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NON-NEWTONIAN FLUID FLOW THROUGH A POROUS MATRIX: MODELING THE CONNECTION BETWEEN POROSITY AND PERMEABILITY USING A MIXTURE THEORY APPROACH

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Abstract. *Among the relevant engineering applications of non-Newtonian fluids flowing through porous media are oilfield applications, in which the fluids exhibit non-Newtonian behavior, such as power-law-like behavior, over some range of shear rates. In this work, the connection between porosity and permeability and its influence in the flow of a power-law fluid through a porous matrix is investigated using a Continuum Theory of Mixtures; which models fluid saturated porous media by considering the fluid and the porous matrix as superimposed continuous constituents of a binary mixture. This theory leads to an apparent independence among the constituents, requiring additional terms (momentum and energy sources) to account for the thermomechanical coupling among the constituents in the balance equations. Thermodynamically consistent constitutive relations connecting porosity, permeability and the interaction between the fluid and the porous matrix are introduced to obtain an adequate model. Special attention is given to the constitutive expression adopted for the interaction force since it is the key to perform a physically realistic connection between porosity and permeability. In order to better understand the coupled influence of porosity and permeability on the flow, the flow of a power-law fluid through a horizontal porous channel is numerically investigated.*

Keywords *Mixture theory, power-law fluid, rigid porous medium, permeability.*

1. INTRODUCTION

Flow of Non-Newtonian fluids through porous media is relevant in many fields of engineering and science. Distinct subjects like Geomechanics – when distinct rock and soil types with varying porosities where subsurface flows occur, Hydrogeology – with problems like soil contamination modeling by pollutants, underground nuclear waste disposal, groundwater modeling and soil drainage, Biological Engineering – with countless problems, since, for instance, blood is non-Newtonian fluid. Another very important field is Petroleum Engineering. In this case, the high demand for oil has induced a massive need for efficient extraction techniques, and thus for advanced simulation methods for enhanced oil recovery, which exhibits non-Newtonian features. A large variety of fluids used in oilfield applications (perforation mud, polymer solutions injected for enhanced oil recovery, etc.) exhibit non-Newtonian behavior, such as power-law-like behavior, over some range of shear rates (Sorbie, 1991). In these cases, the classical Darcy law can be inadequate and, therefore, the knowledge of adequate law for the flow of these kinds of fluids through porous media is of first importance.

In this article the plane saturated porous channel (rigid solid matrix) is modeled using a continuum mixture theory, instead of a volume-averaging technique (Whitaker, 1969) and employed in most of the works dealing with transport in porous media. A comprehensive review of distinct volume averaging models and applications was performed by Alazmi and Vafai (2000). The Mixtures Theory, that generalizes the classical Continuum Mechanics, has been specially developed to describe multiphase phenomena. It models fluid saturated porous media by considering the fluid and the porous matrix as superimposed continuous constituents of a chemically non-reacting binary mixture – each of them occupying its whole volume, but it requires momentum and energy sources to assure the thermomechanical coupling among the constituents in the balance equations.

After some simplifying assumptions, the governing equations give rise to a non-linear system of two-point boundary-value problems in ordinary differential equations, which is simulated using a Runge-Kutta method coupled with a shooting technique, essentially reducing the solution of the problem to finding the root of a real function.

In this work the behavior of generalized Newtonian fluids under the combined effects of porosity and permeability is analyzed, using a Continuum Theory of Mixtures approach. A dimensionless parameter with the connection between porosity and permeability is proposed and its influence in the flow of a power-law fluid through a porous matrix is investigated.

2. MECHANICAL MODEL

Since the solid constituent (representing the porous matrix) is supposed rigid and at rest, it suffices to solve the motion equations for the fluid constituent. The mechanical model combines mass and momentum balance equations for the fluid constituent with constitutive assumptions for the power-law fluid constituent, accounting for the connection between permeability and porosity, proposed in this work. The balance equations (Atkin and Craine, 1976; Rajagopal and Tao, 1995) are given by

$$\frac{\partial \rho_F}{\partial t} + \nabla \cdot (\rho_F \mathbf{v}_F) = 0$$
$$\rho_F \left[\frac{\partial \mathbf{v}_F}{\partial t} + \left(\nabla \mathbf{v}_F \right) \mathbf{v}_F \right] = \nabla \cdot \mathbf{T}_F + \mathbf{m}_F + \rho_F \mathbf{g} \tag{1}$$

in which $\rho_F = \phi \rho$ is the fluid constituent mass density (where the fluid fraction is given by ϕ and the actual fluid density by ρ) $\mathbf{v}_F$ is its velocity, $\mathbf{T}_F$ is the partial stress tensor associated with the fluid constituent and $\mathbf{m}_F$ is an interaction force per unit volume acting on the fluid constituent due to its interaction with the other constituent of the mixture (representing the porous matrix, see Martins-Costa et al., 2000 and references therein). The role of the interaction force is more evident if the Principle of Virtual Power is considered (Costa Mattos et al, 1995) and, in this case, there is a precise framework that allows defining thermodynamically admissible constitutive equations). Constitutive relations for both $\mathbf{T}_F$ and $\mathbf{m}_F$ are required to build the mechanical model. It is important to note that for saturated flows the fluid fraction (ϕ) and the permeability are coincident.

2.1 Constitutive model

The following constitutive equations that characterize the flow of a particular class of non-Newtonian fluids through a rigid porous medium are proposed

$$\mathbf{T}_F = -p\phi \mathbf{I} + \phi^2 \frac{\partial \omega}{\partial \mathbf{D}_F}$$
$$\mathbf{m}_F = -\frac{\phi^2}{\bar{K}} \mathbf{v}_F \tag{2}$$

with ω being a differentiable, strictly convex and positive scalar function such that $\omega(\mathbf{0}) \neq 0$ and $\bar{K}$ a strictly positive function of $\phi, \frac{\partial \phi}{\partial \mathbf{D}_F}, \|\mathbf{v}_F\|$. The constitutive equations (2) can be obtained within a constitutive framework similar to the one obtained by Costa Mattos et al. (1995) and the above conditions are sufficient to assure that a local version of the second law of thermodynamic is always verified, but it is not the goal of the present paper to perform such a theoretical analysis. These constitutive equations will always satisfy a local version of the second law of thermodynamics provided $\alpha > 0$ and ω is a differentiable, strictly convex and positive function, such that $\phi(\mathbf{0}) \neq 0$ (Costa Mattos, 2012).

The function ω for a Generalized Newtonian fluid has the following particular form (Costa Mattos, 1998)

$$\omega(\mathbf{D}) = \hat{\omega}(D_I); \qquad D_I = \mathbf{D}_F \cdot \mathbf{D}_F \quad \Rightarrow \quad \mathbf{T}_F = -p\phi \mathbf{I} + 2\phi^2 \frac{d\hat{\omega}}{d D_I} \mathbf{D}_F$$
$$\frac{1}{\bar{K}} = \alpha \frac{\partial \hat{\omega}}{\partial D_I}, \quad \text{with} \qquad \alpha = \hat{\alpha}\left(\phi, \eta, n, K\right) \quad \Rightarrow \quad \mathbf{m}_F = -\phi^2 \alpha \frac{d\hat{\omega}}{dD_I} \mathbf{v}_F = -\frac{\phi^2}{\bar{K}} \mathbf{v}_F \tag{3}$$

The term ($d\hat{\omega} / dD_I$) is usually called the dynamic viscosity. It allows modelling the dependence of viscosity on the rate of deformation (and temperature, in the case of non isothermal processes). The function $\hat{\alpha}$ is a strictly positive function of $\left(\phi, \eta, n, K\right)$ with K being the porous matrix permeability. While K only depends on the porous medium, $\bar{K}$ also depends on the fluid constituent flow (material, local velocity and its gradient). Permeability and porosity are connected. Porosity is static while permeability can be enhanced. There have been many efforts to establish relations between the permeability and the porosity for both Newtonian and non-Newtonian fluids flowing in porous media. The following simple but effective expression relating K and ϕ is suggested

$$K = \frac{\phi^a b}{1-\phi^a} \tag{4}$$

where a and b are temperature-dependent positive material parameters. Eventually, a and b may also vary with pressure.

Using Eq. (3), expression (4) has a simple physical interpretation within the context of continuous theory of mixtures: the permeability increases and the interaction force became negligible as the porosity decreases while the interaction force increases and the permeability is negligible as the porosity increases (see Eq. (4)).

$$\begin{aligned} &\phi \to 0 \Rightarrow K \to 0 \ \mathbf{m}_F \to \infty \\ &\phi \to 1 \Rightarrow K \to \infty; \ \mathbf{m}_F \to 0 \end{aligned} \tag{5}$$

The parameters a and b in Eq. (4) can be easily obtained from an experimental $K \times \phi$ curve. For two given experimental pairs (ϕ_1, K_1) and (ϕ_2, K_2), it is possible to obtain

$$K_1 = \frac{b(\phi_1)^a}{1-(\phi_1)^a} \qquad K_2 = \frac{b(\phi_2)^a}{1-(\phi_2)^a} \tag{6}$$

Therefore

$$\frac{K_1}{K_2} = \left(\frac{(\phi_1)^a}{1-(\phi_1)^a}\right)\left(\frac{(1-(\phi_2)^a}{(\phi_2)^a}\right) \tag{7}$$

The parameter a is the root of the scalar function $f(x)$ defined as follows

$$f(x) = \frac{K_1}{K_2} - \left(\frac{(\phi_1)^x}{1-(\phi_1)^x}\right)\left(\frac{(1-(\phi_2)^x}{(\phi_2)^x}\right) \quad \Rightarrow \quad f(a) = \frac{K_1}{K_2} - \left(\frac{(\phi_1)^a}{1-(\phi_1)^a}\right)\left(\frac{(1-(\phi_2)^a}{(\phi_2)^a}\right) = 0 \tag{8}$$

Once the parameter a is obtained, the parameter b can be easily identified using one of the relations proposed in Eq. (6). It is easy to verify that Eq. (6) implies a linear relation between $\log(\phi)$ and $\log(K)$.

A power law fluid is a particular class of a generalized Newtonian fluid in which the Cauchy tensor may be stated as $\mathbf{T} = -p\mathbf{I} + 2\eta(\mathbf{D}\cdot\mathbf{D})^n\mathbf{D}$ (Bird et al., 1987). In this equation η and n are the power-law rheological parameters that characterize the fluid behaviour and $\mathbf{D}$is the strain rate tensor acting on the fluid. It is important to note that the usual power-law equation, given by $\boldsymbol{\tau} = 2\kappa\left(\ddot{\gamma}\right)^{m-1}\mathbf{D}$ (Slattery, 1999), in which κ is a consistency index and m a power-law index, could be recovered, making $\eta = 2^{(m-1)/2}\kappa$ and $n = (m-1)/2$. Considering the above-stated Cauchy tensor, the flow of a power-law fluid in an specific (rigid) porous medium can be obtained as a particular generalized Newtonian fluid flow in the proposed theory using the following particular choices of ω and $\hat{\alpha}$ (Martins-Costa et al., 2000; Costa Mattos et al. 1995; Martins-Costa et al., 2013)

$$\hat{\omega}(D_I) = \frac{2^n\eta}{n+1}(D_I)^{n+1} \quad \Rightarrow \quad \mathbf{T}_F = -p\phi\mathbf{I} + 2\eta\phi^2(D_I)^n\mathbf{D}$$

Making $\beta = \eta\phi \quad \Rightarrow \quad \mathbf{T}_F = -p\phi\mathbf{I} + 2\beta\phi(D_I)^n\mathbf{D}$

$$\hat{\alpha}\left(\phi,\eta,n,K\right) = \frac{\eta}{3K}\left(\frac{4n+3}{2n+1}\right)^{2n+1}\left(\frac{\phi}{6K}\right)^n \left\|\mathbf{v}_F\right\|^{2n} \qquad \alpha = \frac{\phi\,\eta}{3K}\left(\frac{4n+3}{2n+1}\right)^{2n+1}\left(\frac{\phi}{6K}\right)^n \tag{9}$$

$$\Rightarrow \quad \mathbf{m}_F = -\frac{\phi^2\eta}{K}\left[\left(\frac{4n+3}{2n+1}\right)^{2n+1}\frac{1}{3}\left(\frac{\phi}{6K}\right)^n \left\|\mathbf{v}_F\right\|^{2n}\right]\mathbf{v}_F = -\phi\alpha\left\|\mathbf{v}_F\right\|^{2n}\mathbf{v}_F$$

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The generalized permeability function $\bar{K}$ is then given by

$$\bar{K} = K\left[\left(\frac{4n+3}{2n+1}\right)^{2n+1}\frac{1}{3}\left(\frac{\phi}{6K}\right)^{n}\|\mathbf{v}_F\|^{2n}\right]^{-1} = \left(\frac{1-\phi^{a}}{\phi^{a}b}\right)\left[\left(\frac{4n+3}{2n+1}\right)^{2n+1}\frac{1}{3}\left(\frac{1-\phi^{a}}{6(\phi^{a-1}b)}\right)^{n}\|\mathbf{v}_F\|^{2n}\right]^{-1} \quad (10)$$

If $n = 0$ (Newtonian fluid), then $\bar{K} = K$.

2.2 Some results for permeability and porosity

Considering the constitutive model proposed in the previous item, particularly concerning Eqs. (4)-(8), some results indicating the behavior of porosity and permeability are now presented. For instance, typically, porosity values for the Gulf Coast (USA) both onshore and offshore range between 10% to 30% while interior basins in the U.S. have much lower porosity values – usually from 5% to 15% (Sorbie, 1991). Many shales have high porosity values reaching 20% to 30%. Taking $\phi_1 = 0.2$, $\phi_2 = 0.3$, $K_1 = 1000\mu D$ and $K_2 = 10000\mu D$ it is possible to obtain $a = 5.68$ and $b = 9334759\ \mu$D . The behavior of $f(x)$ in this example and the identification of the parameter a is depicted in Fig. 1. Figure 2a shows the porosity and permeability trends in this case, while in Fig. 2b the associated log-log graph is presented.

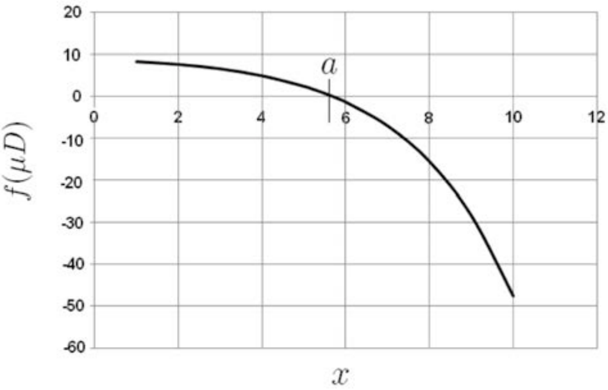
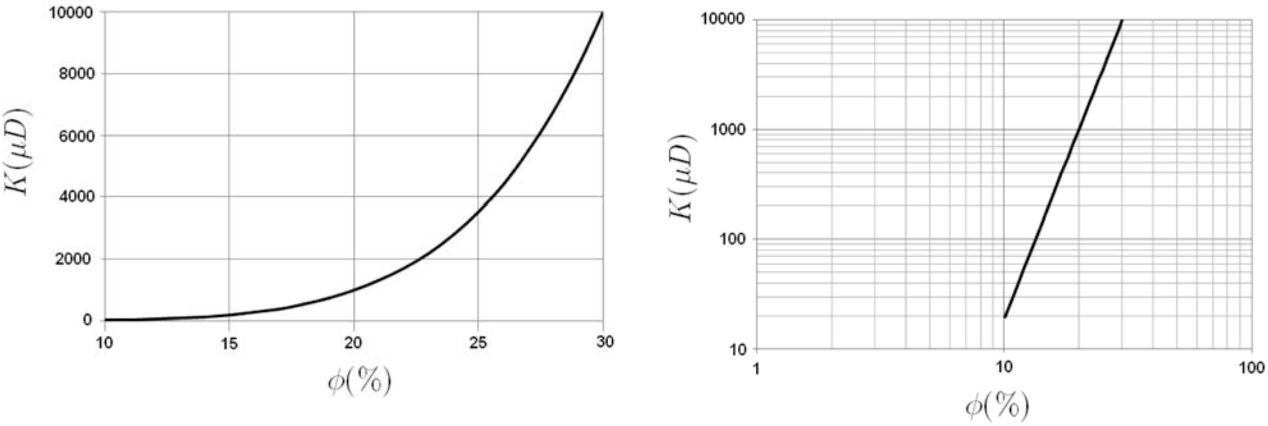


Figure 1. Identification of the parameter a .



(a) (b)

Figure 2. Permeability versus porosity.

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3. PROBLEM STATEMENT

Assuming a steady-state flow of an incompressible fluid constituent though a porous channel with height 2H, defined in Fig. 4, equations (1) and (9) and the no-slip boundary condition give rise to

$$\begin{aligned}&\nabla\cdot\mathbf{v}_F=0\\&\nabla\cdot\left[-p\phi\mathbf{I}+2\phi\beta(\mathbf{D}_F\cdot\mathbf{D}_F)^n\mathbf{D}_F\right]-\phi\alpha\left|\mathbf{v}_F\right|^{2n}\mathbf{v}_F+\rho_F\mathbf{g}=0\\&\mathbf{v}_F=0\qquad\text{on } y=\pm H\end{aligned}\tag{11}$$

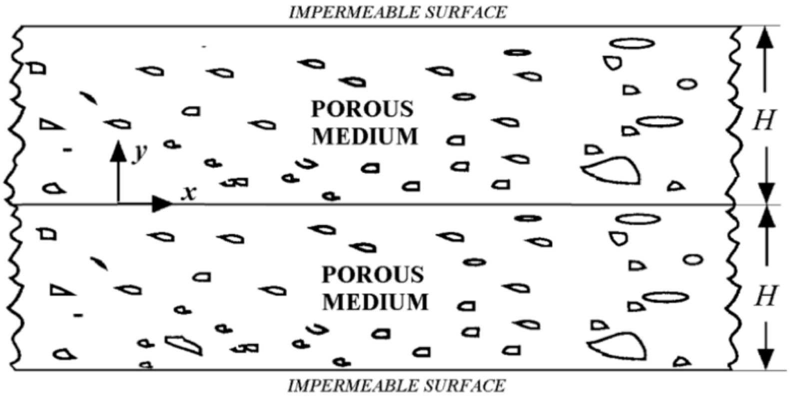

Figure 4. Flow through a plane porous channel.

Neglecting gravitational effects and making $\mathbf{v}_F = v_F\,\mathbf{i}$ with $v_F = w$ in Eq. (11), the fully developed steady-state flow may be expressed as

$$\left.\begin{aligned}&-\frac{dp}{dx}+\frac{2n+1}{2^{2n}}\beta\left|\frac{dw}{dy}\right|^{2n}\frac{d^2w}{dy^2}-\alpha|w|^{2n}w=0 && -H\le y<+H\\&w=0 && \text{at}\quad y=\pm H\end{aligned}\right\rangle\Rightarrow\ w_{\max}=\left(\frac{-1}{\alpha}\frac{dp}{dx}\right)^{\frac{1}{2n+1}}\tag{12}$$

where $w_{\max}$ represents the maximum value of w.

Eq. (12) allows rewriting the pressure drop as

$$\frac{dp}{dx}=\frac{\beta}{2^{2n}}\frac{d}{dy}\left(\left|\frac{dw}{dy}\right|^{2n}\frac{dw}{dy}\right)-\alpha|w|^{2n}w\tag{13}$$

At this point it is convenient to scale the problem. Considering the following dimensionless quantities

$$x^*=\frac{x}{H}\qquad y^*=\frac{y}{H}\qquad w^*=\frac{w}{w_{\max}}\qquad p^*=\frac{p}{\beta\left(w_{\max}/H\right)^{2n+1}}\tag{14}$$

a dimensionless pressure drop may be defined as

$$\frac{dp^*}{dx^*}=\frac{1}{2^{2n}}\frac{d}{dy^*}\left(\left|\frac{dw^*}{dy^*}\right|^{2n}\frac{dw^*}{dy^*}\right)-\underbrace{\frac{\alpha H^{2n+2}}{\beta}}_{1/\chi}\left|w^*\right|^{2n}w^*\tag{15}$$

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In Eq. (15) the parameter χ is a dimensionless number that characterizes the combined effect of porosity and permeability. Using the definitions of the parameters α and β stated in Eq. (9), it comes that

$$\frac{1}{\chi}=\frac{H^{2n+2}}{\lambda K}\left[\left(\frac{4n+3}{2n+1}\right)^{2n+1}\frac{1}{3}\left(\frac{\phi}{6K}\right)^{n}\right] \tag{16}$$

So, the parameter χ may be written as

$$\chi=\frac{\lambda K}{H^{2n+2}}\left[\left(\frac{2n+1}{4n+3}\right)^{2n+1}3\left(\frac{6K}{\phi}\right)^{n}\right] \Rightarrow \chi=\frac{3\lambda}{H^{2n+2}}\left(\frac{2n+1}{4n+3}\right)^{2n+1}\left(\frac{6}{\phi}\right)^{n}K^{n+1} \tag{17}$$

Assuming that the expression relating porous matrix permeability K and its porosity ϕ, proposed in Eq. (4) is valid, then the parameter χ may be represented the as a function of the porosity as

$$\chi=\frac{3\lambda}{H^{2n+2}}\left(\frac{\phi^{a}b}{1-\phi^{a}}\right)\left[\left(\frac{2n+1}{4n+3}\right)^{2n+1}\left(\frac{6(\phi^{a-1}b)}{1-\phi^{a}}\right)^{n}\right] \tag{18}$$

4. NUMERICAL PROCEDURE

Equations (12) form a two-point boundary value problem that can be approximated by using a fourth-order Runge-Kutta methodology coupled with a shooting technique. In order to apply this methodology, the first equation of Eq. (12) may be conveniently rewritten by considering the change of variables: $\mathbf{z}=(z_1,z_2)$, $z_1=w$ and $z_2=dw/dy$, giving rise to the following equivalent system of ordinary differential

$$\frac{dz_1}{dy}=z_2 \quad \text{and} \quad \frac{dz_2}{dy}=\frac{2^n}{(2n+1)\,\beta}|z_2|^{-2n}\left(\frac{dp}{dx}+\alpha|z_1|^{2n}z_1\right) \tag{19}$$

It is important to remark that Eq. (19) is valid only if $z_2\neq 0$. From Eq. (12) it is possible to verify that, if $\frac{dw}{dy}=z_2=0$, and then the value of z_2 is defined, since $w\left|w\right|^{2n}=z_1\left|z_1\right|^{2n}=-\frac{1}{\alpha}\frac{dp}{dx}$. Therefore, the limitation of eq. (19) when $z_2=0$ can be circumvented numerically using the previous relation. The following boundary-value problem approximates the velocity profile at the porous channel: *Find* z_1 *and* z_2*, such that*

$$\begin{gathered}\frac{dz_1}{dy}=z_2\\ \frac{dz_2}{dy}=\frac{2^n}{(2n+1)\,\beta}|z_2|^{-2n}\left(\frac{dp}{dx}+\alpha|z_1|^{2n}z_1\right)\end{gathered} \quad \text{with} \left\{\begin{array}{lll} z_1=0 & \text{at} & y=-H\\ z_1=0 & \text{at} & y=+H\end{array}\right. \tag{20}$$

The problem stated in Eq. (20) is then equivalent to finding the root of a scalar function represented as $\Phi:\mathbb{R}\to\mathbb{R}$; $s\mapsto\Phi(s)=z_1(y=L;s)$, where for a given $s\in\mathbb{R}$, representing an initial estimate, the value $\Phi(s)$ is the value of the variable z_1 at the point $y=+H$, obtained by solving the following initial boundary value problem

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$$\frac{dz_2}{dy} = \frac{2^n}{(2n+1)\,\beta}|z_2|^{-2n}\left(\frac{dp}{dx} + \alpha|z_1|^{2n} z_1\right) \quad \text{for } -H \le y < H$$
$$\frac{dz_1}{dy} = z_2 \tag{21}$$
$$\text{such that} \begin{cases} z_1 = 0 & \text{at} \quad y = -H \\ z_2 = s & \text{at} \quad y = +H \end{cases}$$

Essentially, this procedure is exactly the shooting technique, in which s represents the initial estimate of the derivative (dw/dy) at the point $y = -H$. However, in this case, Φ is a scalar function and uniqueness analysis is reduced to study the sign of this function. The initial boundary value problem may be approximated by many different techniques for initial boundary value problems in ordinary differential equations, such as the Rosembrock methods and its extensions to Runge-Kutta methods (Dahlquist and Bjorc, 1969). The root of the function $\Phi(s)$ is obtained by using unconditionally convergent procedures, such as Regula-Falsi or Bisection methods (Dahlquist and Bjorc, 1969).

5. SOME RESULTS

5.1 Behavior of generalized Newtonian fluids under the combined effects of porosity and permeability

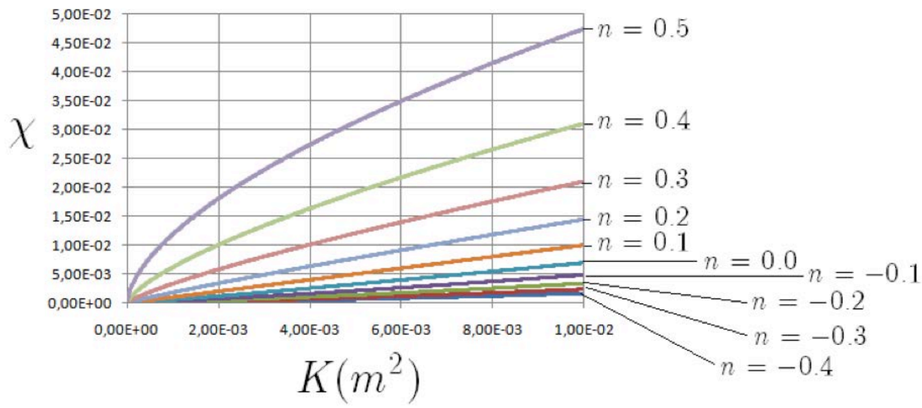

Figure 5. Variation of the parameter χ with the permeability, for distinct power-law indexes.

As stated before, the dimensionless number χ, defined by Eq. (18), characterizes the combined effect of porosity and permeability. For Newtonian fluids ($n = 0$) the parameter χ is reduced to $\chi = \lambda K / H^2$. Fig. 5 shows the variation of χ with the permeability K taking the porosity $\phi = 0.5$ and the channel height $H = 1$. The power-law index was ranged from $n = -0.4$ to $n = +0.5$, covering the so-called shear-thinning, Newtonian and shear-thickening behaviors. It may be noted that for a given power-law index as the permeability increases the dimensionless number χ, defined in this work, also increases. But it is important to note that dimensionless number χ and, thus, the permeability, tends to zero when $n \to -0.5$, no matter the value assumed by all the other parameters.

5.2 Porous channel simulation

It is interesting to note that even when the fluid is Newtonian (n=0) and the equation (12) may be denoted as Brinkman model (see Martins-Costa et al., 2013 and references therein), which may be viewed as a parameter-dependent combination of the porous Darcy flow and the viscous Stokes flow, the mathematical nature of the problem changes radically depending on the ratio of the coefficients of the two velocity terms in the momentum equation (the second equation in Eq. (11)). When the permeability is very high (as in filter applications) the influence of the Darcian term (αw) is almost irrelevant, as expected, since as $\alpha \to 0$ the flow approaches to a flow in a channel without a porous matrix, almost a Stokes flow $(dp/dx \approx \beta(d^2w/dy^2))$. On the other hand, for small permeabilities, as in reservoir simulations, the Darcian term is dominant. So, the choice of the numerical methodology to deal with this equation is very important to assure stability and accuracy of the method for all possible parameter values.

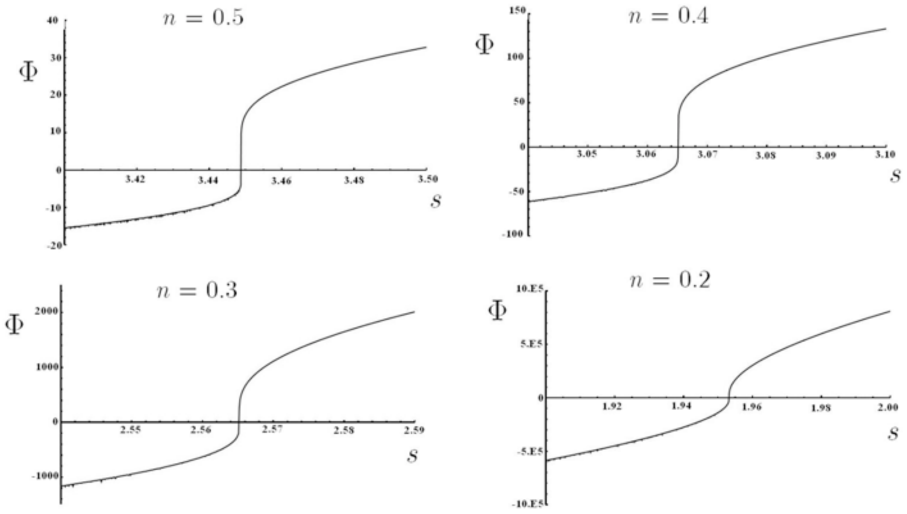

Figure 6. Behavior of the function $\Phi(t)$ for distinct positive values of power-law index.

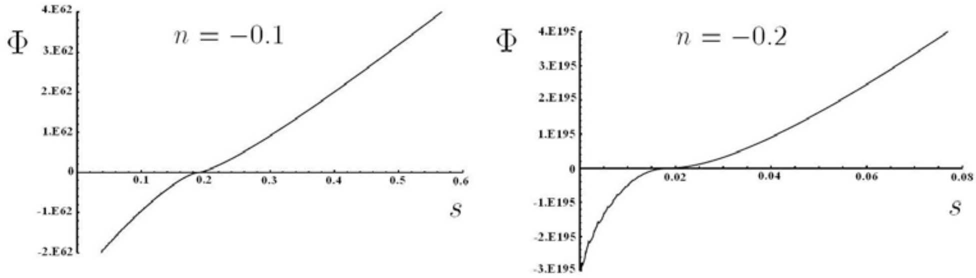

Figure 7. Behavior of the function $\Phi(t)$ for distinct negative values of power-law index.

It is important to note that w_{max} occurs exactly when $z_2 = 0$, giving rise to numerical instabilities in a neighborhood of $y = 0$, in the process of searching for the root of the function $\Phi(s)$, defined in item 4. Due to the behavior of the function $\Phi(s)$, depicted in Figures 6 and 7, Newton-Raphson technique has some convergence shortcomings. Therefore, an unconditionally convergent methodology (such as bisection or regula-falsi) is required to find the root. It is important to observe that distinct scales have been employed in the graphs depicted in these figures. The nonlinear nature of the function $\Phi(s)$ implies that a small variation of the parameter y, for $n > 0$, in the neighborhood of the root, may give rise to a huge variation of the parameter dw/dy, as depicted in Figure 6. Similarly, a small variation of the parameter dw/dy, for $n < 0$ causes a very large variation of the parameter y, in the vicinity of the root, as shown in Figure 7. As expected, the function $\Phi(s)$ for $n = 0$ (Newtonian) is represented by straight line.

Figure 8 depicts the behavior of dimensionless velocity profiles, given by $w^* = w / w_{max}$, as defined in Eq. (12), for distinct values of the power-law index *n*. It may be noted that the smaller the power-law index the flatter the velocity profile.

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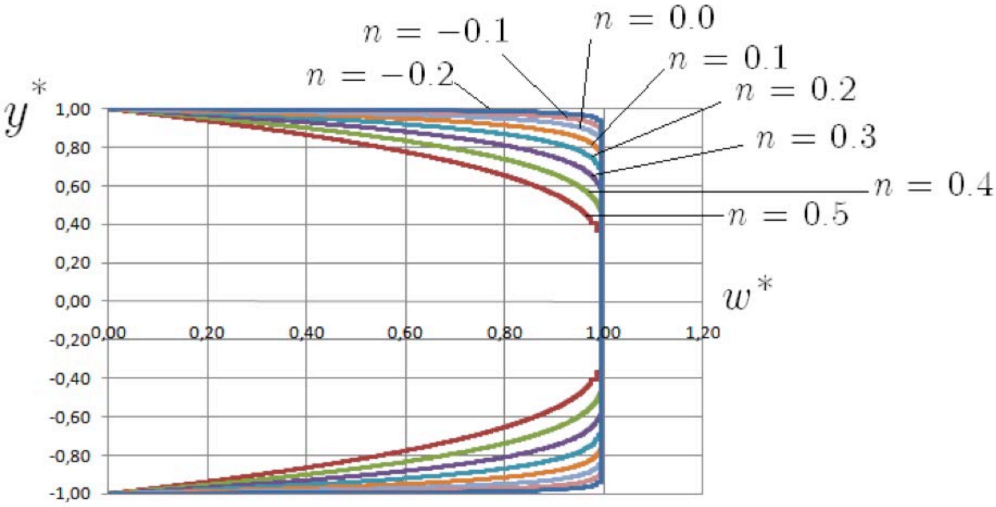

Figure 8. Dimensionless velocity profiles in a porous channel.

Physically the analysis of the problem is easier if dimensions are considered. From now, for the sake of simplicity, the following parameters will be considered in the analysis: $dp/dx = 10^{-2}\,Pa/m$, $\eta = 10^{-3}\,Pa \times s^n$, $\phi = 0.5$, $\beta = \phi\eta = 0.5 \times 10^{-3}\,Pa \times s^n$ and $K = 10^{-3} m^{-2}$.

Figure 9 shows the behavior of the maximum velocity $w_{\max}$ (in *m*/*s*), defined in Eq. (12), for distinct values of the power-law index *n*, obtained considering the material parameters stated above and distinct values of the power-law index, showing that it approaches zero for $n < -0.1$.

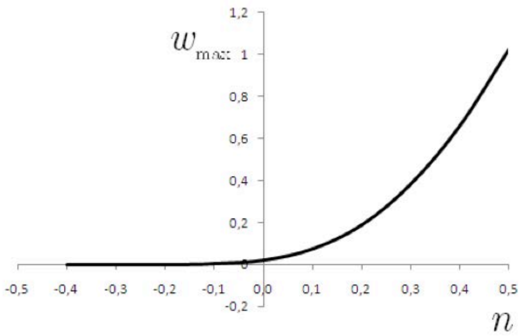

Figure 9. Behavior of the maximum velocity in a porous channel.

As it can be verified, for this value of pressure gradient, the power law model gives physically reasonable results for the velocity $w_{\max}$ only for slightly non-Newtonian fluids, i.e. for small absolute values of parameter *n*. If $n > 0.2$ the value of $w_{\max}$ is too high ($w_{\max} = 1.89 \times 10^{-1}\ m/s \approx 0.68\ km/hour$) and, if $n < -0.2$, the value of $w_{\max}$ is too small ($w_{\max} = 1.1 \times 10^{-4}\ m/s \approx 9.46\ m/day$). Such behavior can be explained from the one-dimensional shear stress versus shear strain curve ($\tau \times\ dw/dy$) depicted in Figure 10.

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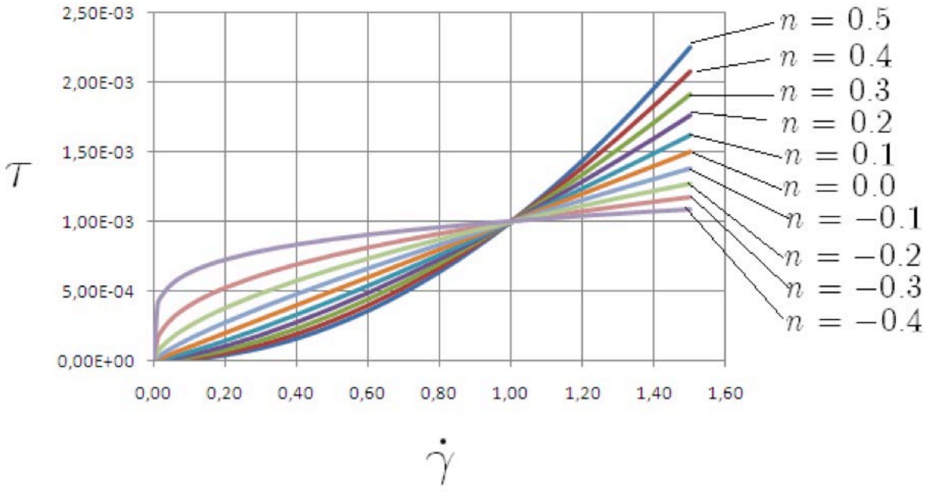

Figure 10. One-dimensional shear stress (τ) versus shear strain ($\dot{\gamma}$).

The behavior is very different depending on the shear rate dw/dy . For higher values of the power-law index *n* ($n \geq 0.2$) the viscosity is very small for smaller strain rates. For smaller values of *n* ($n \leq -0.2$) the viscosity is very high for smaller strain rates. For smaller shear rates ($dw/dy < \dot{\gamma}^*$), the shear stress in a fluid with $n > 0$ is smaller than the obtained for a Newtonian fluid ($n = 0$) while the shear stress is higher if $n < 0$. For higher shear rates ($dw/dy > \dot{\gamma}^*$), the opposite behavior is verified. The fact that a fluid with $n < 0$ requires a much higher pressure to start a flow in a porous medium than a fluid with $n > 0$ can be easily explained by looking at this shear stress versus shear strain curve. The derivative $d\tau/d\dot{\gamma}$ (with $\dot{\gamma} = dw/dy$) for small values of the shear rates tends to infinity if $n < 0$ and it tends to zero if $n > 0$. Besides, the generalized permeability is smaller for smaller values of *n*.

6. FINAL REMARKS

This work studies the flow of a power-law fluid through a porous channel limited by two impermeable flat plates employing a mixture theory approach. An equation was proposed to account for the relation between permeability and porosity. Besides, a dimensionless parameter taking into account the connection between porosity and permeability was proposed and this combined influence on the power-law index in a flow through a porous matrix was investigated.

Some numerical simulations were performed combining a fourth-order Runge-Kutta method with a shooting strategy. The shear-thinning and shear-thickening behavior is discussed for a power-law fluid.

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