ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

Sliding Mode Attitude Control for the ASTER Mission along a Circular Orbit

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Abstract. *ASTER, the first deep space Brazilian mission, targets the exploration of the triple asteroid system known as 2001 SN263. The study intended by the ASTER mission requires an attitude controller robust and capable to deal with the non-linearities and the uncertainties present during the exploration phase, as well as, high amplitude maneuvers. For such requirements, this paper study the applicability of two controllers designed based on sliding mode techniques. One of the controllers also includes an adaptation part used to compensate for the spacecraft's inertia variation. A comparison is also made with a linear controller developed for the same mission in a previous work, which is tested under similar conditions. As a result it was possible to see the usefulness of this type of controllers for this type of mission and a superior performance compared to the linear based controller.*

Keywords: *asteroid mission, non-linear control, attitude, ASTER, sliding mode*

1. INTRODUCTION

ASTER, the first deep space Brazilian mission, targets the group of asteroids in the system know as 2001 SN263, which is composed by a central body named Alpha and two satellites, the largest one orbiting further from the system's center is named Beta, while the smaller asteroid orbiting closer to Alpha is named Gamma. The objective of this mission is to explore this system, taking measurements and pictures from all three asteroids and, if possible, finalize the mission by touching down Alpha, Macau *et al.* (2011); Sarli *et al.* (2012b).

One of the many elements for the success of the mission is the ability of correct pointing and stabilization the spacecraft, which is useful for the orientation of instruments and also for the navigation, which is based on a fixed ion-thruster; therefore, in order to change the trajectory for translation within the system, the attitude of the spacecraft needs to be changed.

A triple asteroid system is highly non-linear due to the nature of the gravity environment generated by the three bodies, allied with the solar radiation pressure. Therefore, a control law based on a linear approximation is not suited, neither for the orbit around the system nor the navigation inside it, because the short maneuver time and the long engine operation. Furthermore, it is required from the spacecraft to perform large angle maneuvers during its operation within the asteroid system, such tracking can not be accurately perform by linear controllers as demonstrated on Sarli *et al.* (2012a), figure 1.

Particularly in this work, only the case where the spacecraft orbits the center of the system will be considered. The navigation issue is much more complex and it will be addressed in a future work. Moreover, the actuators used here are small thrusters for the attitude guidance that can provide an anti-symmetrical setting, which means that the attitude can be controlled without inducing translation.

The control technique chosen for this study is named sliding mode (SM). This formulation is very appealing specially by its invariance to disturbances and model uncertainties, Utkin (1977); Hung *et al.* (1993); Slotine and Li (1991). This characteristic is highly desirable for a mission such as ASTER, where many uncertainties in the model of the asteroid system are present. The chosen SM formulation has another attractive feature for the mission: an adaptive law that is of great importance when using ion-thrusters, which stay on consuming propellant for long periods of time making the inertial matrix vary considerably, Yeh (2010). The focus of this study is to develop the control algorithm, the precise determination of the moment of inertias' values was not the objective. Therefore, the values used in the inertia matrix are hypothetical.

Next sections are divided as follows. Section 2. presents the equations of motion of the spacecraft. Section 3, based in the work of Yeh (2010), presents the formulation of the non-linear controller for the attitude pointing, discussing the sliding surface and control law addressing the problem of chattering, followed by the formulation of the SM adaptive controller, featuring the same important point of the previous controller. The section 4presents simulation results, which

ISSN 2176-5480

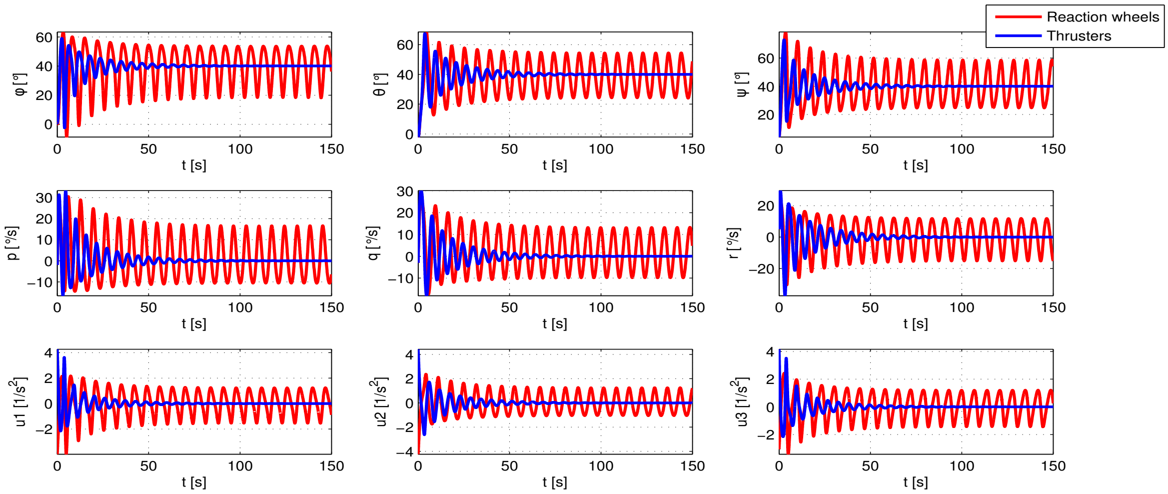

Figure 1. Close loop spacecraft response to step inputs of high amplitudes, Sarli *et al.* (2012a)

are compared with a previous work where the controller was designed using a linear control law, Sarli *et al.* (2012a). Finally, section 5. presents this work conclusions.

2. EQUATIONS OF MOTION

Particularly for a circular orbit, or long orbits, the rotational and translational equations of motion of a spacecraft can be treated independently. For this work the rotational motion will not affect the translational motion, because the gravity force of Alpha around the spacecraft will be constant around the circular orbit and the gravity of Beta and Gamma will be treated as perturbations. However, the perturbing torque or disturbance considered here will come from the gravity gradient torque generated by the three asteroids and their perturbing angular torque, denoted by D, can be calculated as follows, Sarli *et al.* (2012a):

$$\{\vec{g}_c\}_\beta = \sum_{j=1}^{n} 3\left(\frac{GM_j}{R_j^3}\right) c_{i,3} J_x c_{i,3} \rightarrow D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \sum_{j=1}^{n} 3\frac{GM_j}{R_j^3} \begin{bmatrix} (J_{33}-J_{22})\, c_{23,j}c_{33,j} \\ (J_{11}-J_{33})\, c_{33,j}c_{13,j} \\ (J_{22}-J_{11})\, c_{13,j}c_{23,j} \end{bmatrix} \tag{1}$$

where, $c_{i,3}$ is the third column of the transformation matrix from the inertial frame to the body-fixed frame, R_j is the distance between the spacecraft's center of mass and the asteroid's center of mass, J is the inertia matrix, j represents the number of the body: Alpha is 1, Beta 2 and Gamma 3, G is the gravitational constant and M_j is the mass of each asteroid. It is assumed that the values of the secondary terms of the inertia matrix are negligible.

It is important to point out that the evolution of Beta and Gamma around Alpha were studied in Fang *et al.* (2011). However, no official model has been derived yet. Therefore, the ephemeris model used in this work was calculated using the initial conditions from Fang's work and propagated using an Adams integrator for 20 days from June 1st of 2019 until June 20th of the same year, Sarli *et al.* (2012b) . Finally, the solar radiation pressure on the circular orbit is assumed to be a constant affecting only the point where the center of the orbit is located, not its orbital elements.

The rotational equations of motion for the spacecraft as a rigid body can be derived by Euler's formulation, having the following form:

$$J\dot{\Omega} = -\dot{J}\Omega - \Omega \times (J\Omega) + T_b + D \tag{2}$$

where J is the inertia matrix, Ω is the angular velocity, T_b is the moment due to the actuators and D is the disturbance torque.

As mentioned before the spacecraft has a large ion-thrust for trajectory control and a cluster of small thrusters for attitude control. These small thrusters are arranged in such a way that can generate three independent control torques T_x, T_y and T_z around the x, y and z axes of the body, respectively, without inducing translation; then, $T_b = [T_x\ T_y\ T_z]^T$. In practice, these control torques can assume only three levels: zero, minimum negative and minimum positive. However, the control law assumes continuous varying torques. This can be obtained approximately by pulse-width pulse-frequency (PWPF) modulation, Qingleia *et al.* (2011).

ISSN 2176-5480

It is convenient to evaluate the rotational kinematics in quaternions, rather than in Euler angles, in order to avoid singularities. The quaternion formulation is described as it follows:

$$\begin{aligned} Q &= \left[\begin{array}{cccc} q_1 & q_2 & q_3 & q_4 \end{array}\right]^T = \left[\begin{array}{cc} \bar{Q}^T & q_4 \end{array}\right]^T \\ \bar{Q}^T &= \left[\begin{array}{c} q_1 \\ q_2 \\ q_3 \end{array}\right] = U \sin\left(\tfrac{\Phi}{2}\right),\; q_4 = \cos\left(\tfrac{\Phi}{2}\right) \end{aligned} \tag{3}$$

where Q is the quaternion, $\bar{Q}$ is its vectorial part and q_4 is its scalar part. U is the rotation vector and Φ is the rotation angle, that describe a rigid body rotation, according to the Euler's rotation theorem.

The derivative of the quaternion is:

$$\begin{aligned} \dot{\bar{Q}} &= \tfrac{1}{2}\left\langle \bar{Q}_x \right\rangle \Omega + \tfrac{1}{2} q_4 \Omega \\ \left\langle \bar{Q}_x \right\rangle &= \left[\begin{array}{ccc} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{array}\right] \\ \dot{q_4} &= -\tfrac{1}{2}\Omega^T \bar{Q} \end{aligned} \tag{4}$$

with Eq. 2 and Eq. 4 the final form of the spacecraft's dynamic model can be obtained, where e means an error and d means a desired behavior,

$$\begin{aligned} \dot{\bar{Q}}_e &= \tfrac{1}{2}\left\langle \bar{Q}_{ex} \right\rangle \Omega_e + \tfrac{1}{2} q_{e4} \Omega_e \\ \dot{q}_{e4} &= -\tfrac{1}{2}\Omega_e^T \bar{Q}_e \\ J\dot{\Omega} &= -\dot{J}\Omega - \Omega \times (J\Omega) + T_b + D \end{aligned} \tag{5}$$

with,

$$\Omega_e = \Omega - \Omega_d \tag{6}$$

$$Q_e = \left[\begin{array}{cccc} q_{e1} & q_{e2} & q_{e3} & q_{e4} \end{array}\right]^T = \left[\begin{array}{cccc} q_{d4} & q_{d3} & -q_{d2} & -q_{d1} \\ -q_{d3} & q_{d4} & q_{d1} & -q_{d2} \\ q_{d2} & -q_{d1} & q_{d4} & -q_{d3} \\ q_{d1} & q_{d2} & q_{d3} & q_{d4} \end{array}\right] \left[\begin{array}{c} q_1 \\ q_2 \\ q_3 \\ q_4 \end{array}\right] \tag{7}$$

Equation 6 defines an error angular speed with respect to a desired angular speed Ω_d. Equation 7 defines an incremental rotation with respect to a desired attitude Q_d. The desired angular speed and attitude will be considered in the next section, where the problem of attitude tracking control is developed.

3. NONLINEAR ATTITUDE CONTROLLER

This section presents two controllers developed in the reference Yeh (2010). They are sliding mode controllers developed for the attitude control of a spacecraft with thrusters, an application similar to that assumed in this paper.

The first approach is a nonlinear robust controller with respect to external disturbances and parametric uncertainties. The second approach is also a nonlinear robust controller, but with an additional adaptive law to estimate the uncertain inertia matrix.

The formulation of this section is identical to that in the reference Yeh (2010), no changes are introduced in the control structure or tunning methods. However, here, attention is paid in the explanation of the meaning of each design parameter of the controls. By explaining the role of each element in the control law, the designer can understand the impact of such parameters when tunning a specific control law for some application.

3.1 Sliding mode attitude controller design

In the development of a SM controller, one needs to define a sliding surface. This surface is determined in order to obtain a sliding mode that satisfy the requirements of some application, in this case, the tracking of a desired attitude. After the determination of a sliding surface, a reaching condition shall be described, from which the sliding surface can be reached. This condition determines the control law that can make de sliding mode possible and, consequently, the desired behavior, Hung *et al.* (1993); Utkin (1977). The next topics describe each of these steps for the robust control law of Yeh (2010).

3.1.1 Sliding surface

The switching function is defined as:

$$S = P\overline{Q}_e + \Omega_e \tag{8}$$

ISSN 2176-5480

where, P is a positive diagonal matrix.

When $S = 0$, the switching surface is obtained. The path of the system constrained to this surface is the sliding mode. This sliding mode shall represent the desired behavior, which in this case is the tracking of the desired attitude. So, during the sliding mode, one should expect that the errors in the quaternion and angular velocity Q_e and Ω_e tend to zero. The evaluation of this requirement is performed using the Lyapunov stability theory.

During the sliding mode, Eq. 8 tells that:

$$P\overline{Q}_e + \Omega_e = 0 \tag{9}$$

such that the angular velocity and quaternion are constrained by: $\Omega_e = -P\overline{Q}_e$. Applying this result in the Eq. 5, one obtains the differential equation of the sliding mode:

$$\begin{aligned} \dot{\overline{Q}}_e &= -\tfrac{1}{2}\left\langle \bar{Q}_{ex}\right\rangle P\overline{Q}_e - \tfrac{1}{2}q_{e4}P\overline{Q}_e \\ \dot{q}_{e4} &= \tfrac{1}{2}\bar{Q}_e^T P\bar{Q}_e \end{aligned} \tag{10}$$

In order to evaluate the stability of the sliding mode, this Lyapunov function candidate is chosen: $V_e\left(\overline{Q}_e\right) = k\overline{Q}_e^T\overline{Q}_e$ where k is a positive number. Its derivative generates:

$$\begin{aligned} \dot{V}_e\left(\overline{Q}_e\right) &= 2k\overline{Q}_e^T\dot{\overline{Q}}_e \\ \dot{V}_e\left(\overline{Q}_e\right) &= -k\overline{Q}_e^T\left(\left\langle \bar{Q}_{ex}\right\rangle P\overline{Q}_e + q_{e4}P\overline{Q}_e\right) \\ \dot{V}_e\left(\overline{Q}_e\right) &= -kq_{e4}\overline{Q}_e^T P\overline{Q}_e \end{aligned} \tag{11}$$

because $\overline{Q}_e^T\left\langle \bar{Q}_{ex}\right\rangle = 0$.

The sign of the last derivative depends on q_{e4}, the real part of the quaternion. From the definition in Eq. 3, q_{e4} can be always positive or always negative. This arbitrariness depends only in the choice of initial condition: if it is positive, q_{e4} will be also positive, this is the fact assumed here, Yeh (2010).

From the Eq. 10, note that $\dot{q}_{e4}$ is always positive. As its initial condition is also positive, $q_{e4}(t)$ is always positive and growing: $q_{e4}(t) \geq c,\ \forall t$, for some initial value $c > 0$. Moreover, from the fact of unit-norm of the quaternions, the following relation holds:

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \ \Rightarrow\ q_{e4} \leq 1 \tag{12}$$

Hence,

$$c \leq q_{e4} \leq 1, \quad \Rightarrow\ -ck\overline{Q}_e^T P\overline{Q}_e \geq \dot{V}_e\left(\overline{Q}_e\right) \geq -k\overline{Q}_e^T P\overline{Q}_e \tag{13}$$

From the Lyapunov stability theory, the sliding mode is stable, in such a way that the origin of the error dynamics is a stable equilibrium point. Thus, the tracking error converges to zero during the sliding mode:

$$\left(\overline{Q}_e,\ \Omega_e\right) \to \left(0_{3\times1},\ 0_{3\times1}\right),\ t \to 0 \tag{14}$$

This shows that the sliding surface is capable of satisfy the requirement of attitude tracking. In this sense, note the meaning of the positive definite matrix P: from Eq. 13 the decaying rate of the Lyapunov function depends on the magnitude of P, so, increasing the magnitude of the elements of this matrix, one can decrease the convergence time to the origin of the error dynamics.

3.1.2 Control law

This section establishes a control law in order to guarantee that the sliding surface is reached, from any initial condition, and the sliding mode exists. That is, the control shall be determined in order to satisfy a reaching condition, Hung *et al.* (1993). The reaching condition can be specified in a variety of ways, Yeh (2010) chose the usage of the Lyapunov function method. The function candidate is:

$$\begin{aligned} V_s &= \tfrac{1}{2}S^T JS \\ V_s &= S^T J\dot{S} + \tfrac{1}{2}S^T\dot{J}S \end{aligned} \tag{15}$$

Substituting Eq. 5 and Eq. 8 on Eq. 15:

$$\dot{V}_s = S^T\left[-\dot{J}\Omega - \Omega\times(J\Omega) + T_b + D - J\dot{\Omega}_d + JP\left(\frac{1}{2}\left\langle \bar{Q}_{ex}\right\rangle\Omega_e + \frac{1}{2}q_{e4}\Omega_e\right) + \frac{1}{2}\dot{J}S\right] \tag{16}$$

From the above result, the following control torque is assumed:

$$T_b = -K_sS + \dot{J}_0\Omega - \frac{1}{2}\dot{J}_0S - J_0P\left(\frac{1}{2}\left\langle \bar{Q}_{ex}\right\rangle\Omega_e + \frac{1}{2}q_{e4}\Omega_e\right) + \Omega\times(J_0\Omega) - J_0\dot{\Omega}_d + \Lambda_s \tag{17}$$

ISSN 2176-5480

The element K_sS is a feedback of the switching function, $K_s = \text{diag}\begin{bmatrix} k_{s1} & k_{s2} & k_{s3} \end{bmatrix}$ is a 3×3 positive definite matrix, the meaning of K_s will be explained in the following. The element Λ_s is a discontinuous control action, responsible for the generation of the sliding mode:

$$\Lambda_s = \begin{bmatrix} \lambda_{s1} & \lambda_{s2} & \lambda_{s3} \end{bmatrix}^T, \ \lambda_{si} = -c_{si}\left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d\right) \text{sgn}(s_i) \tag{18}$$

the terms $c_{si}\left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d\right)$ are control amplitudes, their meaning will be explained in the following. The element $\text{sgn}(s_i)$ is the sign function:

$$\text{sgn}(s_i) = \begin{cases} 1, \ s_i > 0, \\ 0, \ s_i = 0, \quad i = 1, 2, 3 \\ -1, \ s_i < 0, \end{cases} \tag{19}$$

and $S = \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^T$ is the sliding surface.

The remaining elements in the control torque of Eq. 17 is the *equivalent control*, it is responsible for guarantee that the sliding mode trajectories are tangent to the sliding surface, Hung *et al.* (1993). Note that it is a continuous nonlinear function that depends on the dynamics of the plant.

Substituting the control torque of Eq. 17 in the derivative of Eq. 16, one obtains a condition for the control action to satisfy the reaching condition (negative definite function). This condition is related with the uncertain terms given by the use of a nominal inertia matrix J_0 and derivative $\dot{J}_0$, and disturbance D. This condition guaranties the exponential stability and robustness of the controller for attitude tracking. It is achieved if the following inequality condition is satisfied:

$$c_{si}\left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d\right) > \delta_i^{max}\left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d\right) \geq |\delta_i| \tag{20}$$

where, $\bar{\delta} = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 \end{bmatrix}^T = -\Delta\dot{J}\Omega - \Omega \times (\Delta J\Omega) + D - \Delta J\dot{\Omega}_d + \Delta JP\left(\frac{1}{2}\left\langle\bar{Q}_{ex}\right\rangle\Omega_e + \frac{1}{2}q_{e4}\Omega_e\right) + \frac{1}{2}\Delta\dot{J}S$. These functions depend on the uncertainties $\Delta J = J - J_0$ and $\Delta\dot{J} = \dot{J} - \dot{J}_0$ and the disturbance D. Such quantities are not known, by this way, the maximum amount of uncertainty shall be considered in the Eq. 20.

The proof of condition 20 comes using Eq. 16 with 17 for $S \neq 0$,

$$\dot{V}_s = -S^TK_sS - \textstyle\sum_{i=1}^3 |s_i|\left[c_{si} - \delta_i\text{sgn}(s_i)\right] \leq -S^TK_sS - \sum_{i=1}^3 |s_i|\left[c_{si} - \delta_i^{max}\right] \leq -\sigma_{min}(K_s)|S|^2 < 0 \tag{21}$$

where, σ_{min} is the smallest eigenvalue of K_s.

In this way, guaranteeing the sliding and reaching conditions of the sliding mode $S = 0$. Therefore, by making use of Eq. 8, the switching function equation, and Eq. 17, the control law, one ensures the exponential stability and robustness of the sliding-mode attitude controller.

Special attention shall be paid for the meaning of the control gains $K_s = \text{diag}\begin{bmatrix} k_{s1} & k_{s2} & k_{s3} \end{bmatrix}$ and the control amplitudes $c_{si}\left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d\right)$. From Eq. 21, the amplitude of the gains k_{si} are related to the rate of convergence of the reaching mode, in other words, increasing the magnitude of these gains, the time of convergence to the sliding surface is decreased. In other way, from Eq. 20, the control amplitudes $c_{si}\left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d\right)$ shall be determined in order to guarantee that the reaching condition is satisfied in the presence of the external disturbance and parametric uncertainties. There are a variety of ways for satisfy this, one possibility is consider the worst case scenario, where the modulus of Eq. 20 is taken, with a subsequent substitution of the maximal expected value of each variable, da Silva *et al.* (2008).

3.1.3 Chattering

The chattering problem is originated due to the existence of physical limitations in the implementation of the sign function $\text{sgn}(s_i)$, some problems are delays, dead zones, hysteresis. In order to improve the solution and avoid the problem caused by chattering, the sign function can be replaced by a saturation function; in this way, forcing the system to stay within the limits of the boundary layer $|S_i| < \varepsilon$ (ε represents a positive small scalar value) and no longer exactly on the sliding surface. Such change in the control law will be, of course, followed by a reduction in the accuracy of the desired performance, Slotine and Li (1991). The saturation function is defined as follows:

$$Sat(s_i, \varepsilon) = \begin{cases} 1, \ s_i > \varepsilon \\ \frac{s_i}{\varepsilon} \ |s_i| < \varepsilon \\ -1, \ s_i < \varepsilon \end{cases} \tag{22}$$

where ε is a constant that shall be chosen as small as possible.

ISSN 2176-5480

3.2 Sliding mode adaptive attitude controller design

The variation of the inertia of the spacecraft cannot be neglected if a particular long phase consumes fuel during this entire phase.

As it is too complex to calculate the inertia variation, a very useful solution is to design an adaptive controller that can compensate for the effect of this variation and of the disturbances.

In short, the adaptive controller problem consists in generating a control law and a parameter vector estimation law, such that the tracking error tends exponentially to zero:

1. Given the desired attitude, quaternion Q_d, and angular velocity, Ω_d, with some of all the parameters unknown of J;

2. Derive a control law for the thrusters torques and an estimation law for the unknown parameters, such that Q and Ω precisely track Q_d and Ω_d after the initial adaptation process;

Let A be a constant 6×1 vector containing the unknown parameters and $\hat{A}$ (the time-varying parameter vector estimate); with the error $\tilde{A} = \hat{A} - A$.

The sliding mode surface defined in Eq. 8 remains the same. However, a new Lyapunov function candidate is assumed for the reaching condition:

$$V = \frac{1}{2} S^T J S + \frac{1}{2} \tilde{A}^T \Gamma^{-1} \tilde{A} \tag{23}$$

where, Γ is a positive diagonal matrix, the meaning of which will be discussed further.

The derivative of the function candidate is:

$$\dot{V} = S^T J \dot{S} + \tilde{A}^T \Gamma^{-1} \dot{\tilde{A}} \tag{24}$$

Substituting Eq. 5 and the time derivative of Eq. 8 in Eq. 24 one gets:

$$\dot{V} = S^T \left[-\Omega \times (J\Omega) + J \left(\frac{1}{2} P \left\langle \bar{Q}_{ex} \right\rangle \Omega_e + \frac{1}{2} P q_{e4} \Omega_e - \dot{\Omega}_d \right) \right] + S^T \left(T_b + D \right) + \tilde{A}^T \Gamma^{-1} \dot{\tilde{A}} \tag{25}$$

note that the time derivative of J is assumed *zero*. In such a case, this time derivative is taken as a disturbance inside the vector D. Moreover, see that this implies: $\dot{\tilde{A}} = \dot{\hat{A}} - \dot{A} = \dot{\hat{A}} - 0 = \dot{\hat{A}}$, as long as the vector A contains the elements of the matrix J. As a consequence, in Eq. 25 $\dot{\tilde{A}}$ can be changed by $\dot{\hat{A}}$.

From the Eq. 25, the control torque is chosen as:

$$T_b = \Omega \times \left(\hat{J} \Omega \right) - \hat{J} \left(\frac{1}{2} P \left\langle \bar{Q}_{ex} \right\rangle \Omega_e + \frac{1}{2} P q_{e4} \Omega_e - \dot{\Omega}_d \right) + \Lambda - K_a S \tag{26}$$

where, $K_a = \mathrm{diag} \begin{bmatrix} k_{a1} & k_{a2} & k_{a3} \end{bmatrix}$ is a 3×3 positive definite matrix and $\Lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}^T$ is a vector of discontinuous functions. These terms are analogous to that in Eq. 17, but note that *the nominal matrix J was changed by the estimated matrix $\hat{J}$*.

Then, Eq. 25 can be written as:

$$\begin{aligned} \dot{V} = S^T [&\Omega \times \left(\hat{J} \Omega \right) - \hat{J} \left(\tfrac{1}{2} P \left\langle \bar{Q}_{ex} \right\rangle \Omega_e + \tfrac{1}{2} P q_{e4} \Omega_e - \dot{\Omega}_d \right) + \\ &-\Omega \times (J\Omega) + J \left(\tfrac{1}{2} P \left\langle \bar{Q}_{ex} \right\rangle \Omega_e + \tfrac{1}{2} P q_{e4} \Omega_e - \dot{\Omega}_d \right)] - S^T K_a S + S^T \left(\Lambda + D \right) + \tilde{A}^T \Gamma^{-1} \dot{\hat{A}} \end{aligned} \tag{27}$$

By some algebraic manipulation one has:

$$\Omega \times \left(\hat{J} \Omega \right) - \hat{J} \left(\frac{1}{2} P \left\langle \bar{Q}_{ex} \right\rangle \Omega_e + \frac{1}{2} P q_{e4} \Omega_e - \dot{\Omega}_d \right) = Y \left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d \right) \hat{A} \tag{28}$$

where $Y \left(Q, \Omega, Q_d, \dot{Q}_d, \ddot{Q}_d \right)$ is a 3×6 matrix.

So, Eq. 27 becomes:

$$\dot{V} = S^T Y \tilde{A} - S^T K_a S + \Sigma_{i=1}^{3} s_i \left(\lambda_i + d_i \right) + \tilde{A}^T \Gamma^{-1} \dot{\hat{A}} \tag{29}$$

note that the subtle but important relation was taken: $\tilde{A} = \hat{A} - A$.

With $\lambda_i = -k_i \mathrm{sgn}(s_i)$ (k_i are amplitudes to be determined), the above equation becomes:

$$\dot{V} = S^T Y \tilde{A} - S^T K_a S + \Sigma_{i=1}^{3} \left| s_i \right| \left(-k_i + d_i \mathrm{sgn}(s_i) \right) + \tilde{A}^T \Gamma^{-1} \dot{\hat{A}} \tag{30}$$

ISSN 2176-5480

Note that in the last equation one can write: $S^T Y \tilde{A} + \tilde{A}^T \Gamma^{-1} \dot{\hat{A}} = \tilde{A}^T Y^T S + \tilde{A}^T \Gamma^{-1} \dot{\hat{A}} = \tilde{A}^T \left(Y^T S + \Gamma^{-1} \dot{\hat{A}} \right)$. So, the following adaptive law can be chosen:

$$Y^T S + \Gamma^{-1} \dot{\hat{A}} = 0, \Rightarrow \dot{\hat{A}} = -\Gamma Y^T S \tag{31}$$

from this assumption, Eq. 30 becomes:

$$\dot{V} = -S^T K_a S + \Sigma_{i=1}^{3} |s_i| \left(-k_i + d_i \text{sgn}(s_i)\right) \tag{32}$$

Choosing the following control amplitudes for disturbance rejection:

$$k_i > d_i^{max} \geq |d_i| \ (i = 1, 2, 3) \tag{33}$$

then Eq. 32 becomes:

$$\dot{V} \leq -S^T K_a S \tag{34}$$

This guarantees the global stability of the attitude tracking system provided by the sliding mode adaptive controller. In Yeh (2010), it is shown that this controller also provide the convergence of the states $\bar{Q}_e$ and Ω_e and parameter estimation error $\tilde{A}$. The demonstration involves the Lyapunov stability theory and the Barbalat's lemma. This proves that the adaptive controller can also solve the attitude tracking problem.

Again, special attention shall be paid for the meaning of the control gains $K_a = \text{diag} \begin{bmatrix} k_{a1} & k_{a2} & k_{a3} \end{bmatrix}$, the control amplitudes k_i and the elements $\Gamma = \text{diag} \left[\gamma_1 \ \gamma_2 \ \ldots \ \gamma_6\right]$. From Eq. 34, the amplitude of the gains k_{ai} are related to the rate of convergence of the reaching mode, in other words, by increasing the magnitude of these gains, the time of convergence to the sliding surface is decreased. In other way, from Eq. 33, the control amplitudes k_i shall be determined in order to guarantee that the reaching condition is satisfied in the presence of the disturbance D, which is related to the external perturbations and the uncertainties in the inertia matrix. Finally, from Eq. 31, one shall note that the elements of Γ are related with the convergence rate of the parameter estimation law.

3.2.1 Chattering suppression

In the same way of the non adaptive controller, the sign function shall be changed by a saturation function in order to avoid chattering.

4. Simulations

Two simulations were performed with the two controllers described at section 3. As the ASTER is still being prepared and final values for most of the variables used here are yet to be evaluated and are constantly changing; the values for the variables used in this work were based on the references Sarli *et al.* (2012b); Yeh (2010); Sarli *et al.* (2012a). The orbital conditions were chosen in a way to put the spacecraft under the most severe conditions with respect to the perturbations; the initial orbit is a circular, on Alpha's ecliptic plane between Beta and Gamma, with a radius of 11.9 km. The simulation time is set as approximately 12.5 second. In order to obtain a more demanding condition in which the control has to quickly perform the correction maneuvers, the disturbance were evaluated in a different time frame; while the simulation time is counted in seconds the disturbance time is counted in days, that means that the disturbances and, by extension, the orbits of the planets and the orbit of the spacecraft are evaluated at 12.5 days, figure 2. Therefore, all the disturbance faced in two full orbits (approximately 12.5 days) are compressed in an interval of 12.5 seconds in which the spacecraft has to correctly steer its attitude and maintain stability. This rather unusual condition is done so not only to make more challenging but also to place the attitude and the orbit conditions in the same frequency, if not, the time necessary to perform an attitude maneuver would be much faster than the translation of the system; it would appear that the asteroids and the spacecraft didn't even move during the time of the attitude maneuver.

For both simulations the initial attitude conditions are the same as the ones used in the previous work related with ASTER's linear attitude control Sarli *et al.* (2012a), were the initial attitude is represented by the Euler angles with values of $\begin{bmatrix} 1^\circ & -2^\circ & 4^\circ \end{bmatrix}^T$, which converted to quaternions are $Q(0) = \begin{bmatrix} 0.0081 & -0.0171 & 0.0350 & 0.9992 \end{bmatrix}^T$, the angular velocity is $\Omega(0) = \begin{bmatrix} -2^\circ/s & -3^\circ/s & 5^\circ/s \end{bmatrix}^T$ and the desired conditions are the high amplitude conditions for the Euler angles $\begin{bmatrix} 45^\circ & 45^\circ & 45^\circ \end{bmatrix}^T$, which in quaternions is $Q_d(t) = \begin{bmatrix} 0.4119 & 0.4119 & 0.1913 & 0.7325 \end{bmatrix}^T$, $\Omega_d(t) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and $\dot{\Omega}_d(t) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$.

The inertia conditions of the spacecraft will be

$$J_0 = \begin{bmatrix} 19.4 & 0.1 & 3 \\ 0.1 & 25.7 & 0.5 \\ 3 & 0.5 & 18.4 \end{bmatrix}, \ \dot{J}_0 = \frac{-J_0}{1000}, \ \Delta J = \frac{J_0}{10}, \ \Delta \dot{J} = \frac{\dot{J}_0}{10} \tag{35}$$

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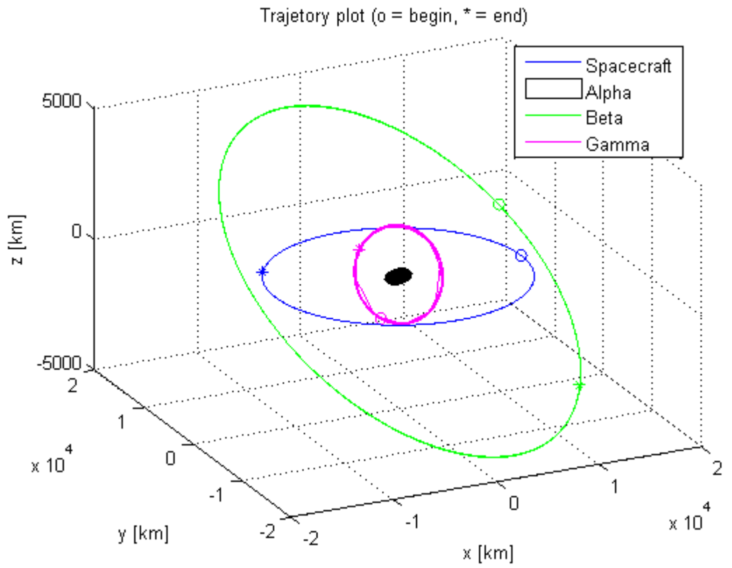

Figure 2. Spacecraft trajectory

For the non-adaptive controller the data for the sliding surface are $c_{s1} = c_{s2} = c_{s3} = 0.2$ and

$$P = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 12 & 0 \\ 0 & 0 & 12 \end{bmatrix}, K_s = \begin{bmatrix} 14 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 14 \end{bmatrix} \tag{36}$$

Finally, for the adaptive controller: $k_1 = k_2 = k_3 = 0.2$,

$$P = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{bmatrix}, \Gamma = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, K_a = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \tag{37}$$

The above values for both controllers were adjusted by trial and error methods aiming a compromise between the sliding mode and switching surface's convergence speeds represented by P and K respectively, the value of c_s's and k's were based on the expected uncertainty in the inertia matrix which has its largest value of 2.57 and, finally, it was chosen and exponential convergence for the saturation function used in the controller.

The results of both simulations for the full non-linear system can be seen in the next figures, where 3, 4 and 5 show respectively the Euler angles, quaternions and angular velocity evolution, 6 shows the control used in each direction, 7 shows the disturbance over the spacecraft, 8 presents the variation of the inertia matrix and 9 shows the estimation error in the adaptive controller; on all figures blue represents the non-adaptive and red the adaptive controller.

From the figures it is possible to see that both controllers perform well even in face of a very demanding perturbations and commands, with both controllers converging in less than 8 seconds. A slightly faster convergence rate can be seen for the non-adaptive controller; however, this can be considered negligible when compared with the total translation time, 12.5 days. The adaptive controller is more demanding in the controls. The error the inertia matrix is quickly corrected and it converges to zero in less than 4 seconds.

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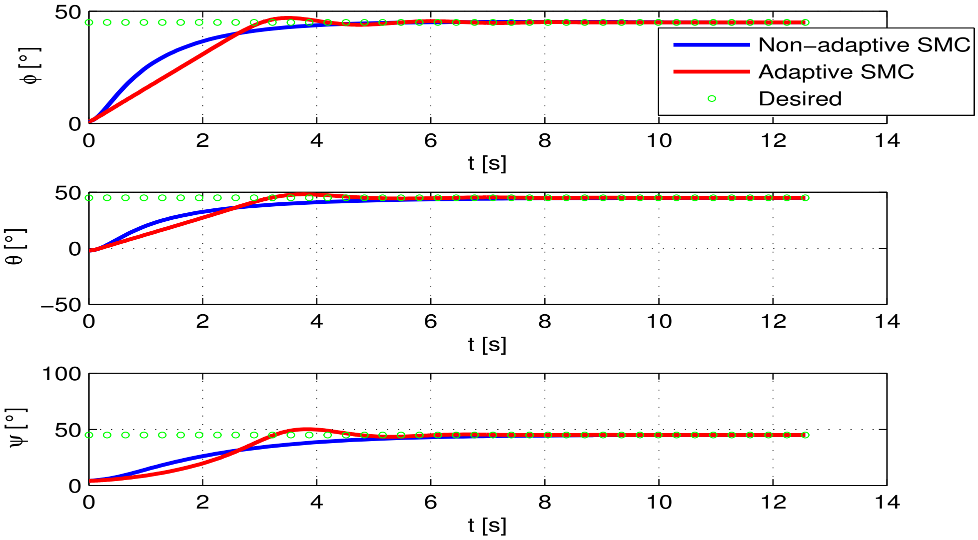

Figure 3. Attitude Euler angles evolution

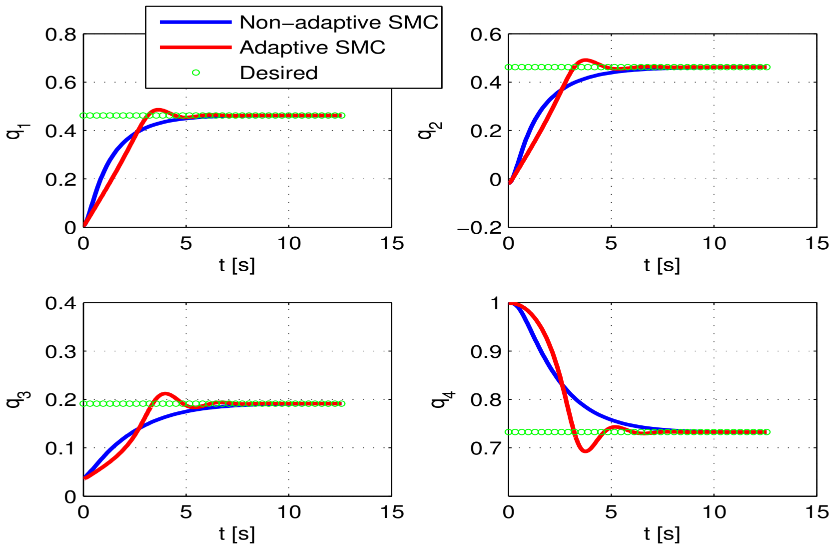

Figure 4. Attitude quaternion evolution

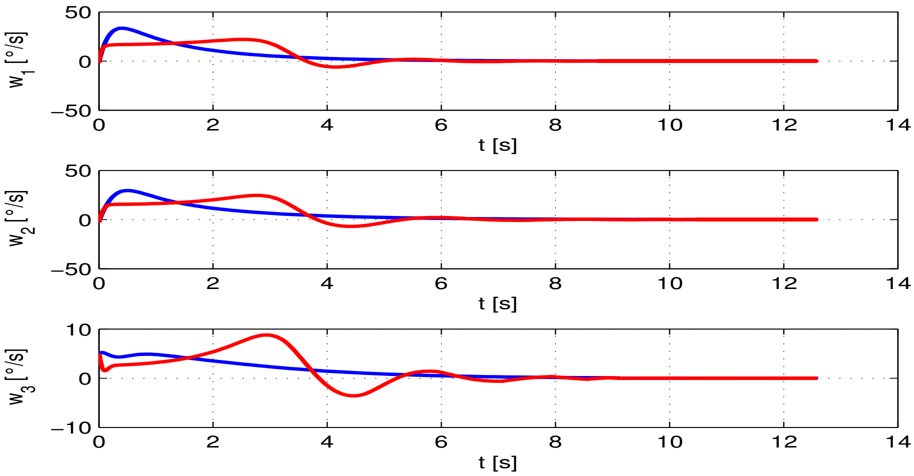

Figure 5. Attitude angular velocity evolution

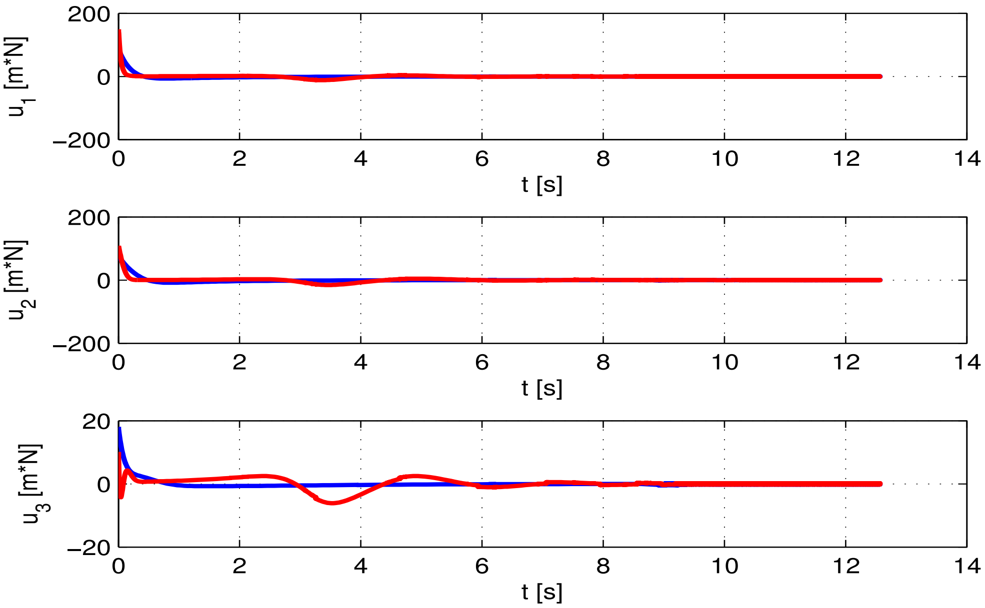

Figure 6. Thrusters torque profile

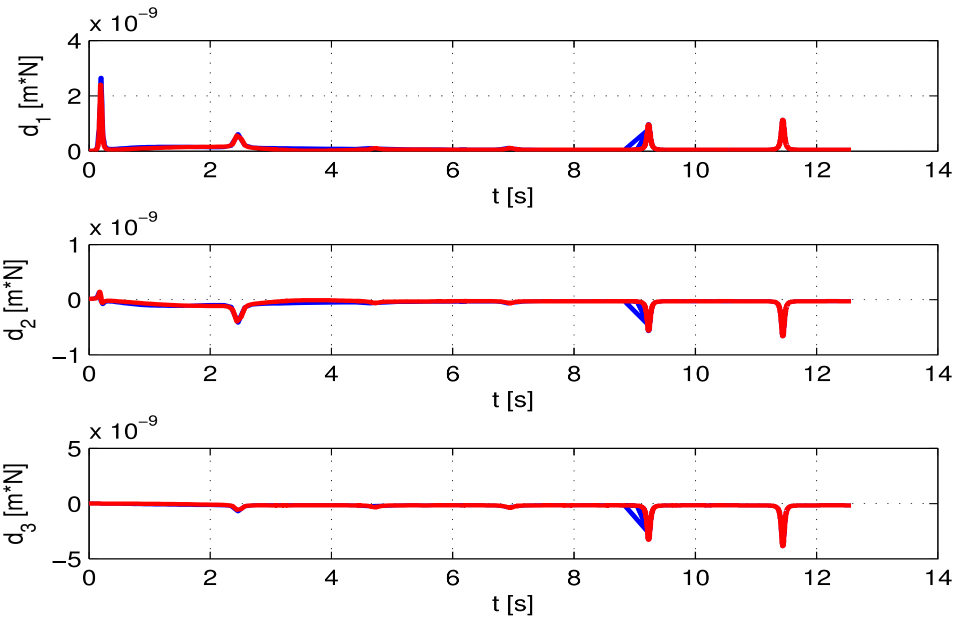

Figure 7. Orbit gravitational perturbation on the spacecraft

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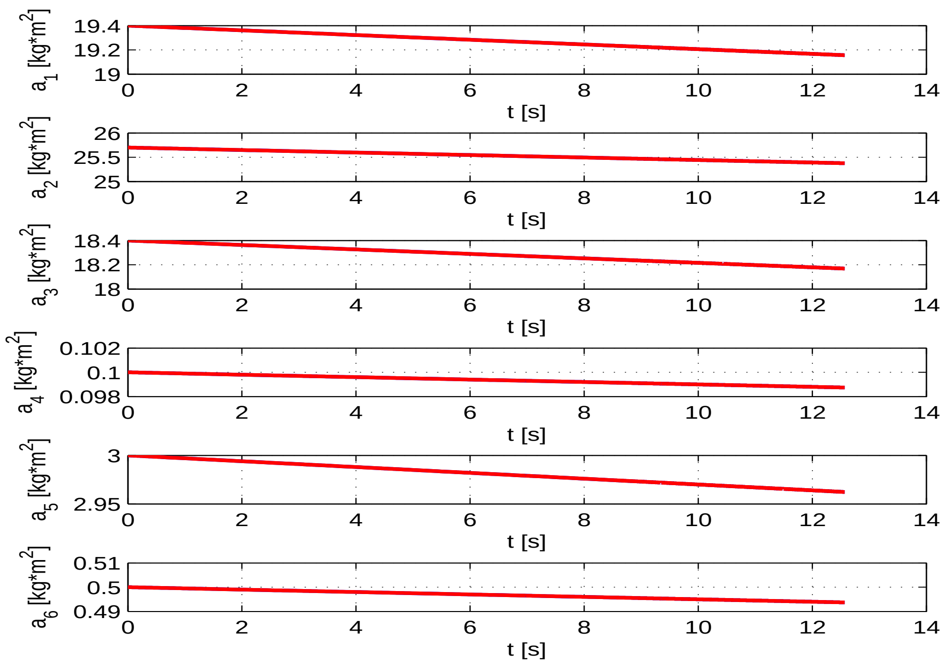

Figure 8. Inertia matrix variation

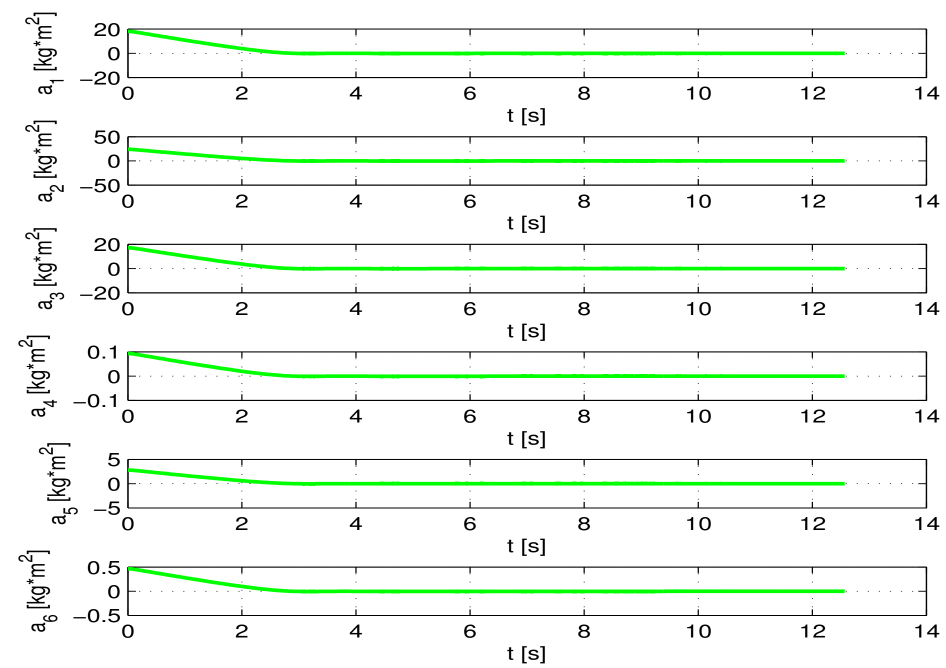

Figure 9. Inertia estimation error

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5. CONCLUSIONS

During this work an attitude controller for the ASTER mission was design based on sliding mode techniques, the need of a non-linear invariant controller comes from the nature of the problem, a spiral escape and heliocentric transfer with posterior navigation on a complex gravitational asteroid system, and uncertainties found in the assessment of the environment. The adaptability necessity comes from the lack of accurate inertia measurement during the course of the mission, since it makes use of a low-thrust propulsion system which continuously consumes fuel changing the spacecraft's inertia matrix.

The results show that this type of controller is, in fact, suitable for the mission's requirements both with non-adaptive and adaptive concepts. Nevertheless, the adaptive simulations presents a few advantages as it is more realistic because, as already explained, the inertia matrix cannot be considered constant over the whole trajectory. Compared to a linear controller for the same conditions the non-linear controllers developed in this work can operate in much more severe conditions and do not have convergence problems with high amplitude maneuvers, different from what was observed with the linear controller that at high amplitude maneuvers was degraded.

The robustness, adaptability, capacity to perform under demanding environment and at high amplitude maneuvers point out the importance of the use of a non-linear controller and particularly the sliding mode controller for this type of missions.

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7. RESPONSIBILITY NOTICE

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8. ACKNOWLEDGMENTS

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