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MODELING AND LQR/LQG CONTROL OF A CANTILEVER BEAM USING PIEZOELECTRIC MATERIAL

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Abstract. : *This paper address the development of a control scheme to be used to attenuate mechanical vibrations of an elastic aluminum cantilever beam that is modeled analytically according to the theory of Euler-Bernoulli. In the modeling procedure, the piezoelectric actuators apply concentrated moments in part of the structure when subjected to an electric potential in (d_{31}) mode. Two different algorithms, a LQR (Linear Quadratic regulator) and a LQG (Linear Quadratic Gaussian) were tested in the control design. One aspect that contributes to the effectiveness of these controllers is the correct determination of the weighting matrices of the state (Q) and control (R), which influence the gain matrix. Therefore, it was developed a methodology for choosing the values of these matrices. The proposed methodology is based in the construction of a map that establishes the compromise between the settling time of the system and the control signal applied in terms of electric potential with the aim of determining the appropriate weighting matrices that respect the real actuator maximum voltages. The simulations to an unitary module impulsive force allows to conclude that the control strategy using the LQG controller presents better performance in terms of settling time, damping and control signal energy when compared to the LQR controller.*

Keywords: *Piezoelectric Actuator, Vibration Control, LQR and LQG Control.*

1. INTRODUCTION

This paper presents a LQR and LQG mechanical vibrations control design for a cantilever beam modeled according the elastic theory of Euler-Bernoulli. The piezoelectric actuators are embedded in the structure and apply concentrated moments in the structure when an electric potential is applied in the (d_{31}) mode. In the areas of structural and control engineering, the search for innovation is constant. In structural engineering, one of main goal is to design structures that can withstand several loads and dynamics, absorb energy and transmit them without collapse or vibrate excessively during their lifetime. In control engineering, an issue that is focused, due to the possibility of generating losses to industrial processes, is the excess of undesirable vibrations that can be transmitted to the engine, instrument or equipment. The attenuation of these vibrations using smart materials or more sophisticated controllers is, probably, the subject of study and major investments in the control vibration area.

The growing need for lighter and adaptable structures especially in applications such as aerospace, automotive and robotics show the importance of advanced methods for structural optimization and active control. Smart Structures employ three basic elements: sensors, that record internal and external information; actuators, that apply forces; and control systems, that make decisions. These structures have numerous applications, for example, in spacecraft, aircraft, automobiles, ships and robots.

Some papers present studies about the modeling and optimal location of actuators and sensors in smart structures and in the vibration control of these structures. Oliveira, 2008, present a study about the positioning of piezoelectric actuators in smart structures using measures of modal space controllability, obtained by finite element method and singular values analysis. These values are used to obtain an index that quantifies the system controllability so as to position the actuator while minimizing the driver effort. Agrawal and Treanor, 1999, present analytical and experimental results on the optimal placement of piezoelectric actuators for beam structures. They determine the

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actuator voltage and size that minimize the error between the desired and obtained deformed shape using the Euler-Bernoulli model.

Kumar and Narayanan, 2008, consider the optimal location of piezoelectric actuator-sensor pairs, placed in a flexible beam. In this paper, genetic algorithms were used to optimize the LQR performance index. Silveira and Fonseca, 2010, present a simultaneous design for structural topology and the location of actuators. The topology optimization problem is formulated for three material phases (two solid and one empty): a non-piezoelectric isotropic elastic material forms the structural part, while a piezoelectric material forms the active part. It was proposed a nested solution approach, where the main loop optimization distributes solid material and empty space, and a subprocess defines where the material must have piezoelectric properties by optimizing a control law.

Vasques and Rodrigues, 2006, presented a numerical study on the active beam vibration control with piezoelectric material. The article presents a comparison between classical control strategies (constant gain and velocity feedback) and optimal control strategy (LQR and LQG).

The present paper is organized as follows: Section 2 presents the modeling of the beam with piezoelectric material. The theory of LQR and LQG controller is depicted in section 3. Section 4 presents the methodology for choosing the weighting matrices Q and R used in control projects. The results, in terms of the beam displacement behavior, and the discussion about the controllers performance are presented in section 5. Finally, in Section 6, the conclusion of the present work is outlined.

2. CANTILEVER BEAM MODEL WITH PIEZOELECTRIC ACTUATOR

The present model uses a cantilever beam with a piezoelectric actuator that applies a concentrated moment in the structure (Fig. 1). According to the Euler-Bernoulli theory, we obtain the dynamic equation using moment induced and moment flexion in function of the transverse displacement (Dimitriadis et al., 1991), resulting in Eq. (1):

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + YI \frac{\partial^4 w(x,t)}{\partial x^4} = f(x,t) + \frac{\partial^2 m_x(x,t)}{\partial x^2} \tag{1}$$

where ρ is the density, A is the transverse area, Y is the Young module, I is the inertia moment, f is the external force, w is the displacement transverse and m is the moment induced by piezoelectric. According to Wang et al., 2001, the aplication of a electric potential in the piezoelectric element results in the following relation between strain (σ_x) and stress (ε_x) in the beam:

$$\sigma_x = Y_{pe}\, \varepsilon_x - e_{31} \frac{\phi}{h_{pe}} \tag{2}$$

with:

$$e_{31} = Y_{pe}\, d_{31} \tag{3}$$

where Y_{pe} is the piezoelectric Young module, h_{pe} is the piezoelectric thickness, e_{31} is the piezoelectric constant, ϕ is the electric potential and d_{31} is the coefficient piezoelectric.

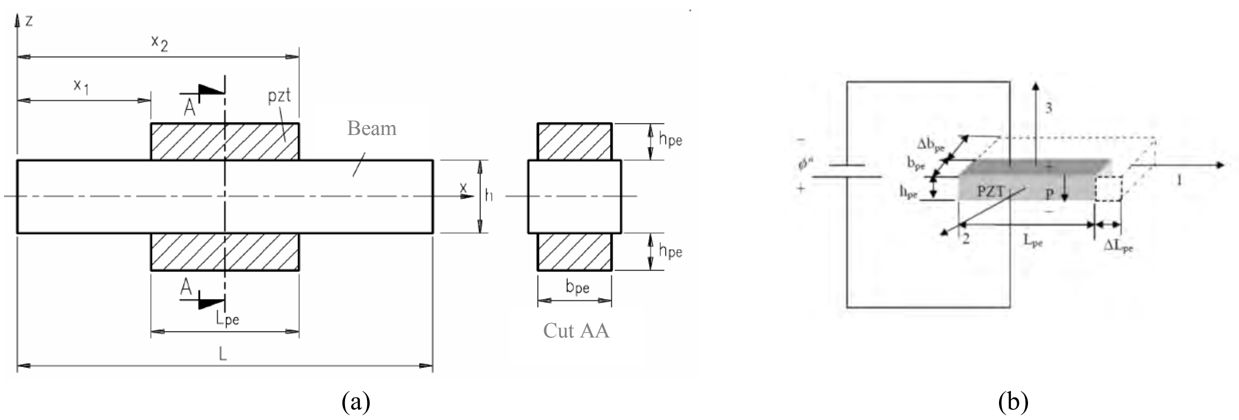

Figure 1. (a) Beam with piezoelectric element, (b) piezoelectric effect d_{31}

To calculate the moment applied in the structure due the piezoelectric effect, we can use the relation between flexion strain and moment according the elastic theory. The inertia moment of area of the piezoelectric material is transferred to the axis x through the parallels axis theory. Due to the little thickness, we do not use the portion referring the piezoelectric element inertia. Using these considerations, the moment applied in structure due the piezoelectric effect d_{31} can be expressed by Eq. (4).

$$m_x = Y_{pe} b_{pe} h_{pe} \left(\varepsilon_x - d_{31} \frac{\phi}{h_{pe}} \right) \frac{1}{2} \left(h + h_{pe} \right) \tag{4}$$

where b_{pe} is the width of piezoelectric and h is the thicknesse of beam. The transverse structural displacement ($w(x,t)$) can be related with the stress by

$$\varepsilon_x = -\frac{h}{2} \frac{\partial^2 w(x,t)}{\partial x^2} \tag{5}$$

Thus, substituting Eq. (5) in Eq. (4):

$$m_x = Y_{pe} b_{pe} h_{pe} \left(-\frac{h}{2} \frac{\partial^2 w(x,t)}{\partial x^2} - d_{31} \frac{\phi}{h_{pe}} \right) \frac{1}{2} \left(h + h_{pe} \right) \tag{6}$$

Due to the fact that the actuator is located in a specific part of the structure, we limit the interval of actuator action (between x_1 and x_2), using the Heaviside function defined as,

$$H(x - x_1) = \begin{cases} 1 & x \geq x_1 \\ 0 & x < x_1 \end{cases} \tag{7}$$

Multiplying the Heaviside equation in Eq. (6) and replacing it in Eq. (1), we have the dinamyc behavior of a cantilever beam with the piezoelectric element:

$$\begin{aligned} \rho A \frac{\partial^2 w(x,t)}{\partial t^2} + YI \frac{\partial^4 w(x,t)}{\partial x^4} + \left[Y_{pe} b_{pe} \frac{h}{2} \left(\frac{h + h_{pe}}{2} \right) \frac{\partial^4 w(x,t)}{\partial x^4} \right] [H(x - x_1) - H(x - x_2)] \\ = f(x,t) - \left[Y_{pe} d_{31} b_{pe} \phi \left(\frac{h + h_{pe}}{2} \right) \right] \frac{\partial^2}{\partial x^2} [H(x - x_1) - H(x - x_2)] \end{aligned} \tag{8}$$

Equation (8) has its analytic solution given through the use of the modal expansion expressed by Eq. (9):

$$w(x,t) = \sum_{i=1}^{n} \chi_i(x) \eta_i(t) \qquad i = 1,2,3\ldots n \tag{9}$$

where n is the vibration mode number, η modal coordinate in the time and χ is the vibration modes. Replacing the Eq. (9) for one particular vibration mode i in Eq. (8), we obtain:

$$\rho A \frac{\partial^2 \chi_i(x) q_i(t)}{\partial t^2} + YI \frac{\partial^4 \chi_i(x) \eta_i(t)}{\partial x^4} + C_5 \frac{\partial^4 \chi_i(x) \eta_i(t)}{\partial x^4} [H(x - x_1) - H(x - x_2)] = f(x,t) + C_6 \frac{\partial^2}{\partial x^2} [H(x - x_1) - H(x - x_2)], \tag{10}$$

$$\begin{aligned} C_5 &= Y_{pe} b_{pe} \frac{h}{2} \left(\frac{h + h_{pe}}{2} \right) \\ C_6 &= -Y_{pe} d_{31} b_{pe} \phi \left(\frac{h + h_{pe}}{2} \right) \end{aligned} \tag{11}$$

Differentiating the Eq. (10), multiplying by function $\chi(x)$, integrating over the length of the beam, and using the modes orthogonally principle (Meirovitch, 1984), we obtain,

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = f_i(x,t) + C_6 \left[\frac{d}{dx} \chi_i(x_1) - \frac{d}{dx} \chi_i(x_2) \right] \tag{12}$$

where:

$$f_i(x,t) = \int_0^L \chi_i(x) f(x,t) dx$$
$$\chi_i(x_1) = \chi_i(x)\, H(x - x_1)$$
$$C_5 q_i(t) \int_0^L \chi_j(x) \frac{d^4 \chi_i(x)}{dx^4} [H(x - x_1) - H(x - x_2)] dx = 0 \tag{13}$$
$$C_6 \int_0^L \chi_j(x) \frac{d^2}{dx^2} [H(x - x_1) - H(x - x_2)] dx = C_6 \left[\frac{d}{dx}\chi_i(x_1) - \frac{d}{dx}\chi_i(x_2)\right]$$

In the case with the presence of more than one actuator on the beam, Eq. (14) assume the following generic way:

$$\begin{aligned}\ddot{q}_i(t) + \omega_i^2 q_i(t) &= f_i(x,t) + C_6 \left[\frac{d}{dx}\chi_i(x_{11}) - \frac{d}{dx}\chi_i(x_{12})\right] + C_6 \left[\frac{d}{dx}\chi_i(x_{21}) - \frac{d}{dx}\chi_i(x_{22})\right] + \cdots \\ &+ C_6 \left[\frac{d}{dx}\chi_i(x_{p1}) - \frac{d}{dx}\chi_i(x_{p2})\right]\end{aligned} \tag{14}$$

To use the LQR and LQG controller, it is necessary to write the dynamic equation of the beam in a space state representation. Therefore, it is necessary to preform the adition of the following state vector (Kwon, 1997):

$$\{z\} = \begin{Bmatrix} \{\eta_i(t)\} \\ \{\dot{\eta}_i(t)\} \end{Bmatrix} \tag{15}$$

where

$$\{q\} = [\eta_1 \;\; \eta_2 \;\; \dots \;\; \eta_n]^T \tag{16}$$

and

$$\{\dot{q}\} = [\dot{\eta}_1 \;\; \dot{\eta}_2 \;\; \dots \;\; \dot{\eta}_n]^T \tag{17}$$

Considering p the actuators number, then:

$$\{u\} = \left[k_a \phi_1 \;\;\; k_a \phi_2 \; \dots \;\; k_a \phi_p\right]^T \tag{18}$$

with:

$$k_a = -Y_{pe}\, d_{31}\, b_{pe} \left(\frac{h + h_{pe}}{2}\right) \tag{19}$$

Equation (20) is in the space state form considering the applied forces by actuator and the external disturbances, where x is the state vector, $\boldsymbol{A}$ is the dynamic matrix, $\boldsymbol{B}$ is the input matrix, u is the control vector and f_e is the external force vector:

$$\{\dot{x}\} = \boldsymbol{A}x + \boldsymbol{B}u + f_e(t) \tag{20}$$

with:

$$\boldsymbol{A} = \begin{bmatrix} 0 & I \\ \Omega & \Delta \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \omega_1^2 & 0 & \cdots & 0 \\ 0 & \omega_2^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \omega_n^2 \end{bmatrix} \tag{21}$$

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$$\Delta = \begin{bmatrix} -2\zeta_1\omega_1 & 0 & \cdots & 0 \\ 0 & -2\zeta_2\omega_2 & \cdots & 0 \\ \vdots \; \vdots & & \ddots & \vdots \\ 0 \;\; 0 & & \cdots & -2\zeta_n\omega_n \end{bmatrix}$$

and ζ_n is the damping ratio and ω_n is the natural frequency of the vibration mode n.

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ B_a \end{bmatrix}$$

$$\boldsymbol{B}_a = \begin{bmatrix} B_{a1}^1 & B_{a2}^1 & \cdots & B_{ap}^1 \\ B_{a1}^2 & B_{a2}^2 & \cdots & B_{ap}^2 \\ \vdots & \vdots & \ddots & \vdots \\ B_{a1}^n & B_{a2}^n & \cdots & B_{ap}^n \end{bmatrix} \tag{22}$$

$$B_{ap}^n = \left[\frac{d}{dx}\chi_n(x_{p1}) - \frac{d}{dx}\chi_n(x_{p2})\right] \tag{23}$$

The external forces vector is described as:

$$\{f_e(t)\} = [f_1(t) \quad f_e(t) \quad \cdots \quad f_n(t)]^T \tag{24}$$

3. LQR AND LQG OPTIMAL CONTROL

The use of a controller is required for a device that has an ideal behavior, i.e., the controller must manage and to ensure an adequate dynamic behavior. According to Vasques and Rodrigues, 2006, the feedback gains are chosen to change the dynamics of the system, aiming to reduce the motion of the mechanical system, acting as a regulator.

The LQR method is based on the minimization of a quadratic performance index that is associated with the energy of the state variables and control signals. The goal of LQR controller design is to establish a compromise between the energy state and control by minimizing a cost function defined by the Eq. (25).

$$J = \int_0^{t_f} (\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{u}^T\boldsymbol{R}\boldsymbol{u})dt \tag{25}$$

where $\boldsymbol{Q}$ is a real symmetric or Hermitian matrix positive definite or positive semidefinite, and expresses the weight of the state variables and $\boldsymbol{R}$ is a Hermitian or real symmetric positive definite matrix and express energy expenditure derived by the control signal (Preumont, 2002) and $\boldsymbol{x}$ is the state vector. It is assumed in this problem that the control vector $\boldsymbol{u}(t)$ is unrestricted. A well-constructed project must take into account a consistent choice for these matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$, where a widely used choice is the identity matrix, or a multiple thereof.

According Ogata, 1998, the linear control law given by Eq. (26) is the optimal control law. Consequently, if the matrix of elements $\boldsymbol{G}$ are determined to minimize the performance index (Eq. (25)), then $\boldsymbol{u}(t)$ is suitable for whatever initial state $x(0)$.

$$\boldsymbol{u} = -\boldsymbol{G}\boldsymbol{x} \tag{26}$$

The optimal gain matrix is expressed by $\boldsymbol{G} = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P}$, where $\boldsymbol{P}$ is the solution of the Riccati equation given by:

$$\boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{Q} = \boldsymbol{0} \tag{27}$$

Considering the state feedback and the feedback gains matrix, the open-loop state equation is given by:

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} - \boldsymbol{B}\boldsymbol{G})\boldsymbol{x} \tag{28}$$

As a hypothesis, it is assumed that all states are completely observable and that they are related with the outputs (Ogata, 1998; Burl, 1999 and Preumont, 2002).

Considering that the plant and the output measurements are subject to Gaussian noise, the LQG regulator uses the Kalman filter to estimate the states in an optimal way, while the gains are determined by the mean square error criterion. Regarding the controller, the LQG follows the same principles of the LQR controller.

The LQG strategy can be definite as a problem where is necessary to project a control law that keeps the system stable and minimize a criterion based on squared errors (Maciejowski, 1989). Considering the time-invariant linear system completely controllable and observable, this problem can be formulated as:

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u + w, \\ y = \boldsymbol{C}x + v, \tag{29}$$

where $\boldsymbol{x}$ is the state vector, $\boldsymbol{u}$ is the control vector and $\boldsymbol{y}$ is the outputs vector contamined by v; w e v are modeled as white noise, featuring Gaussian stochastic processes with zero mean. It is considered that w e v are not correlated. The output vector is the system states, and the output matrix is an identity matrix.

$$E\{ww^T\} = \boldsymbol{R_f} \geq 0 \qquad E\{vv^T\} = \boldsymbol{Q_f} > 0 \qquad E\{wv^T\} = 0 \tag{30}$$

In the LQG problem we wish to minimize the cost function:

$$J = \int_0^{t_f} (\boldsymbol{x}'\boldsymbol{Qx} + \boldsymbol{u}'\boldsymbol{Ru})dt, \tag{31}$$

where the matrices Q and R are the same as defined in Eq. (25).

The solution of the problem is a constant linear feedback gain, where G and the solution of the problem LQR and $\hat{\boldsymbol{x}}$ is the state obtained with the Kalman-Bucy filter.

$$\boldsymbol{u} = -\boldsymbol{K}\hat{\boldsymbol{x}}. \tag{32}$$

According to Preumont, 2002, since the error covariance matrix $\boldsymbol{K}$ depends on the gain matrix of the observer, it look for a optimum effort of $\boldsymbol{K}$ which minimizes the quadratic function given by Eq. (31). This problem is solved by using a Kalman-Bucy ignoring completely the problem of control,

$$J = E[(a^T e)^2 = a^T E[ee^T]a = a^T \boldsymbol{P}a], \tag{33}$$

where a is a vector of arbitrary coefficients. There is a choice of $\boldsymbol{K}$ which J is the minimum for all a,

$$\boldsymbol{K} = \boldsymbol{PC^T R_f^{-1}}, \tag{34}$$

where $\boldsymbol{P}$ is the covariance matrix of the optimal observer given by the solution of the Riccati equation expressed by Eq. (35).

$$\boldsymbol{AP} + \boldsymbol{PA^T} + \boldsymbol{Q_f} - \boldsymbol{PC^T R_f^{-1} CP} = \boldsymbol{0}. \tag{35}$$

It is possible to show that the eigenvalues of the full system (filter eigenvalues summed to the eigenvalues of the controller) are comprised by the addition of the eigenvalues of the filter and the LQR (Kwakernaak and Sivan, 1972).

Combining the Kalman-Bucy with the LQR controller, this control is known as LQG and is related to the system dynamics expressed by Eq. (36).

$$\begin{Bmatrix} \dot{x} \\ \dot{e} \end{Bmatrix} = \begin{bmatrix} \boldsymbol{A} - \boldsymbol{BG} & \boldsymbol{BG} \\ \boldsymbol{0} & \boldsymbol{A} - \boldsymbol{KC} \end{bmatrix} \begin{Bmatrix} x \\ e \end{Bmatrix} + \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{I} & -\boldsymbol{K} \end{bmatrix} \begin{Bmatrix} w \\ v \end{Bmatrix}. \tag{36}$$

The triangular shape implies in the separation principle. The eigenvalues of the closed-loop system consist of two uncoupled sets that match the LQR and Kalman observer.

4. METHODOLOGY FOR THE CHOICE OF THE MATRICES Q AND R

One issue that contributes to the effectiveness of the LQR controller is the correct determination of the weighting of the state matrix **Q** and of the control matrix **R** seeking to satisfy certain conditions of control design. The determination of these matrices has a direct influence on the calculation of the control gains. An arbitrarily reduction can be achieved at the expense of an increase in the control signal amplitude, implying, in some cases, in practical impossibility to implement such a solution. On the other hand, an arbitrarily large reduction in the control can cause an increasing of the state values, situation often undesirable in certain control processes. Therefore, the objective is to determine the values

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that best meet a prescribed criterion, such as the percentage of response, the maximum stabilization control or the stabilization time, that, when achieved, reflect a better system performance, in order to determine a value near the reality of the problem.

The methodology developed is based on a scan between **Q** and **R** predetermined values, generating a map of compromise that takes into account the settling time of the system versus the energy used in the control. The method includes the following steps:

1. Choosing the values for scanning **Q** and **R**.
2. Reading the values of **Q** and **R** defined in step one (each possible combination is used in the dynamic system and used to the LQR control).
3. Identification of the control signal used for each of the combinations between the prescribed values of **Q** and **R**.
4. Presentation of the settlement time map of the system and the control signal for each possible combinations of weighting matrices.
5. Analyzing the map generated in stage 4. With the results of the analysis it is possible to choose which is the best set of values of **Q** and **R** to a given specification of the problem.

Suggested values for the scan of $\boldsymbol{R}$ are in the range of $\boldsymbol{R} = 0.1{:}\,0.1{:}\,1$, while for the values of $\boldsymbol{Q}$ it can be used $\boldsymbol{Q} = 10^{(n-1)}$ (with $n = 1,2 \ldots 10$).

5. RESULTS

In this section the results for the use of proposed methodology for choosing the values of the matrix Q e R are presented. The strategy was applied to the the beam model with piezoelectric material using LQR and LQG controllers. The simulation was performed using MATLAB software. The cantilever beam damping ratio was taken as $\zeta = 0.01$ for all the vibration modes and the excitation applied in the structure consists of a unit module impulsive force. The aim is to compare the LQR and LQG control performances for the first and the second vibration mode. Table 1 shows the geometry and material properties of the aluminum. Table 2 shows the geometry and material properties of the piezoelectric material.

Table 1. Geometry and Material Properties of Aluminum.

Properties	Value	Unit
Length (L)	1,5	*m*
Width (b)	0,075	*m*
Thickness (h)	0,01	*m*
Density (ρ)	7800	*kg/m³*
Elastic Modulus (Y)	210x10^9	*N/m²*
Area (A)	7,5x10^{-4}	*m²*
Inertia Moment (I)	6,25x10^{-9}	*m⁴*

Table 2. Geometry and Material Properties of Piezoelectric

Properties	Value	Unit
Length (L_{pe})	0,15	*m*
Width (b_{pe})	0,05	*m*
Thickness (h_{pe})	0,0075	*m*
Elastic Modulus (Y_{pe})	139x10^9	*N/m²*
Constant Piezoelectric (e_{31})	-6.8	*C/m²*
Constant Dielectric (d_{31})	-4.89x10^{11}	*m/V*

The piezoelectric actuator is located near the tip cantilever due the highest controllability index of this region for the first two vibration mode. The actuator used have a working range between $\pm$ 2000 v, and the methodology proposed in this work is used to choose the best matrices Q and R that achieve the desired control performance concomitantly guaranteeing that the control signal keeps confined in the working range of the actuator. In the present case, the value nearer of actuator limit was used to choose the matrices.

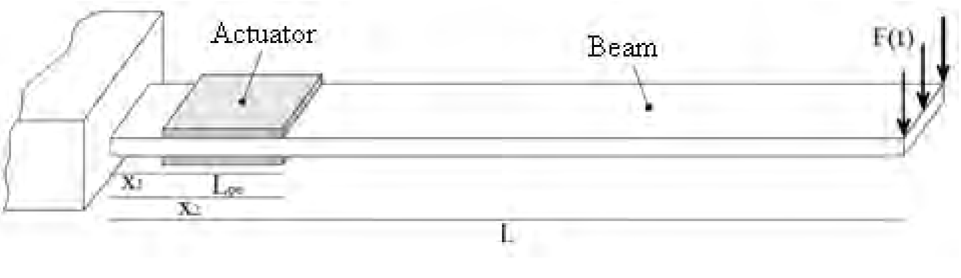
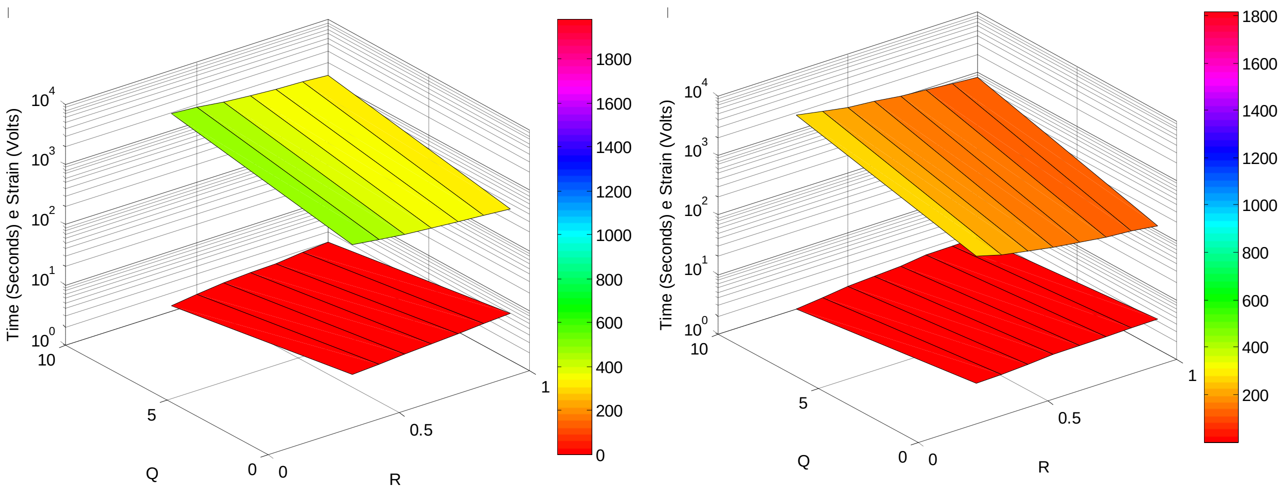


Figure 2. Cantilever Beam with piezoelectric material.

According to Fig. 2, the geometric dimensions of the actuator assume the following values: $x_1 = 0.1\ m$, $x_2 = x_1 + L_{pe}$.

5.1 Control of the First and Second Vibration Mode.

It was first used the methodology for choosing matrices **Q** and **R**, later it was generated the map of possibilities for which the control signal remains within the actuator working range. Figure 3 (a) presents the map of first vibration mode and Fig. 3(b) presents the map of the second vibration mode.



(a) (b)

Figure 3. Analysis of the control signal and stabilization time (axis Z) for the set of possible matrices Q and R (plane XY).

Trough of the map presented in the Figure 3, it is possible choice the best set of values of **Q** and **R** to a given specification of the problem, such as stabilization time or control signal. In the superior surface is presented the work voltage maximum of the actuator to each possible combination (according Step 2). On the other hand, in the inferior surface is presented the respective stabilization time. In this work, it was used how choice criteria, the combination nearer of the working limit of the actuator. The superior surface (Figure 3) is observed for this choice.

The values for the matrices nearer of the working range limit of the actuator control signal, for first and second vibration mode respectively, are:

$$\mathbf{Q} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \qquad \mathbf{R} = [0.4] \tag{37}$$

and

$$\mathbf{Q} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \qquad \mathbf{R} = [0.3] \tag{38}$$

Using the values of the equation (37) and (38), it was made the control simulations with the LQR and LQG control schemes. Figure 4 shows the result of the beam displacement in the first vibration mode (Fig. 4 (a)) and in the second vibration mode (Fig. 4(b)) with and without control actions. It is possible to observe that the LQG control has one stabilization time smaller and with smaller oscillations amplitudes. However, both controllers have stabilization time similar, around 0,33 seconds for the first vibration mode and, for the second vibration mode, around 0,15 seconds for LQG and 0,35 seconds for the LQR controller.

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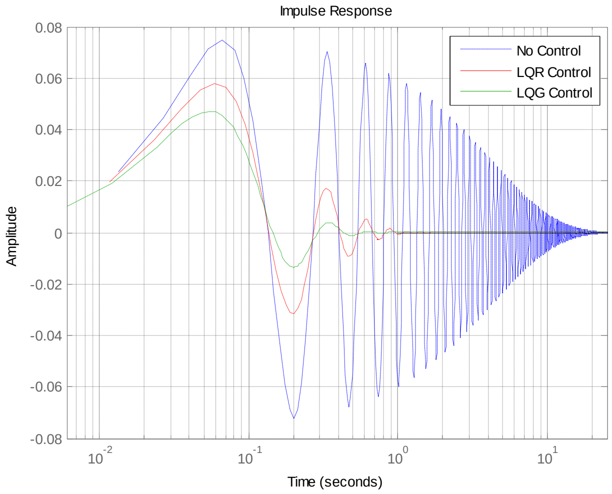
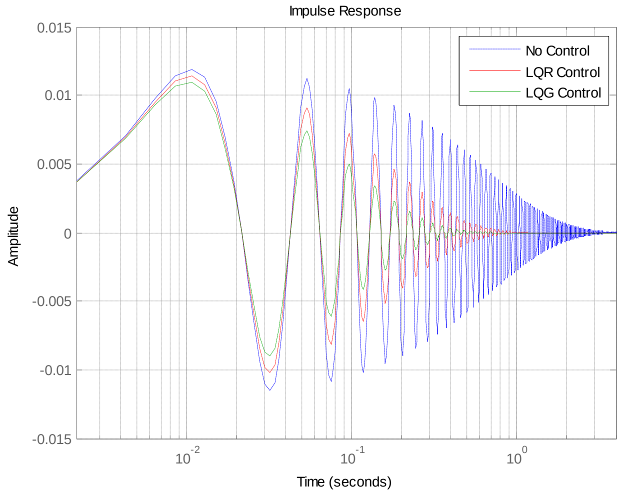


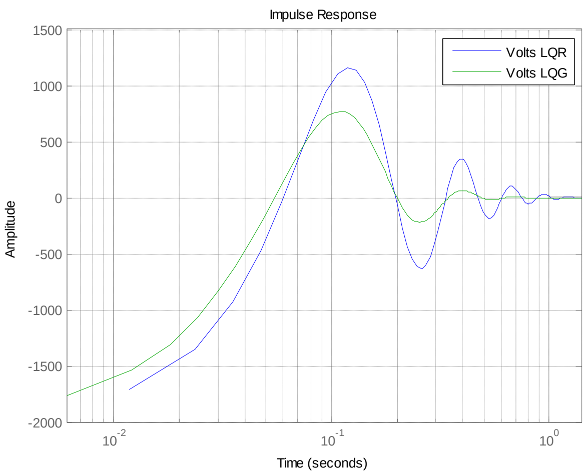
(a) first vibration mode



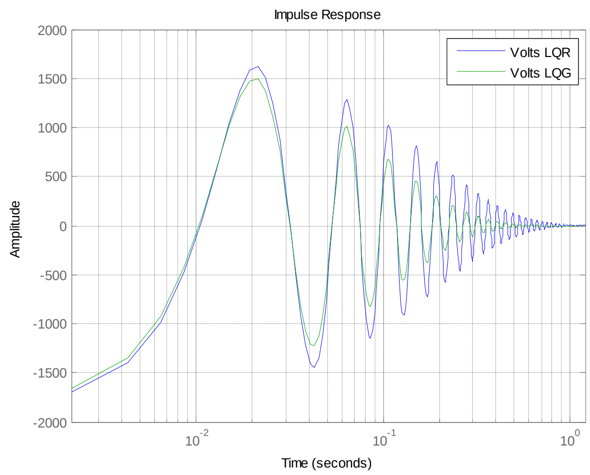
(b) second vibration mode

Figure 4. Results using the LQR and LQG control.

The control signal is presented in Fig. 5. It is possible to observe that for both vibrations mode the signal is within of the actuator working range. It can also be seen that, when using the LQR control, the beam has smaller oscillations and the signal necessary for the control is smaller than in the LQR control scheme.



(a) First vibration mode.



(b) Second vibration mode

Figure 5. Control Signal (Volt), using the LQR and LQG control.

5. CONCLUSION

This paper presents the proposition of a methodology for selection of the weighting matrices Q and R used in the design of controllers LQR and LQG for beam vibration control using piezoelectric actuators. The methodology is based on the construction of a compromising map between the response settling time and the amplitude of the control signal. Analyzing the map, it is possible to choose the values of Q and R according to the desired characteristics of the system with respect to these two parameters.

To validate the proposed methodology, it was performed simulation of an aluminum cantilever beam modeled analytically according to the theory of Euler-Bernoulli where it was applied an impulsive force. Two cases were simulated: with only one piezoelectric actuator and with two actuators acting simultaneously in the structure. In both cases, the objective was to compare the controlled system through of the control strategy using the LQR and LQG controller for the first two modes of vibration of the structure.

The results demonstrated that the proposed methodology is a suitable tool for helping the definition of matrices Q and R, in a way that the prescribed closed loop behavior can b accomplished while the actuator control limits are preserved. Also, by means of the results analysis, it was possible to verify that, in all simulated cases, the strategy using the LQG controller showed better results in terms of settling time, structure oscillations and control signal level when compared to the LQR controller.

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7. RESPONSIBILITY NOTICE

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