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SYNTHESIS AND ANALYSIS OF TRANSLATING CAM-FOLLOWER MECHANISM

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Abstract. *Cam-follower mechanisms are extremely important in modern equipments, widely used in automotive engines and machinery that requires complex motion with repeatability and reliability. A cam is a solid body and its shape in conjunction of follower's type defines a movement that can be too difficult or even impossible to reproduce with others mechanisms. The objective of this project is the kinematic studying, which is divided into: synthesis and analysis for both flat faced and offset roller follower in translating motion. For a specified displacement function, it is possible to design the cam profile and this process is denominated synthesis. Analysis is the study of displacement, velocity and acceleration of the mechanism. Furthermore, radius of curvature, pressure angle and Hertz contact stress are also determined. Cycloidal, harmonic and eighth grade polynomial are well known functions applied to cam-follower pairs. However, the application of classical sixth order spline is an alternative to the cam design. Splines are curves made up of polynomial parts and the main advantage of this choice is the possibility to determine the value of some points – called knots – around the motion. On the other hand, their behavior outside the knots is not predictable, which can cause serious problems. In order to prevent the follower jumping as well as vibrations and impacts, the first and the second derivatives of the displacement function must be continuous during the entire interval. This continuity makes the third derivative - jerk - finite, which leads to smoother transitions of velocity and acceleration. Each kind of function has its own peculiarity. Therefore, analyzing the requirements of each project is important to make the best choice and meeting expectations for displacement, velocity and acceleration of the follower, cam size and contact stress. Moreover, the kinematic study is essential to introduce the dynamic of cam-follower system, which has gained more importance in recent years due to increase of machinery's speed.*

Keywords: *cam-follower, synthesis and analysis, spline curves, kinematics.*

1. INTRODUCTION

Mechanism is a combination of rigid bodies, which are connected to move in a dependent way with a defined relative movement. One or more mechanisms that transmit force and execute an able work form a machine. The cam-follower pair is part of the group of mechanisms and its application is more exponent in internal combustion engines, but it is also present in other machinery, such as mechanical computers, machine tools and instruments, especially those where high repeatability and reliability are required.

Figure 1. Different types of cams. (Rothbart, 2004)

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There is a quite long list of cam and follower types. The cam may translate, oscillate, rotate or be stationary and the follower may have translating, oscillating or indexing movement (Fig. 1). The subjects of study in the present paper are the flat-faced and the roller cam-follower, both in translating motion. The flat-faced follower is cheaper and well known as the one applied in internal combustion engines. On the other hand, the roller follower is the most popular design to accomplish a large speedy range, since it leads to lower contact stress.

According to Chen (1977), the first few studies on cam-followers date back to late of years 20. Before, cam design was based mainly on empirical knowledge and prototype tests. Graphical techniques and hand calculation used to be the only way to determine the cam profile. However, only after the late 60's, the computer has become a fundamental instrument to cams design (Norton, 2009).

Thus, with the computer progress, mathematical functions started to be used to describe the cam movement. The development of kinematics was based in finding the best function that would be applied in any design case. The first curves were geometric as arc segments and parabolic. The harmonic, cycloidal, trapezoidal and polynomial were elected as better options than the geometrics because of the less rate of force application. In recent years, others curves are being studied as good alternatives to design such as Fourier series function and the spline (Chavan and Joshi, 2011; Gordon and Norton, 2002; Mandal and Naskar; 2009).

The objective of this paper is to analyze some factors that influence the kinematics of cam mechanisms. Furthermore, the kinematics is the first step to introduce the cam dynamics, which has gained attention in the past few years because of machinery's speed increasing and fatigue problems.

2. METHODOLOGY

According to Mabie (1986), a cam may be designed in two different ways: synthesis and analysis of mechanism. The synthesis consists in designing the cam profile for a desired motion. Nevertheless, it may be too difficult to manufacture this contour. On the other hand, the cam profile can be previously decided and afterwards, displacement, velocity and acceleration of the follower are determined.

2.1 Cam curves

There are several mathematical functions used to define the motion of the follower and they are still in development and improvement. Some of the well known cam curves are cycloidal, harmonic, polynomial and also their variations. The main functions studied in this paper are represented in Figures 2, 3 and 4.

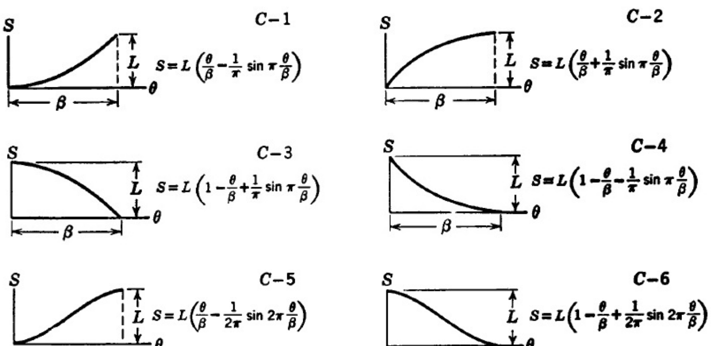


Figure 2: Cycloidal functions used in numerical simulations. (Mabie, 1986)

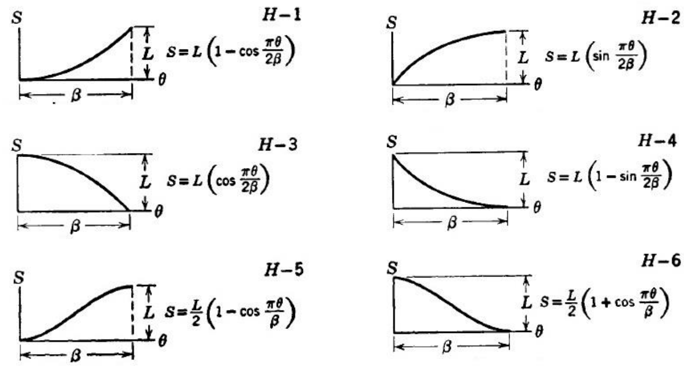


Figure 3: Harmonic functions used in numerical simulations. (Mabie, 1986)

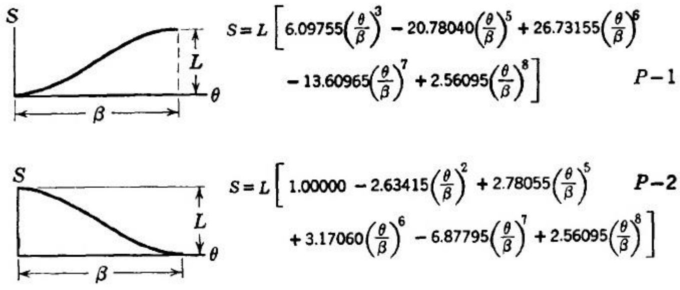


Figure 4: Eighth grade polynomial functions used in numerical simulations. (Mabie, 1986)

On the other hand, the application of spline is an alternative to cam design due to its versatility. There is a quite long list of spline's types. As displacement, velocity and acceleration need to be continuous during the entire interval to avoid losing contact between cam and follower, there are six boundary conditions. Thus, Norton (2009) suggests the classical sixth order spline.

Considering the interval *[a, b]* and that there is a knot in each extreme, the *k* knots are:

$$a = \theta_1 \le \theta_2 \le \cdots \theta_k = b \tag{1}$$

In addition, the corresponding displacements are:

$$s(a) = s_1, s_2 \cdots s_{k-1}, s_k = s(b) \tag{2}$$

According to Norton (2009), a classical spline of *m* order and *k* knots is a curve made up of polynomial pieces with degree *m-1* and its derivative is continuous until *m-2* order. For $\theta_j \le \theta \le \theta_{j+1}$ and $j = 1,2,\ldots,k-1$, the polynomials of the sixth order spline are:

$$A_j(\theta-\theta_j)^5 + B_j(\theta-\theta_j)^4 + C_j(\theta-\theta_j)^3 + D_j(\theta-\theta_j)^2 + E_j(\theta-\theta_j) + F_j \tag{3}$$

For this type of spline, the values of each interior knot, $a \ne \theta \ne b$, need to interpolate:

$$F_j = s_j \;\; para \;\; j = 2,3,\cdots,k-1 \tag{4}$$

Ensuring the continuity of the spline until the fourth derivative, let:

$$h_j = \theta_{j+1} - \theta_j \;\; para \;\; j = 1,2,\cdots,k-1 \tag{5}$$

From Eq. (3), (4) and (5):

$$A_j h_j^{\,5} + B_j h_j^{\,4} + C_j h_j^{\,3} + D_j h_j^{\,2} + E_j h_j + F_j = F_{j+1} \quad \text{(Displacement)} \tag{6}$$

$$5A_j h_j^{\,4} + 4B_j h_j^{\,3} + 3C_j h_j^{\,2} + 2D_j h_j + E_j = E_{j+1} \quad \text{(First derivative)} \tag{7}$$

$$10A_j h_j^{\,3} + 6B_j h_j^{\,2} + 3C_j h_j + D_j = D_{j+1} \quad \text{(Second derivative)} \tag{8}$$

$$10A_j h_j^{\,2} + 4B_j h_j + C_j = C_{j+1} \quad \text{(Third derivative or jerk)} \tag{9}$$

$$5A_j h_j + B_j = B_{j+1} \quad \text{(Fourth derivative or ping)} \tag{10}$$

Rearranging Eq. (6), (7) and (8), the coefficients A_j, B_j and C_j can be found out in function of D_j, E_j and F_j for *j=1, 2, …, k*:

$$A_j = \frac{1}{h_j^{\,4}}\left(-h_j D_j + h_j D_{j+1} - 3E_j - 3E_{j+1} + 6\frac{F_{j+1} - F_j}{h_j}\right) \tag{11}$$

$$B_j = \frac{1}{h_j^{\,3}}\left(3h_j D_j - 2h_j D_{j+1} + 8E_j + 7E_{j+1} - 15\frac{F_{j+1} - F_j}{h_j}\right) \tag{12}$$

$$C_j = \frac{1}{h_j^{\,2}}\left(-3h_j D_j + h_j D_{j+1} - 6E_j - 4E_{j+1} + 10\frac{F_{j+1} - F_j}{h_j}\right) \tag{13}$$

Substituting Eq. (11), (12) and (13) in Eq. (9) and (10):

$$-\frac{1}{2h_{j-1}}D_{j-1} + \frac{3}{2}\left(\frac{1}{h_{j-1}} + \frac{1}{h_j}\right)D_j - \frac{1}{2h_j}D_{j+1} - \frac{2}{h_{j-1}^{\,2}}E_{j-1} - 3\left(\frac{1}{h_{j-1}^{\,2}} - \frac{1}{h_j^{\,2}}\right)E_j + \frac{2}{h_j^{\,2}}E_{j+1} =$$
$$= 5\left[\left(\frac{F_{j-1}}{h_{j-1}^{\,3}} + \frac{F_{j+1}}{h_j^{\,3}}\right) - \left(\frac{F_j}{h_{j-1}^{\,3}} + \frac{F_j}{h_j^{\,3}}\right)\right] \tag{14}$$

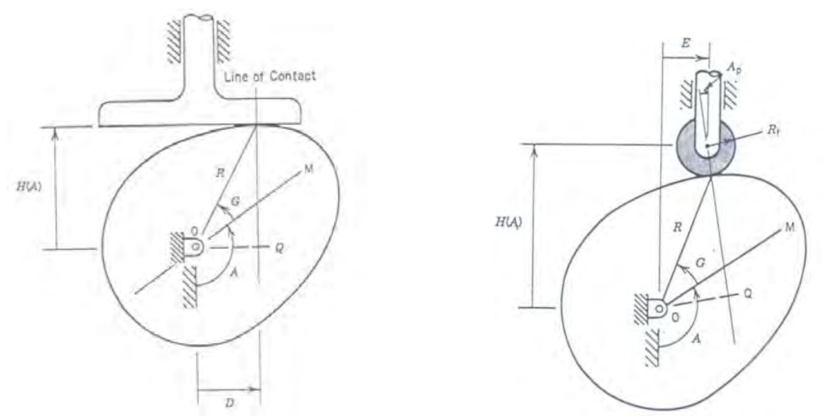
$$\frac{2}{h_{j-1}^{\,2}}D_{j-1} - 3\left(\frac{1}{h_{j-1}^{\,2}} + \frac{1}{h_j^{\,2}}\right)D_j - \frac{1}{h_j^{\,2}}D_{j+1} + \frac{7}{h_{j-1}^{\,3}}E_{j-1} + 8\left(\frac{1}{h_{j-1}^{\,3}} - \frac{1}{h_j^{\,3}}\right)E_j + \frac{7}{h_j^{\,3}}E_{j+1} = 15\left(\frac{F_{j+1}}{h_j^{\,4}} - \frac{F_{j-1}}{h_{j-1}^{\,4}}\right) \tag{15}$$

Thus, by solving Eq. (14) and (15), it is possible to determine the coefficients D_j, E_j and F_j. Afterwards, it is simple to evaluate A_j, B_j and C_j by Eq. (11), (12) and (13) respectively. Nevertheless, the values in the internal knots as well as the boundary conditions must be given:

$$F_1 = s(a); \qquad E_1 = s'(a); \qquad D_1 = s''(a) \tag{16}$$

$$F_k = s(b); \qquad E_k = s'(b); \qquad D_k = s''(b) \tag{17}$$

2.2 Displacement, velocity and acceleration of the follower



(A) (B)

Figure 5. Cam with translating follower (Doughty, 1988). (A) Flat-faced follower. (B) Roller follower.

H(A) denotes the position of the cam follower in function of its rotation angle *A*. For the flat-faced follower (Fig. 5 (A)), the initial displacement H_0 is the base circle radius R_0. Therefore, *H(A)* is a simple sum of R_0 and the displacement function *f(A)* chosen (Eq. (18)).

$$H(A) = H_0 + f(A) = R_0 + f(A) \tag{18}$$

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On the other hand, the position of the roller follower (Fig. 5 (B)) must consider the offset:

$$H(A) = H_0 + f(A) = \sqrt{R_{po}^2 - E^2} + f(A) \tag{19}$$

Where R_{po} is the prime circle radius and E, the offset.

Appling the chain rule for differentiation, the velocity $\dot{H}(A)$ and the acceleration $\ddot{H}(A)$ are respectively given by:

$$\dot{H}(A) = \frac{df(A)}{dA}\frac{dA}{dt} = f'(A)\cdot\dot{A} \tag{20}$$

$$\ddot{H}(A) = \ddot{A}\cdot f'(A) + \dot{A}^2\cdot f''(A) \tag{21}$$

Where the notation prime denotes the differentiation of a function with respect to its argument (A) and the dot indicates time derivative.

2.3 Cam profile

In the past, the graphical techniques were the only way to design cam profile. With the development of computers, new design process makes headway. Giving the polar coordinates (R, G) of the contact point, it is simple to specify the cam profile. After some geometric considerations, the polar coordinates of translating cam with flat-faced follower are (Doughty, 1988):

$$R = \sqrt{f'^2 + (R + f)^2} \tag{22}$$

$$G = -A + \arctan_2\{f'(A), -[R_0 + f(A)]\} \tag{23}$$

And the polar coordinates of cam with roller radius are:

$$R = \sqrt{\left[H(A) - R_f\cos A_p\right]^2 + \left(E + R_f\sin A_p\right)^2} \tag{24}$$

$$G = -\pi - A + \arctan\left[\frac{E + R_f\sin A_p}{H(A) - R_f\cos A_p}\right] \tag{25}$$

Where R_f is the follower radius and A_p is the angle of pressure given by:

$$A_p = \arctan\left[\frac{f'(A) - E}{\sqrt{R_{po}^2 - E^2} + f(A)}\right] \tag{26}$$

The pressure angle is an essential parameter for cam design, also for size determination. It must not exceed 30º for translating cam-follower pairs, since greater values can increase excessive side load and the follower sliding.

2.4 Some important parameters of cam determination

Any curve has an instantaneous radius of curvature, which is a mathematical property of a function according to Norton (2008). If the radius of curvature is zero or infinite, there is a cusp or corner and the surface is not smooth enough, leading to stress critical condition. Translating cam with flat-faced and roller followers have the respectively radius of curvature (Doughty, 1988):

$$P = R_0 + f(A) + f''(A) \tag{27}$$

$$P = \frac{[H^2 + (f' - E^2)]^{3/2}}{H^2 - Hf'' + (f' - E)(2f' - E)} - R_f \tag{28}$$

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The cam usually works under elastohydrodynamic lubrication, mainly around the nose (pointed region) where the lubricant entrainment is small. However, if considering dry contact, the maximum normal stress (Hertz contact stress) is:

$$\sigma_0 = \sqrt{\frac{F \cdot E_1 \cdot E_2 \cdot (P_1 + P_2)}{\pi \cdot t \cdot P_1 \cdot P_2 \cdot (E_1 + E_2)}} \tag{29}$$

Where F is the force between the bodies, E_1 and E_2 are the Young's modulii for both cam and follower materials, t is the thickness of the cam. If the follower is flat, the contact surface stress simplifies to:

$$\sigma_0 = \sqrt{\frac{F \cdot E_1 \cdot E_2}{\pi \cdot t \cdot P(E_1 + E_2)}} \tag{30}$$

3. RESULTS AND GENERAL COMMENTS

The input parameters considered in the numerical simulations are presented in Tab. 1.

Table 1. Input cam-follower parameter considered in the simulations.

Total Lift (L)			20 mm
Profile thickness (t)			10 mm
Young's modulus (E_1) - cam			200 Mpa
Young's modulus (E_2) - follower			200 MPa
Force (F)			3000 N
Rotation (ω)			180 rpm
Flat-faced		Roller	
Base circle radius (R_0)	60 mm	Prime circle radius (R_{po})	60 mm
		Offset (E)	10 mm
		Roller radius (R_f)	15 mm

Every cam function has its own peculiarity and during years, there was a discussion about the optimized curve. However, there is no general agreement in scientific community yet. The cycloidal curve (Fig. 6), for example, presents null second derivatives at the boundary limits, which makes the third derivative – called jerk – finite in motions with double-dwell, single-dwell and no dwell. The fact that the jerk is finite across the entire interval is essential in cam design, because it provides smooth transitions of velocity and acceleration since there are no discontinuities. Although the advantage of cycloidal curves regarding variety of combinations, cycloidal curves present undesirable high peaks of accelerations and then, high contact forces and, consequently, high contact surface stresses.

The harmonic presents lower peaks of acceleration than cycloidal and eighth grade polynomial, because its second derivatives are not null at the boundaries, this function is indicated for cases when the acceleration at the boundary is equaled with the other boundary's acceleration. Moreover, these curves can be used without dwell too. If the motion is double-dwell, the jerk may be infinite in some points and consequently, there will be discontinuities on the acceleration (see Fig. 7).

The polynomial curves present the intermediary peaks of acceleration compared with the previous ones (Fig. 8). The rise function P-1 has null second derivative at the beginning, but it is different from zero at the end. The fall function P-2 is the opposite: null at the end and nonzero at the beginning. Thus, they are recommended in motions with rise-fall-dwell or just rise-fall.

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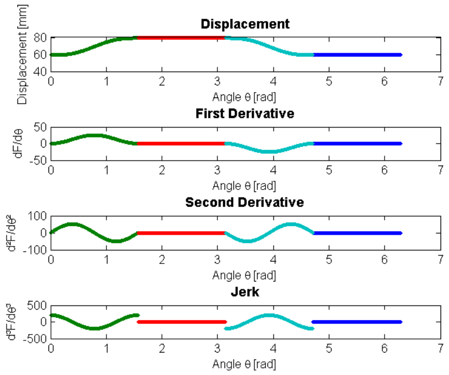

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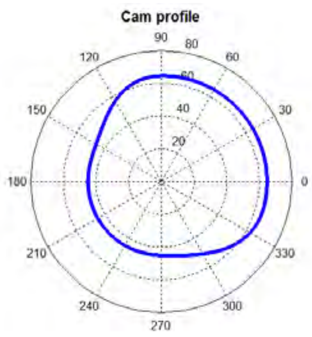

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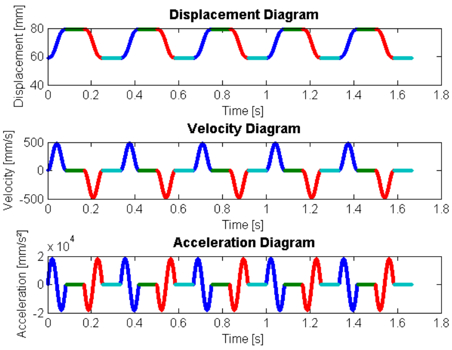
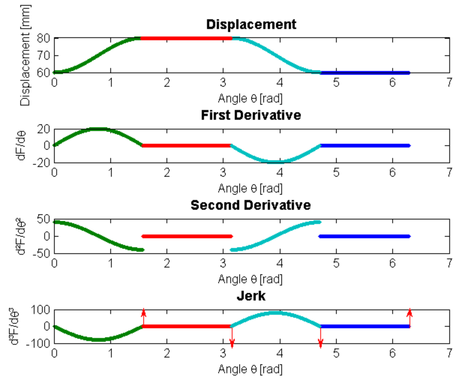
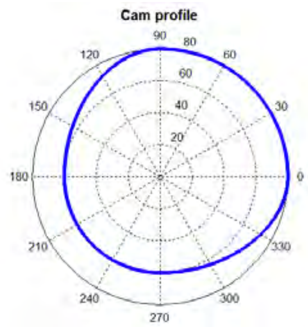


(C)

Figure 6. Double-dwell motion with curves C-5 and C-6 for roller follower. (A) Derivatives. (B) Cam profile. (C) Kinematics diagram.



(A)



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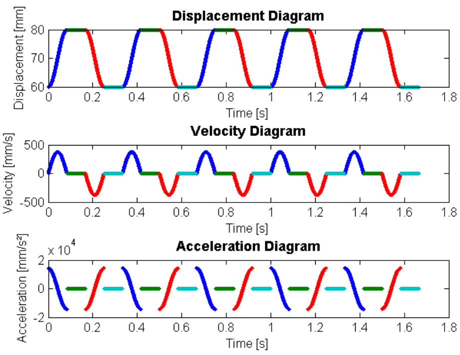

(C)

Figure 7. Double-dwell motion with curves H-5 and H-6 for flat-faced follower. (A) Derivatives. (B) Cam profile. (C) Kinematics diagram.

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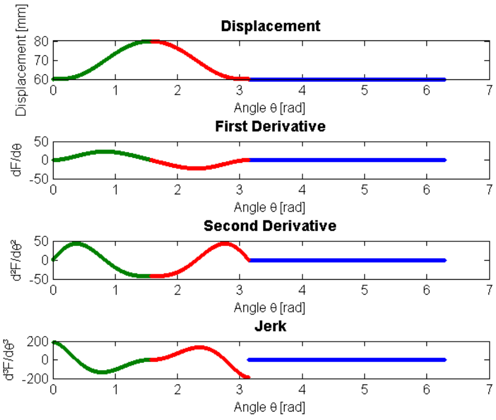
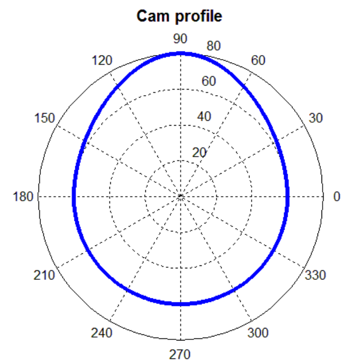


(A)



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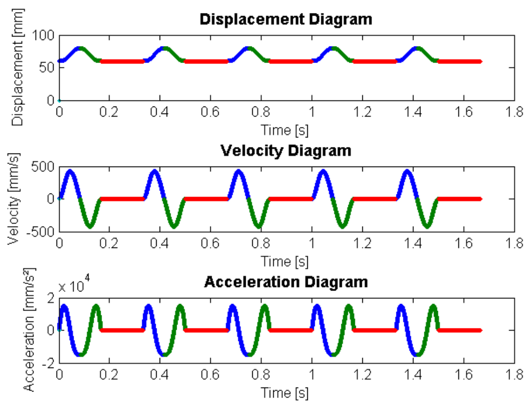

(C)

Figure 8. Double-dwell motion with curves P-1 and P-2 for flat-faced follower. (A) Derivatives. (B) Cam profile. (C) Kinematics diagram.

The radius of curvature is one of the main factors considered when sizing a cam. The profile must have a tangent continuous and smooth with no sharp corners, which could create a stress concentration area. For the curves in the study, it was noticed that there is only discontinuity in curvature radius when it occurs in the first or second derivative, i.e., when the jerk is infinite (Fig. 9 (A)).

Pressure angle is another relevant factor in sizing cams with roller follower. It is defined as the angle between the direction of follower's velocity and the axis of transmission. Therefore, when this angle is equal to zero, the total force is transmitted to the follower. When it is equal to 90˚, the force leads to maximize the slip speed, which is not desired. According Uicker (2003), if the pressure angle is too high, the follower can be stuck in the trajectory. Making it too small results in the increasing of the size of the cam, having influence in the stability at high speeds. Therefore, it is used to attempt the pressure angle between 0˚and 30˚to translating cam-followers.

Figure 9 gives a better understanding of the influence of the radius of curvature in the contact stress. The larger the radius of curvature, the lower the Hertz stress, since there is more surface area to distribute the contact force. When the radius of curvature is constant, the stress does not change. Therefore, it is important to maintain the continuity of the first and second derivatives of the displacement function and, consequently, of the radius of curvature to avoid vibration problems and loss of contact.

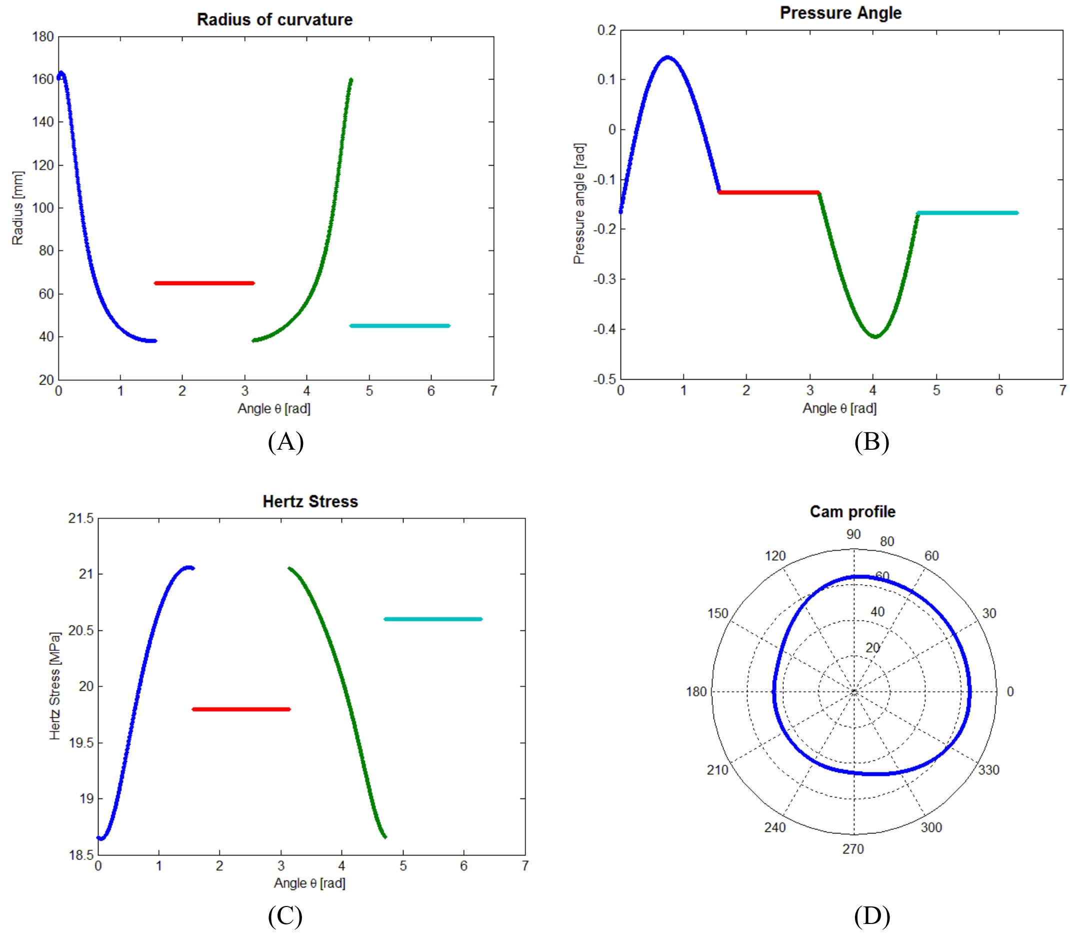

Figure 9. Roller cam-follower mechanism with rise-dwell-fall-dwell motion. H1/H2 and H3/H4 curves. (A) Radius of curvature. (B) Pressure angle. (C) Hertz contact stress. (D) Cam profile.

The introduction of splines as an option for the curves used in cam-follower pair was mainly due to the possibility of expanding the choice of the displacement function. Position, velocity and acceleration at the boundaries must be given as an input. The routine also allows choosing between the following options that should be given to design the curves:

• three equidistant points of displacement as intermediate knots;
• two points of displacement and speed as intermediate knots;
• one displacement, one speed and one acceleration as intermediate knots.

Thus, displacements that are not possible with the other functions may be design with splines. The use of the sixth order leads the fourth derivative (ping) continuous, handily ensuring the necessary condition of the third derivative (jerk) be finite to avoid discontinuities in the velocity and acceleration of the follower. Moreover, the use of splines allows selecting the velocity and acceleration at some point to design the curve, which can be interesting for cases where a given moment is the most important to be specified in a project. (Fig. 10)

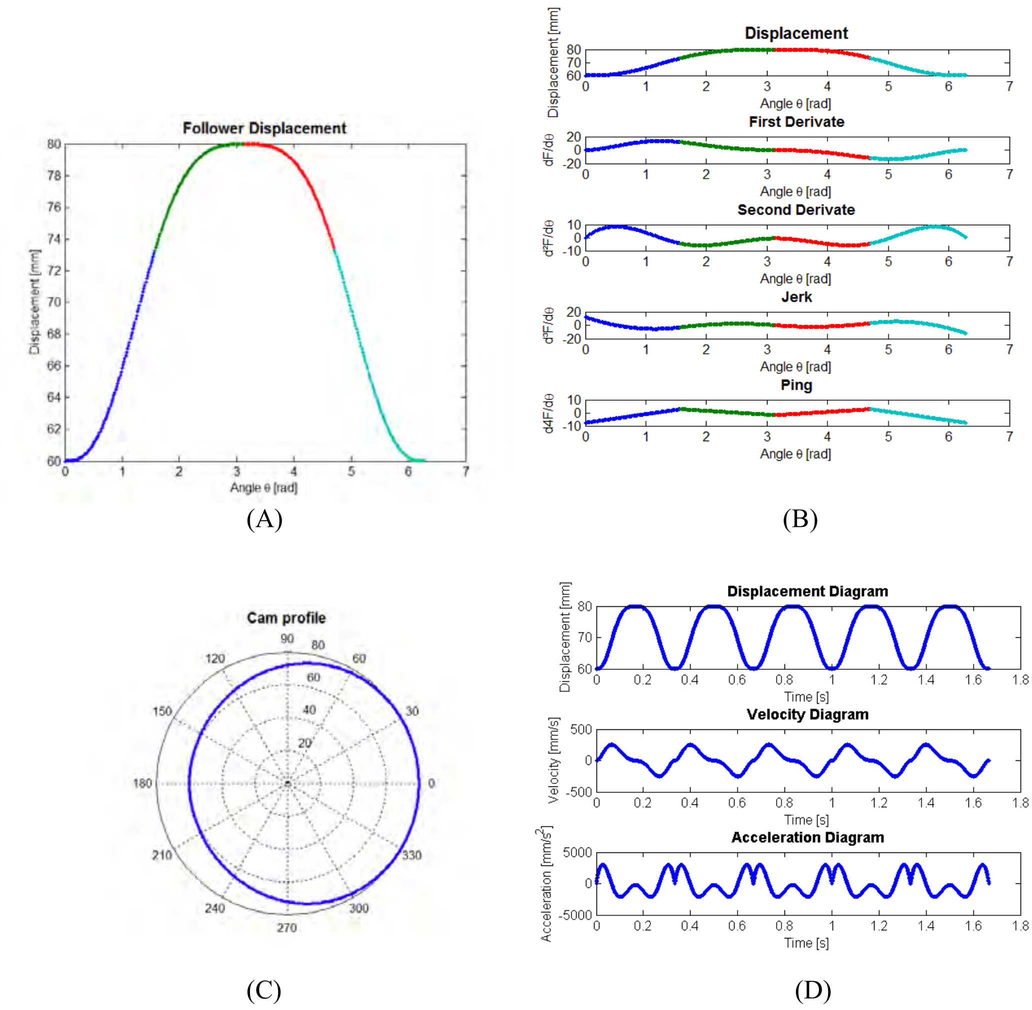

Figure 10. Flat-faced cam-follower motion applying splines. Boundary knots: position and velocity equal zero, acceleration 200 mm/s². At $\theta = \pi$, the displacement is 20 mm, the velocity is zero and the acceleration is -200 mm/s². (A) Displacement function *H(θ).* (B) Derivatives. (C) Cam profile. (D) Kinematics.

In contrast the range of splines' application, these are very unstable and the behavior between the knots can not be predictable. Keeping all the boundary conditions used in the Figure 10 except for the velocity at average knot which was changed to 100 mm/s, the displacement function considerably changes (Fig. 11) and even loses the symmetry.

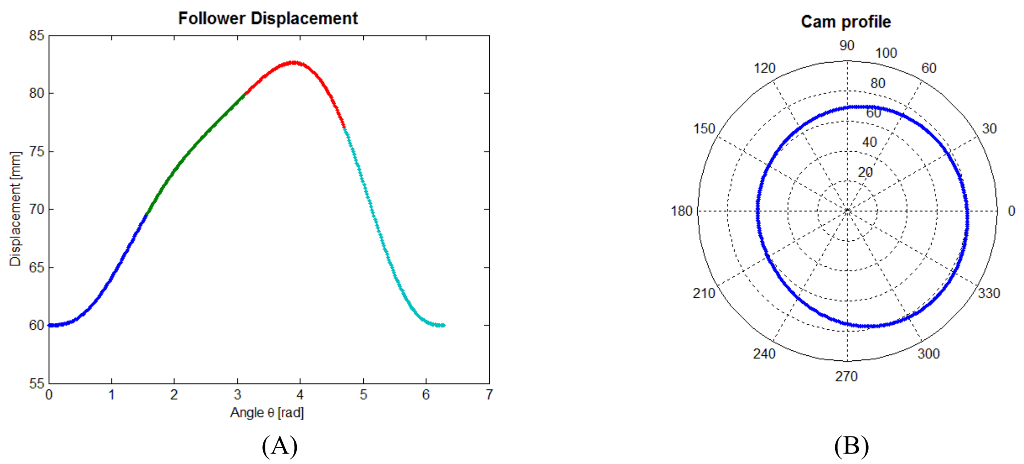

Figure 11. Input datas equal to Fig. 10, except for the velocity at the middle knot. (A) Displacement function. (B) Flat-faced cam-follower profile.

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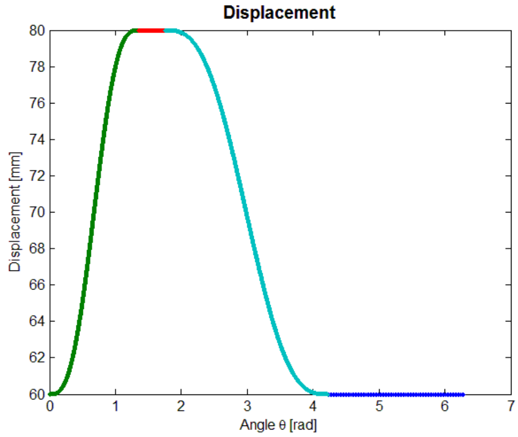
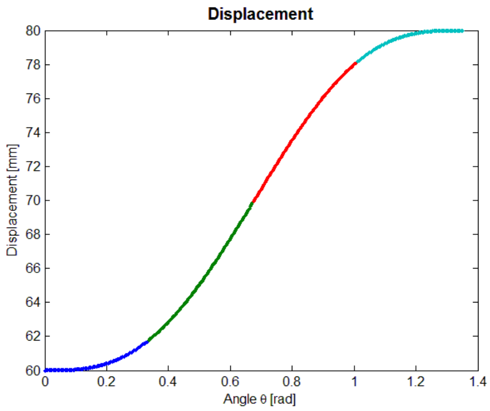
When dwells are required, the sixth order spline will not predict the linear behavior between the knots. In these cases, an alternative is to use others mathematical functions. Another option is separate the rise and fall curves and use one spline for each and after, combine them (Fig. 12).



(A)



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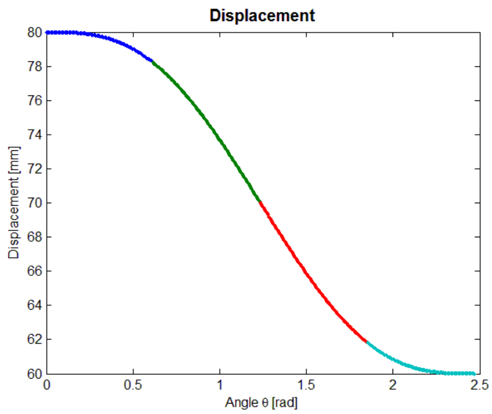

(C)

Figure 12. (A) Displcamente function with double-dwell using C1/C2 and C3/C4. (B) Rise curve similar to rise motion of (A) applying splines. (C) Fall curve similar to fall motion of (A) applying splines.

4. CONCLUSIONS

The design of a mechanical system involves many variables to consider. In the case of kinematic cam-follower pair, it is desired a specified displacement and, simultaneously, minimize factors such as vibration and undesirable stress caused by this movement. Thus, it is important to select properly the cam function, since each curve has its own peculiarities.

The application of spline functions is an alternative to others classic cam curves (cycloidal, harmonic and eighth order polynomial). The type of spline used in this project is the sixth order polynomial suggested in the literature. The main advantage of the spline is the possibility to select the values in some points – called knots. In contrast, it is not possible to predict their behavior outside the knots, which can cause project difficulties.

For a smooth velocity and acceleration transition as possible, the third derivative of the function (jerk) must to be finite across the entire rotation. Such condition makes the first and second derivatives continuous and therefore, there is no abrupt change or discontinuity in the velocity and acceleration of the follower. Another way to verify is the smoothness of the motion is analyzing the continuity of the radius of curvature, since it is dependent of the input data and derivatives.

Furthermore, the radius of curvature proved great importance in sizing the mechanism and in the contact stresses between the cam and the follower. The pressure angle must also be studied because if it is too small leads to larger dimensions and if it is too large, it may cause vibrations.

Hence, these are some factors that tangent the kinematics study of translating mechanism with flat-faced and roller cam-follower as well as the dynamic of those system when considered in the mechanism design.

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