ANALYSIS OF FORCED CONVECTION IN TWO-DIMENSIONAL LAMINAR FLOW IN RECTANGULAR DUCTS BY GITT PRIMITIVE WITH VARIABLE.

Thiago Andrade Fernandes
Carlos Antônio Cabral dos Santos
UFPB, Cidade Universitária, CEP: 58051-9000,João Pessoa-PB,Brasil
cabral@les.ufpb.br

Kleber Lima Cezar
UFPB, Cidade Universitária, CEP: 58051-9000,João Pessoa-PB,Brasil
kleber_21m4@hotmail.com

Marcos Cézar Lima Cordeiro
UFPB, Cidade Universitária, CEP: 58051-9000,João Pessoa-PB,Brasil
marcoscezarfine@hotmail.com

Abstract. *In the present study, forced convection flow in two-dimensional laminar simultaneous development pipeline with rectangular geometry, is analyzed analytically using the Navier-Stokes, Poisson and Energy in its primitive form. These equations are solved by the Generalized Integral Transform Technique, which is characteristic hybrid-numeric. The development work is composed primarily by restructuring the problem, so that, it can be solved by using the subroutine Fortran, IMSL Library, DBVPFD. Then, the results obtained are considered of practical interest, such as the velocity field, the temperature profile, the behavior of the average temperature, the Nusselt number and the friction factor, with the goal of providing a better parameter for construction equipment. These parameters are assessed both longitudinally and horizontally. Finally, the results are validated through convergence charts in order to comparisons with literature articles.*
Keywords: *Forced Convection; GITT; Navier-Stokes; Variables Primitives ;Laminar)*

1. INTRODUCTION

The growth of technology, is directly correlated to the importance of heat transfer in the daily lives of people. We have as examples the fridge that we use daily, oven, electric shower and others. But this issue extends to both macro proportions, in the space field, the constructions of ships and interactions with the media, as the micro field, in IT for example, where more and more electronics and processors is decreasing ratio .

This work has as main goal, and more specifically, the forced convection. Where this is when the flow is caused by external means, for example, a fan or a pump or by wind in the atmosphere (Incropera, 2008).

The study of forced convection has been the subject of interest of many researchers due to its importance in engineering practice. The knowledge of certain parameters enables the development, optimization and construction of new equipment, as examples of these parameters can cite the friction factor, average speed, pressure, temperature and average Nusselt where these indicators enables the analysis of equipment, making with us to avoid undesirable effects, such as failure or reduction of the thermal performance of the same.

Another important factor in this study is the increasing need for exact solutions to engineering problems, increasingly complex, in a short time. For this reason the techniques of numerical approximations have been gaining more space for experimentation and classical analytical methods. This is because experimentation generally be time consuming and, depending on the test onerous, since for each new experiment has been spent on new use of other equipment, new measurements of equipment for the adaptation of the new situation. Recalling that one of the most important reasons of numerical technique is required is related to the fact that computers present processing capacity increasing, so has advanced considerably in the simulation of problems in fluid mechanics and heat transfer, allowing a smaller cost and minimizing time spent on the job.

The purely numerical methods are used a lot, but it has been used in recent years the Generalized Integral Transform Technique, known by its acronym in English as GITT (Generalized Integral Transform Technique), which is an analytical method for handling which presents solutions hybrid analysis.

The Generalized Integral Transform Technique, presents some advantages over purely numerical methods, since it retains the same characteristics of an analytical solution, not requiring domain discretization or meshing in a multidimensional geometry, in addition to keeping an automatic control over the relative error of the results, which is established a priori. Besides its excellent performace is confirmed by numerous studies in fluid mechanics (Cotta, 1993a) and heat transfer (Santos et al., 2002; Perreira and Pérez Guerrero, 2002).

Thiago Andrade Fernandes; Carlos Cabral Santos; Kleber Lima Cezar; Marcos Lima Cordeiro
Analysis Of Forced Convection In Two-Dimensional Laminar Flow In Rectangular Ducts By Gitt Primitive With Variable.

This work aims a theoretical study of the simultaneous determination of the hydrodynamic profiles and energy, in the region of a channel parallel flat plates. The numerical simulation is based on the solution of the Navier-Stokes equations for the flow and energy equation, given in a two-dimensional space, transforming it in terms of primitive variables by use of the methodology used in the Generalized Integral Transform Technique.

2. PROBLEM DEFINITION

We consider the problem of simultaneous development of the field of velocity, pressure and temperature geometry of a channel with parallel plates, operating at steady state in which the boundary conditions are specified in the input and output thereof, as illustrated in Figure 1
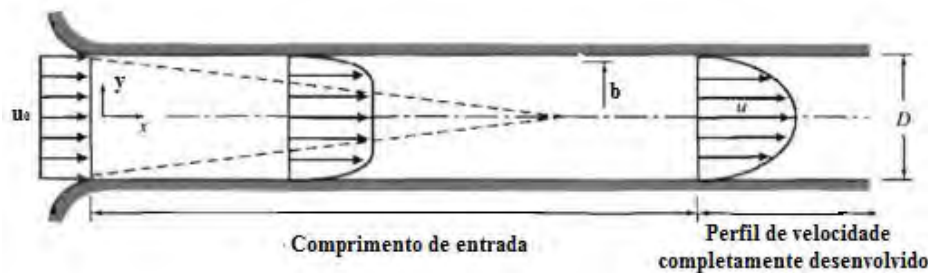

Figura 1 – Development in parallel pipelines

The mathematical formulation of the physical problem is obtained from the following restrictive assumptions: incompressible flow, waterproof and non-slip on the walls of the duct; Newtonian fluid, viscous dissipation negligible; physical constants; laminar flow, two-dimensional, steady. With the aim of determining the hydrodynamic characteristics of flow and heat transfer analysis along the apparatus.

2.1. HYDRODYNAMIC

For the solution of the hydrodynamic problem, the flow in the channel parallel plates is governed by the equations of continuity, Navier-Stokes, Poisson and submitted to the dimensionless groups:

$$X = \frac{x^{\bullet}}{b} \qquad\qquad Y = \frac{y^{\bullet}}{b} \qquad\qquad (1) - (2)$$

$$U(X,Y) = \frac{u^{**}(x^*, y^*)}{u_0} \qquad\qquad V(X,Y) = \frac{v^*(x^*, y^*)}{u_0} \qquad\qquad (3) - (4)$$

$$P(X,Y) = \frac{p^{**}(x^*, y^*) - p_0}{\rho u_0^2} \qquad\qquad Re = \frac{u_0 b}{v} \qquad\qquad (5) - (6)$$

Therefore the equations cited are now valid in the domain $0 < Y < 1$, $X > 0$ and its boundary conditions are rewritten in the following dimensionless form:

$$\frac{\partial U(X,Y)}{\partial X} + \frac{\partial V(X,Y)}{\partial Y} = 0 \qquad\qquad (7)$$

$$U(X,Y)\frac{\partial U(X,Y)}{\partial X} + V(X,Y)\frac{\partial U(X,Y)}{\partial Y} = -\frac{\partial P(X,Y)}{\partial X} + \frac{1}{Re}\left[\frac{\partial^2 U(X,Y)}{\partial X^2} + \frac{\partial^2 U(X,Y)}{\partial Y^2}\right] \qquad (8)$$

$$U(X,Y)\frac{\partial V(X,Y)}{\partial X} + V(X,Y)\frac{\partial V(X,Y)}{\partial Y} = -\frac{\partial P(X,Y)}{\partial Y} + \frac{1}{Re}\left[\frac{\partial^2 V(X,Y)}{\partial X^2} + \frac{\partial^2 V(X,Y)}{\partial Y^2}\right]$$ (9)

$$\frac{\partial^2 P(X,Y)}{\partial X^2} + \frac{\partial^2 P(X,Y)}{\partial Y^2} = 2\left[\frac{\partial U(X,Y)}{\partial X}\frac{\partial V(X,Y)}{\partial Y} - \frac{\partial V(X,Y)}{\partial X}\frac{\partial U(X,Y)}{\partial Y}\right]$$ (10)

With the boundary conditions of the form:

- Entry ($X=0$):
$$U(X,Y) = 1 \qquad\qquad\qquad P(X,Y) = 0$$ (11) - (12)

$$V(X,Y) = 0$$ (13)

- In the center and the symmetry ($Y=0$):
$$\frac{\partial U(X,Y)}{\partial Y} = 0 \qquad\qquad \frac{\partial P(X,Y)}{\partial Y} = 0$$ (14) – (15)

$$V(X,Y) = 0$$ (16)

- On exit ($X \rightarrow \infty$):
$$U(X,Y) = U_\infty(y) \qquad\qquad \frac{\partial P(X,Y)}{\partial X} = \frac{1}{Re}\frac{\partial^2 U_\infty(Y)}{\partial Y^2}$$ (17) – (18)

$$V(X,Y) = 0$$ (19)

- No slippage and tightness in the wall ($Y=1$):
$$U(X,Y) = 0 \qquad\qquad \frac{\partial P(X,Y)}{\partial Y} = \frac{1}{Re}\frac{\partial^2 V(X,Y)}{\partial Y^2}$$ (20) – (21)

$$V(X,Y) = 0$$ (22)

2.1.1 SOLUTION OF THE PROBLEM BY HYDRODYNAMIC GITT

For the proper implementation of GITT and improve their performance computing is convenient to a homogenization of the boundary conditions. This is done because according to Machado (1992) separation speed in two stages, where the first is under development according to the two dimensions and the second is already developed, depending only on the thickness of the channel greatly improves the convergence of solutions differential equations processed in this order:

$$U(X,Y) = U_\infty(Y) + U^*(X,Y)$$ (22)

Where $U_\infty(Y)$ is the velocity field is fully developed and $U^*(X,Y)$ the velocity field in development, to be discovered.

The expression representing the developed part is given by:

$$U_\infty(Y) = \frac{3}{2}(1 - y^2)$$ (23)

A filter pressure is needed to make a homogeneous boundary conditions. Therefore, the solution pressure is given as follows:

Thiago Andrade Fernandes; Carlos Cabral Santos; Kleber Lima Cezar; Marcos Lima Cordeiro
Analysis Of Forced Convection In Two-Dimensional Laminar Flow In Rectangular Ducts By Gitt Primitive With Variable.

$$P(X,Y) = P_F(X,Y) + P^*(X,Y) \tag{24}$$

Which gives:

$$P_F(X,Y) = P_0 + \frac{1}{Re}\frac{\partial V}{\partial Y} \qquad ou \qquad P_F(X,Y) = P_0 - \frac{1}{Re}\frac{\partial U^*}{\partial X} \tag{25}$$

With the boundary conditions for the filter defined as:

$$X = 0 \rightarrow \quad P_F(X,Y) = P_0 \tag{26}$$

$$Y = 0 \rightarrow \quad \frac{\partial P_F(X,Y)}{\partial Y} = -\frac{1}{Re}\frac{\partial^2 U}{\partial X \partial Y} = 0 \tag{27}$$

$$X = \infty \rightarrow \quad \frac{\partial P_F(X,Y)}{\partial X} = 0 \tag{28}$$

$$Y = 1 \rightarrow \quad \frac{\partial P_F(X,Y)}{\partial Y} = \frac{1}{Re}\frac{\partial^2 V}{\partial Y^2}\bigg|_{y=1} \tag{29}$$

With this technique adequately applying the GITT, or by choosing an auxiliary problem and determining a transformation pair of series and its inverse, can transform the hydrodynamic problem, which consisted of an EDP in a series of ODEs. Thus the governing equations are of the form:

$$\frac{d^2\bar{U}_i^*(X)}{dX^2} = \frac{Re}{2}\left\{\sum_{k=1}^{\infty}\left[\sum_{j=1}^{\infty}AB_{ijk}\bar{U}_j^*(X) + AB_{ik\infty}\right]\frac{d\bar{U}_k^*(X)}{dX} + \sum_{n=1}^{\infty}C_{in}\frac{d\bar{P}_n^*(X)}{dX}\right\} + \frac{\mu_i^2\bar{U}_i^*(X)}{2} - \frac{D_{if}}{2} \tag{30}$$

$$\sum_{n=1}^{\infty}P_{mat}(i,n)\frac{d^2\bar{P}_n^*(X)}{dX^2} = \bar{G}_i + \frac{1}{2}\sum_{m=1}^{\infty}G_{im}\cdot A_m \tag{31}$$

where:

$$\bar{G}_i = \beta_i^2\bar{P}_n + 2\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}E_{ijk}\frac{d\bar{U}_j^*(X)}{dX}\frac{d\bar{U}_k^*(X)}{dX} - 2\sum_{k=1}^{\infty}\left[\sum_{j=1}^{\infty}F_{ijk}\bar{U}_j^*(X) + F_{ik\infty}\right]\frac{d^2\bar{U}_k^*(X)}{dX^2} \tag{32}$$

$$A_m = \sum_{k=1}^{\infty}\sum_{j=1}^{\infty}AB_{mjk}\frac{d\bar{U}_j^*(X)}{dX}\frac{d\bar{U}_k^*(X)}{dX} + \sum_{k=1}^{\infty}\left[\sum_{j=1}^{\infty}AB_{mjk}\bar{U}_j^*(X) + AB_{mk\infty}\right]\frac{d^2\bar{U}_k^*(X)}{dX^2} + \frac{1}{Re}\mu_m^2\frac{d\bar{U}_m^*(X)}{dX} \tag{33}$$

$$P_{mat}(i,n) = \delta_{in} - \frac{1}{2}\sum_{i=1}^{\infty}G_{im}\cdot C_{mn} \tag{34}$$

Where the coefficients are calculated analytically from the integral:

$$AB_{ijk} = \int_0^1 \tilde{\phi}_i(Y)\tilde{\phi}_j(Y)\tilde{\phi}_k(Y)\,dY + \int_0^1 \tilde{\phi}_i(Y)\frac{d\tilde{\phi}_j(Y)}{dY}\bar{F}_k(Y)\,dY \tag{35}$$

$$AB_{ij\infty} = \int_0^1 U_\infty(Y)\tilde{\phi}_i(Y)\tilde{\phi}_k(Y)\,dY + \int_0^1 \tilde{\phi}_i(Y)\frac{dU_\infty}{dY}\bar{F}_k(Y)\,dY \tag{36}$$

$$C_{in} = \int_0^1 \tilde{\Phi}_i(Y)\,\tilde{\Psi}_n(Y)\,dY \tag{37}$$

$$D_{i\infty} = \int_0^1 \tilde{\Phi}_i(Y)\,\frac{d^2 U_\infty}{dY^2}\,dY \tag{38}$$

$$\overline{F}_k(Y) = \int_y^1 \tilde{\Phi}_k(Y)\,dY \tag{39}$$

$$E_{ijk} = \int_0^1 \tilde{\Psi}_i(Y)\,\tilde{\Phi}_j(Y)\,\frac{d\overline{F}_k(Y)}{dY}\,dY \tag{40}$$

$$F_{ijk} = \int_0^1 \tilde{\Psi}_i(Y)\,\frac{d\tilde{\Phi}_j(Y)}{dY}\,\overline{F}_k(Y)\,dY \tag{41}$$

$$F_{ik\infty} = \int_0^1 \tilde{\Psi}_i(Y)\,\frac{dU_\infty(Y)}{dY}\,\overline{F}_k(Y)\,dY \tag{42}$$

$$G_{im} = \int_0^1 \tilde{\Psi}_i(Y)\,\tilde{\Phi}_m(Y)\,dY \tag{43}$$

The same procedure is also applied to the boundary conditions:

$$\overline{U}_i^*(0) = \int_0^1 \tilde{\Phi}_i(Y)\,dY - \int_0^1 \tilde{\Phi}_i(Y)\,U_\infty(Y)\,dY \tag{44}$$

$$\overline{P}_i^*(0) = 0 \tag{45}$$

$$\overline{U}_i^*(\infty) = 0 \tag{46}$$

$$\frac{d\overline{P}_i^*(\infty)}{dX} = -\frac{3}{Re}\int_0^1 \tilde{\Psi}_i(Y)\,dY \tag{47}$$

Thus, the partial differential problem has been transformed into a series of common nonlinear equations with the boundary conditions at two points.

2.2. PROBLEM OF HEAT TRANSFER

Using the dimensionless parameters already mentioned, along with defined below:

$$T(X,Y) = \frac{T^*(x^*,y^*) - T_\infty}{\Delta T^*} \qquad\qquad Pr = \frac{\nu}{\alpha} \tag{48 – 49}$$

$$Pe = Re.Pr \qquad\qquad Bi = \frac{h_e b}{k_s} \tag{50 – 51}$$

Adimensionalizando equation (XX) and applying the filter velocity, we obtain:

-Energy Equation

$$[U_\infty(Y) + U^*(X,Y)]\frac{\partial T(X,Y)}{\partial X} + V(X,Y)\frac{\partial T(X,Y)}{\partial Y} = \frac{1}{Pe}\left[\frac{\partial^2 T(X,Y)}{\partial X^2} + \frac{\partial^2 T(X,Y)}{\partial Y^2}\right] \tag{52}$$

And similarly the equations of the conditions of entry and contour becomes:

$$T(X,Y) = 0 \qquad\qquad\qquad , com\ X = 0 \tag{53}$$

Thiago Andrade Fernandes; Carlos Cabral Santos; Kleber Lima Cezar; Marcos Lima Cordeiro
Analysis Of Forced Convection In Two-Dimensional Laminar Flow In Rectangular Ducts By Gitt Primitive With Variable.

$$T(X,Y) = 1 \qquad\qquad , com \; X \to \infty \qquad (54)$$

$$\frac{\partial T(X,Y)}{\partial Y} = 0 \qquad\qquad , com \; Y = 0 \qquad (55)$$

$$BiT(X,Y) + \frac{\partial T(X,Y)}{\partial Y} = 0 \qquad\qquad , com \; Y = 1 \qquad (56)$$

Using the technique adequately, ie selecting one auxiliary problem and determining a transformation pair of series and its inverse, can transform the energy problem, which consisted of an EDP's in a series of ODE's. Thus the governing equation is of the form:

$$\frac{d^2 \bar{T}_i^{*}(X)}{dX^2} = Pe \left\{ \sum_{j=1}^{\infty} \left[\sum_{k=1}^{\infty} AA_{ijk} \, \bar{U}_k^{*}(X) + P_{ij} \right] \frac{d\bar{T}_j^{*}(X)}{dX} + \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} CC_{ijk} \frac{d\bar{U}_k^{*}(X)}{dX} \bar{T}_j^{*}(X) \right\} + \gamma_i^2 \bar{T}_i^{*}(X) \qquad (57)$$

Where the coefficients are calculated analytically from the integral:

$$AA_{ijk} = \int_0^1 \hat{\Gamma}_i(y) \hat{\Gamma}_j(y) \tilde{\phi}_k(Y) dY \qquad (58)$$

$$P_{ij} = \frac{3}{2} \left[\delta_{ij} - \int_0^1 Y^2 \hat{\Gamma}_i(Y) \hat{\Gamma}_j(Y) dY \right] \qquad (59)$$

$$CC_{ijk} = \int_0^1 \hat{\Gamma}_i(Y) \frac{d\hat{\Gamma}_j(Y)}{dY} \bar{F}_k(Y) dY \qquad (60)$$

$$\bar{F}_k(Y) = \int_y^1 \tilde{\phi}_k(Y) \, dY \qquad (61)$$

3. SOLUTION OF ORDINARY DIFFERENTIAL SYSTEM

Because you do not have analytical solution of the ordinary second-order equations (30), (31) and (57), will be addressed from numerical procedures. The Generalized Integral Transform Technique, has as one of its important feature, ensuring convergence of solutions to increasing order of the number of terms of the series. The GITT method is also known for its controlled precision factor, as determined with this amount of terms of the series guarantees the convergence of the solution. Due to this control set in the order truncation of the expansion, which can be automatically determined during the solution process, similar to the technique rather than a purely analytical one.

As the boundary conditions for the channel output are specified at infinity do a contraction of scale, ie, we make a coordinate transformation such that one can map the semi-infinite domain in a finite domain. This change in channel length has been used by Perez Guerrero (1995), which is of the form:

$$\eta = 1 - e^{-cx} \qquad (62)$$

And its derivative,

$$\frac{d\eta}{dX} = c(1 - \eta) \qquad (63)$$

Where the constant c is the scale factor.

All coefficients were manipulated and then conferred by MATHEMATICA 8.0 (Wolfram, 2011), to validate the proposed problem, which is revolved by Fortran 90. The Jacobian was analyzed numerically by subroutine FDJAC IMSL library, which uses the finite difference method for obtaining the Jacobian matrix of the given function.

4. RESULTS AND DISCUSSION

The computational code was built in Fortran 90 and run on a working platform Intel Core i5 first generation of Solar Energy Laboratory of the DEM / UFPB.

In order to solve the problem of energy transfer associated with the hydrodynamic problem is determined and analyzed the behavior of the pressure and velocity fields for the system formulation of equations in terms of primitive variables. It was built the computational code where ever the problem is solved for Reynolds equal to 300, Prandtl equal to 0.7 and a domain by finite transformations of the domain (62). It is also important to note that all calculations were made with the same amounts of eigenfunctions, ie, NU equal to NP, which is equal to NT, we will deal with them by NN. The shrinkage factor worth 0:55.

In table 4.1 the results are shown in several works with results for Reynolds 300, which is worth remembering that it comes from the hydraulic Reynolds.

It is noted that the present results are consistent with literature and is consistent with the results of both Lime (2002) working with the primitive variables, and for Perez Guerrero (1995), working with input conditions and irrotational those of Wang and longwell (1994) and McDonald et al. (1972), these methods using finite differences and finite elements. The data are compared regarding Reynolds equal to 300, which are considered 60 terms in the series expansion.

Table 1 - Comparison table of the longitudinal velocity in the center of the pipe () to Reynolds 300.

	Position x			
Reference	0.2083	0.8333	3.3333	7.5000
Present Work	1.053	1.171	1.340	1.473
Lima (2002)	1.052	1.176	1.325	1.443
Peréz Guerrero (1995)	1.052	1.170	1.337	1.444
Wang e Longwell (1994)	1.058	1.188	1.357	1.451
Mac Donald et al, (1972)	1.050	1.170	1.340	1.440

In all tables developed observed that for small numbers of terms convergence is not achieved. It is also remarkable to note in regions near the entrance to the duct, the convergence is slow and is only achieved for higher numbers of terms of speed and pressure at the end of the duct convergence is faster speed due to filters and pressure applied the problem. The convergence rate improves along the pipeline, the measure that reaches the fully developed flow. The function $U_\infty(Y)$ used as a filter, the velocity profile is fully developed $U(X,Y)$, so when advancing the flow along the duct, tends to $U_\infty(Y)$ the speed $U(X,Y)$, requiring less term in the expansion.
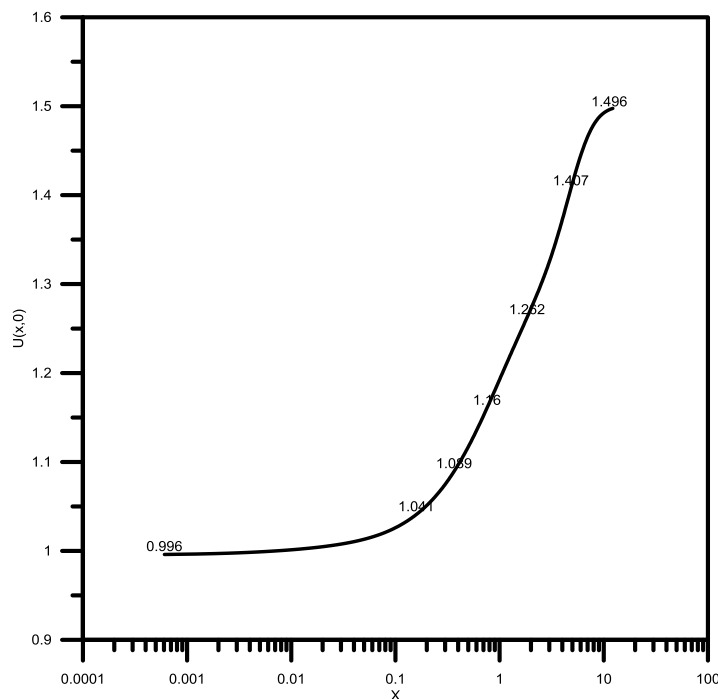
.

Figure 2 - Development of the longitudinal component of the speed. $U(X,0)$

Through the figure (2) we can observe the behavior of the longitudinal velocity in the center of the duct, where it can be seen that conforms with the conditions imposed. Note that the entrance to the duct velocity this next one (1), but

Thiago Andrade Fernandes; Carlos Cabral Santos; Kleber Lima Cezar; Marcos Lima Cordeiro
Analysis Of Forced Convection In Two-Dimensional Laminar Flow In Rectangular Ducts By Gitt Primitive With Variable.

at the end of it, where the problem is developed, the speed tends to 1.5 as default, since by equation (23) with Y equal to 0 (zero), the value is U_∞ equal 1.5.

Figure 4.3 shows the behavior of the relationship f.Re for the flow in a channel parallel plate obtained by the formulation in primitive variables. Where it is found that the result was satisfactory in this respect, since, to a value considerably large input channel, the relationship tends to 24, as can be confronted with the vast literature on the subject.
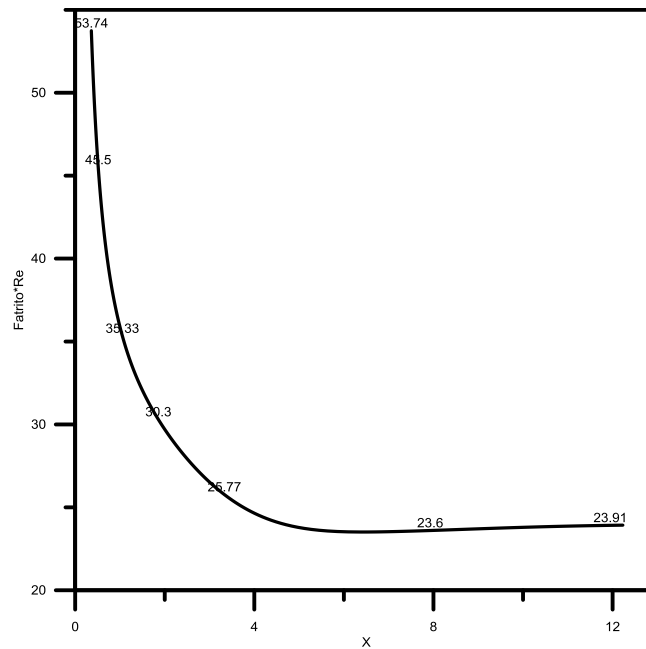

Figure 3 - Behavior of the relationship f.Re along the component X.

The following figures deal with the velocity profile in the duct at different axial positions in the case of Reynolds equal to 300.
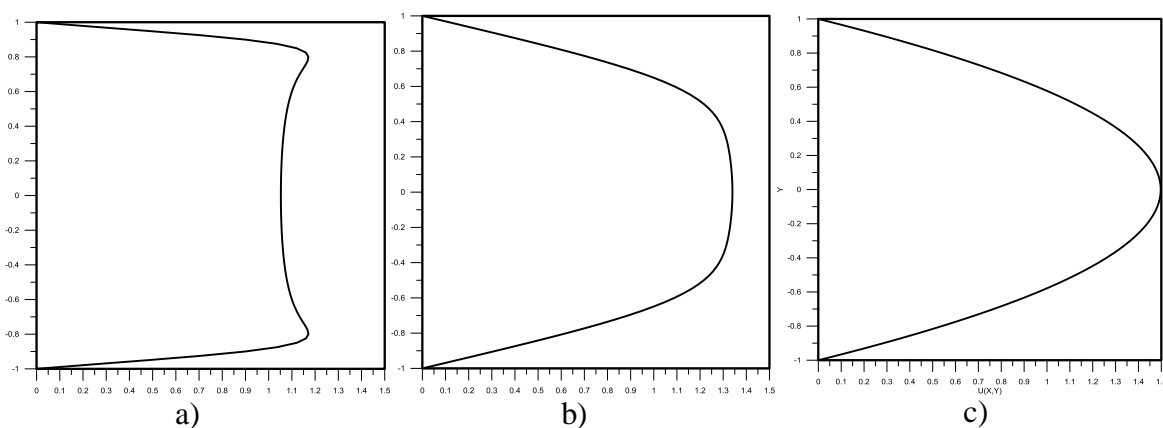

Figure 4 - Velocity Profile in the axial position (a) X = 0.208, (b) X = 3.333 and (c) X = 12.228.

The results show that as it advances into the duct, the parabolic profile is most evident, featuring fully developed flow.

The following figures deal with the pressure profile in the duct at different axial positions in the case of Reynolds equal to 300. It is observed that the behavior of the pressure is inversely proportional to the velocity, ie the higher the speed the lower the pressure, such as due, and conditions are imposed to agreements with the problem.
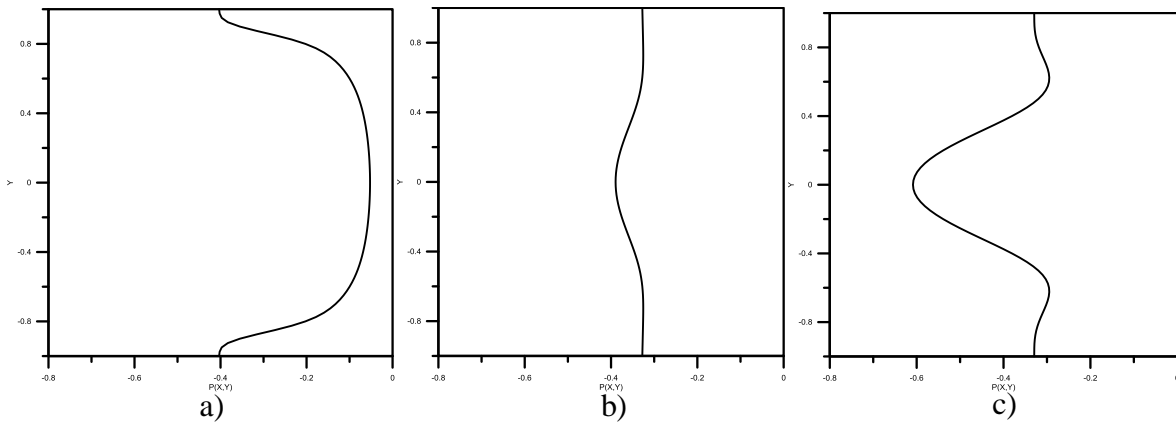
Figure 5 - Pressure Profile in the axial position (a) X = 0.208, (b) X = 3.333 and (c) X = 12.228.

For figure (6) it is observed that according to the conditions where the temperature at the beginning of the duct remains at 1 (one), however it along the pipeline decreases tending to 0 (zero). This implies that when the temperature development tends to infinity approaches T_∞.
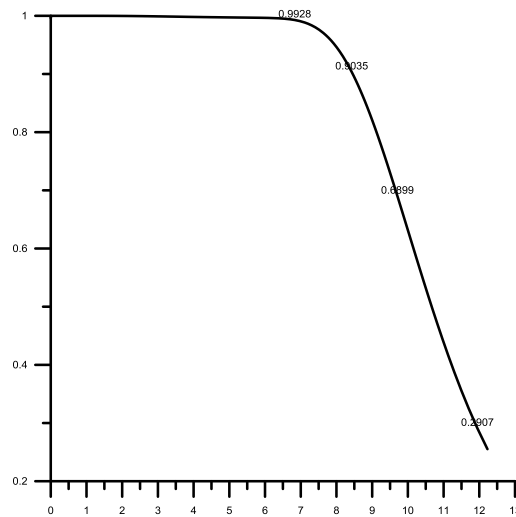

Figure 6 - Behavior of temperature at the center of the duct along the same Reynolds to 300.

The following figures deal with the temperature profile in the duct at different axial positions in the case of Reynolds equal to 300.
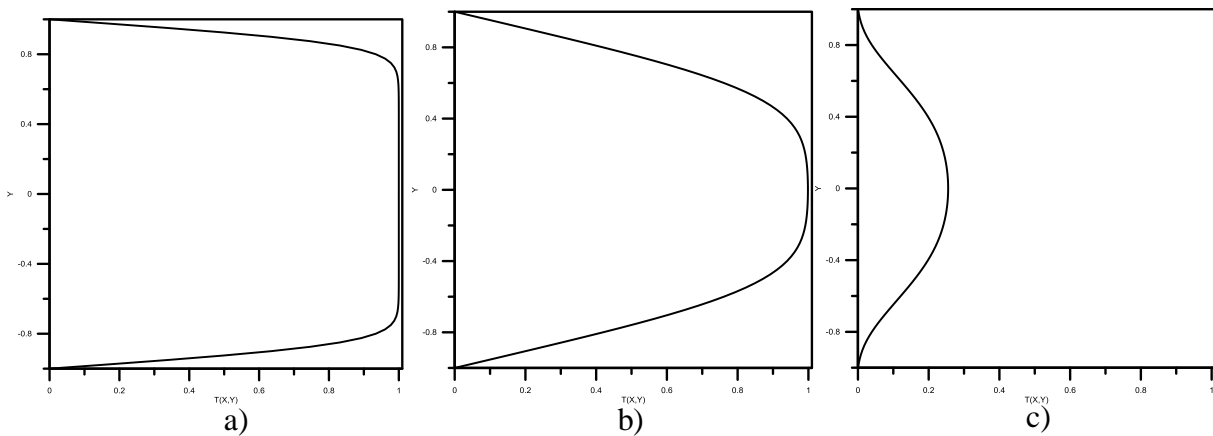

Figure 7 - Temperature Profile in axial direction (a) X = 0.208, (b) X = 3.333 and (c) X = 12.228.

Thiago Andrade Fernandes; Carlos Cabral Santos; Kleber Lima Cezar; Marcos Lima Cordeiro
Analysis Of Forced Convection In Two-Dimensional Laminar Flow In Rectangular Ducts By Gitt Primitive With Variable.

The graphics of the temperature profile satisfies the boundary conditions. Where the beginning of the temperature tends to channel 1 and channel develop in this tends to 0. And it is noticeable that the temperature behavior evolves much faster than the speed due to the Prandtl number is less than one.
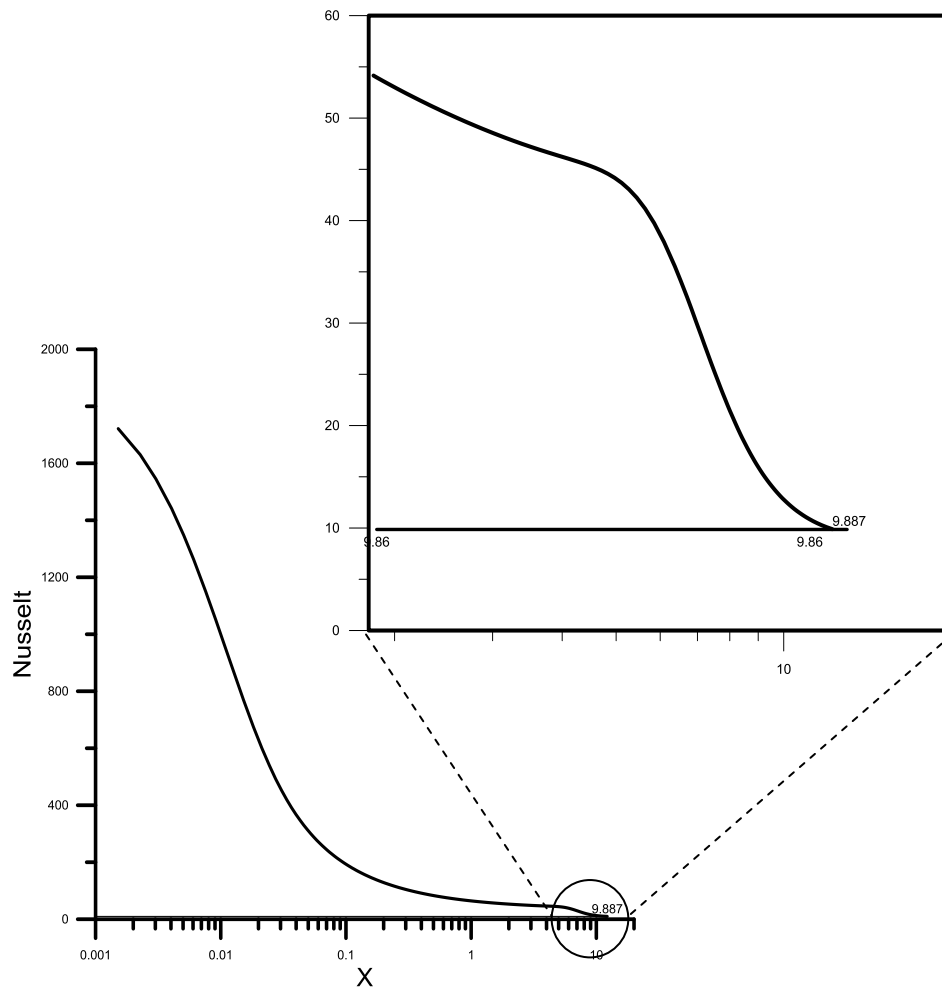

Figure 8 - Behavior Nusselt along the X component for Reynolds 300, with emphasis on the end of the flow.

In figure (8) observed downward trend of Nusselt, where it tends to stabilize around 9.88. The result obtained this consistent with the literature, where KAKAÇ (1994) states that the value of Nusselt, for our type of problem tends to 9.8696.

5. CONCLUSION

This work at first tried to address the Navier-Stokes equations coupled with the Poisson equation simultaneously, in order to determine the velocity field and pressure, so studying the important parameters of literature, the second time addressed the equation energy to obtain the temperature field, where it possible to analyze the behavior of Nusselt. These equations were solved using the technique of generalized integral transformation (GITT) in primitive form, considering the Newtonian fluid, incompressible and physical constants.

The solution in primitive variables allows the lifting of the velocity field and pressure, can be used to control leakage in pipelines and industrial. With this, the present results are reliable at work, therefore, are consistent with the previous work. The use of filters, with the main goal to homogenize the boundary conditions, is enhanced both for speed and for pressure, because that enhances the convergence and improves computational performance. Solve the problem by primitive variables facilitated the construction schedule with reference to the difficulty we would have if we used the current function, however, it is noteworthy that the latter provides continuity automatically.

The temperature field behaved stable, its development was quite satisfactory, with convergence to low numbers of eigenvalues, much of it due to the Prandtl number is less than 1, which allowed more rapid development to temperature.

The results obtained for these parameters had good behavior, where the friction factor multiplied by Reynolds converged to 23.91 where the vast literature on the subject shows that this must be close to 24, for the proposed problem. Already Nusselt number is validated by the book KAKAÇ (1995) where it says that, with conditions identical to the present work, tends to converge to 9.8698, where we obtained 9.88 for 70 eigenvalues.

With all this above, it is found that transformation technique Generalized Integral, which is used in this work was found to be effective in solving the problem of a drain, simultaneous, two-dimensional, permanent, considering the velocity, pressure and temperature, also showing excellent agreement with other previous works.

6. REFERENCES

ÇENGEL, A.Y. e CIMBALA J. M. Mecânica dos Fluidos, (2007). Fundamentos e Aplicações. Mc Graw Hill Editora. 6ª edição, São Paulo.

CORRÊIA, E.J., COTTA, R.M. e ORLANDE, H.R.B., 1997, "On the reduction of Computational Costs in Eigenfunctions Expansions of Multidimensional Diffusion Problems" International Journal Numerical Method Heat & Fluid Flow, Vol. 7, pp. 675-695.

COTTA, R. M., 1993. Integral Transform in Computational Heat and Fuid Flow. CRC Press, Boca Raton, FL.

COTTA, R.M., 1993ª, "A Review of the integral Transform Method In Nonlinear Heat and fluid Flow Problems", Proc. of the 1st Ibero-American Congress of Mechenical Engineering, Madri, Espanha

IMSL library - MATH/LIB, (1989). Houston, Texas.

INCROPERA, FRANK P., DE Witt, DAVID P., BERGMAN, Theodore L., LAVINE, Adrienne S., 2008, Fundamentos de Transferência de Calor e Massa, 6ª ed. LTC.

KAKAÇ, S. e YENNER, Y., 1995. Convective Heat Transfer. 2ª edição, CRC Press, Boca Raton, FL.

LEAL, M.A., 1996, "Convecção Natural em Cavidades para Regimes Permanentes e Transientes – O Método da Transformação Integral", Tese de Doutorado, COPPE/UFRJ, Rio de Janeiro, RJ, Brasil.

LIMA, J.A., 1995, "Soluções das Equações de navier-Stokes Médias para Escoamento turbulento via Transformação Integral", Dissertação de Mestrado, COPPE/UFRJ, Rio de Janeiro, RJ, Brasil.

LIMA, J.A., 2000, "Escoamento Turbulento em Canais de Placas planas e paralelas: Análise via Transformação Integral e Modelos Algébricos e K-L de Turbulência", Tese de Doutorado, CT/UFPB, João Pessoa, PB, Brasil.

LIMA, G.G.C., 2002, " Análise Teórica de Fotoreatores Utilizando a Técnica da Transformação Integral Generalizada.", Tese de Doutorado, CT/UFPB, João Pessoa, PB, Brasil.

MACÊDO, E.N., Cotta, R.M., e Orlande, H.R.B., 1999, "Local Instantaneous Filtering in the Integral Transform Solution of Non-Linear Diffusion Problems", Computational Mechanics, Vol. 23, pp. 524-532.

MACHADO, H.A., 1998, "Soluções em convecção Térmica com Propiedades variáveis via Transformação Integral", Tese de Doutorado, PEM/COPPE, Rio de Janeiro, RJ, Brasil.

MIKHAILOV, M. D. e ÖZISIK, M. N., (1984). Unified Analysis and Solutions of Heat and Mass Diffusion, John Wiley, New York.

PEREIRA, L.M., e Pérez Guerrero, J.S., 2002, "Navier-Stokes Formulation Laminar Flow" in: Convective Heat Transfer in Ducts, the integral transform Approach, Santos C.A.C., Quaresma J.N.N. e Lima J.A. (eds), E-Papers Serviços Editoriais, Rio de Janeiro, pp. 217-236.

PERREZ GUERRERO, J.S. e Cotta, R.M., 1996, "Bechmark Integral Transform Results for Flow over a Backward-facing Step", Computer & Fluids, Vol. 25, pp. 527-540.

QUARESMA, J.N.N. e Cotta R.M., 1997, "Integral Transform Method for Navier-Stokes Equations i Steady Three-Dimensional Flow, Proceedings of the 10th International Symposium on Transport Phenomena (ISTP-10), Vol 1, pp. 281-287, Kyoto, Japan.

SANTOS, C.A.C., Gondim R.R., e Cotta R.M., 2002, Finned Channells", In Convective Heat Tranfer in Ducts, the Integral transform Approach, Santos C.A.C., Quaresma J.N.N. e Lima J.A. (eds), E-Papers Serviços Editoriais, Rio de Janeiro, pp. 81-90.

FERNANDES, Thiago Andrade., 2012, "Análise da convecção forçada no escoamento bidimensional laminar em dutos retangulares via GITT com variáveis primitivas", Dissertação de Mestrado, CT/UFPB, João Pessoa, PB, Brasil.

VERONESE, J. P., 2008. "Análise do Escoamento Bidimensional de Fluidos Não -Newtonianos com Formulação em Variáveis Primitivas Via GITT". Tese de Doutorado. UFPB, Abril.

WOLFRAM, S,, 1996, MATHEMATIC, A System for Doing Mathematics by Computer, The Advanced Book Program, Addison-Wesley Publisshing Co,, Redwood City, California,