ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
November 3-7, 2013, Ribeirão Preto, SP, Brazil

DESIGN OF RADIANT ENCLOSURES USING TOPOLOGY OPTIMIZATION

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Abstract. *Structures subjected to high absolute temperatures or to natural convection, as well structures that exchange heat in the absence of a physical medium have significant heat transfer through thermal radiation. This phenomenon is important for several applications and processes, such as in the operation of solar collectors, satellites, industrial furnaces, combustion engines and nuclear plants. The present work shows the application of topology optimization to the design of structures that exchange heat substantially by thermal radiation within an enclosure. However, the design of such radiant enclosures is not trivial and it is necessary to use robust and systematic design tools, such as optimization techniques. Topology optimization is a numerical method which allows finding the layout, or topology, of a structure such that a prescribed objective is maximized or minimized subjected to design constraints. The optimization algorithm, based on the Method of Moving Asymptotes (MMA), and the Finite Element Method for analysis of the phenomenon of radiation in enclosures, are implemented using Matlab. The developed formulation is verified through analytical models of the problem. As an example, it is presented the optimized distribution of reflective material on the surface of structures that build the interior of a furnace, subjected to prescribed boundary conditions of temperature and heat flux, in order to maximize the irradiation in a specific region of the design domain.*

Keywords: *Radiative heat transfer, Topology optimization, View factors, Finite element analysis*

1. INTRODUCTION

Radiant enclosures are widely presented in industrial applications such as blast furnaces, satellites and spacecrafts, and the theory that describes this phenomenon has been consistently discussed in the literature (Sparrow and Cess, 1978; Siegel and Howell, 2002). The current demands of quality and efficiency in the design of these structures requires a better tool than the tradicional "trial-and-error" method. Under that perspective, topology optimization represents a robust and reliable option to accelerate the interior design of enclosures, once it allows a systemized distribution of reflective material over that design domain, such that a prescribed objective function is maximized or minimized, subjected to design constraints. However, the bibliography that reports the application of topology optimization to problems of radiation is scarce. Bruns (2007) wrote short and specific observations regarding a possible treatment of radiation as a linearized convective boundary condition in the design of micro-cooling fins. Despite the lack of studies that show the application of topology optimization to the design of radiant enclosures, an extensively discussion of other optimization methods is encountered in literature.

Tortorelli *et al.* (1989) used Lagrange multipliers and convolution theory in order to formulate sensitivities for a functional that characterizes the performance of transient and nonlinear thermal systems, which includes heat fluxes originated from radiant exchange. Fedorov *et al.* (1998) proposed a gradient based method to design the heater settings of a material processing furnace. In (Daun *et al.*, 2003b), a gradient based optimization technique is presented for the geometry determination of an enclosure with diffuse walls and in (Daun *et al.*, 2003a) an application result of this technique is compared with results obtained by using inverse method. The design of time-dependent temperature ovens, such as the ovens used in the automotive industry for paint curing, are addressed in (Ashrafizadeh *et al.*, 2012) as a dynamic optimization problem, due to motion of the load and the variation of the radiation exchange factors during the process. Recently, Rauch (2011) discussed the solar radiation inclusion in sensitivity analysis for conjugating heat transfer problems with the purpose of calculation of the uncertainties parameters in the model.

The methodology presented in this work makes possible the investigation of the influence of a variation in the reflective properties from the interior enclosure structures in order to maximize the irradiation heat flux in a specific area of that inner domain, which is subjected to boundary conditions of prescribed temperatures and heat fluxes. This is done trough an oriented distribution of reflective material and represents a modern demand, constantly found in the layout of various types of furnaces, for instance.

This paper is organized as follows: in section 2, the analytical governing equations of the radiative heat transfer within an enclosure is presented. In section 3, the numerical formulation of the problem using the finite element method is described. In section 4, the topology optimization formulation, with the definition of the objective function and its derivatives, is provided. In section 5, a numerical example demonstrating the influence of the distribution of reflective material in the maximization of the irradiation over an specific area of the domain is shown. Finally, in section 6, some conclusions are inferred.

2. GOVERNING EQUATIONS

The radiative heat transfer is modeled trough a transparent media in this work. It is admitted that the superficies of the enclosures are isothermal and the radiosities are uniform over them. It is also assumed that theses structures are opaque, diffuse and grey.

An accurate determination of the radiation within enclosures depends on the precise calculation of the view factors, as noted by Feingold (1966). Therefore, the percentage of radiation that leaves the surface i and reaches the surface j is determined trough the contour double integral formula (Sparrow, 1963):

$$F_{ij} = \frac{1}{2\pi A_i} \oint_{C_i} \oint_{C_j} \left[\ln(r_{ij})\, dx_i dx_j + \ln(r_{ij})\, dy_i dy_j + \ln(r_{ij})\, dz_i dz_j \right] \tag{1}$$

where C_i and C_j are contours that bound areas A_i and A_j, r_{ij} is the distance between two line elements on each contour and dx, dy and dz correspond to the differential length of that line elements.

The radiosity over each surface of the enclosure needs to be defined, in order to make possible the determination of the irradiosities that are used as objective function. Thus, for surfaces with prescribed temperature:

$$J_i = \varepsilon_i \sigma T_i^4 + (1 - \varepsilon_i) \sum_{j=1}^{N} F_{ij} J_j \tag{2}$$

where T_i is the known temperature, ε_i is the emissivity and N is the total number of surfaces that build the enclosure. For surfaces that have prescribed heat flux, it is necessary to calculate the radiosities as follow:

$$J_i = \frac{Q_i}{A_i} + G_i = \frac{Q_i}{A_i} + \sum_{j=1}^{N} F_{ij} J_j \tag{3}$$

where Q_i is the known heat rate of surface i. By the understanding of Eq. (3), it is possible to define the values of the irradiosities as:

$$G_i = \sum_{j=1}^{N} F_{ij} J_j \tag{4}$$

Through the net-radiation method (Siegel and Howell, 2002) the unknown temperatures and heat fluxes over the surfaces are organized in a system of N equations, which is represented by:

$$\sum_{j=1}^{N} \left[\frac{\delta_{ij}}{\varepsilon_j} - F_{ij} \left(\frac{1 - \varepsilon_j}{\varepsilon_j} \right) \right] \frac{Q_j}{A_j} = \sum_{j=1}^{N} (\delta_{ij} - F_{ij}) \sigma T_j^4 = \sum_{j=1}^{N} F_{ij} \sigma \left(T_i^4 - {T_j}^4 \right) \tag{5}$$

where δ_{ij} is the Kronecker Delta and σ the Stefan-Boltzmann constant.

3. FINITE ELEMENT METHOD FOR RADIANT ENCLOSURES

In this work, the view factors are determined numerically by a Gauss quadrature over the line contours that bound the finite elements (Checchi *et al.*, 1991) as:

$$F_{ij} = \frac{1}{2\pi A_i} \sum_{el_i=1}^{N_i} \sum_{p_i=1}^{N_{pg}} W_{p_i} J_{el_i} \sum_{el_j=1}^{N_j} \sum_{p_j=1}^{N_{pg}} W_{p_j} J_{el_j} \ln \left[r_{p_i p_j}(z_{p_i}, z_{p_j}) \right] \hat{r}_i . \hat{r}_j \tag{6}$$

where N_i and N_j represent the amount of contours, W_{p_i} and W_{p_j} are the weights of the integration, J_{el_i} and J_{el_j} are the Jacobians calculated in the elements el_i and el_j, $r_{p_i p_j}$ refers to the distance between the coordinates of the Gauss points p_i and p_j and $\hat{r}_i . \hat{r}_j$ is the scalar product between the versors that guide the parameterization of the contours C_i and C_j.

The finite element method for the calculation of the radiosities is defined similarly as done by Cohen and Greenberg (1985). Thus:

$$\mathbf{K_R J} = \mathbf{E} \tag{7}$$

ISSN 2176-5480

22nd International Congress of Mechanical Engineering (COBEM 2013)
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where $\mathbf{K_R}$ is the stiffness matriz, $\mathbf{J}$ the radiosities vector and $\mathbf{E}$ the emissions vector. The essential difference between this work and (Cohen and Greenberg, 1985) consists in the approach used to assemble $\mathbf{K_R}$ and $\mathbf{E}$ wich is divided in two types, as in section 2. Thus, for elements with prescribed temperature:

$$\begin{cases} K_{R_{ij}} = \dfrac{\delta_{ij} - (1-\varepsilon_j)\, F_{ij}}{\varepsilon_j} & (8) \\ E_i = \sigma {T_i}^4 & (9) \end{cases}$$

and for elements with prescribed heat flux:

$$\begin{cases} K_{R_{ij}} = \delta_{ij} - F_{ij} & (10) \\ E_i = q_i & (11) \end{cases}$$

where the view factor value F_{ij} is organized into a matrix $\mathbf{F}$, so that the sum of its lines is approximately one. After the determination of the radiosities it is possible to calculate the irradiosities as suggested by Eq. (4):

$$\mathbf{G} = \mathbf{FJ} \quad (12)$$

For the determination of the unknown heat fluxes and temperatures, the finite element method can be defined as (Reddy and Gartling, 2010):

$$\{(\mathbf{I} - \mathbf{F})\,[\mathbf{I} - (\mathbf{I} - diag\,(\varepsilon))\,\mathbf{F}]^{-1}\, diag\,(\varepsilon)\,\sigma\}\overline{\mathbf{T}} = \mathbf{q} \quad (13)$$

where $\mathbf{I}$ is an identity matrix, $diag\,(\varepsilon)$ is a diagonal matrix for the emissivities of the inner structures, $\mathbf{q}$ is a vector for the heat fluxes and the temperature vector $\overline{\mathbf{T}}$ is given as:

$$\overline{\mathbf{T}} = \begin{bmatrix} {T_1}^4 \\ {T_2}^4 \\ \vdots \\ {T_N}^4 \end{bmatrix} \quad (14)$$

The approach used to solve the system of linear equations represented by Eq. (13) begins with the labeling of the known variables as showed below:

$$\begin{cases} \theta : set\ of\ elements\ with\ prescribed\ temperatures \\ h : set\ of\ elements\ with\ prescribed\ heat\ fluxes \end{cases} \quad (15)$$

In other words, the finite elements that are referenced by θ have unknown heat fluxes and the elements referenced by h have unknown temperatures. The correct allocation of these sets in the stiffness matrix and in the vector of heat fluxes allows the calculation of the unknown temperatures. Therefore:

$$\overline{\mathbf{T}}(h) = \{\mathbf{K}(h,h)^{-1}[\mathbf{q}(h) - \mathbf{K}(h,\theta)\overline{\mathbf{T}}(\theta)]\} \quad (16)$$

where the unknown temperatures vector $\mathbf{T}(h)$, calculated over a set of elements with unknown heat fluxes of dimension N_h, is given as:

$$\mathbf{T}(h) = \begin{bmatrix} \overline{T}(h_1)^{\frac{1}{4}} \\ \overline{T}(h_2)^{\frac{1}{4}} \\ \vdots \\ \overline{T}(h_{N_h})^{\frac{1}{4}} \end{bmatrix} \quad (17)$$

Only after obtaining the unknown temperatures $\mathbf{T}(h)$ it is possible to calculate the unknown heat fluxes. Thus:

$$\mathbf{q}(\theta) = \mathbf{K}(\theta,:)\overline{\mathbf{T}} \quad (18)$$

where the colon means that all columns are considered.

4. TOPOLOGY OPTIMIZATION

The topology optimization, in the case addressed in this work, consists of distributing material over structures that build the enclosure to maximize a desired objective function. The steps of a conceptual topology optimization of two structures inside a enclosure are shown in Fig. 1.

Boundary conditions are applied to the design domain, which is discretized into finite elements. Iteratively, the physical behavior evaluation of the superficies is carried out and, through an optimization algorithm, the best distribution of reflective material that meets the thermal design requirements is searched. In the next step, the post-processing is performed and then the results are once more verified and post-processed.

ISSN 2176-5480

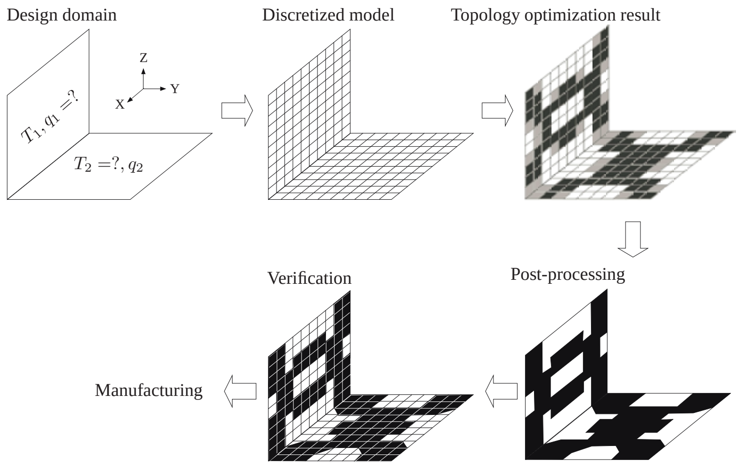

Figure 1. Steps of a inner radiative enclosure design by using topology optimization.

4.1 Material Model

The material model is a mathematical resource that allows the relaxation of the material distribution problem inside the design domain, so that it is possible to work with a discrete problem in a continuous form (Bendsøe and Kikuchi, 1988). Thus, instead of working with design variables that have discrete values (0 and 1), the relaxation allows working with intermediate values of these variables. In this article, the topology optimization formulation makes use of a material model, based on SIMP (Solid Isotropic Material with Penalization) (Bendsøe and Sigmund, 2003), that interpolates the reflective material values ρ_i over the elements as:

$$\rho_i = {\gamma_i}^p \rho_m + \left(1 - {\gamma_i}^p\right) \rho_0 \tag{19}$$

where ρ_m is the reflectivity of the material that the designer wishes to distribute, ρ_0 is the original value of the structure reflectivity and p is penalty factor used to minimize undesired intermediate values, problem known as grey scale (Bendsøe and Sigmund, 2003). The design variable γ_i describes the amount of material at each location of the domain and can assume values between 0 and 1.

5. NUMERICAL IMPLEMENTATION

This work aims to maximize the irradiosity heat flux over a specific location of the design domain. So, the discrete optimization problem can be stated as:

$$\begin{aligned} &\underset{\gamma}{Maximize}: \quad G \\ &subjected\,to: equilibrium\,equation \\ &\qquad\qquad\quad 0 \leq \boldsymbol{\gamma_i} \leq 1, \; i = 1, \ldots, N \\ &\qquad\qquad\quad V(\boldsymbol{\gamma}) = \sum_{i=1}^{N} \boldsymbol{\gamma_i} d\Omega \leq V_s \end{aligned} \tag{20}$$

where the irradiosities are numerically calculated as in Eq. (12), through the substitution of the radiosities calculated by Eq. (7) and the view factors determined by Eq. (6). The optimization problem is solved by the MMA (Method of Moving Asymptotes) (Svanberg, 1987) solver.

5.1 Sensitivities

The sensitivities of the objective function in relation to the design variables is a necessary information to be provided to the MMA solver. In this sense, these derivatives can be calculated as follow:

$$\frac{\partial \mathbf{G}}{\partial \gamma_i} = \mathbf{F} \frac{\partial \mathbf{J}}{\partial \gamma_i} \tag{21}$$

once the enclosure geometry is considered fixed. The derivatives of the radiosities used in the Eq. (21) can be stated as:

$$\frac{\partial \mathbf{J}}{\partial \gamma_i} = \mathbf{K_R}^{-1} \left(\frac{\partial \mathbf{E}}{\partial \gamma_i} - \frac{\partial \mathbf{K_R}}{\partial \gamma_i} \mathbf{J} \right) \tag{22}$$

ISSN 2176-5480

Thus, after the replacement of Eq. (22) in Eq. (21) and after the application of the Adjoint Method, one can write the irradiation derivatives as:

$$\frac{\partial \mathbf{G}}{\partial \gamma_i} = \boldsymbol{\lambda_R} \left(\frac{\partial \mathbf{E}}{\partial \gamma_i} - \frac{\partial \mathbf{K_R}}{\partial \gamma_i} \mathbf{J} \right) \tag{23}$$

where the column vector $\boldsymbol{\lambda_R}$ is given by:

$$\boldsymbol{\lambda_R} = (\mathbf{K_R}^T)^{-1} (\mathbf{f}_D \mathbf{F})^T \tag{24}$$

The line vector $\mathbf{f}_D$ is of dimension $1 \times N$, with one in the elements positions that the designer wants to maximize the irradiosity and zero in the other positions.

Since emission derivatives for elements with prescribed heat flux are equal to zero, it is required to calculate these sensitivities only for elements with prescribed temperature. Thus:

$$\frac{\partial \mathbf{E}}{\partial \gamma_i} = - \left(p {\gamma_i}^{p-1} \rho_m - p {\gamma_i}^{p-1} \rho_0 \right) \sigma \mathbf{T}(\theta)^{\mathbf{4}} \tag{25}$$

For the stiffness matrix, the derivatives are written as:

$$\frac{\partial \mathbf{K_R}}{\partial \gamma_i} = - \left(p {\gamma_i}^{p-1} \rho_m - p {\gamma_i}^{p-1} \rho_0 \right) \mathbf{F} \tag{26}$$

The problem described in this work is solved through a software implemented in MATLAB. A flowchart of the integration between finite element analysis, objective function calculation, sensitivities determination and the design variables optimization is shown in Fig. 2.

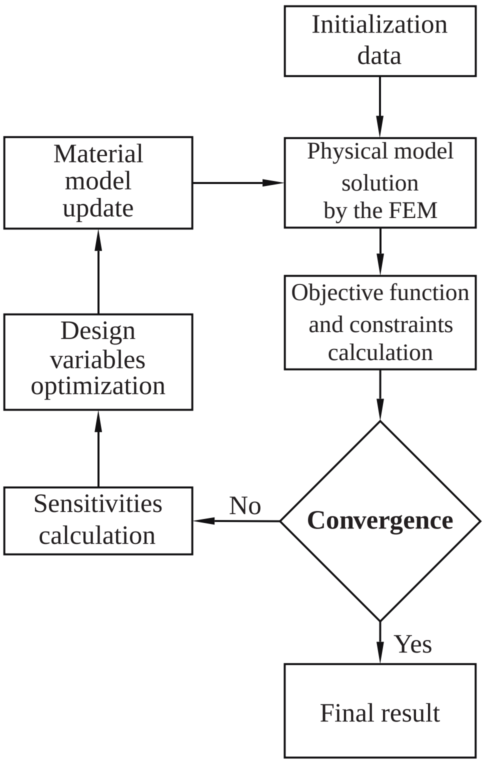

Figure 2. Steps followed during the optimization process.

6. NUMERICAL EXAMPLE

The numerical example analyzed in this article is concerned about the inner design of a furnace, which has a design domain similar to the represented by Fig. 3. The inner furnace surfaces are subjected to boundary conditions of temperature and heat flux. In the last case, the surface can be, eventually, treated as a heater. A target area where the designer wishes to maximize the irradiation flux is also specified in Fig. 3.

For result viewing purposes, a furnace, whose dimensions are $0.4 \times 0.5 \times 0.3\ m$, is unfolded as shown in Fig. 4.

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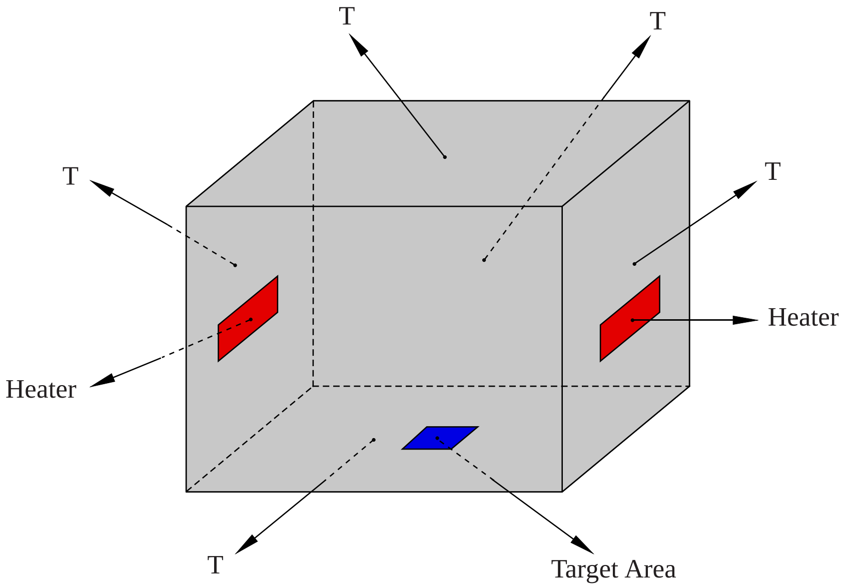

Figure 3. Example of a design domain with its boundary conditions.

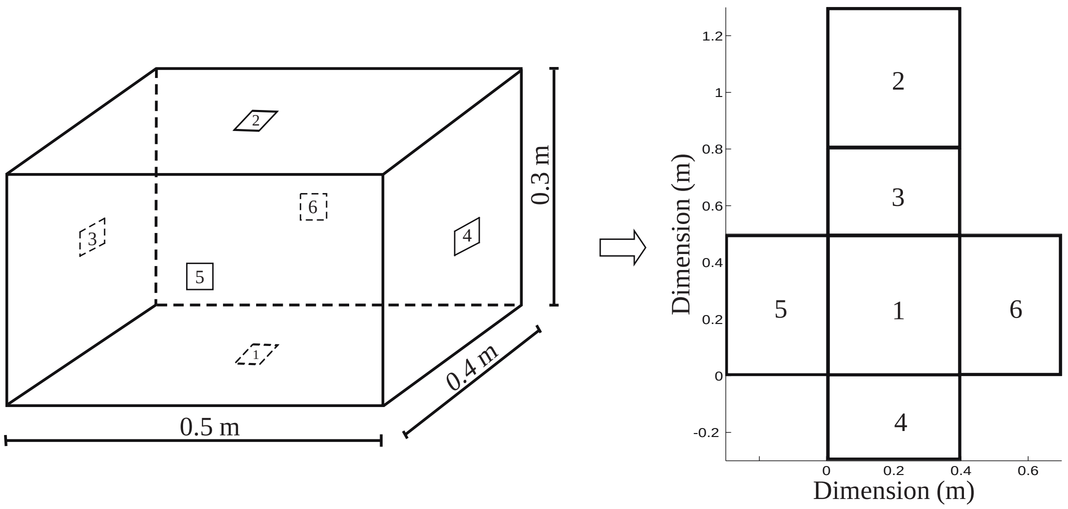

Figure 4. Unfolded furnace for result viewing purposes.

6.1 Case 1

In the first case, the design domain is meshed with 15,000 retangular elements. The initial reflectivity values of the furnace surfaces, the boundary conditions and the value of the reflective material that it is distributed through the optimization process are given by:

$$\begin{cases} \rho_1 = 0.10 \\ \rho_2 = 0.30 \\ \rho_3 = 0.20 \\ \rho_4 = 0.70 \\ \rho_5 = 0.10 \\ \rho_6 = 0.10 \end{cases} \qquad \begin{cases} T_1 = 500K \\ T_2 = 800K \\ T_3 = 1000K \\ T_4 = 1200K \end{cases} \qquad \begin{cases} Q_5 = 0 \\ Q_6 = 0 \end{cases} \qquad \rho_m = 0.8$$

The target area where it is desired to maximize the irradiation heat flux is specified in the first surface, between $0.1 - 0.3\ m$ in the X direction and between $0.1 - 0.4\ m$ in the Y direction, as shown in Fig. 5a.

A convergence curve and the design variables last iteration of the topology optimization process can be viewed in Fig. 6. The convergence is achieved with just a few steps and the final iteration has little gray. The optimized material distribution is represented in Fig. 7 and it becomes clear that no volume constraint is necessary for such problem. The idea of covering all the inner furnace surface with the material design ρ_m, just because it has greater reflectivity value, it proved to be false for the optimization objective of maximizing the irradiation heat flux over the red area. Through the

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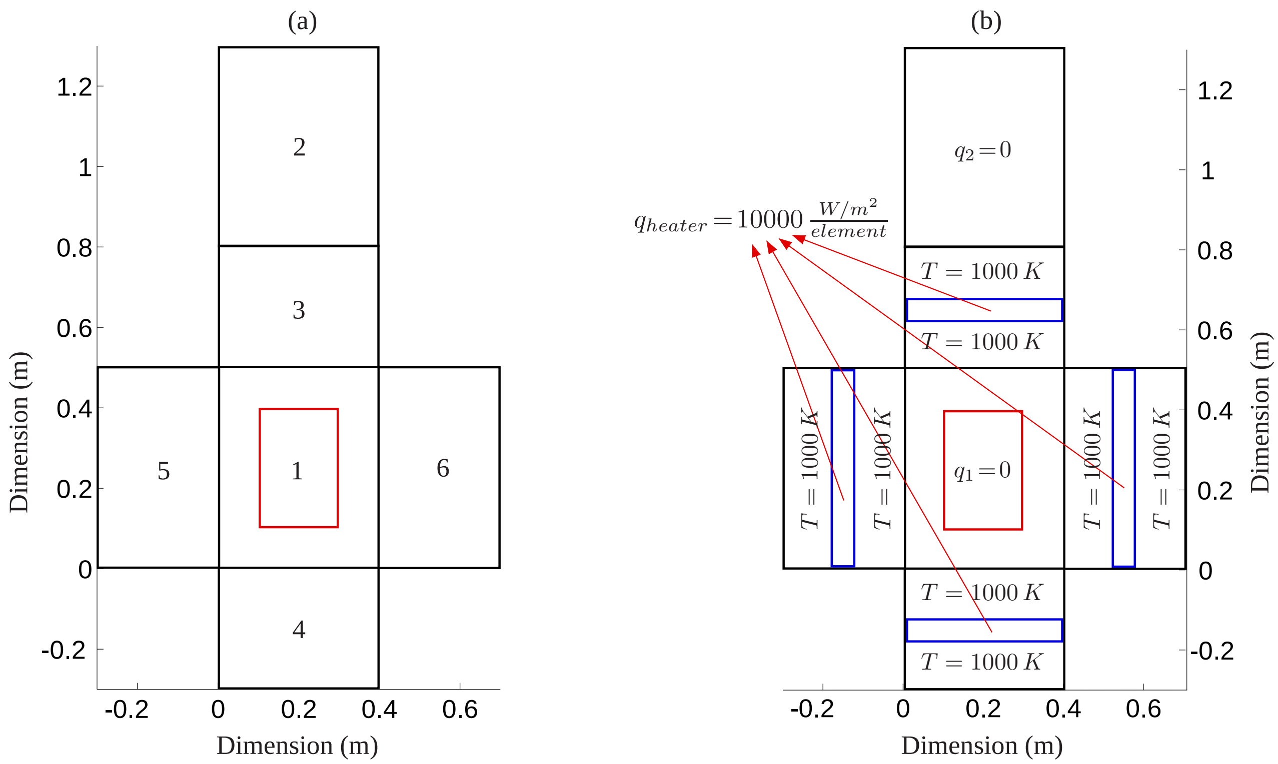

Figure 5. Furnaces design domains: (a) Case 1 (b) Case 2.

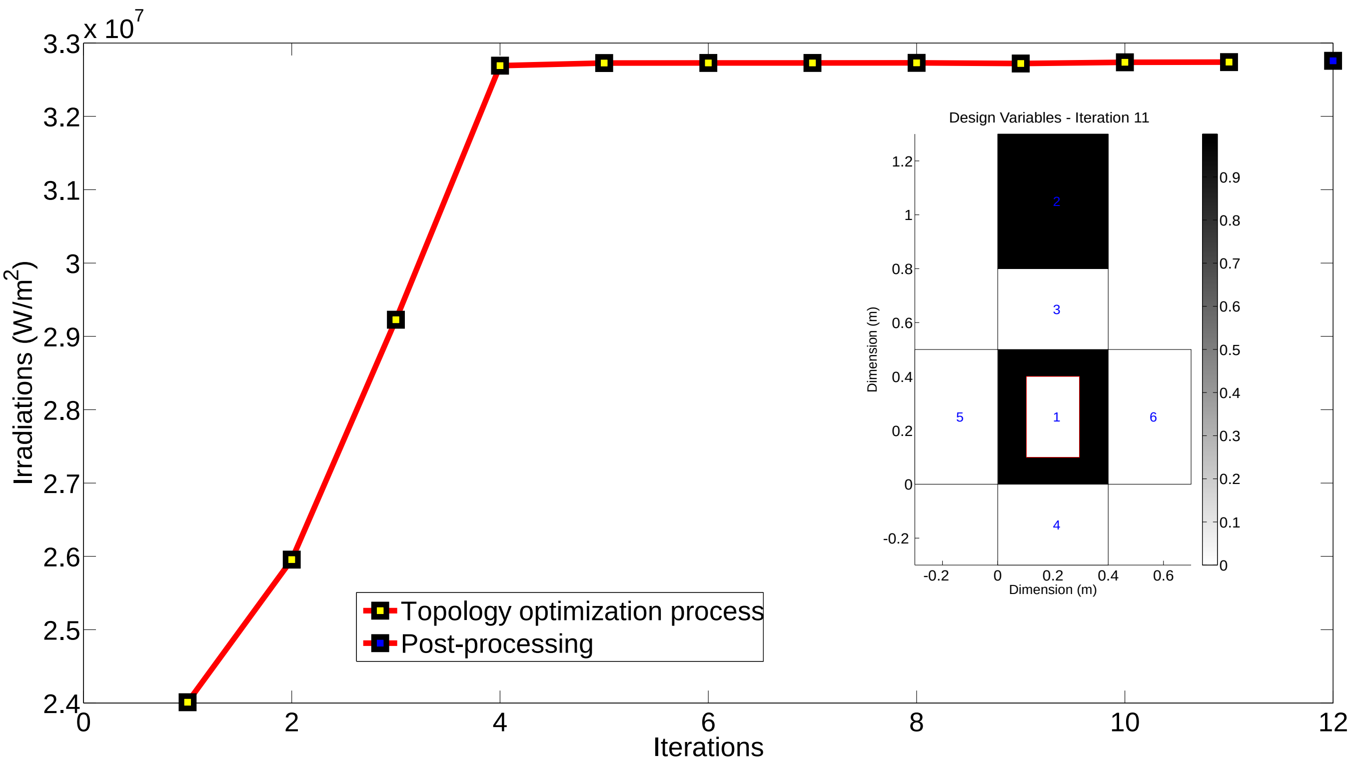

Figure 6. Convergence curve for the first case.

distribution showed in Fig. 7, the irradiation incidence over the red area is increased by approximately 36.4%, as Fig. 8 shows. That amount represents an economy of the energy use, since the furnace can heat the red area more efficiently.

6.2 Case 2

The furnace design domain of the second case is schematically represented in Fig. 5b with its boundaries conditions. This domain is meshed with 15,000 retangular elements and the surfaces comprising that enclosure have initial reflectivities equal to the first case, as well as the reflectivity value of the material that is distributed in the optimization process an

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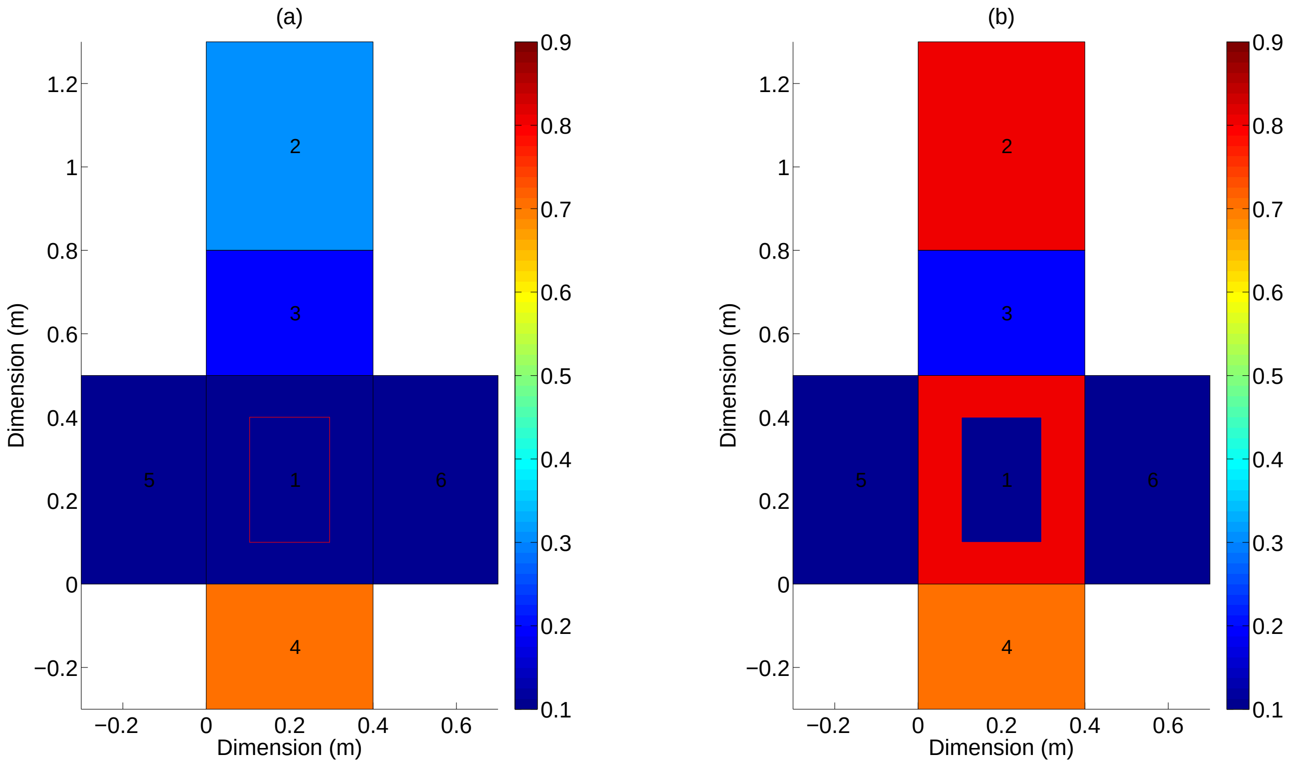

Figure 7. Material distribution for the first case: (a) initial furnace design domain (b) optimized furnace design domain.

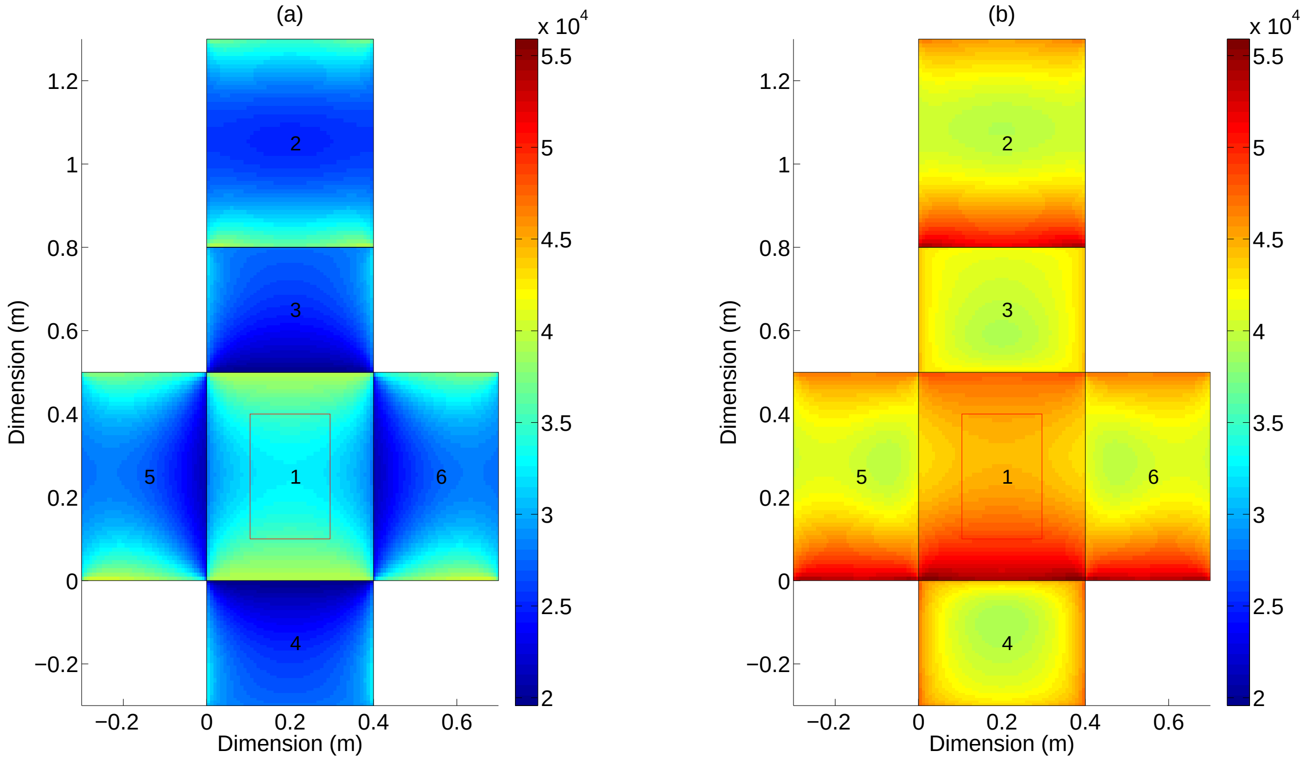

Figure 8. Irradiation heat flux (W/m^2) distribution for the first case: (a) initial furnace design domain (b) optimized furnace design domain.

the target area where it is desired to maximize the irradiation heat flux still the same.

The convergence curve and the last iteration design variables of the topology optimization process can be viewed in Fig. 9. In the final iteration, the design variables are practically binaries, with a very small presence of undesired

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intermediate values (gray scale). The optimized material distribution is represented in Fig. 10 and, unlike the first case,

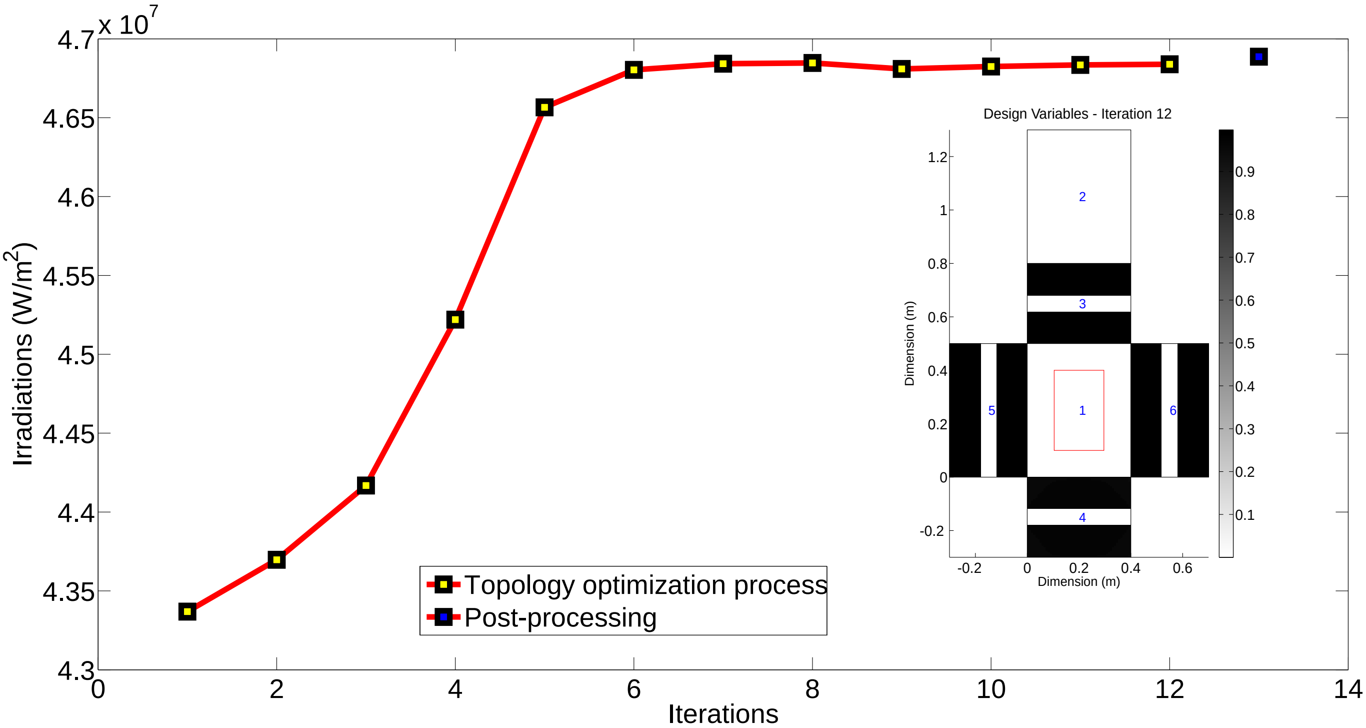

Figure 9. Convergence curve for the second case.

the designer can now impose a volume constraint, since the material is distributed over all the possible surfaces (i.e., surfaces with boundary condition of prescribed temperature). The idea of covering all the inner enclosure surface with the material design ρ_m, just because it has greater reflectivity value, it proved to be true for the optimization objective of maximizing the irradiation over the red area. The distribution showed in Fig. 10 increased the irradiation incidence

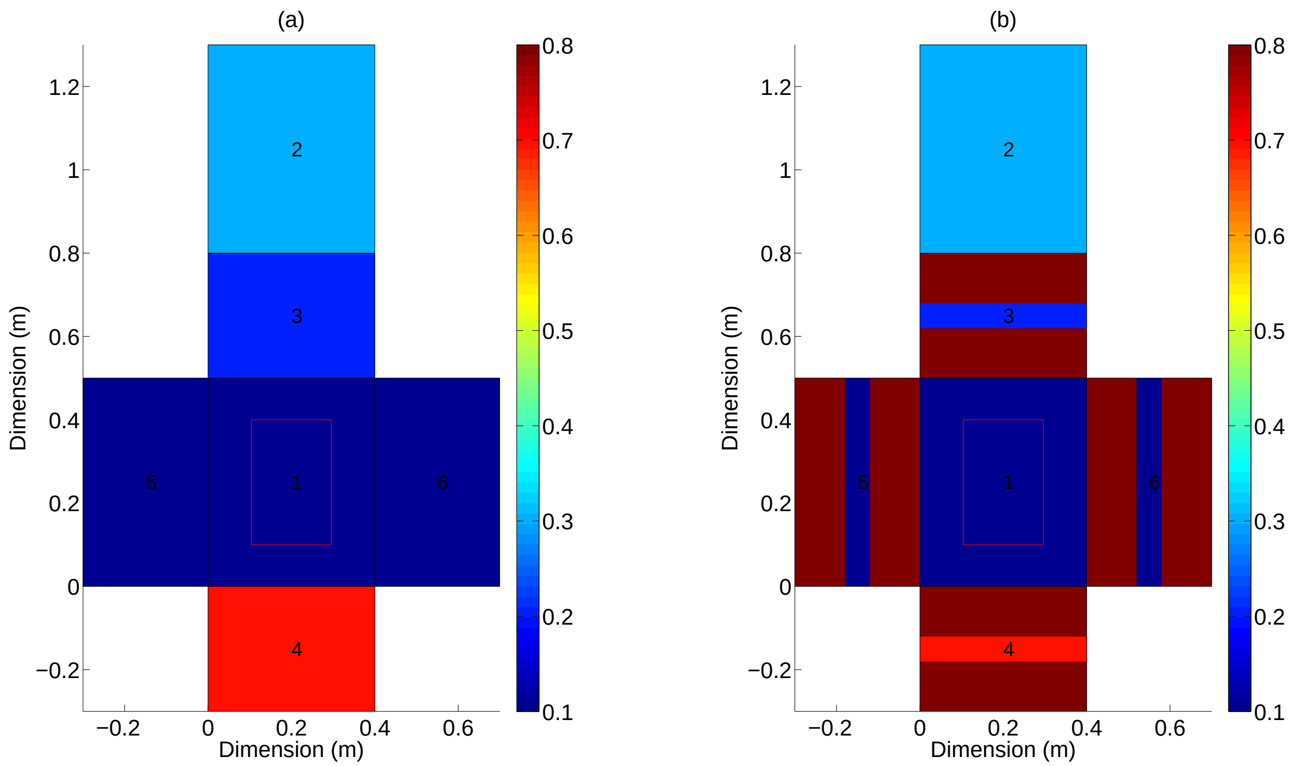

Figure 10. Material distribution for the second case: (a) initial furnace design domain (b) optimized furnace design domain.

over the red area by approximately 8.1%, as Fig. 11 shows. That amount represents an economy in energy use, since the furnace can heat the red area more efficiently.

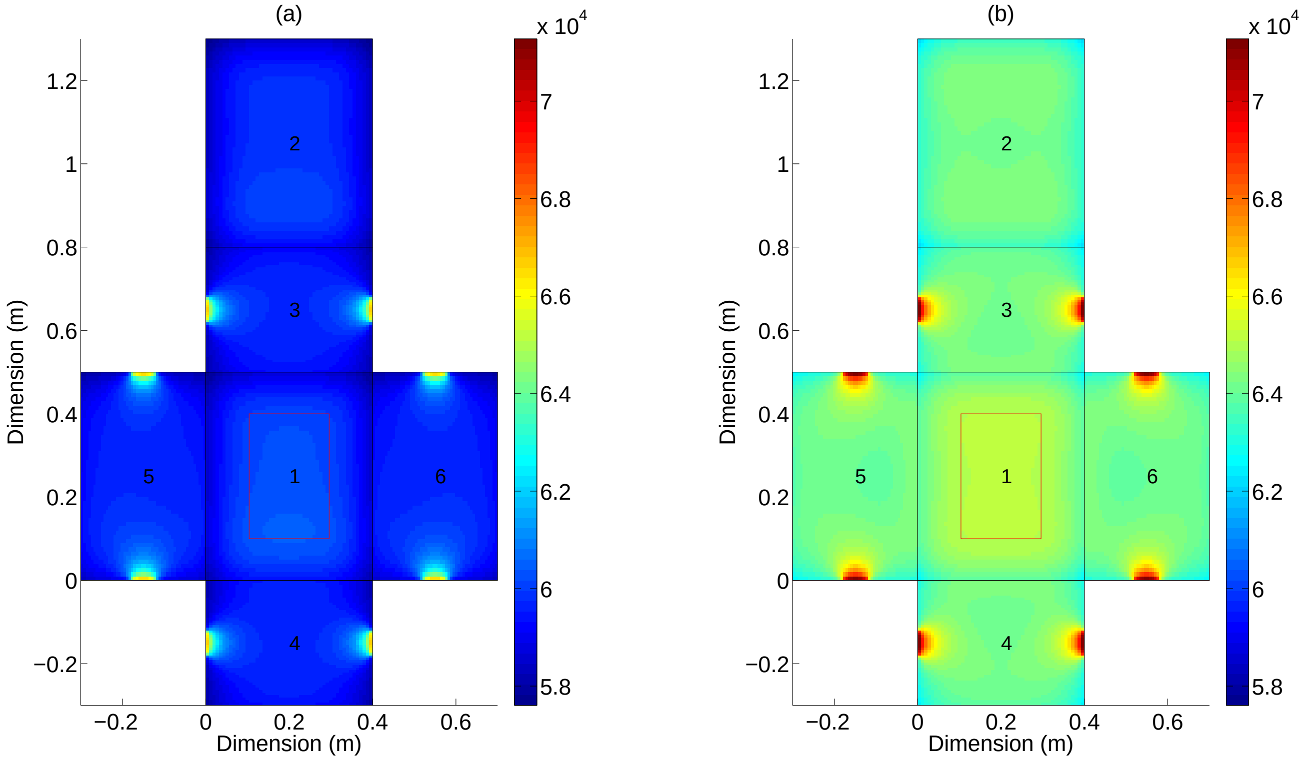

Figure 11. Irradiation heat flux distribution (W/m^2) for the second case: (a) initial design domain (b) optimized design domain.

7. CONCLUDING REMARKS

This paper studies the influence of the reflective material distribution over radiant enclosure inner surfaces with the objective of maximizing the irradiation heat flux in a specified area of that design domain. From the numerical results presented it can be seen that this task has not always obvious solutions. The percentage gains obtained with the topology optimization process also shows that this tool is highly recommended for a systematically design of such enclosures.

8. ACKNOWLEDGEMENTS

The first author thanks CNPq (National Counsel of Technological and Scientific Development) by supporting him in his graduate studies through the fellowship No. 132369/2011-7. The second author thanks FAPESP (São Paulo Research Foundation) by supporting him in his graduate studies through the fellowship No. 2012/14576-9. The authors would like also to thank Prof. Svanberg for providing the MATLAB source code of the Method of Moving Asymptotes (MMA).

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