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A CRANK-SHAFT-SLIDER DEVICE FOR ENERGY HARVEST OF SEA WAVES

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Abstract. *We derive a mathematical model of a crank-shaft-slider mechanism proposed as a device for energy harvest of sea waves. The slider is a floating body free to undergo vertical motions as exited by sea waves. This support motion v(t) is the forcing term of our model. The L long shaft acts upon the crank mechanism, of radius R, that turns an electrical generator mounted on a spring-damper suspension. The equation of motion is derived via a Lagrange's Equations approach. A preliminary simpler model where the motor axis is supposed fixed is also analyzed. The sole generalized coordinate is the angular displacement of the generator. The mathematical model is quite involved due to the highly non linear kinematics of the problem. Preliminary simulations display a rich dynamical behavior. From the engineering point of view, an important parameter is the amplitude of the support motion. If it is less than the radius R of the crank, the device will stuck and will be unable to generate a rotation of the crank. If it is larger, it will deform the generator's suspension.*

Keywords: *energy harvest, crank-shaft-slider, sea waves, nonlinear dynamics.*

1. INTRODUCTION

The world renewable energy sources may potentially, in the near future, substitute a large part of the fossil and nuclear fuels consumption. One of the most promising ideas is the harvest of the energy available in ambient vibrations such as winds, ocean waves, transit of vehicles, human motions etc. In this paper, the main aim is to investigate some devices capable of extracting alternative energy from the base excitation due to sea motion, a relatively untapped resource. An interesting feature of this idea is that these apparatus can be coupled to already existing offshore oil rigs and eolic energy generation offshore farms.

One of the most studied and well established of these devices are floating pendula that under certain circumstances can develop steady rotations that may be used to generate electric energy. Details are to be found in recent work by Horton *et al*. (2011), Lenci *et al*. (2008), Litak *et al*. (2010), Nandakumar *et al*. (2012) and Xu *et al*. (2007).

In this ongoing research, we propose a mathematical model of a crank-shaft-slider device for energy harvest of sea waves. Such mechanisms are commonly used in other mechanical applications (Silva *et al*, 2013) but, as far as we know, has not yet being studied in the context o energy harvest of sea waves.

The slider is a floating body free to undergo vertical motions as exited by sea waves. This support motion *v(t)* is the forcing term of our model. The *L* long shaft acts upon the crank mechanism, of radius *R*, that turns an electrical generator mounted on a spring-damper suspension. The equation of motion is derived via a Lagrange's Equations approach, whose sole generalized coordinate is the angular displacement of the generator. The mathematical model is quite involved due to the highly non linear kinematics of the problem. Preliminary simulations display a rich dynamical behavior. From the engineering point of view, an important parameter is the amplitude v_0 of the support motion. If it is less than the radius *R* of the crank, the device will stuck and will be unable to generate a rotation of the crank. If it is larger, it will deform the generator's suspension. As the sea motions are random in nature, we intend, in future work, to simulate them as series of time histories generated by superposition of harmonic components whose amplitudes are computed from standard sea spectra, with pseudo-randomly set phase angles, a procedure parallel to the so called Synthetic Wind Method, in the context of Civil Engineering applications, due to Franco (1997) and work by Spanos *et al*. (1990) in Aerospace Engineering.

2. THE MATHEMATICAL MODEL

2.1 A first simpler model

A preliminary model, displayed in Fig. 1, makes fixed the position of the axis of the electric generator, no visco-elastic suspension considered. Thus, the state of the crank-shaft-slider system is completely defined by the time function *v(t)* that denotes the vertical motion of the floating slider. Angle φ in the triangle formed by shaft and the crank is given by the law of cosines in the form:

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$$\varphi = \arccos\left[\frac{-v(L+R)+R(L+R)+\dfrac{v^2}{2}}{RL+R^2-vR}\right] \quad (1)$$

were L is the length of the shaft, R the radius of the crank and φ its angular displacement. The velocity is given by

$$\dot{\varphi} = \frac{\dot{v}\left(2L^2+2LR-2Lv-2Rv+v^2\right)}{R\left(L+R-v\right)^2\sqrt{\dfrac{v(2L-v)(4LR-2Lv+4R^2-4Rv+v^2)}{R^2\left(L+R-v\right)^2}}} \quad (2)$$

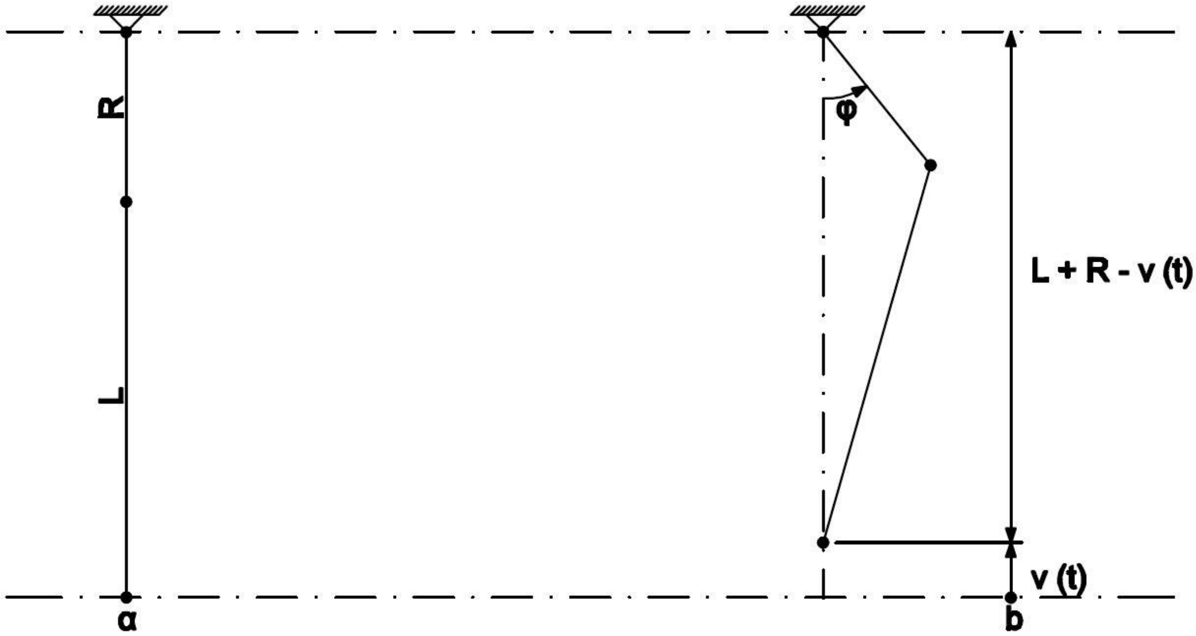

Figure 1. A first simpler model

2.2 The full model

Our full model is displayed in Fig. 2. It is a crank-shaft-slider device where the slider is a floating body free to undergo vertical motions as exited by sea waves. This support motion *v(t)* is the forcing term of our model. The L long shaft acts upon the crank mechanism, of radius R, that turns an electrical generator of mass m mounted on a suspension composed of a spring with stiffness k . The rotation angle of the crank is φ and y is the vertical motion of the generator.

Next, we derive the equations of motion using Lagrange's equation

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}}\right) - \frac{\partial T}{\partial \varphi} + \frac{\partial V}{\partial \varphi} = N_c \quad (3)$$

In Eq. 3, T is the kinetic energy,

$$T = \frac{1}{2}\left(m\dot{y}^2 + J\dot{\varphi}^2\right) \quad (4)$$

J being the moment of inertia of the crank and the generator rotor, $V = U - W_c$ is the total potential energy, where U is the strain energy of the spring element

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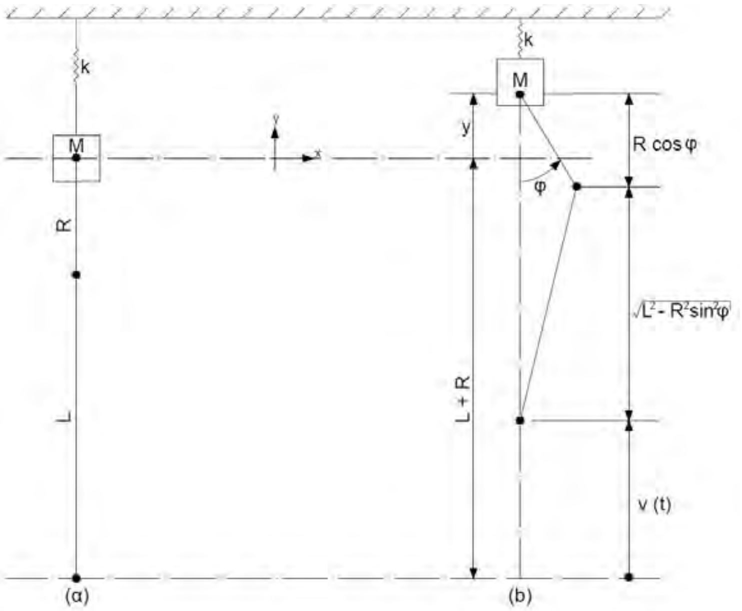

Figure 2. The full model

$$U = \frac{1}{2} k y^2 \tag{5}$$

and W_c the work of the conservative forces, the weight of the generator, in this case,

$$W_c = -mg\, y \tag{6}$$

The only non conservative force considered is an adopted linear viscous damping

$$N_c = -c\dot{\varphi}. \tag{7}$$

The kinematics of the model is given by

$$y + L + R = v + \sqrt{L^2 - R^2 (\sin\varphi)^2} + R\cos\varphi \tag{8}$$

so that

$$\dot{y} = \dot{v} - \frac{R^2 \sin\varphi \cos\varphi}{\sqrt{-R^2(\sin\varphi)^2 + L^2}} - R\dot{\varphi}\sin\varphi \tag{9}$$

Differentiation of the kinetic energy with respect to φ and $\dot{\varphi}$ renders

$$\frac{\partial T}{\partial \varphi} = m\left[-\frac{\dot{\varphi}R^2 \sin\varphi\cos\varphi}{\sqrt{-R^2(\sin\varphi)^2 + L^2}} - \dot{\varphi}R\sin\varphi + \dot{v} \right]\left[\begin{array}{l} -\dfrac{\dot{\varphi}R^4(\sin\varphi)^2(\cos\varphi)^2}{\left(-R^2(\sin\varphi)^2 + L^2\right)^{3/2}} + \dfrac{\dot{\varphi}R^2(\sin\varphi)^2}{\sqrt{-R^2(\sin\varphi)^2 + L^2}} \\ -\dfrac{\dot{\varphi}R^2(\cos\varphi)^2}{\sqrt{-R^2(\sin\varphi)^2 + L^2}} - \dot{\varphi}R\cos\varphi \end{array} \right] \tag{10}$$

$$\frac{\partial T}{\partial \dot{\varphi}} = m\left[-\frac{\dot{v}R^2 \sin\varphi\cos\varphi}{\sqrt{-R^2(\sin\varphi)^2 + L^2}} + \frac{R^4\dot{\varphi}(\sin\varphi)^2(\cos\varphi)^2}{-R^2(\sin\varphi)^2 + L^2} + \frac{2\dot{\varphi}R^3(\sin\varphi)^2\cos\varphi}{\sqrt{-R^2(\sin\varphi)^2 + L^2}} - R\dot{v}\sin\varphi + R^2\dot{\varphi}(\sin\varphi)^2 \right] + J\dot{\varphi} \tag{11}$$

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Differentiation of the total potential energy with respect to φ gives

$$\frac{\partial V}{\partial \varphi} = -k\left[\sqrt{-R^2(\sin\varphi)^2 \_ L^2} + R(\cos\varphi - 1) + v - L\right]\left[\frac{R^2\sin\varphi\cos\varphi}{\sqrt{-R^2(\sin\varphi)^2 + L^2}} + R\sin\varphi\right] + mg\left[\frac{-R^2\sin\varphi\cos\varphi}{\sqrt{L^2 - R^2(\sin\varphi)^2}} - R\sin\varphi\right] \quad (12)$$

Substitution of Eqs. 7, 10, 11 and 12 in Eq. 3 gives the complete nonlinear equation of motion of the model:

$$\begin{aligned} & J\ddot{\varphi} + \sin\varphi\left[kR^2 + R^2\ddot{\varphi}m\sin\varphi + kLR - kRv - R\ddot{v}m - kR\sqrt{L^2 - R^2(\sin\varphi)^2} - Rgm\right] + \sin\varphi\cos\varphi[-2kR^2 + R^2\dot{\varphi}^2 m] \\ & + \frac{kR^3\cos\varphi\sin\varphi}{\sqrt{L^2 - R^2(\sin\varphi)^2}}\left[-\cos\varphi + 1 - \frac{v}{R} - \frac{\ddot{v}m}{Rk} + \frac{2\dot{\varphi}^2 m\cos\varphi}{k} + \frac{L}{R} - \frac{\dot{\varphi}^2 m(\sin\varphi)^2}{k\cos\varphi} + \frac{2\ddot{\varphi}m\sin\varphi}{k} - \frac{gm}{kR}\right] \\ & + \frac{R^4\dot{\varphi}^2 m}{-L^2 + R^2(\sin\varphi)^2}\left[\cos\varphi(\sin\varphi)^3 - (\cos\varphi)^3\sin\varphi - \frac{\ddot{\varphi}}{\dot{\varphi}^2}(\cos\varphi)^2(\sin\varphi)^2\right] \\ & + \frac{R^4\dot{\varphi}m}{\left(L^2 - R^2(\sin\varphi)^2\right)^{3/2}}\left[\dot{\varphi}(\cos\varphi)^2(\sin\varphi)^3 R + 2\dot{v}(\cos\varphi)^2(\sin\varphi)^2\right] \\ & + \frac{R^6\dot{\varphi}^2 m(\cos\varphi\sin\varphi)^3}{L^4 - 2L^2R^2(\sin\varphi)^2 + R^4(\sin\varphi)^4} = -c\dot{\varphi} \end{aligned} \quad (13)$$

3. PRELIMINARY RESULTS

3.1 A first simpler model

Figure 3 displays a response time history for the first simpler model for L = 1 m and R = 0.5 m, and a support motion given by

$$v(t) = R(1 - \cos t). \quad (14)$$

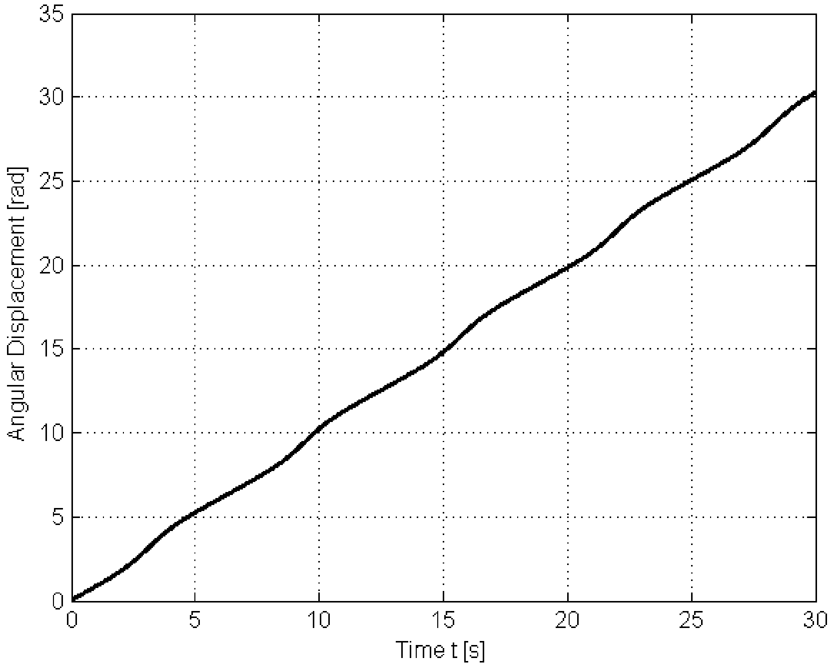

Figure 3. Response time history, first simpler model

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Figure 4 shows the corresponding phase plane.

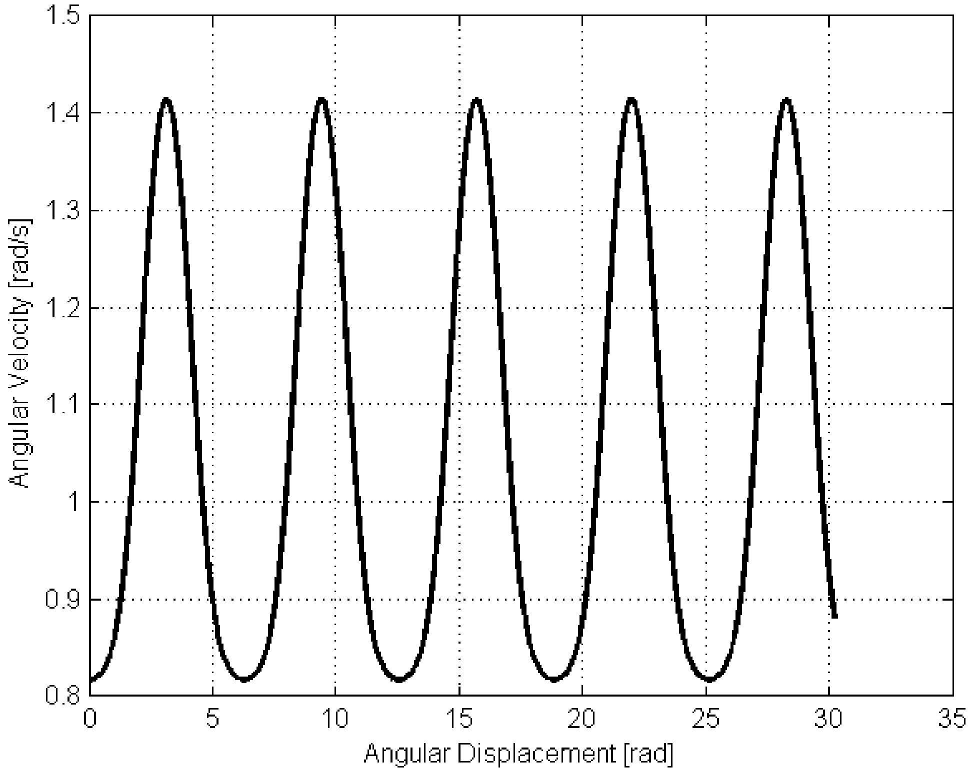

Figure 4. Response phase plane, first simpler model

3.2 The full model

Figure 5 displays a numerically integrated response time history for the full model, according to Eq. 13, for the following set of parameters: $L = 1$ m and $R = 0.5$ m, and a support motion given by Eq. 14.

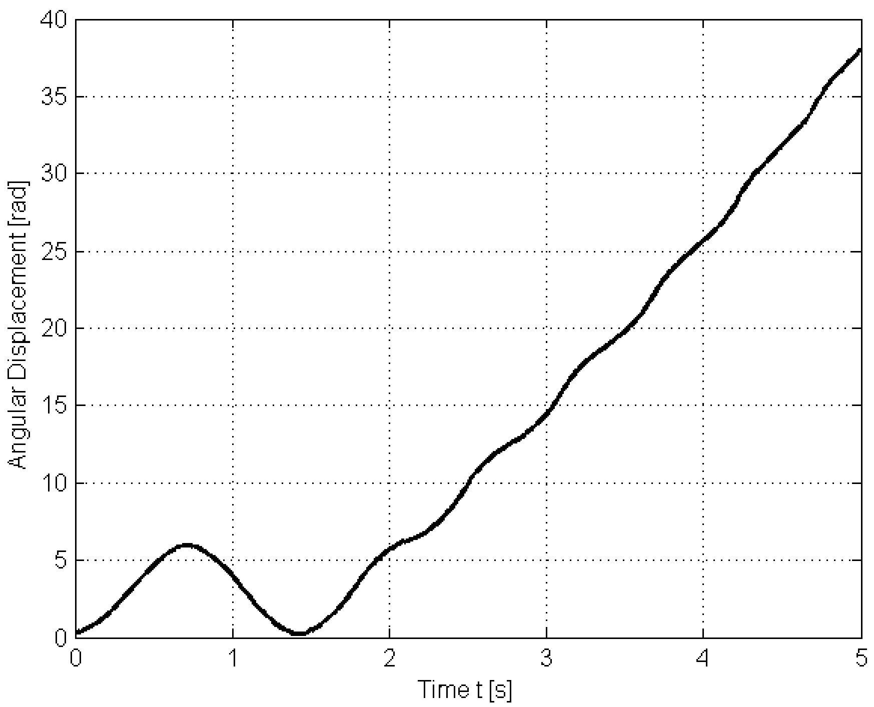

Figure 5. Response time history for the full model

Figure 6 shows the corresponding phase plane.

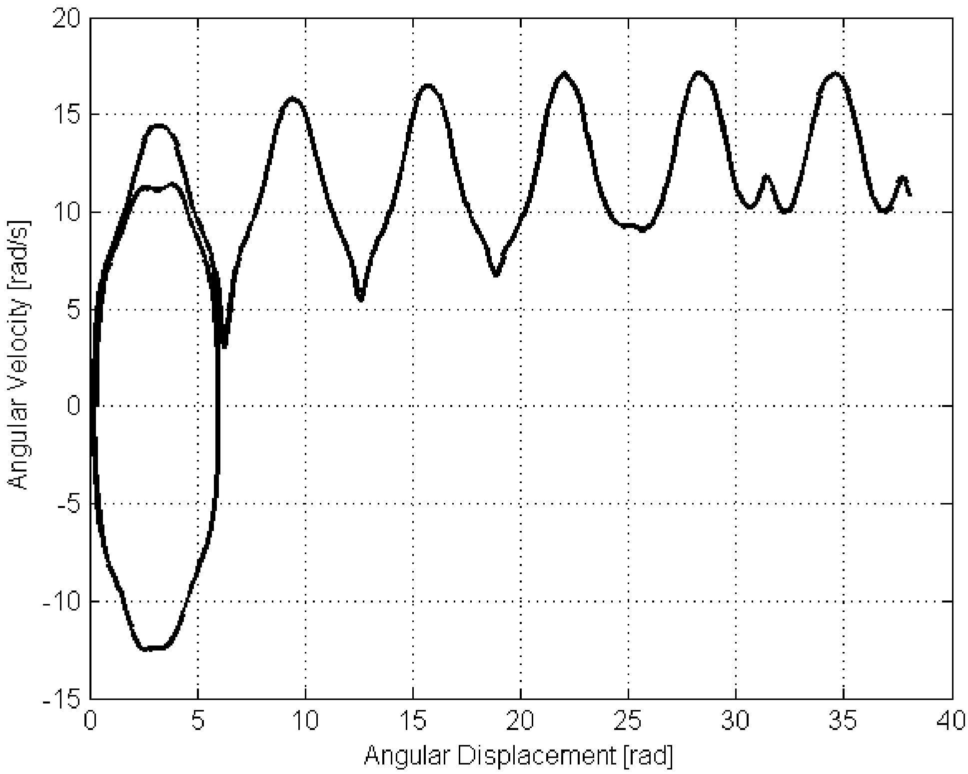

Figure 6. Response phase plane, full model

4. NEXT STEPS IN THE RESEARCH

Due to the large number of parameters involved, a large scale parametric study will be our next main goal. Special attention will be given to cases when the base excitation amplitude is different from the crank radius. Situations as these may lead to motion halting.

As the sea motions are random in nature, we intend to simulate them as series of time histories generated by superposition of harmonic components whose amplitudes are computed from standard sea spectra, such as the Pierson-Moskowitz spectrum, with pseudo-randomly set phase angles. This procedure is parallel to the so called Synthetic Wind Method, proposed by Franco (1997), in the context of Civil Engineering applications, and Spanos *et al* (1990) in Aerospace Engineering. If a sufficiently large number of response time histories are available, statistical handling of them will unable engineering assessment of the energy efficiency of the proposed device. Such a procedure is akin to the Monte Carlo techniques.

5. CONCLUSIONS

In this ongoing research, we present the derivation, via Lagrange's equation, of mathematical models of a crank-shaft-slide mechanism proposed as a sea waves energy harvest device. Two levels of models are studied: a simpler one considering the generator axis fixed and a fuller one considering the generator mounted on a visco-elastic suspension. Some preliminary results are also presented hinting of a rich nonlinear dynamic behavior to be explored in future research continuation.

As sea motions are random in nature we consider it necessary to implement an algorithm of harmonic components synthesis based on a sea wave spectrum.

6. ACKNOWLEDGEMENTS

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8. RESPONSIBILITY NOTICE

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