. This region can be found using the Jacobi's equation (Eq. (3)) for $\mathrm{C}_{\mathrm{J}}$ (Jacobi constant) and the movement is limited for the zero velocities curves.

$$
\begin{equation*}
C_{J}=n^{2}\left(x^{2}+y^{2}\right)+2\left(\frac{\mu_{1}}{r_{1}}+\frac{\mu_{2}}{r_{2}}\right)-\dot{x}^{2}-\dot{y}^{2}-\dot{z}^{2} \tag{3}
\end{equation*}
$$

Through simulations and tests, can be observed that the movement around Mercury, perturbed by the Sun, is only possible for $\mathrm{C}_{\mathrm{J}}$ between 2.9 and 9.7. Figure 3 illustrate the space regions where the clean areas are the places are the movement is possible and the dark areas are the places where the movement is not possible.


Figure 3. Zero velocity curves for restricted three-body problem (Sun and Mercury). (a) $C_{J}=2.9$, (b) $C_{J}=3.0$, (c) $C_{J}=$

$$
3.1, \text { (d) } \mathrm{C}_{\mathrm{J}}=3.2
$$

Another important concept is sphere of influence definition. Is has been proposed by Laplace and consist in a space region where the effect of a primary is major about an orbit body compared if other primary that is over the sphere (CHOBOTOV, 2002). The general expression for calculus of the radio of the sphere of influence can be found in Barrabés et.Al (2004) paper, and is given by Eq. (4)

$$
\begin{equation*}
\left(m_{1}+m_{2}\right) m_{1} r^{2}\left|\frac{\vec{r}_{12}}{r_{12}^{3}}-\frac{\vec{\rho}}{\rho^{3}}\right|=\left(m_{2}+m\right) m_{2} \rho^{2}\left|\frac{\vec{r}_{12}}{r_{12}^{3}}-\frac{\vec{r}}{r^{3}}\right| \tag{4}
\end{equation*}
$$

in that $r_{12}$ is the distance between the Sun and Mercury. As the Mercury mass is much more inferior compared with the Sun mass Eq. (4) can be simplified and became the expression given by Eq. (5).

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$$
\begin{equation*}
\rho_{m}=r_{12}\left[\frac{m_{1}}{m_{2}}\right]^{2 / 5} \tag{5}
\end{equation*}
$$

Been the Sun and Mercury masses given by $198910010^{30} \mathrm{~kg}$ and $330210^{23} \mathrm{~kg}$ respectively, and the distance $\mathrm{r}_{12}$ given by $57.9110^{6} \mathrm{~km}$, the sphere of influence of Mercury adopts a value of $1.124110^{5} \mathrm{~km}$.

The simulations for the gravitational capture start with the assumption that the entry angle and position in the sphere of influence in the crossing, after the BepiColombo cruise, is not known.

The model gives by the set of Eq. (1) represented by state variables is given by the set of Eqs. (6). If the orbital velocity value in the initial time from simulation is obtain by Vis-Viva equation (Eq.(7)), then its value in the pericenter is $3.8476 \mathrm{~km} / \mathrm{s}$ and in the apocenter is $2.8167 \mathrm{~km} / \mathrm{s}$. The limit case that make possible the entry is when the orbital energy is zero, in others words, the orbital trajectory is not elliptical and the modules cross the sphere influence in few time. For this case the velocity at pericenter is $3.8787 \mathrm{~km} / \mathrm{s}$ and at apocenter is $3.3448 \mathrm{~km} / \mathrm{s}$. The total time for simulation is 50 hours. In the cases that the modules cross the sphere of influence before the end of simulation, this interval is not figure. By the fact that the problem adopted is symmetric in relation with the x -axis, the orbital trajectory for direct and retrogrades orbits are mirror.

$$
\begin{align*}
& \dot{x}(1)=x(4) \\
& \dot{x}(2)=x(5) \\
& \dot{x}(3)=x(6) \\
& \dot{x}(4)=-\frac{\mu_{1}[x(1)+a]}{\left[(x(1)+a)^{2}+x(2)^{2}+x(3)^{2}\right]^{3 / 2}}-\frac{\mu_{1}[x(2)]}{\left[(x(1)-b)^{2}+x(2)^{2}+x(3)^{2}\right]^{3 / 2}}  \tag{6}\\
& \dot{x}(5)=-\frac{\mu_{2}[x(1)-b]}{\left[(x(1)+a)^{2}+x(2)^{2}+x(3)^{2}\right]^{3 / 2}}-\frac{\mu_{2}[x(2)]}{\left[(x(1)-b)^{2}+x(2)^{2}+x(3)^{2}\right]^{3 / 2}} \\
& \dot{x}(6)=-\frac{\mu_{1}[x(3)]}{\left[(x(1)+a)^{2}+x(2)^{2}+x(3)^{2}\right]^{3 / 2}}-\frac{\mu_{2}[x(3)]}{\left[(x(1)-b)^{2}+x(2)^{2}+x(3)^{2}\right]^{3 / 2}}
\end{align*}
$$

$$
\begin{equation*}
V_{0}=\sqrt{2\left(\frac{\mu}{r}-\frac{\mu}{2 a}\right)} \tag{7}
\end{equation*}
$$

Increasing slowly the velocity at pericenter from $3.8476 \mathrm{~km} / \mathrm{s}$ (value for elliptical orbit) to $3.8787 \mathrm{~km} / \mathrm{s}$ (value for zero energy) and analyzing the total capture time, is possible to observe that with this increase the both total capture time and time for escape from sphere of influence are reduced. The difference between these values refers to the parameter analyzed. The total capture time consist in all the interval time that the orbit presents negative energy. How in fact the orbit takes the null energy before cross the sphere of influence, is interesting to know how long it takes to abandon the sphere, what consist in another quantity. The results of simulation with pericenter velocity variation are presented on Tab. 1. The fields in that there is no value specified means that the modules do not leave the sphere.

Now, if the energy value is considered fix at zero and the position of insertion in the temporary orbit (pericenter direction vector) is modified in a way that it points to one of the six directions on the orthonormal Cartesian system, the total capture time and the time for abandon the sphere of influence are presented in Tab. 2 For some positions in the space there is no condition for gravitational capture and the modules go direct for the limit of the sphere of influence in few time.

At least, is considered the case in that the orbit is orthonormal in relation with the line connecting the Sun and Mercury (Tab. 3). In this case the perturbation is normal to the central field and there is any kind of resistance to the perturbation once in any moment there are movement in the other direction of the perturbation. So the total capture time is always zero because the simulation starts with null energy and it always increase with perturbation and the modules go direct for the limit of the sphere of influence in few time again.

Table 1. Total capture time for MPO final orbit with pericenter velocity variation.

| Case | Orbit type | Initial conditions$\mathbf{R}_{0}(\mathrm{~km}), \mathbf{V}_{0}(\mathrm{~km} / \mathrm{s})$ |  | Total capture time | Time for abandon the sphere of influence |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Direct | $\mathrm{R}_{0}=\left[\begin{array}{llll}\mathrm{b}-490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & -3.8476 & 0\end{array}\right]$ | - | - |
| 2 |  | $\mathrm{R}_{0}=\left[\begin{array}{lllll}\mathrm{b}-490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & -3.8576 & 0\end{array}\right]$ | 28h 17m 48s | 43h 56m 32s |
| 3 |  | $\mathrm{R}_{0}=\left[\begin{array}{lllll}\mathrm{b}-490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & -3.8676 & 0\end{array}\right]$ | $\begin{gathered} 13 \mathrm{~h} 46 \mathrm{~m} 43 \mathrm{~s} * / \\ 27 \mathrm{~h} 02 \mathrm{~m} 00 \mathrm{~s} \end{gathered}$ | 41h 48m 56s |
| 4 |  | $\mathrm{R}_{0}=\left[\begin{array}{lllll}\mathrm{b}-490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & -3.8787\end{array}\right]$ | 13 h 58 m 08 s | 35h 39 m 05 s |
| 5 | Retrograde | $\mathrm{R}_{0}=\left[\begin{array}{lllll}\mathrm{b}-490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 3.8476 & 0\end{array}\right]$ | - | - |
| 6 |  | $\mathrm{R}_{0}=\left[\begin{array}{lllll}\mathrm{b}-490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{lll}0 & 3.8576 & 0\end{array}\right]$ | 13h 58m 08s | 43h 56m 32s |
| 7 |  | $\mathrm{R}_{0}=\left[\begin{array}{llll}\mathrm{b}-490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{lll}0 & 3.8676 & 0\end{array}\right]$ | $\begin{gathered} 13 \mathrm{~h} 46 \mathrm{~m} \mathrm{43s} * / \\ 27 \mathrm{~h} 02 \mathrm{~m} \mathrm{00s} \end{gathered}$ | 41h 48m 56s |
| 8 |  | $\mathrm{R}_{0}=\left[\begin{array}{lllll}\mathrm{b}-490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{lll}0 & 3.87870\end{array}\right]$ | $13 \mathrm{~h} 58 \mathrm{~m} \mathrm{08s}$ | 35h 39 m 05 s |

*Temporary energy transition instant, however with return to an elliptical orbit.

Table 2. Total capture time for MPO final orbit with energy $\mathrm{E}_{2}=0 \mathrm{~J}$.

| Case | Director vectors | Initial conditions $\mathbf{R}_{\mathbf{0}}(\mathrm{km}), \mathbf{V}_{\mathbf{0}}(\mathrm{km} / \mathrm{s})$ |  | Total capture time | Time for abandon the sphere of |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\begin{aligned} & \mathrm{d}_{1}=\left[\begin{array}{lll} -1 & 0 & 0 \end{array}\right] \\ & \mathrm{d}_{2}=\left[\begin{array}{lll} 0 & -1 & 0 \end{array}\right] \end{aligned}$ | $\mathrm{R}_{0}=\left[\begin{array}{llll}\mathrm{b} & -490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0-3.8787 & 0\end{array}\right]$ | 13h 58m 08s | 35h 13m 35s |
| 2 |  | $\mathrm{R}_{0}=\left[\begin{array}{lllll}\text { b } & 500 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 3.3448 & 0\end{array}\right]$ | 0 | 14h 18m 29s |
| 3 | $\begin{aligned} & \mathrm{d}_{1}=\left[\begin{array}{lll} 1 & 0 & 0 \end{array}\right] \\ & \mathrm{d}_{2}=\left[\begin{array}{lll} 0 & 1 & 0 \end{array}\right] \end{aligned}$ | $\mathrm{R}_{0}=\left[\begin{array}{llllll}\text { b } & 490 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{lll}0 & 3.8787 & 0\end{array}\right]$ | 0 | 13h 59m 10s |
| 4 |  | $\mathrm{R}_{0}=\left[\begin{array}{llllll}\text { - } & 500 & 0 & 0\end{array}\right]$ | $\mathrm{V}_{0}=[0-3.34480]$ | 16h 38m 47s | $31 \mathrm{~h} \mathrm{14m} \mathrm{04s}$ |
| 5 | $\begin{aligned} \mathrm{d}_{1} & =\left[\begin{array}{lll} 0 & -1 & 0 \end{array}\right] \\ \mathrm{d}_{2} & =\left[\begin{array}{lll} 1 & 0 & 0 \end{array}\right] \end{aligned}$ | $\mathrm{R}_{0}=[\mathrm{b}-49000]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}3.8787 & 0 & 0\end{array}\right]$ | 8h 27 m 00 s | 22h 55m 02s |
| 6 |  | $\mathrm{R}_{0}=[\mathrm{b} 150000]$ | $\mathrm{V}_{0}=\left[\begin{array}{lllll}-3.3448 & 0 & 0\end{array}\right]$ | 0 | 14h 31m 45s |
| 7 | $\begin{aligned} & \mathrm{d}_{1}=\left[\begin{array}{lll} 0 & 1 & 0 \end{array}\right] \\ & \mathrm{d}_{2}=\left[\begin{array}{lll} -1 & 0 & 0 \end{array}\right] \end{aligned}$ | $\mathrm{R}_{0}=[\mathrm{b} 49000]$ | $\mathrm{V}_{0}=\left[\begin{array}{lllll}-3.8787 & 0 & 0\end{array}\right]$ | 0 | 14h 46m 45s |
| 8 |  | $\mathrm{R}_{0}=[\mathrm{b}-15000]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 3.3448 & 0\end{array}\right]$ | 8h 50m 53s | 23h 19m 00s |
| 9 | $\begin{aligned} & \mathrm{d}_{1}=\left[\begin{array}{lll} 0 & 0 & -1 \end{array}\right] \\ & \mathrm{d}_{2}=\left[\begin{array}{lll} 1 & 0 & 0 \end{array}\right] \end{aligned}$ | $\mathrm{R}_{0}=\left[\begin{array}{llll}\mathrm{b} & 0 & -490\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}3.8787 & 0 & 0\end{array}\right]$ | 8h 27 m 00 s | 22h 55m 02s |
| 10 |  | $\mathrm{R}_{0}=\left[\begin{array}{lll}\mathrm{b} & 0 & 1500\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 3.3448 & 0\end{array}\right]$ | 0 | 14h 31m 45s |
| 11 | $\begin{aligned} \mathrm{d}_{1} & =\left[\begin{array}{lll} 0 & 0 & 1 \end{array}\right] \\ \mathrm{d}_{2} & =\left[\begin{array}{lll} -1 & 0 & 0 \end{array}\right] \end{aligned}$ | $\mathrm{R}_{0}=\left[\begin{array}{lll}\mathrm{b} & 0 & 490\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{lll}0 & -3.8787 & 0\end{array}\right]$ | 0 | 14h 46m 45s |
| 12 |  | $\mathrm{R}_{0}=\left[\begin{array}{lll}\mathrm{b} & -1500\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 3.3448\end{array}\right]$ | 8h 50m 53s | 23h 19m 00s |

Table 3. Total capture time for MPO final orbit with energy $\mathrm{E}_{2}=0 \mathrm{~J}$ and with angular momentum vector parallel with the apsides line.

| Case | Director vectors | Initial conditions $\mathbf{R}_{0}(\mathrm{~km}), \mathbf{V}_{\mathbf{0}}(\mathrm{km} / \mathrm{s})$ |  | Total capture time | Time for abandon the sphere of |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | $\mathrm{d}_{1}=\left[\begin{array}{lll}0 & 0 & 1\end{array}\right]$ | $\mathrm{R}_{0}=[\mathrm{b} 49000]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 0 & 3.8787\end{array}\right]$ | 0 | 18h $52 \mathrm{~m} \mathrm{00s}$ |
| 14 | $\mathrm{d}_{2}=\left[\begin{array}{lll}0 & 1 & 0\end{array}\right]$ | $\mathrm{R}_{0}=[\mathrm{b}-15000]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 0 & -3.3448\end{array}\right]$ | 0 | 18h 58m 02s |
| 15 | $\mathrm{d}_{1}=\left[\begin{array}{lll}0 & 0 & -1\end{array}\right]$ | $\mathrm{R}_{0}=[\mathrm{b}-49000]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 0 & -3.8787\end{array}\right]$ | 0 | 18h 52m 00s |
| 16 | $\mathrm{d}_{2}=\left[\begin{array}{lll}0 & -1 & 0\end{array}\right]$ | $\mathrm{R}_{0}=\left[\begin{array}{llll}\mathrm{b} & 1500 & 0\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & 0 & 3.3448\end{array}\right]$ | 0 | 18h 58m 02s |
| 17 | $\mathrm{d}_{1}=\left[\begin{array}{lll}0 & 1 & 0\end{array}\right]$ | $\mathrm{R}_{0}=\left[\begin{array}{lll}\mathrm{b} & 0 & 490\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & -3.8787 & 0\end{array}\right]$ | 0 | 18h 52m 00s |
| 18 | $\mathrm{d}_{2}=\left[\begin{array}{lll}0 & 0 & 1\end{array}\right]$ | $\mathrm{R}_{0}=\left[\begin{array}{lll}\mathrm{b} & 0 & -1500\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{lll}0 & 3.3448 & 0\end{array}\right]$ | 0 | 18h 58m 02s |
| 19 | $\mathrm{d}_{1}=\left[\begin{array}{lll}0 & -1 & 0\end{array}\right]$ | $\mathrm{R}_{0}=\left[\begin{array}{lll}\mathrm{b} & 0 & -490\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{lll}0 & 3.87870\end{array}\right]$ | 0 | 18h $52 \mathrm{~m} \mathrm{00s}$ |
| 20 |  | $\mathrm{R}_{0}=\left[\begin{array}{lll}\mathrm{b} & 0 & 1500\end{array}\right]$ | $\mathrm{V}_{0}=\left[\begin{array}{llll}0 & -3.3448 & 0\end{array}\right]$ | 0 | 18h 58m 02s |

Based in the effects of the variation of velocity in the initial conditions and the variation of initial position, is finally considered the official parameters of the Mission. The arrival of modules in the sphere of influence Mercury will be in the aphelion, so the initial position of the modules is in the pericenter of the temporary orbit (ESA, 2013). Now considering this contour condition (start of simulation at pericenter of temporary entry orbit) and realizing a numerical integration with negative step the trajectory illustrated in Fig. 4 is describe. The modules cross the sphere of influence and are captured by the planet (moments in that the total orbit energy is negative, as show in Fig. 5). By this analysis it is possible to identify the capture gravitational parameters, illustrated in Fig. 2.b, in other words, the entry angle ( $\beta$ ) equals to $193.9137^{\circ}$, the arrival angle in orbit insertion ( $\sigma$ ) equals to $0^{\circ}$ (periapice condition), vector velocity in the sphere equals to $\left[\begin{array}{lll}2.3561 & 0.49140\end{array}\right]^{\mathrm{T}} \mathrm{km} / \mathrm{s}$ and vector position equals to $[5.78-0.00270]^{\mathrm{T}} 10^{7} \mathrm{~km}$.


Figure 4. Gravitational capture of the modules.


Figure 5. Temporal evolution of orbit total energy during the gravitational capture

## 3. MPO ATITUDE CONTROL

### 3.1 MPO RIGID-BODY MODEL

The MPO rigid-body model is derived by the Lagragian formalism. The Lagrangian of the system (L) is given by the algebraic sum of the potential energy given by Eq.(8) and the kinetic energy given by Eq. (9).

$$
\begin{align*}
& V=\frac{1}{2} K\left(\theta_{y}-\theta\right)^{2}+\frac{1}{2} K\left(\alpha_{1}-\theta_{y}\right)^{2}+\frac{1}{2} K\left(\alpha_{2}-\alpha_{1}\right)^{2}+\frac{1}{2} K\left(\alpha_{3}-\alpha_{2}\right)^{2}  \tag{8}\\
& T=\frac{1}{2} J_{t} \dot{\theta}_{s}^{2}+\frac{1}{2} J_{t} \dot{\theta}_{y}^{2}+\frac{1}{2} J_{t} \dot{\alpha}_{1}^{2}+\frac{1}{2} J_{t} \dot{\alpha}_{2}^{2}+\frac{1}{2} J_{t} \dot{\alpha}_{3}^{2}+\frac{1}{2} m_{y} \dot{\vec{r}}_{y}^{2}+\frac{1}{2} m_{1} \dot{\vec{r}}_{1}^{2}+\frac{1}{2} m_{2} \dot{\vec{r}}_{2}^{2}+\frac{1}{2} m_{3} \dot{\vec{r}}_{3}^{2} \tag{9}
\end{align*}
$$

The equations of movement is derived by the Euler-Lagrange equation (Eq.(10)) for each general coordinate (freedom degrees), in other words, the attitude angle $\left(\theta_{\mathrm{s}}\right)$ and rigid solar panel angles ( $\theta_{\mathrm{y}}$ for yoke angle, $\alpha_{1}$ for the interior panel, $\alpha_{2}$ for the central panel, $\alpha_{3}$ for the exterior panel)

$$
\begin{equation*}
\frac{d}{d t} \frac{\partial L}{\partial \dot{q}}-\frac{\partial L}{\partial q}=\tau_{\theta_{y}} \tag{10}
\end{equation*}
$$

So, the five linear differential equations that govern the attitude movement of MPO and its solar panel vibration are given by Eq. (11), Eq. (12), Eq.(13), Eq.(14) and Eq.(15), in that $m_{y}, m_{1}, m_{2}$ and $m_{3}$ are the yoke, interior panel, central panel and exterior panel masses respectively, R is the main dimension of the cubic satellite, $\mathrm{J}_{\mathrm{t}}$ is the satellite inertia, $1_{\mathrm{y}}$ is the yoke length, $l_{p}$ is the panels length.

$$
\begin{align*}
& J_{t} \ddot{\theta}_{s}+\left(m_{y}+m_{1}+m_{2}+m_{3}\right) R^{2} \ddot{\theta}_{s}+\left(\frac{m_{y}}{2}+m_{1}+m_{2}+m_{3}\right) R l_{y} \ddot{\theta}_{y}+ \\
& +\left(\frac{m_{1}}{2}+m_{2}+m_{3}\right) R l_{p} \ddot{\alpha}_{1}+\left(\frac{m_{2}}{2}+m_{3}\right) R l_{p} \ddot{\alpha}_{2}+\left(\frac{m_{3}}{2}\right) R l_{p} \ddot{\alpha}_{3}-k_{y} \theta_{y}+k_{y} \theta_{s}=T_{\theta_{s}} \tag{11}
\end{align*}
$$

$$
\begin{align*}
& J_{y} \ddot{\theta}_{y}+\left(\frac{m_{y}}{2}+m_{1}+m_{2}+m_{3}\right) R l_{y} \ddot{\theta}_{s}+\left(m_{1}+m_{2}+m_{3}\right) l_{y}^{2} \ddot{\theta}_{y}+  \tag{12}\\
& +\left(\frac{m_{1}}{2}+m_{2}+m_{3}\right) l_{y} l_{p} \ddot{\alpha}_{1}+\left(\frac{m_{2}}{2}+m_{3}\right) l_{y} l_{p} \ddot{\alpha}_{2}+\left(\frac{m_{3}}{2}\right) l_{y} l_{p} \ddot{\alpha}_{3}+\left(k_{y}+k_{1}\right) \theta_{y}-k_{y} \theta_{s}-k_{1} \alpha_{1}=T_{\theta_{y}} \\
& J_{1} \ddot{\alpha}_{1}+\left(\frac{m_{1}}{2}+m_{2}+m_{3}\right) R l_{p} \ddot{\theta}_{s}+\left(\frac{m_{1}}{2}+m_{2}+m_{3}\right) l_{y} l_{p} \ddot{\theta}_{y}+  \tag{13}\\
& +\left(\frac{m_{1}}{4}+m_{2}+m_{3}\right) l_{p}^{2} \ddot{\alpha}_{1}+\left(\frac{m_{2}}{2}+m_{3}\right) l_{p}^{2} \ddot{\alpha}_{2}+\left(\frac{m_{3}}{2}\right) l_{p}^{2} \ddot{\alpha}_{3}-k_{1} \theta_{y}-k_{y} \theta_{s}+\left(k_{1}+k_{2}\right) \alpha_{1}-k_{2} \alpha_{2}=T_{1} \\
& J_{2} \ddot{\alpha}_{2}+\left(\frac{m_{2}}{2}+m_{3}\right) R l_{p} \ddot{\theta}_{s}+\left(\frac{m_{2}}{2}+m_{3}\right) l_{y} l_{p} \ddot{\theta}_{y}+ \\
& +\left(\frac{m_{2}}{2}+m_{3}\right) l_{p}^{2} \ddot{\alpha}_{1}+\left(\frac{m_{2}}{4}+m_{3}\right) l_{p}^{2} \ddot{\alpha}_{2}+\left(\frac{m_{3}}{2}\right) l_{p}^{2} \ddot{\alpha}_{3}-k_{2} \alpha_{1}+\left(k_{2}+k_{3}\right) \alpha_{2}-k_{3} \alpha_{3}=T_{2}  \tag{14}\\
& J_{3} \ddot{\alpha}_{3}+\left(\frac{m_{3}}{2}\right) R l_{p} \ddot{\theta}_{s}+\left(\frac{m_{3}}{2}\right) l_{y} l_{p} \ddot{\theta}_{y}+\left(\frac{m_{3}}{2}\right) l_{p}^{2} \ddot{\alpha}_{1}+\left(\frac{m_{3}}{2}\right) l_{p}^{2} \ddot{\alpha}_{2}+\left(\frac{m_{3}}{4}\right) l_{p}^{2} \ddot{\alpha}_{3}-k_{3} \alpha_{2}+k_{3} \alpha_{3}=T_{3} \tag{15}
\end{align*}
$$

### 3.2 MPO ATTITUDE CONTROL

In this section is applied a PID control for the MPO attitude control considering three different cases. In the first case is consider a reaction wheel for the control, the second case is consider a gas jet and the last both actuators together. The reaction wheel present a slow dynamic compared to the gas jet, however the attitude error presents a smooth curve. The parameters of the reaction wheel are the inertia, the viscous friction and the Coulumb friction. These parameters were estimated using a SunSpace Reaction Wheel allocated in the Laboratory of Simulation (LabSim) in the National Institute for Space Research (INPE) whose values are $1.510^{-3} \mathrm{~kg} . \mathrm{m}^{2}$ for the reaction wheel inertia, $5.99410^{-6}$ $\mathrm{Nm} . \mathrm{s}$ for the viscous friction and 0.00104 Nm for the Coulomb friction.

Considering the values for the PID gains as $\mathrm{K}_{\mathrm{p}}=30, \mathrm{~K}_{\mathrm{d}}=30$ and $\mathrm{K}_{\mathrm{i}}=0$, and using as reference for the controller the attitude that make the module points to nadir in all instant of time and for all position in the orbit, the attitude error evolution in the time using only reaction wheel are illustrated in Fig. 6. The maximum error is almost $10^{\circ}$, what is consider a high level of error. Due to the reaction wheel saturation (maximum torque available equals to 0.06842 Nm ) the controller is not able to hold the attitude precisely. The time evolution of the torques applied in the MPO is illustrated in Fig. 6.

Using the same gains for the PID control and the same performance requirements but with a gas jat actuator, the time evolution of the attitude error and the time evolution of the torques applied are illustrated in Fig. 7. The case in that the controls are coupled is illustrated in Fig. 8.


Figure 6. MPO attitude error (controlled by reaction wheel) and torques generate.


Figure 7. MPO attitude errors (controlled by jats) and torques generate.


Figure 8. MPO attitude error (controlled by reaction wheel and jats) and torques generate.

## 4. CONCLUSIONS

Through the analysis of the gravitational capture model, based on the restricted problem of three bodies, it follows that, after the cruise of the BepiColombo mission modules, they can be gravitationally captured by the planet Mercury and join the ideal transfer orbit (assigned by ESA) intersect the sphere of influence of the planet with a speed of 2.40 $\mathrm{km} / \mathrm{s}$ and submit an entry angle $(\beta)$ of $193.9137^{\circ}$ degrees from the line connecting the Sun and Mercury.

After entering the sphere of influence, it was showed that the attitude control by reaction wheels allows that the attitude not diverge indefinitely, but showing slow response. The amplitude of the attitude error with the use of gas jets was smaller in relation to the wheels. Finally the coupled control wheel-jets had satisfactory performance.

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## 6. RESPONSIBILITY NOTICE

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