



## NONLINEAR BACKSTEPPING TECHNIQUE APPLIED TO A LATERAL FLIGHT ENVELOPE PROTECTION OF AN AIRCRAFT

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**Abstract.** Nowadays advanced flight control systems from military and commercial aircrafts have resources to warn pilots about flight out of its safe envelope, this characteristic is well known as Flight Envelope Protection (FEP). The Fly-By-Wire technology is used in flight control laws to protect and limit the aircraft's operations and maintain inside their safe set. The purpose of this paper is to present a Backstepping control law for a lateral aircraft FEP in order to protect it against high bank angle operation. Two controllers in parallel, commuted by a switching technique are proposed to protect the bank angle, one is a first order Backstepping to track the roll rate and the other, a second order Backstepping to track bank angle using the roll rate as an intermediate virtual control. Each control block will generate the aileron and rudder commands. The original motivation was to apply this architecture to general aviation aircraft. However, in lack of good aerodynamic data of such aircraft over the whole envelope, an F-16 model is chosen by its availability. Beyond an introduction, an aircraft math model, Backstepping controller architecture, switching technique, results and conclusions are presented.

**Keywords:** Flight Envelope Protection, Nonlinear Control, Backstepping, Lateral Dynamics, Bank Angle.

### 1. INTRODUCTION

The technological advancement in the aviation sector has greatly increased due to the development and manufacture of military aircraft. The major contribution of this development, within the flight mechanics, was the implementation of Fly-By-Wire technology in commercial transport aircraft, giving way to research this technology in small aircraft, using low cost Fly-By-Wire, similar to that used in the automotive industry (Falkena, 2011). Near future, it is expected growth of this industry, and as a consequence, the likelihood of an increase in accidents related to this sector, due to maneuverability factors and/or little experience of the pilots, which is summarized in a loss of control (LOC). Technically, the LOC is associated with the operation of the aircraft outside its normal flight envelope (and/or insurance). In a study conducted by NASA Langley Research and the Boeing Company, were determined quantitatively evaluation criteria for the LOC of an aircraft. Such criteria were classified into 5 envelopes of flight: Adverse Aerodynamics Envelope, Unusual Attitude Envelope; Structural Integrity Envelope, Dynamics Pitch Control Envelope and Dynamics Roll Control Envelope (Wilborn, 2004).

Nowadays, the flight control systems to envelope protection are designed using linear models at each operating points, needing to use the Gain Scheduling technique, in order to consider all possible situations of operation (Oosterom, 2006).

In this work a nonlinear Backstepping technique is presented and studied as an alternative to Lateral Flight Envelope Protection, thus replacing several linear projects by a single nonlinear control. Another motivation for using the Backstepping technique is the higher non-linearity threshold points of the aircraft model, and also the greater difficulty of linearization in this sector. In Section 2 is present the aircraft equations and its necessary reorganization for the application of Backstepping technique which will be detailed in section 4, defining previously in Section 3, the control structure of flight envelope protection, to finally present the results of simulations in section 5.

### 2. AIRCRAFT MODEL

The initial motivation was to use a transport aircraft, but due to limitations of the model's availability and fidelity, the Fighter F-16 was used by its high fidelity and availability. This study can be applied to any aircraft.

The F-16 mathematical model used in this work was presented in (Lewis and Stevens, 2003). The dynamic nature of this aircraft is unstable, which allows to reach high bank angle and roll rate values, this is why it was made the limitation on their maximum value, like a common transport aircraft.

The rearranged equations of motion expressed in the lateral body system are (Lee T, Kim Y. 2001):

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$$\begin{aligned} \begin{bmatrix} \dot{\beta} \\ \dot{\phi} \end{bmatrix} &= \frac{1}{m \cdot V} \begin{bmatrix} -\cos \alpha_e \sin \beta [T + C_x(\alpha_e) \bar{q} S] + \cos \beta C_y(\beta) \bar{q} S - \sin \alpha_e \sin \beta C_z(\alpha_e, \beta) \bar{q} S \\ 0 \end{bmatrix} \\ &+ \begin{bmatrix} \sin \alpha_e & -\cos \alpha_e \\ 1 & \cos \phi \tan \theta_e \end{bmatrix} \begin{bmatrix} p \\ r \end{bmatrix} + \frac{\rho \cdot S}{4 \cdot m} \begin{bmatrix} \cos \beta \cdot C_{y_p}(\alpha_e) \cdot b & \cos \beta \cdot C_{y_p}(\alpha_e) \cdot b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ r \end{bmatrix} \\ &+ \frac{\rho \cdot V \cdot S}{2 \cdot m} \begin{bmatrix} \cos \beta \cdot C_{y_{\delta_a}}(\beta) & \cos \beta \cdot C_{y_{\delta_r}}(\beta) \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \\ &+ \frac{g}{V} \begin{bmatrix} \cos \alpha_e \sin \beta \sin \theta_e + \cos \beta \cos \theta_e \sin \phi - \sin \alpha_e \sin \beta \cos \phi \cos \theta_e \\ 0 \end{bmatrix} \end{aligned} \quad (1)$$

$$\begin{aligned} \begin{bmatrix} \dot{p} \\ \dot{r} \end{bmatrix} &= \begin{bmatrix} I_2 \cdot p \cdot q_e + I_1 \cdot q_e \cdot r \\ -I_2 \cdot q_e \cdot r + I_8 \cdot p \cdot q_e \end{bmatrix} + \begin{bmatrix} I_3 \cdot C_l(\alpha_e, \beta) \bar{q} \cdot S \cdot b + I_4 \cdot C_n(\alpha_e, \beta) \bar{q} \cdot S \cdot b \\ I_4 \cdot C_l(\alpha_e, \beta) \bar{q} \cdot S \cdot b + I_9 \cdot C_n(\alpha_e, \beta) \bar{q} \cdot S \cdot b \end{bmatrix} \\ &+ \frac{\rho \cdot V \cdot S}{4} \begin{bmatrix} I_3 C_{l_p}(\alpha_e) b + I_4 C_{n_p}(\alpha_e) b & I_3 C_{l_r}(\alpha_e) b + I_4 C_{n_r}(\alpha_e) b \\ I_4 C_{l_p}(\alpha_e) b + I_9 C_{n_p}(\alpha_e) b & I_4 C_{l_r}(\alpha_e) b + I_9 C_{n_r}(\alpha_e) b \end{bmatrix} \begin{bmatrix} p \\ r \end{bmatrix} \\ &+ \bar{q} \cdot S \begin{bmatrix} I_3 C_{l_{\delta_a}}(\alpha_e, \beta) b + I_4 C_{n_{\delta_a}}(\alpha_e, \beta) b & I_3 C_{l_{\delta_r}}(\alpha_e, \beta) b + I_4 C_{n_{\delta_r}}(\alpha_e, \beta) b \\ I_4 C_{l_{\delta_a}}(\alpha_e, \beta) b + I_9 C_{n_{\delta_a}}(\alpha_e, \beta) b & I_4 C_{l_{\delta_r}}(\alpha_e, \beta) b + I_9 C_{n_{\delta_r}}(\alpha_e, \beta) b \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \end{aligned} \quad (2)$$

$$\dot{\psi} = \begin{bmatrix} 0 & \frac{\cos \phi}{\cos \theta_e} \end{bmatrix} \begin{bmatrix} p \\ r \end{bmatrix} \quad (3)$$

with,

$$I_2 = -\frac{I_{xz} \cdot (I_x - I_y + I_z)}{I_x \cdot I_z - I_{xz}^2}; \quad I_3 = -\frac{I_z}{I_x \cdot I_z - I_{xz}^2};$$

$$I_4 = -\frac{I_{xz}}{I_x \cdot I_z - I_{xz}^2}; \quad I_9 = -\frac{I_x}{I_x \cdot I_z - I_{xz}^2}$$

and the dynamic pressure as:  $\bar{q} = \frac{1}{2} \cdot \rho \cdot V^2$

The state variables are respectively:  $(\beta, \phi, p, r, \psi)$  sideslip angle, bank angle, roll rate, yaw rate, yaw angle. The aerodynamic coefficients approximation was based on mathematic model presented by (Morelli E. - NASA, 1998).

### 3. CONTROLLER ARCHITECTURE FOR FLIGHT ENVELOPE PROTECTION

A lateral sidestick command corresponds to a roll rate tracking controller. Maximum and minimum roll rates can easily be incorporated.

The bank angle protection is realized in a similar way as for the Airbus aircraft. Let us assume there is a “normal” positive maximum bank angle of  $\phi_{\max 1}$  and a “non-normal” maximum bank angle of  $\phi_{\max 2}$  with  $\phi_{\max 2} > \phi_{\max 1}$ . As soon as is approached the commanded roll rate is reduced from its value to zero. In case the stick input is retained at its maximum value the roll rate controller is switched automatically to a bank angle tracking controller.

This controller then has protections against the maximum bank angle  $\phi_{\max 2}$ . In case the stick input is returned to neutral the bank angle controller commands  $\phi_{\max 1}$ . Whenever the stick input generates commands in the opposite direction the bank angle controller is switched back to the roll rate controller (Well K, 2006).

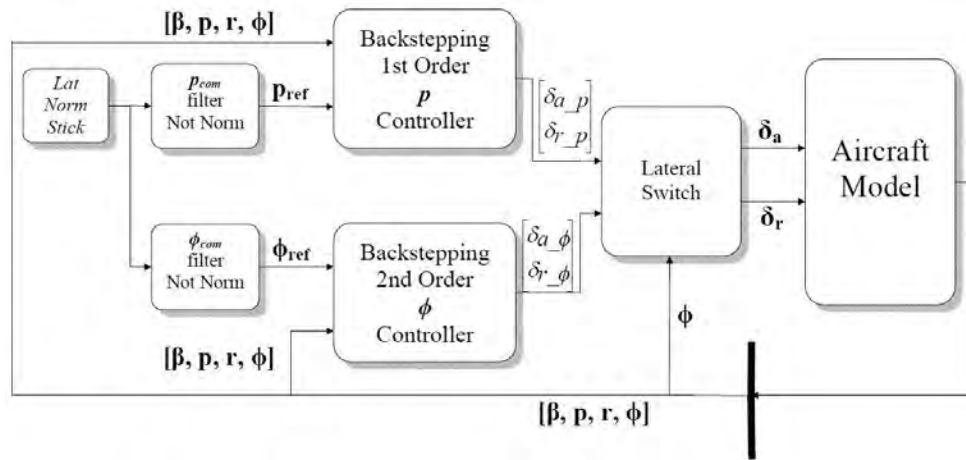


Figure1. Structure Lateral Flight Envelope Protection Controller

The Figure 1 shows the structure of the lateral flight envelope protection controller and it has two controller blocks running in parallel. Both lateral controllers produce input commands for aileron and rudder deflections and they are switched by the switch block.

The lateral normalize stick input signal is on the interval  $[-1, 1]$ . It is filtered and converted in an input signal reference for roll rate ( $p$ ) and bank angle ( $\phi$ ). This conversion was made using the Eq. (4) and Eq. (5):

$$p_{ref} = p_{max} \cdot u_{Stick-Lat} \quad (4)$$

$$\begin{aligned} \phi_{ref} &= (\phi_{max2} - \phi_{max1}) \cdot u_{stick-Lat} + \phi_{max1} \quad Se \quad u_{Stick-Lat} \geq 0 \\ \phi_{ref} &= (\phi_{min1} - \phi_{min2}) \cdot u_{stick-Lat} + \phi_{min1} \quad Se \quad u_{Stick-Lat} \leq 0 \end{aligned} \quad (5)$$

Where  $p_{max}$ ,  $\phi_{max1}$  and  $\phi_{max2}$  are the safe operation limits inside the flight envelope.

As both controllers work in parallel, the switching function receive the output from each controller block as an input, and using a switching function, it generate the final input signal to the aircraft. Grouping the manipulated variables as  $u_p = [\delta_{a-p}, \delta_{r-p}]^T$ ,  $u_\phi = [\delta_{a-\phi}, \delta_{r-\phi}]^T$  and the real actuator  $u_{lat} = [u_p, u_\phi]^T$ , the switching function is defined on the Eq. (6).

$$u_{lat} = S_\phi \cdot u_p + (1 - S_\phi) \cdot u_\phi \quad (6)$$

with  $s_\phi$  as a function depending from bank angle ( $\phi$ ) defined at Eq. (7), Eq. (8) and Eq. (9).

$$S_\phi = 1 \quad Se \quad \phi_{min1} < \phi < \phi_{max1} \quad (7)$$

$$S_\phi = \max\left(\frac{\phi - \phi_{min2}}{\phi_{min1} - \phi_{min2}}, 0\right) \quad Se \quad \phi \leq \phi_{min1} \quad (8)$$

$$S_\phi = \max\left(1 - \frac{\phi - \phi_{max1}}{\phi_{max2} - \phi_{max1}}, 0\right) \quad Se \quad \phi \geq \phi_{max1} \quad (9)$$

#### 4. LATERAL BACKSTEPPING CONTROLLER

Backstepping is a non-linear controller technique. It is an alternative between feedback linearization and dynamic inversion techniques (Van Oort E, 2011). Backstepping is based on Lyapunov second theory, using Lyapunov's candidate functions (LCF) expressed in terms of energy, where its derivative must be negative defined in order to guarantee the global stability (Harkegard O, 2003).

The purpose of this work is to design a first order Backstepping controller for the roll rate (because it has a direct relation with the actuators and it does not require a virtual controller) and a second order Backstepping controller for the bank angle (using the roll rate as a virtual controller).

Redefining the states from Eq. (1), Eq. (2), and Eq. (3) as:  $x_1, x_2 \in \mathfrak{R}^2$  and  $x_3 \in \mathfrak{R} / x_1 = [\beta, \phi]^T$ ,  $x_2 = [p, r]^T$ ,  $x_3 = \psi$  and the controller  $u = [\delta_a, \delta_r]^T$ . The new lateral mathematic model will be:

$$\dot{x}_1 = g_1(\alpha_e, \beta) + g_2(\alpha_e, \phi, \theta_e) \cdot x_2 + g_3(\alpha_e, \beta) \cdot x_2 + g_4(\beta) \cdot u + g_5(\alpha_e, \beta, \theta_e, \phi) \tag{10}$$

$$\dot{x}_2 = g_6(\alpha_e, \beta, p, q_e, r) + g_7(\alpha_e) \cdot x_2 + g_8(\alpha_e, \beta) \cdot u \tag{11}$$

$$\dot{x}_3 = g_9(\theta_e, \phi) \cdot x_2 \tag{12}$$

with the  $g_i$   $i = 1 \dots 9$  are the terms from the Eq. (1), Eq. (2) and Eq. (3). The  $g_6$  function groups two initial terms. In order to apply the technique, it must necessary consider the assumptions below.

*Assumption1:* The desire trajectories  $x_1^d = [\beta_{ref}, \phi_{ref}]^T$  and  $x_2^{ref} = [p_{ref}, r_{ref}]^T$  are bounded as

$$\left\| \begin{bmatrix} x_1^d, \dot{x}_1^d, \ddot{x}_1^d \\ x_2^{ref}, \dot{x}_2^{ref}, \ddot{x}_2^{ref} \end{bmatrix} \right\| \leq c_d$$

$$\left\| \begin{bmatrix} x_2^{ref}, \dot{x}_2^{ref}, \ddot{x}_2^{ref} \end{bmatrix} \right\| \leq m_d$$

where  $c_d$  and  $m_d \in \mathfrak{R}$  are known positive constants.

*Assumption2:* The total velocity and the dynamic pressure are constant.

$$\dot{V} = 0, \quad \dot{q} = 0$$

*Assumption3:* The control surface deflection has no effects on the aerodynamic force component:

$$g_4(\beta) = 0$$

#### 4.1 First Order Backstepping for roll rate

The tracking controller design for roll rate (and tracking the yaw rate) was develop using the structure show at Fig. 2.

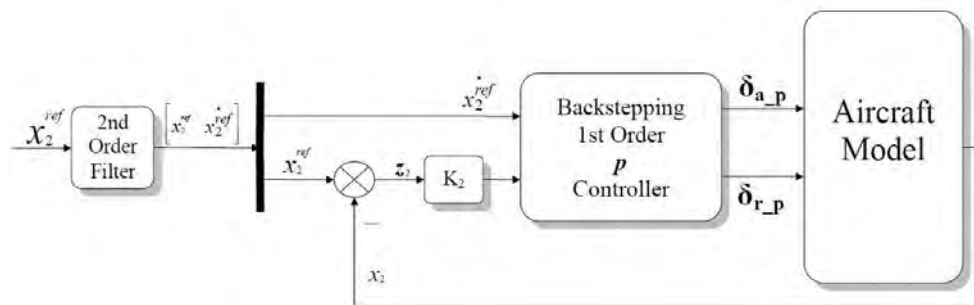


Figure2. Structure roll rate (p) Controller

The procedure to determinate the non-linear control law are defined on the Eq. (13) to Eq. (17). With the virtual variable as error  $z_2 = x_2 - x_2^{ref}$  and its derivative.

$$z_2 = x_2 - x_2^{ref} \tag{13}$$

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{x}_2^{ref} \\ &= g_6(\cdot) + g_7(\cdot) \cdot x_2 + g_8(\cdot) \cdot u - \dot{x}_2^{ref} \end{aligned} \tag{14}$$

where  $(\cdot)$  represent the function parameters.

Choosing a positive defined LCF and its derivative:

$$V_p = \frac{1}{2} z_2^2 \quad (15)$$

$$\begin{aligned} \dot{V}_p &= z_2 \cdot \dot{z}_2 \\ &= z_2 [g_6(\cdot) + g_7(\cdot) \cdot x_2 + g_8(\cdot) \cdot u - \dot{x}_2^{ref}] \end{aligned} \quad (16)$$

In order to guarantee  $\dot{V}_p$  will be negative defined, from the Eq. (16) the second term must be equal:

$$g_6(\cdot) + g_7(\cdot) \cdot x_2 + g_8(\cdot) \cdot u - \dot{x}_2^{ref} = -K_{zp} \cdot z_2 \quad (17)$$

From Eq. (17), the final control law for this block will be:

$$u = g_8^{-1}(\alpha_e, \beta) [-K_{zp} \cdot z_2 - g_6(\alpha_e, \beta, p, q_e, r) - g_7(\alpha_e) \cdot x_2 + \dot{x}_2^{ref}] \quad (18)$$

#### 4.2 Second order Backstepping for bank angle

The roll rate ( $\phi$ ) and the sideslip tracking are made together; one must follow a trajectory bounded by its maximum or minimum value. The second must keep on close to zero all time. The second order structure is show on the Fig. 3, there it has two blocks, one is the inner loop controller (it controls the fast dynamic  $x_2$ ) and the other is the outer loop controller (it controls the slow dynamic  $x_1$ ).

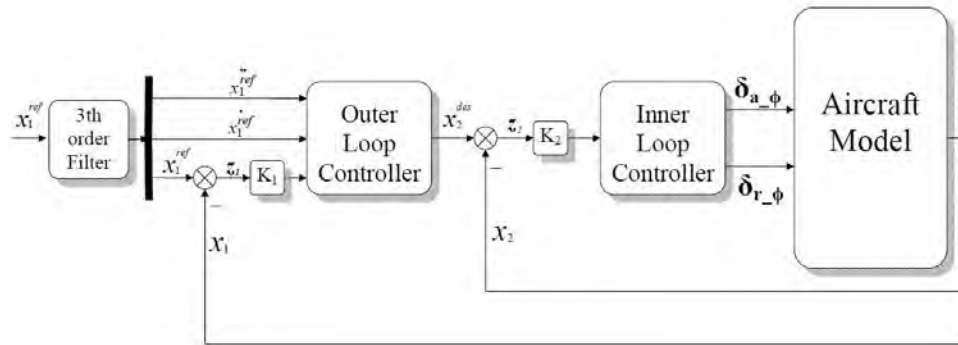


Figure3. Structure bank angle ( $\phi$ ) Controller

The second order Backstepping starts again defining the error variables as:

$$z_1 = x_1 - x_1^d \quad (19)$$

$$z_2 = x_2 - x_2^d \quad (20)$$

Deriving and replacing we have:

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{x}_1^d \\ &= g_1(\alpha_e, \beta) + g_2(\alpha_e, \phi, \theta_e) \cdot x_2 + g_3(\alpha_e, \beta) \cdot x_2 + g_5(\alpha_e, \beta, \theta_e, \phi) - \dot{x}_1^d \end{aligned} \quad (21)$$

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 - \dot{x}_2^d \\ &= g_6(\alpha_e, \beta, p, q_e, r) + g_7(\alpha_e) \cdot x_2 + g_8(\alpha_e, \beta) \cdot u - \dot{x}_2^d \end{aligned} \quad (22)$$

First, the virtual controller  $x_2^d$  must be found, to do this, the positive defined FCL must be selected as:

$$V_1 = \frac{1}{2}z_1^2 \quad (23)$$

$$\begin{aligned} \dot{V}_1 &= z_1 \cdot \dot{z}_1 \\ &= z_1[g_1(\cdot) + g_2(\cdot) \cdot x_2 + g_3(\cdot) \cdot x_2 + g_5(\cdot) - \dot{x}_1^d] \end{aligned} \quad (24)$$

In order to guarantee  $\dot{V}_1$  will be negative defined, from the Eq. (24) the second term must be equal:

$$g_1(\cdot) + g_2(\cdot) \cdot x_2 + g_3(\cdot) \cdot x_2 + g_5(\cdot) - \dot{x}_1^d = -K_{1\phi} \cdot z_1 \quad (25)$$

With the Eq. (25), virtual controller law  $x_2^d = x_2$  is used as a signal command to inner loop block.

$$x_2^d = (g_2(\cdot) + g_3(\cdot))^{-1}[-K_{1\phi} \cdot z_1 - g_1(\cdot) - g_5(\cdot) + \dot{x}_1^d] \quad (26)$$

Using  $x_2^d = x_2$ , the  $\dot{z}_1$  will be rewrite as follow:

$$\begin{aligned} \dot{z}_1 &= g_1(\cdot) + g_2(\cdot)(z_2 + x_2^d) + g_3(\cdot) \cdot (z_2 + x_2^d) + g_5(\cdot) - \dot{x}_1^d \\ &= g_1(\cdot) + (g_2 + g_3) \cdot z_2 + (g_2 + g_3(\cdot)) \cdot x_2^d + g_5(\cdot) - \dot{x}_1^d \\ &= g_1 + (g_2 + g_3) \cdot z_2 + (g_2 + g_3) \cdot (g_2 + g_3)^{-1}[-K_{1\phi} \cdot z_1 - g_1 - g_5 + \dot{x}_1^d] + g_5 - \dot{x}_1^d \\ \dot{z}_1 &= (g_2(\cdot) + g_3(\cdot))z_2 - K_{1\phi} \cdot z_1 \end{aligned} \quad (27)$$

Now the LCF is defined as:

$$V_2 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \quad (28)$$

$$\begin{aligned} \dot{V}_2 &= z_1 \cdot \dot{z}_1 + z_2 \cdot \dot{z}_2 \\ &= z_1[(g_2(\cdot) + g_3(\cdot))z_2 - K_{1\phi} \cdot z_1] + z_2[g_6(\cdot) + g_7(\cdot) \cdot x_2 + g_8(\cdot) \cdot u - \dot{x}_2^d] \\ &= -K_{1\phi} \cdot z_1^2 + z_2[z_1(g_2(\cdot) + g_3(\cdot)) + g_6(\cdot) + g_7(\cdot) \cdot x_2 + g_8(\cdot) \cdot u - \dot{x}_2^d] \end{aligned} \quad (29)$$

In order to  $\dot{V}_2$  will be negative defined the follow equation must be accomplished:

$$z_1(g_2(\cdot) + g_3(\cdot)) + g_6(\cdot) + g_7(\cdot) \cdot x_2 + g_8(\cdot) \cdot u - \dot{x}_2^d = -K_{2\phi} \cdot z_2 \quad (30)$$

From (30) the final control law for the bank angle will be:

$$u = g_8^{-1}(\alpha_e, \beta)[-K_{2\phi} \cdot z_2 - z_1(g_2(\alpha_e, \phi, \theta_e) + g_3(\alpha_e, \beta)) - g_6(\alpha_e, \beta, p, q_e, r) - g_7(\alpha_e) \cdot x_2 + \dot{x}_2^d] \quad (31)$$

Where, the  $K_i$  gain must be positive.

## 5. NUMERICAL SIMULATIONS

In this section the results and simulations are presented using the MatLab/Simulink tools.

For this effect the aircraft model F-16 was used at the equilibrium point of  $V = 50ft/s$ ,  $H = 20000ft$ . The controller gains are:  $K_{zp} = 25$ ,  $K_{1\phi} = 3$ ,  $K_{2\phi} = 6$ .

The protection values used was obtained from (Well K, 2006) :

$$\begin{aligned} \phi_{max1} &= 50^\circ, \phi_{min1} = -50^\circ \\ \phi_{max2} &= 60^\circ, \phi_{min2} = -60^\circ \\ p_{max/min} &= \pm 60(^\circ/s) \end{aligned}$$

The input stick is showed at the Fig. 4.

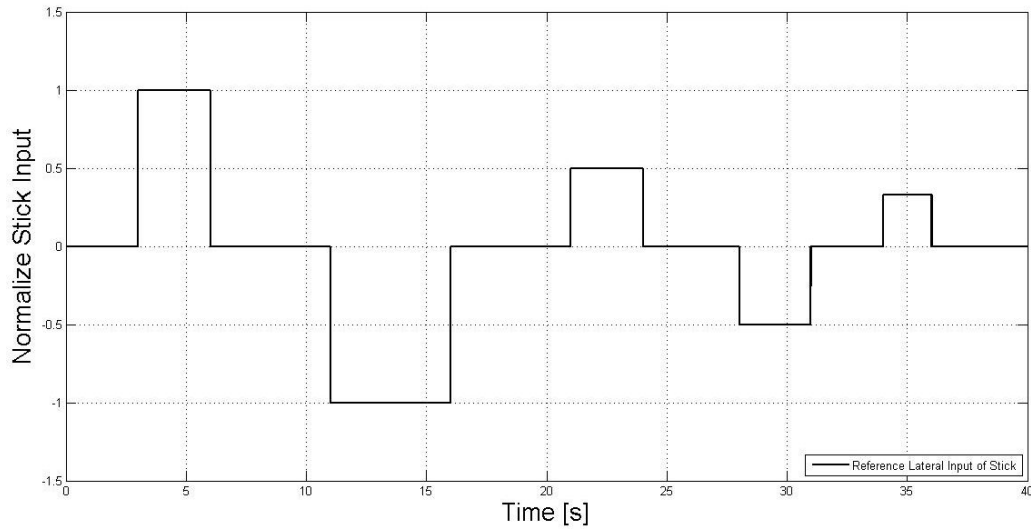


Figure 4. Lateral stick input from pilot

The roll rate response is showed at the Fig. 5, and the bank angle response is showed at the Fig. 6.

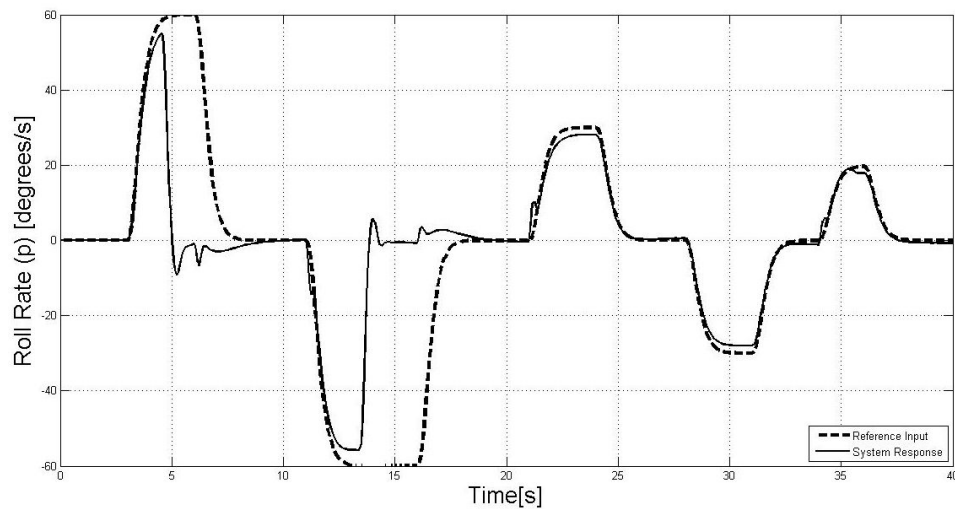


Figure 5. Roll rate response

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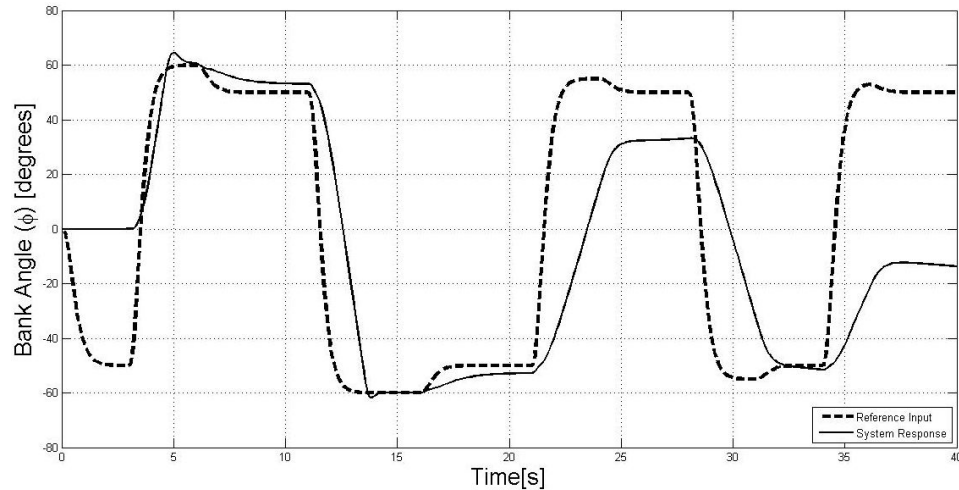


Figure 6. Bank angle response

Observing the Figs.5 and 6, we can see that the system begin tracking the roll rate, and when the bank angle is near the maximum or minimum value (value of protections), the switch function is activate, switching from the roll rate controller to bank angle controller. It occurs at time 4 and 12 s. When the bank angle value return to its safe envelope values and the pilot give to the stick an opposite command, the system do switch from the bank angle controller to roll rate controller.

The Fig. 7 show the actuator responses.

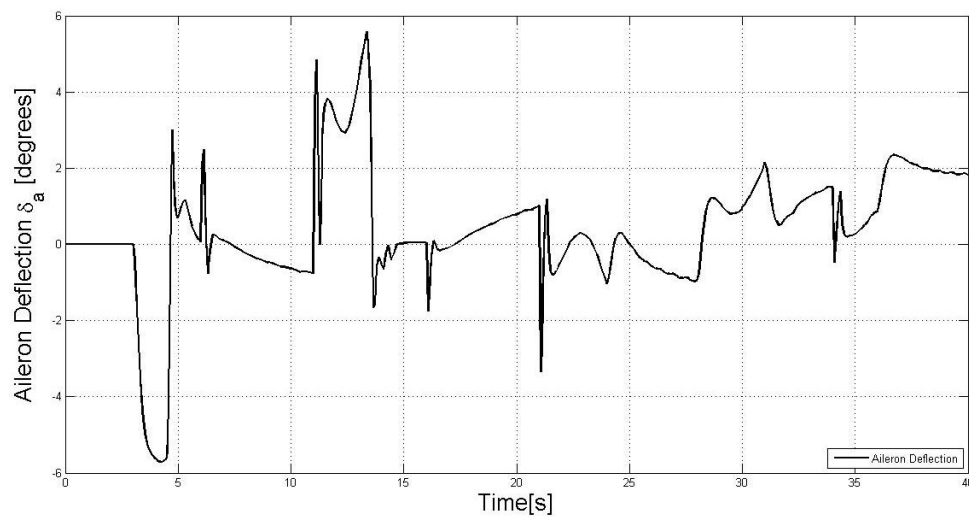


Figure 7. Aileron Deflection response



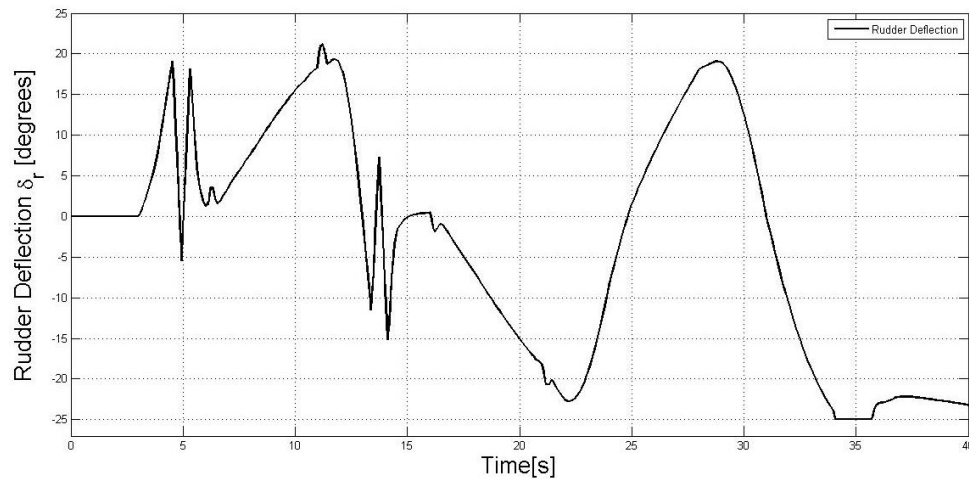


Figure 8. Rudder Deflection response

Both actuators have a reasonable response, and nobody reach saturation values.

## 6. CONCLUSIONS

In this paper, the flight envelope protection for a lateral motion of an aircraft F-16 was presented, using the non-linear Backstepping technique.

The results showed that both controllers works so good during the commutation between them. If the control surface deflection has no effects on the aerodynamic force component, this technique will be a good alternative to use for tracking some trajectories, deleting uniquely the bad non-linearities.

The results represent an alternative to the linear controllers, mainly in regions with high non-linearities and operations near the limits from an envelope.

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