



## OPTIMUM HYDRODYNAMIC DESIGN FOR HYDROKINETIC TURBINES BLADES WITH CAVITATION

**Léo Daiki Shinomiya**

**Jerson Rogério Pinheiro Vaz**

Faculdade de Engenharia Mecânica, Instituto de Tecnologia, Universidade Federal do Pará, Rua Augusto Corrêa, 01, CEP.: 66075-110, Guamá, Belém, Pará. , Fone: (91) 3201-7962

leo.shinomiya@itec.ufpa.br

jerson@ufpa.br

**André Luiz Amarante Mesquita**

Faculdade de Engenharia Mecânica, Instituto de Tecnologia, Universidade Federal do Pará, Rua Augusto Corrêa, 01, CEP.: 66075-110, Guamá, Belém, Pará. , Fone: (91) 3201-7962

andream@ufpa.br

**Déborah Aline Tavares Dias do Rio Vaz**

Programa de Pós-Graduação em Engenharia de Recursos Naturais da Amazônia, Universidade Federal do Pará, Rua Augusto Corrêa, 01, CEP.: 66075-110, Guamá, Belém, Pará, Fone: (91) 3201-7962

deborahvaz@ufpa.br

**Abstract.** *This paper aims to propose a mathematical model applied to optimization of horizontal axis hydrokinetic turbines blades considering the effect of cavitation. The employee uses the minimum pressure coefficient criterion as a limit for a flow without cavitation on the blade hydraulic profile. The proposed model corrects the chord and the twist angle in each section of the blade by a modification of the local thrust coefficient, in order to be considered the effect of cavitation on the hydrodynamic shape of the rotor. The results are compared with data from hydrokinetic turbines designed using the Glauert optimization model without cavitation. The results shows that the model provides good performance and can be used in the efficient hydrokinetic turbines design.*

**Keywords:** *Horizontal Axis Hydrokinetics Turbines, Cavitation, minimum pressure coefficient criterion.*

### 1. INTRODUCTION

The optimization models of hydrokinetic turbine blade in general, are based on the BEM method (Blade Element Momentum), considering only the kinetic energy transported by the flow for calculating the hydrodynamic shape of the blade. Such models are direct applications of the approaches developed for the case of wind turbines, where the main limitation does not consider the effect of cavitation, because in the case of wind turbines this phenomenon not occur. However, for the hydrokinetic turbines design, it is important to consider this effect, since, in water, is frequent the emergence of the pressure fields very close to the vapor pressure, causing the undesirable effect of erosion on the rotor blades. In the literature there are few publications that consider the cavitation in the hydrokinetic turbines design. Sale et. al. (2009) show a method of optimizing hydrokinetic blade that considers the effect of cavitation based on genetic algorithms coupled to the BEM method. This method gives good results, but the computational cost is high, and in general is not easy to implement. Thus, this paper proposes a methodology for optimization of hydrokinetic rotors to low computational cost and easy implementation, in addition to considering the criterion of minimum pressure coefficient as a boundary for a flow without cavitation on hydraulic profile. The proposed approach is an adaptation of Glauert optimization model to consider the effect of cavitation. This adaptation is the correction coefficient of thrust (or load factor) in such a way that is considered the minimum pressure coefficient in the calculation of the distributions of chord and twist angle of the hydraulic blade. The results are discussed and compared with optimization models of the literature showing good results.

### 2. MATHEMATICAL MODEL

#### 2.1 The Cavitation Effect

The cavitation effect is a phenomenon that have been analyzed carefully in hydrokinetic turbines because of cavitation causes structural damage in the turbine blade, causing vibrations and reducing their performance. This effect presumably originated in the blade section where the pressure decreases below the vapor pressure of the fluid. Due to this fact, the liquid vaporizes instantly forming a cavity vapor altering the condition of the flow, changing the flow passage, as a result of changing the local pressure of the fluid, influencing the shape and size of the vapor cavity. When the cavity vapor implodes, the surface pressure increases to the point location of the blades erode the material causing

damage. The failures caused on the blades, reduce its lift and increase their aerodynamic drag, changing its ratio  $Cl/Cd$ , making it less efficient.

The possibility of this phenomenon occur increases with increasing flow velocity and the local flow. The cavitation can be minimized or even reduced to zero if a minimum pressure maintained above the vapor pressure of the operating fluid on the surface of the blades, also, the origin of cavitation can predicted by comparing the minimum pressure coefficient with the number of cavitation shown below.

$$\sigma = \frac{p_0 - p_v}{0,5\rho W^2} \quad (1)$$

The cavitation will occur if the local minimum pressure coefficient is smaller than the Thomas coefficient shown in equation (1), and the objective of design, is to operate the rotary equipment submerged in liquid fluids at a boundary where there is no such event.

The pressure coefficient is an important parameter that allows us to observe the aerodynamic loading on the profile of the blades. This parameter was defined as:

$$C_p = \frac{p - p_0}{\frac{1}{2}\rho W_\infty^2} \quad (2)$$

where  $p$  is the static pressure of the blade profile and  $p_0$  is the pressure reference. The coefficient of minimum suction pressure was defined as the minimum value of the pressure coefficient on the suction side of the airfoil. This coefficient can be used as a criterion for aerodynamic loading.

## 2.2 Optimum Hydrodynamic Design of Hydrokinetic Blade

This proposal corresponds to a simplified model, compared with the formulations described by Sale *et al* (2009) in which, starting from the cavitation criterion is given by:

$$\sigma + C_{p,min} \geq 0 \quad (3)$$

where  $\sigma$  is the cavitation number and  $C_{p,min}$  is the minimum pressure coefficient. In the case of the hydrokinetic turbines, the cavitation number is (Sale et al., 2009):

$$\sigma = \frac{P_{atm} + \rho gh + \frac{1}{2}\rho V_0^2 a(2-a) - \frac{1}{2}\rho(\Omega r a')^2 - P_v}{\frac{1}{2}\rho W^2} \quad (4)$$

The relative velocity, according to the velocity diagram of Figure 1 is given by:

$$W = \sqrt{[V_0(1-a)]^2 + [\Omega r(1+a')]^2} \quad (5)$$

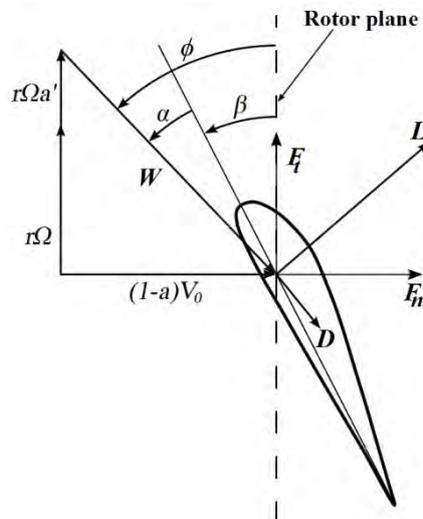


Figure 1. Velocity diagram for the section of the rotor blade.

Sale *et al.* (2009) suggest that the criterion to avoiding cavitation is to make the relative velocity ( $W$ ) in each section of the blade should be always smaller than the speed which causes cavitation ( $V_{cav}$ ).

$$V_{cav} = \sqrt{\frac{P_{atm} + \rho gh + \frac{1}{2}\rho V_0^2 a(2-a) - \frac{1}{2}\rho(\Omega r a')^2 - P_v}{-\frac{1}{2}\rho C_{pmin}}} \quad (6)$$

In the present work, makes the following proposal: as  $W < V_{cav}$ , then, is assigned:

$$W = (1 - f_s)V_{cav} \quad (7)$$

where  $f_s$  is a factor, which will always maintain the relative velocity below the velocity of cavitation. The equation (5) was replaced in the formulation for the thrust coefficient ( $C_T$ ). For this, it has, the velocity diagram in Figure 1:

$$F_n = L \cos \phi + D \sin \phi \quad (8)$$

The normal force coefficient is define by:

$$C_n = \frac{F_n}{\frac{1}{2}\rho W^2 c} = C_L \cos \phi + C_D \sin \phi \quad (9)$$

The thrust over the turbine as a function of the rotor geometry can be determined, using the normal force exerted on a section of the blade.

$$dT = B F_n dr \quad (10)$$

Combining the equations (8), (9) and (10), we get:

$$dT = \frac{1}{2}\rho B W^2 c C_n dr \quad (11)$$

Using the Equation (11), we obtain the thrust coefficient as function of relative velocity

$$C_T = \frac{dT}{\frac{1}{2}\rho V_0^2 A} = \left(\frac{W}{V_0}\right)^2 \frac{Bc}{2\pi r} C_n \quad (12)$$

Detailing the chord in equation (12), we have:

$$c = \frac{2\pi r}{B} \frac{C_T}{C_n} \left(\frac{V_0}{W}\right)^2 \quad (13)$$

Introducing the equation (7) in (13), we obtain:

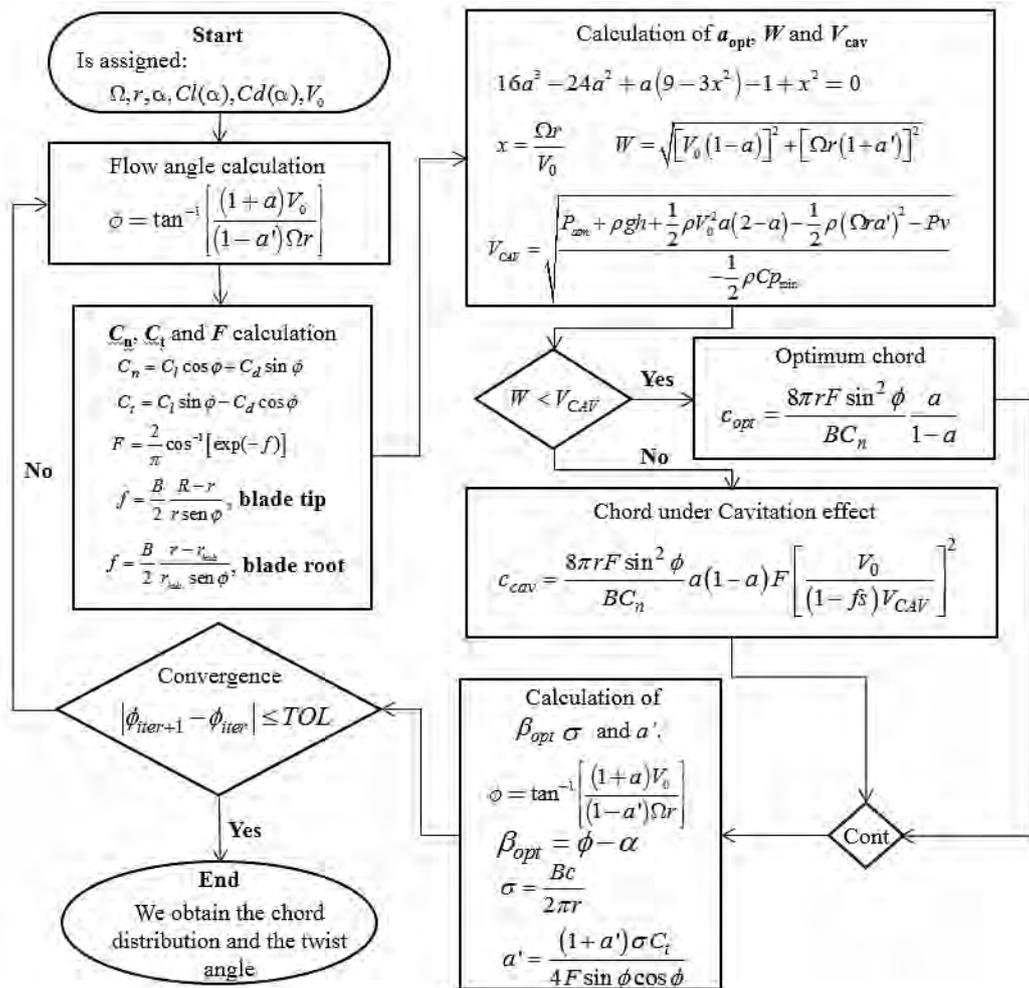
$$c = \frac{2\pi r}{B} \frac{C_T}{C_n} \left[\frac{V_0}{(1-f_s)V_{cav}}\right]^2 \quad (14)$$

where

$$C_T = 4a(1-a)F \quad (15)$$

$F$  is the Prandtl factor.

Equation (14) at first instance can be used in any optimization model. In general, the goal of these models is optimize the axial induction factor “ $a$ ”. With this equation is possible to calculate of the chord and twist angle distributions.



### 3. RESULTS AND DISCUSSIONS

In order to evaluate the performance of the optimization model with cavitation was considered two hydrokinetic turbines of horizontal axis using the hydrofoil NACA 65<sub>3</sub>-618, where the design parameters are described in Table 1. The hydrodynamic parameters such as the lift, drag and minimum pressure coefficients were obtained using the free software XFOIL.

Table 1. The design parameters for horizontal axis hydrokinetic turbines.

<i>Parâmetros</i>	<i>Turbina 1</i>	<i>Turbina 2</i>
<i>Diameter (D)</i>	0,6 m	6,0 m
<i>Hub Diameter</i>	0,06 m	0,6 m
<i>Nº of blades</i>	4	3
<i>Flow Velocity (V<sub>0</sub>)</i>	1,5 m/s	2,5 m/s
<i>H</i>	1,1m	4 m
<i>Patm</i>	10 <sup>5</sup> Pa	10 <sup>5</sup> Pa
<i>Pv</i>	3,17x10 <sup>3</sup> Pa	3,17x10 <sup>3</sup> Pa
<i>Gravity (g)</i>	9,81 m/s <sup>2</sup>	9,81 m/s <sup>2</sup>
<i>Especific Mass (ρ)</i>	997 kg/m <sup>3</sup>	997 kg/m <sup>3</sup>
<i>Security factor</i>	5%	5%

In order to determine the optimal rotation curve was generated for optimal power coefficient as a function of rotation axis.

Figure 1 shows the optimum efficiency of the turbines as a function of the shaft rotation. In this case, the aim is obtain the rotation of design for an operating point with the highest possible power coefficient without considering the effect of cavitation. The cavitation threshold for the second turbine is given to a rotation of approximately 40 RPM, from which the turbine is susceptible to cavitation.

In the case of the first turbine, the cavitation not occurred in the rotation shown in figure 2. This can occur because the small diameter of the turbine, resulting in a decrease of the relative velocity ( $W$ ) on the blade, so that it is smaller than the speed cavitation ( $V_{CAV}$ ) shown in Figure 3. The operating conditions for the turbines are shown in table 1.

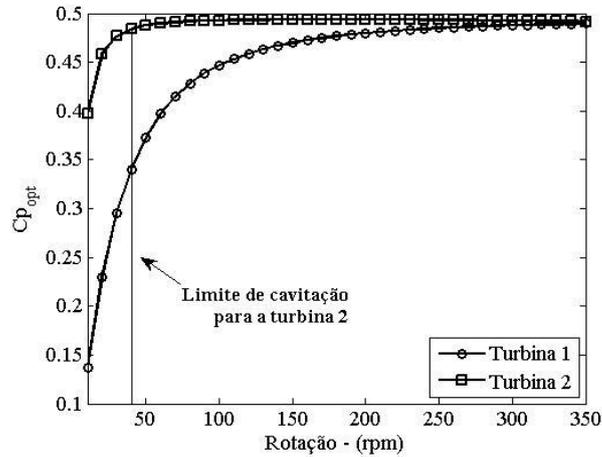


Figure 2. Power coefficient as a function of the shaft rotation.

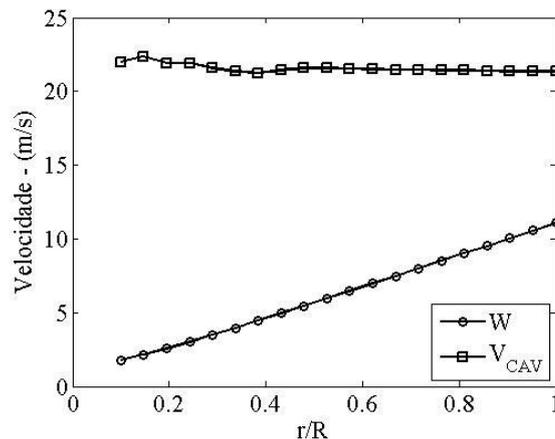


Figure 3. Relative and cavitation velocity as function of the radial position in rotation of 350 RPM (Turbine 1).

Still for the first turbine, as given in table 2, the minimum pressure coefficient as a function of the Reynolds number for each blade section for a rotation of 350 RPM. In this condition, the relative velocity along the blade is less than the speed of cavitation, as shown in figure 3. Therefore, if there is no cavitation inception for the rotation of 350 RPM, the turbine will not cavitate for any rotation below of 350 RPM, because the relative velocity decreases when the shaft rotation decreases.

Table 2. Reynolds and minimum pressure coefficient to rotation of 350 RPM for turbine 1.

350 RPM		
Radial Position	Re	Cp_min
2.999	710467	-0.448
4.421	812790	-0.432
5.842	866780	-0.449
7.263	897640	-0.449
8.684	916584	-0.462

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0.101	928927	-0.472
0.115	937368	-0.476
0.129	943376	-0.468
0.143	947796	-0.460
0.157	951137	-0.460
0.172	953723	-0.461
0.186	955764	-0.461
0.200	957403	-0.462
0.214	958738	-0.462
0.228	959841	-0.463
0.243	960763	-0.463
0.257	961540	-0.464
0.271	962202	-0.464
0.285	962771	-0.464
0.300	963263	-0.464

The rotation design project in case of the first turbine can be any rotation starting in 100 RPM, where the range of maximum power is from 44 to 48%. However, it is necessary to evaluate the mechanical strength of the rotor blades, whereas the higher is the shaft rotation in the same design condition, less is the chord distribution, as shown in figure 4.

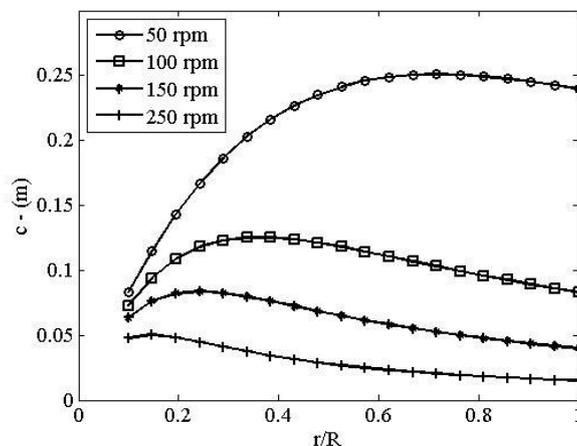


Figure 4. Distribution of chord as function of radial position in various rotations (Turbine 1).

In the case of second turbine, the cavitation limit is 40 RPM (see figure 4). The probability cavitation inception increases, because of diameter of the second rotor is bigger than de first rotor (6 m). This might happen because the relative velocity on the tip of the blade.

Table 3 shows the minimum pressure coefficient and the Reynolds number for each radial position of turbine, and figure 5 shows the result of relative velocity compared with the cavitation velocity, and the figure give us the possibility to observe the relative velocity approximates of cavitation velocity, and this velocity thought to decreases because of the minimum pressure coefficient on the tip of the blade decreases.

Table 3. Reynolds number and minimum pressure coefficient to rotation of 40 RPM for turbine 1.

<b>40 RPM</b>		
<b>Posição Radial</b>	<b>Re</b>	<b>Cp_min</b>
0.300	191172	-1.135
0.442	231255	-1.138

0.584	255896	-1.140
0.726	271559	-1.141
0.868	281902	-1.141
1.010	288991	-1.142
1.152	294016	-1.142
1.294	297687	-1.142
1.436	300440	-1.142
1.578	302552	-1.142
1.721	304204	-1.142
1.863	305520	-1.143
2.005	306584	-1.143
2.147	307455	-1.143
2.289	308178	-1.143
2.431	308784	-1.143
2.573	309297	-1.143
2.715	309734	-1.143
2.857	310111	-1.143
3.000	310437	-1.143

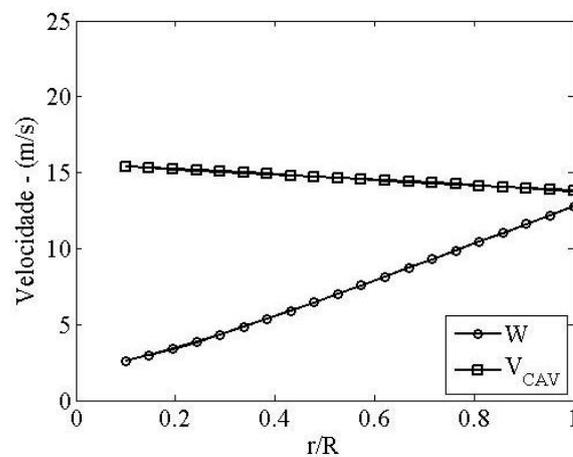


Figure 5. Relative and cavitation Velocity as function of the radial position in rotation of 40 RPM (turbine 2).

To evaluate the behavior of the model in the region where cavitation occurs, the rotations were taken from 50 RPM to 75 RPM. Table 4 shows the minimum pressure coefficient and Reynolds number in relation to the radial position of the second turbine evaluated in this work. Figure 6 shows the turbine cavitating at 88% of the blade operating at 50 RPM and 61% of the blade operating at 75 RPM.

Table 4. Representing the minimum pressure coefficient to rotation of 75 RPM for second turbine.

Radial Position	50 RPM		75 RPM	
	Re	Cp_min	Re	Cp_min
0.300	171836	-1.133	155302	-1.131
0.442	201095	-1.136	176949	-1.133
0.584	217524	-1.137	188245	-1.135
0.726	227307	-1.137	194656	-1.135

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0.868	233476	-1.138	198574	-1.136
1.010	237565	-1.138	201119	-1.136
1.152	240392	-1.138	202857	-1.136
1.294	242411	-1.138	204092	-1.136
1.436	243882	-1.139	204999	-1.136
1.578	244952	-1.139	205685	-1.136
1.721	245690	-1.139	206215	-1.136
1.863	246087	-1.139	206633	-1.136
2.005	246040	-1.139	206969	-1.136
2.147	245277	-1.139	207243	-1.136
2.289	243224	-1.138	207469	-1.136
2.431	238716	-1.138	207657	-1.136
2.573	229416	-1.138	207817	-1.136
2.715	210529	-1.137	207952	-1.136
2.857	170980	-1.133	208069	-1.136
3.000	592641	-1.133	208169	-1.136

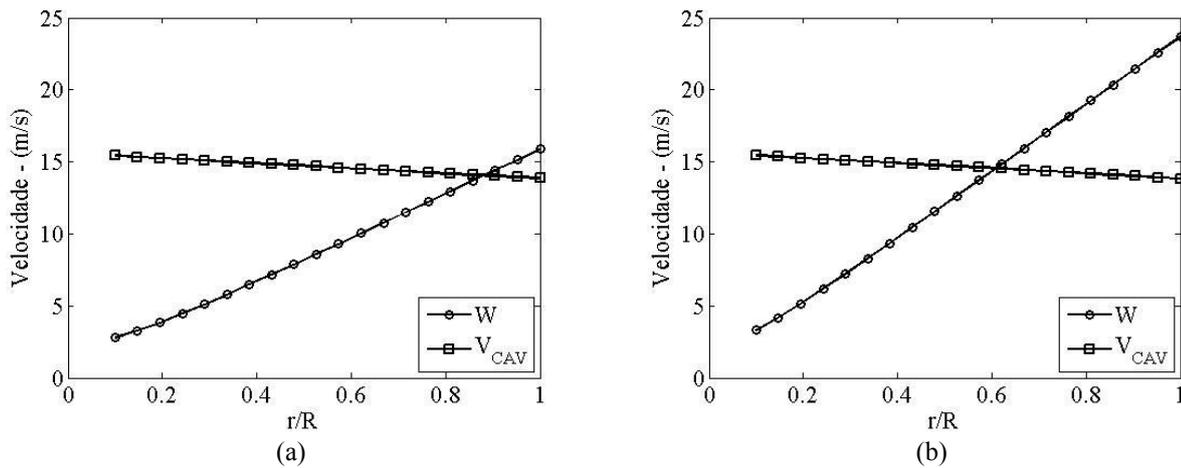


Figure 6. Relative velocity and cavitation velocity in relation of the radial position to rotation of (a) 50 and (b) 75 RPM.

Figure 7 shows the model developed to correct the chord distribution to avoid the inception of cavitation. The adjusted shapes of the blades have relative velocities below the cavitation velocity, as shown in figure 8.

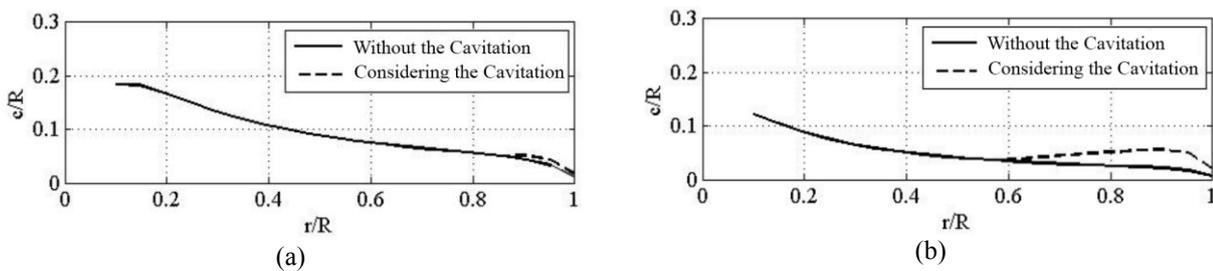


Figure 7. Chord distribution with correction at (a) 50 and (b) 75 RPM.

Therefore, the model corrects radial positions in the chord where the relative velocity is greater than the cavitation velocity. After correcting, the relative velocity takes values always lower than  $V_{CAV}$ . It is emphasized that in this case there is no change in the twist angle all over the chord.

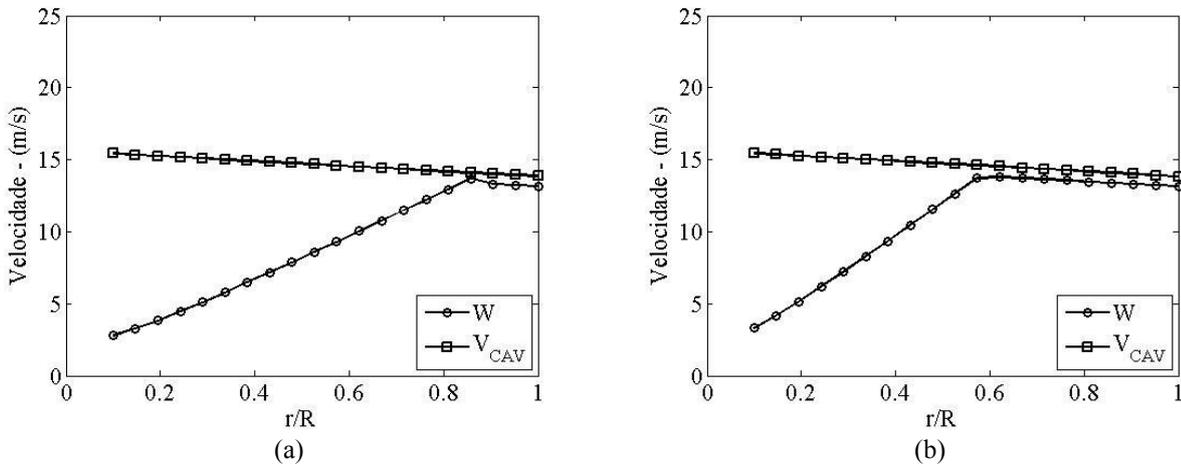


Figure 8. Relative velocity and cavitation velocity in relation of the radial position to rotation of (a) 50 and (b) 75 RPM with correction.

Figure 9 shows a comparison for the cases with and without correction due the cavitation effect. It is observed that the power coefficient is a little larger than the corrected case. For the output power there is no difference. This occurs because the correction of chord moves the machine curve efficiency due to the increased chord promoted by the mathematical model. This effect was expected since there was a change in the geometry of the rotor.

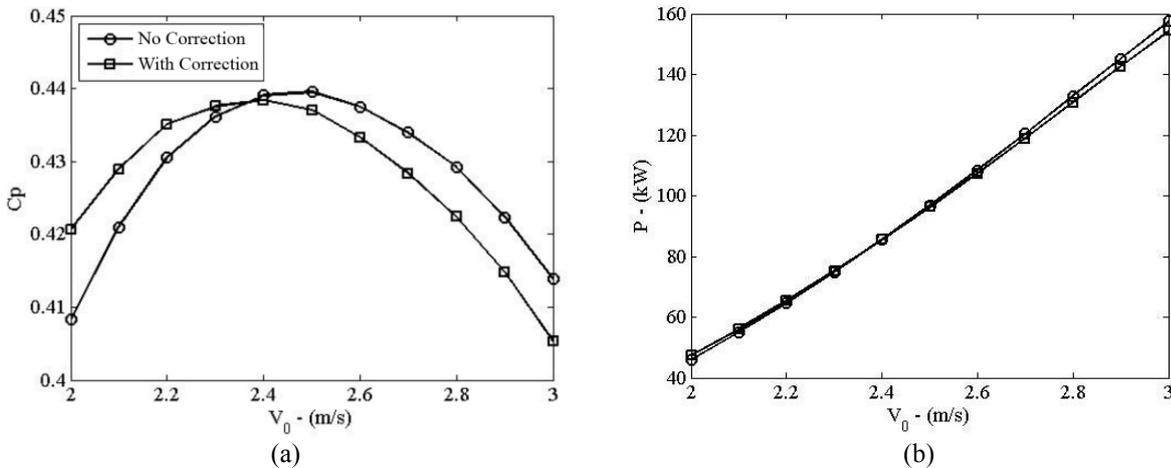


Figure 9. (a) Power coefficient and (b) generated power in function of the flow velocity of operations turbine in 50 RPM.

#### 4. CONCLUSIONS

The presented model has good indicators to be used in design stage of hydrokinetic turbines, which makes corrections in the geometric shape of the blade, in aim to prevent the emergence of cavitation. This methodology has demonstrated efficiency when compared to the optimal model of Glauert (1926), which is not considered in this model the effect of cavitation. However, some still limitations should be analyzed carefully, as promoting the model for hydrodynamic analysis of the rotor out of the project condition, in addition to consider the grid effect for hydrodynamic parameters calculation. In addition, the influence of electrical torque on turbine rotation, also deserves attention, because the imposition of electrical load caused by the generator, there is a decrease in turbine rotation.

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