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Design of a Set of Reaction Wheels for Satellite Attitude Control Simulation

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Abstract. *At Instituto Tecnológico de Aeronáutica, we are working in conception, design and development of a tri-dimensional attitude control experiment. It consists of an aluminum hollow sphere, that floats on a micro-metrical air layer supported by an aerostatic bearing. The sphere, denominated here Satellite Attitude Simulator (SAS), can turn freely without friction, reproducing in this way, in laboratory, the satellite in its orbit in the space. The sphere contains the sensors, in this case the three-axis gyroscopes, and the actuators, consists of three orthogonally installed reaction wheels. An external computer system communicates with the mentioned devices to perform an attitude control action. This work reports the design, mathematical modeling and parameters definition for the reaction wheels, used as actuators in this Satellite Attitude Simulator. The first issue addressed in this work is the definition of the minimum requirements for the three actuators of the SAS system. Then, the reaction wheels are dimensioned, and with their design parameters the mathematical modeling is performed, and by iteratively repeated simulation of the mathematical model with change of parameters, the selection of the reaction wheels is optimized. With this iterative simulation, the goal is to test actuators of different dimensions, that met the design requirements of the SAS, but also minimize its weight and energy consumption. This mathematical model is also used to evaluate the performance of control algorithms that make use of the sensors as inputs and generates actuation signal as outputs to activate the reaction wheels. It will be effective also for discussions about the treatment of disturbances caused by nonlinearities in the reaction wheels. The simulations results obtained with the mathematical model are compared with experimental data, to validate the model.*

Keywords: *Reaction wheels, Simulation, Satellite, Attitude control*

1. INTRODUCTION

The attitude control of satellites is a major focus of studies of Instituto Tecnológico de Aeronáutica - ITA. The Institute prepare engineers to work with aerospace technology and offers courses in the area of dynamics and control of satellites. Satellites dynamics and control are challenges to students' comprehension of the subject. In order to facilitate the understanding of the students, a platform for Satellite Attitude Simulator (SAS) has been conceived and designed, with hardware, software, sensors and actuators similar to real satellites.

Rotational Air Bearing system was chosen for the implementation of SAS. Rotational air bearings can operate by rotating a spherical object above a concave structure that matches the geometry of the sphere. Rotational air bearings also use compressed air gas to float the spherical object just above the surface of the concave support structure. Though constrained in all three translational degrees of freedom, rotational air bearings provide three rotational degrees of freedom (Crowell, 2011). Pressurized air passes through small holes in the grounded section of the bearing and establishes a thin film that supports the weight of the moving section. This slow-moving air imparts virtually no shear between the two sections of the bearing. Thus, the air film is an effective lubricant. An air bearing that can support a payload weighting several thousand pounds may require air pressurized to only about 100 psi with a low rate of only a few cubic feet per minute. A familiar example of such a device is an air-hockey table (Schwartz *et al.*, 2003). Figure 1 shows the Rotational Air Bearing.

1.1 Satellite Simulator Systems

Air bearings have been used for spacecraft attitude determination and control hardware verification and software development for nearly 45 years, virtually coincident with the beginnings of the Space Race. Facilities vary widely, ranging from prodigious government laboratories to simple university test beds. Spherical air bearings are one of the most common devices used in spacecraft attitude dynamics research because (ideally) they provide unconstrained rotational motion. As the name implies, the two sections of the bearing are portions of concentric spheres,machined and lapped to small tolerances.(Schwartz *et al.*, 2003)

Spherical air-bearing test beds appear in three primary styles: dumbbell, tabletop and umbrella. Dumbbell style allows for nearly 360 degree motion about the roll and yaw axes, tabletop style allows for 360 degree motion about the yaw-axis

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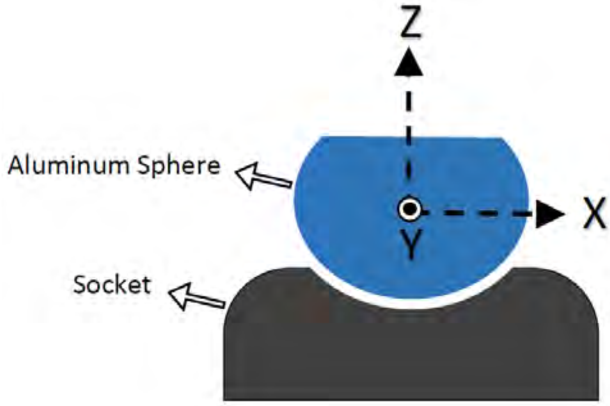

Figure 1. Rotational air-bearing and socket illustration

and limited motion about the pitch and roll axes, umbrella is similar to that of the tabletop-style air-bearing, umbrella-style test beds allow for 360 degree motion about the yaw-axis and limited motion about the pitch and roll axes, Umbrella-style spherical air-bearings are assembled by extruding a rod from a fully spherical bearing. The work (Snider, 2010) shows a graphical representation of these three types of air-bearing are shown in Figures 2, 3 and 4.

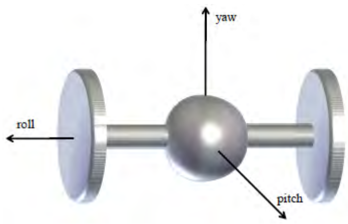

Figure 2. Dumbbell (Crowell, 2011)

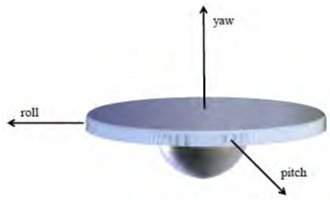

Figure 3. Tabletop (Crowell, 2011)

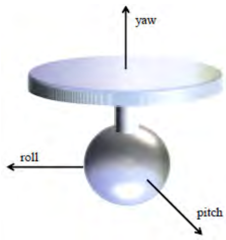

Figure 4. Umbrella (Crowell, 2011)

The attitude control of satellites is an essential task essential for the realization of space missions. Different types of actuators are used to attitude control. Reaction wheels, magnetic coils and gas jets are among the most used.

Reaction wheels consist of a flywheel (also referred to as a rotor) and an electric motor. The flywheel's axis of rotation is fixed relative to the spacecraft body. Torque is applied to the flywheel by the motor, altering its rotational speed, and thus its angular momentum, which in turn changes the angular velocity of the spacecraft to maintain the total angular momentum of the vehicle. Three or more reaction wheels, properly configured, for 3-axis control of the spacecraft. Reaction wheels are commonly used on spacecraft as they provide more accurate control than thrusters and do not require fuel (McChesney, 2011). The angular momentum given by the control law is divided into components in line with each of the reaction wheels thus making the implementation of the control system simple. Figure 5 represents this system.

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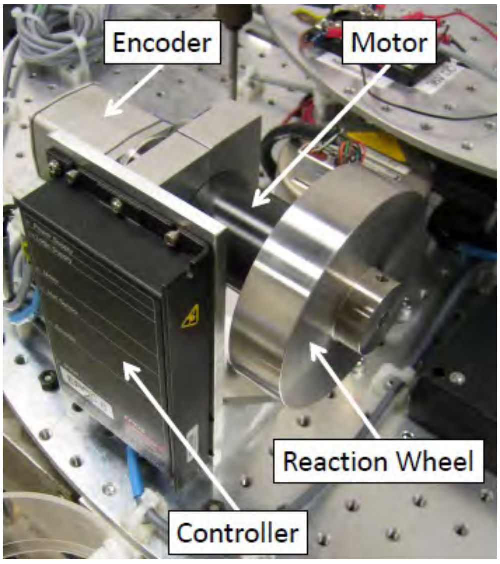

Figure 5. Reaction Whell (Snider, 2010)

1.2 Problem

This paper aims to raise the parameters and to develop the mathematical model of the Satellite Attitude Simulator installed in ITA. These actuator parameters will be used to design the spherical air bearing and the embedded electronics, in such a way that the resultant system will be useful to evaluate various control techniques. Thus, the model will be able to be compared with data from experiments conducted with the platform in order to achieve a fit that allows to study the nonlinearities of the system, and also test several control techniques, using it for the information extracted from harvested inertial sensors installed in the field. The dynamic model should be adjusted to match the real dynamics of the sphere.

2. Satellite Attitude Simulation Environment

The final architecture chosen was the spherical air-bearing tabletop to simulate the dynamics of the satellite with three reaction wheels, accelerometers and gyroscopes with three axes of measurement for angular velocities and positions.

2.1 Design

At this stage of the project the dynamic model of the platform was created, the main causes of disturbances were identified and the main dimensioning factors of the actuators were identified. For this phase the parameters estimated for initial sizing of the SAS platform actuators are presented in Table 1.

Table 1. Estimated Parameters

Parameter	Value	Symbol
Mass SAS	$15\ Kg$	M
Misalignment	$5 * 10^{-6}\ m$	δ
Simulation time	$20\ s$	Δt
Maneuvering Speed	$20°/min$	ω_{max}
Inertia XX	$0.107\ Kgm^2$	I_{xx}
Inertia YY	$0.108\ Kgm^2$	I_{xx}
Inertia ZZ	$0.116\ Kgm^2$	I_{xx}

The inertia parameters (I_{xx}, I_{yy}, I_{zz}) were estimated through a model in CAD software, with leading inserted components, such as batteries, motors and on-board computer.

2.2 Coordinate System

The air bearing system has three principal coordinate systems, which are the: fixed inertial reference (FIR) coordinate system, the air bearing body fixed (ABBF) coordinate system, and the reaction wheel coordinate system (RWCS). The first to be defined is the fixed inertial reference coordinate system that is fixed with respect to the laboratory. The origin of this

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coordinate system is located in the center of rotation of the bearing. The second coordinate system to be defined is the air bearing body fixed coordinate system, initially this system has axes aligned with the FIR system. During the simulations the system rotates in relation to the FIR system. The third coordinate system to be defined is the reaction wheel fixed coordinate system. It is aligned with the rotation axes of the three orthogonal reaction wheels, which coincides with the axes of the ABBF system.

2.3 Dynamic Model

Equation 1 is the angular representation of Newton's second law of motion, which states that a mass subject to a force undergoes an acceleration. In this case, a mass, represented by its second moment of inertia I, subject to a torque undergoes an angular acceleration, $\dot{\omega}$.The inertia times the angular acceleration is also equal to the change in angular momentum, $\dot{H}$. (Greenwood, 1987)

$$T_{ext} = I\dot{\omega} = \dot{H} \quad (1)$$

The angular acceleration and angular momentum relationship given in Equation 1 is assumes a fixed inertial reference frame. In order to represent the motion of the air bearing in the ABBF coordinate system that moves with respect to the FIR coordinate system, the additional cross product of the angular velocity vector $\dot{\omega}$ with the angular momentum vector H given in Equation 2 must be included.(Greenwood, 1987)

$$T_{ext} = \dot{H} + \omega \text{ x } H \quad (2)$$

The angular momentum vector H represents the total angular momentum of the air bearing system, which consists of the angular momentum of the air bearing itself and the angular momentum of the reaction wheels. Equation 3 shows the total angular momentum.

$$H_{total} = H_{AB} + H_{RW} \quad (3)$$

Substituting Equation 3 in Equation 2 gives Equation 4.

$$T_{ext} = \dot{H_{AB}} + \omega \text{ x } H_{AB} + \dot{H_{RW}} + \omega \text{ x } H_{RW} \quad (4)$$

In Equation 1, change in angular momentum is equal to inertia times angular acceleration. Making this substitution into Equation 4 gives Equation 5, which represents the complete vector equation of rotational motion for the air bearing system.

$$T_{ext} = I_{AB}\dot{\omega} + \omega \text{ x } I_{AB}\omega + DI_{RW}\dot{\Omega} + \omega \text{ x } DI_{RW}\Omega \quad (5)$$

The angular velocity of the air bearing ω, and the angular velocity of the reaction wheels, Ω will be used for reaction wheel angular velocity, I_{AB} and I_{RW} are the inertia matrices respectively air bearing and reaction wheels, D is the direction cosine matrix (DCM). Equation 4 can be used to prove the important concept that reaction wheels are simply angular momentum storage devices. They have the capability to transfer angular momentum to and from the vehicle to which they are attached.

The symbolic inertia matrix of the complete air bearing I_{AB} is given in Equation 6.

$$I_{AB} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} kg * m^2 \quad (6)$$

The inertia matrix of the reaction wheels I_{RW} is given in Equation 7.

$$I_{RW} = I_{flywheel} + I_{motor} = \begin{bmatrix} I_a & 0 & 0 \\ 0 & I_a & 0 \\ 0 & 0 & I_a \end{bmatrix} kg * m^2 \quad (7)$$

The reaction wheel inertia matrix is a diagonal matrix, because the wheels are aligned with the axes of the system ABBF.

Constant term D in Equation 5 represents the DCM between the reaction wheel coordinate system and the ABBF coordinate system. For this case, the coordinate system of reaction wheels are aligned with the system ABBF, in our case DCM matrix and a diagonal matrix.

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The final model in state space equations under linearized conditions for equilibrium are presented in the Equations 8 and 9. This model was built based on the work of (Crowell, 2011).

$$\begin{bmatrix} \dot{\theta}_x \\ \dot{\theta}_y \\ \dot{\theta}_z \\ \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{I_a}{I_{xx}} & 0 & 0 \\ 0 & \frac{I_a}{I_{yy}} & 0 \\ 0 & 0 & \frac{I_a}{I_{zz}} \end{bmatrix} \begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_x \\ \theta_y \\ \theta_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \tag{9}$$

The mathematical model of reaction wheels were developed based on direct current DC motor shown in Fig 6.

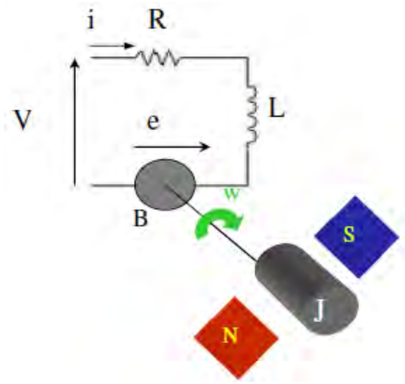

Figure 6. Diagram of direct current Motor (Fonseca, 2001)

Equation 10 gives Kirchoff's voltage law applied to the reaction wheel system.

$$V - V_R - V_L - V_C = 0 \tag{10}$$

Here, V_R, V_L and V_C can be described respectively by $V_R = i * R$, $V_L = \dot{i} * L$ and $V_C = K_e * \Omega$. Substituting these equations into Kirchoff's voltage law gives Equation 11, which is the dynamic equation representing the electrical characteristics of the reaction wheel (Altas., 2007).

$$\dot{i} = -\frac{K_e}{L}\Omega - \frac{R}{L}i + \frac{V}{L} \tag{11}$$

Based on Newton's second law for rotational systems that states the sum of the torques is equal to inertia multiplied by the angular acceleration. The present torques T_e and T_Ω expressed by $T_e = K_t * i$ and $T_\Omega = b * \Omega$, the mechanical dynamics can be expressed by the Equation 12.

$$\dot{\Omega} = -\frac{b}{J}\Omega - \frac{K_t}{J}i \tag{12}$$

The electrical and mechanical dynamic equations can be combined to model the motion of the three reaction wheels on the air bearing given voltage inputs from the motor controllers. The combined model of the electrical and mechanical dynamics of the reaction wheels, are presented in Equations 13 and 14.

$$\begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \\ \dot{i}_x \\ \dot{i}_y \\ \dot{i}_z \end{bmatrix} = \begin{bmatrix} -\frac{b}{J} & 0 & 0 & \frac{K_t}{J} & 0 & 0 \\ 0 & -\frac{b}{J} & 0 & 0 & \frac{K_t}{J} & 0 \\ 0 & 0 & -\frac{b}{J} & 0 & 0 & \frac{K_t}{J} \\ -\frac{K_e}{L} & 0 & 0 & -\frac{R}{L} & 0 & 0 \\ 0 & -\frac{K_e}{L} & 0 & 0 & -\frac{R}{L} & 0 \\ 0 & 0 & -\frac{K_e}{L} & 0 & 0 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \\ \dot{i}_x \\ \dot{i}_y \\ \dot{i}_z \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{L} & 0 & 0 \\ 0 & \frac{1}{L} & 0 \\ 0 & 0 & \frac{1}{L} \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} \tag{13}$$

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$$\begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\Omega}_x \\ \dot{\Omega}_y \\ \dot{\Omega}_z \\ \dot{i}_x \\ \dot{i}_y \\ \dot{i}_z \end{bmatrix} \tag{14}$$

2.4 Disturbances

This system of spherical air bearing suffers the influence of two major disturbances, aerodynamic torque and unbalance between the center of pressure and center of mass. The aerodynamic disturbance is difficult to be measured. It is caused by friction between the sphere and the aluminum layer of air. The torque imbalance occurs by misalignment of the pressure point with the center of mass of the sphere, causing the weight the sphere generate a torque disturbance. Figures 7 and 8 illustrate these phenomena.

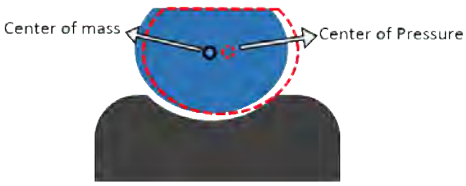

Figure 7. Unbalance between center of mass and center of pressure.

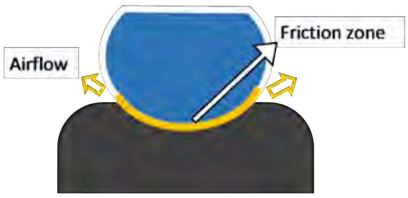

Figure 8. Airflow friction.

2.5 Design of actuators

The main function of the actuators is to stabilize the platform and perform maneuvers appointment. To accomplish these tasks, the reaction wheels must generate a control torque that can counteract the disturbances and perform maneuvers.

The reaction wheels must be dimensioned so that the gravitational torque does not cause the residual saturation of the wheels during simulations. The projected time duration of simulations is approximately 5 minutes. The deviation of the center of pressure and the center of mass of the sphere is established for the design of the wheels is $e = 5 * 10^{-6}$ and speed variation of $\Delta\omega = 2000RPM$. Therefore, the reaction wheel inertia can be calculated as follows:

$$T_{GR} = M * g * e = \dot{L} \tag{15}$$

T_{GR} is the residual gravitational torque, M is the mass of the SAS system, g the acceleration of gravity e e is the deviation of the center of pressure.

For the condition of equilibrium we have:

$$\dot{L} = I_{RR} * \alpha_{RR} = T_{GR} \tag{16}$$

$$I_{RR} = \frac{\Delta t * e * M * g}{\Delta\omega} = I_{RR} = 0.0011Kgm^2 \tag{17}$$

I_{RR} represents the inertia of the reaction wheels, α_{RR} is the acceleration of reaction wheels, Δt is the simulation time, and $\Delta\omega$ is the variation speed allowed by the wheels reaction.

Thus we can calculate the required torque to be generated by the engines as follows:

$$T_{RW} = M * g * e = 7.35 * 10^-04Nm \tag{18}$$

Starting with these parameters it was possible to select the best suited motors to the SAS system actuator. The chosen motors are 2232-BX brushless motor, manufactured by Faulhaber. The motor parameters are shown in Table 2.

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Table 2. Reaction Wheel Motor Constants

Parameter	Name	Value	Units
J	Inertia	$9.60e-5$	$Kg*m^2$
K_e	Voltage Constant	0.0036	$V/rad/s$
K_t	Torque Constant	$3.46e-3$	Nm/A
b	Damping Ration	$2.4828e-08$	$Nm/rad/s$
L	Inductance	$590e-6$	H
R	Resistance	22	Ω

3. Simulation

Matlab/Simulink simulations were made, to implement the dynamic model of the reaction wheels and actuators. The system was simulated to assess whether the parameters selected for actuators would meet the design requirements. Figure 9 shows how the model was implemented in Simulink and then will be presented the responses of angular positions.

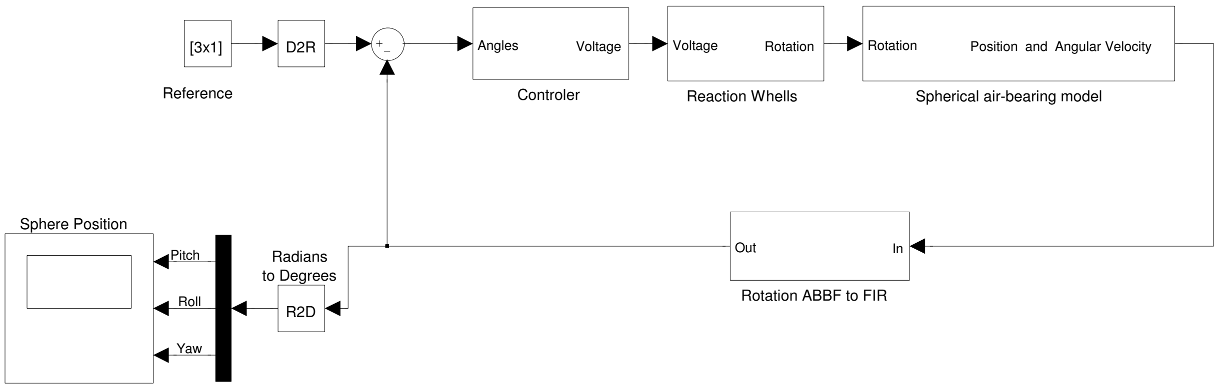

Figure 9. Simulink Diagram

The reference block generates a reference signal position for the SAS system. Subtracting the feedback to this signal, the error signal is obtained, which is sent to the control block. The control block was developed to stabilize the system and make it meets the design requirements. To design the control system the tool SISOTOOL part of the package MATLAB was used.

The reaction wheel block receives three voltage inputs from the controller block within the control module. Each voltage input corresponds to each of the three reaction wheels. The Spherical air-bearing block receives speeds of the three reaction wheels and returns the angular position to the ABBF system. The rotation block transforms the coordinates ABBF for FIR system.

Simulations were performed, using the parameters previously mentioned. Simulations results for the angular velocities are shown in Figure 10. The simulation presented was obtained exciting the system with a signal step from 0 to 20 degrees at time $t = 0$. This reference signal was chosen to evaluate the requirement for maneuvering speed of the sphere. Simulation results showed that a new stable position is attained after 50 seconds for the three axes, verifying that the requirement was achieved for all axes, the response obtained was similar for pitch, roll and yaw as the system has inertia approximately equal to the axes x, y and z.

4. Conclusion and Future Works

This work develops the mathematical model for the SAS platform and applying simulation, performs the design of the reaction wheel, besides the proof that the actuators meet the design requirements, furthermore this work is a first step to study and implement controls techniques in SAS platform and also an important step for developing software-in-the-loop and hardware-in-the-loop simulations with the SAS platform.

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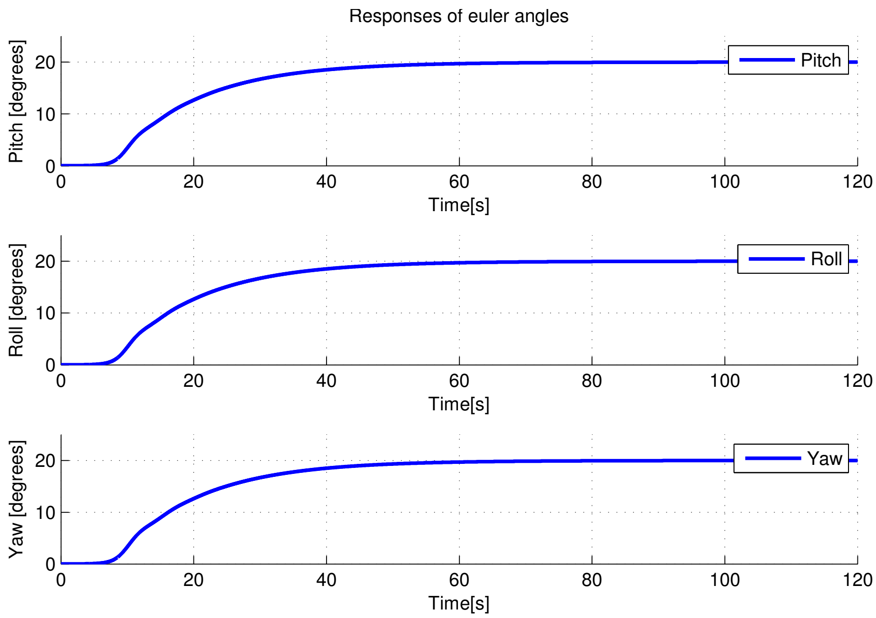

Figure 10. Response of euler angles: Pitch, Roll, and Yaw.

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