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NATURAL FREQUENCY ESTIMATION FOR THE HULL GIRDER VERTICAL VIBRATION OF A BULK CARRIER NAVIGATING IN SHALLOW WATER

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Abstract. *One of the requirements of ship design is to minimize the vibration levels of the structure and prevent the operation of the ship near resonance conditions, which may happen due to the interaction between the ship structure and the main excitation forces. To evaluate the vibration levels, it is necessary to analyze the forced vibration response of the structure, which requires knowing the structural damping, information that is not always available in the preliminary design phase. On the other hand, the evaluation of hull girder resonance condition requires the natural frequencies of the structure to be compared with the excitation frequencies. In this paper, two methods are used to estimate the natural frequencies for the hull girder vertical vibration of a bulk carrier operating in shallows waters: the first, is to perform a modal analysis of the structure's 3D finite element model (FEM); the second, is to perform a modal analysis of the structure's 1D FEM model, taking into account only some of the ship's cross section properties. The results will be compared with the excitation frequencies of the ship and the pros and cons of each methodology will be discussed.*

Keywords: *Ship Vibration, natural frequency, shallow water.*

1. INTRODUCTION

In the world, inland waterways is one of the most economical means for transporting loads within countries and continents, because it demands a much lower energy consumption compared to modal road and even in some cases the rail. The waterway is heavily used to transport large volumes of low-value cargo such as grain and agricultural inputs.

The navigation of vessels through rivers presents problems due to the low depth, resulting in the restriction of the draft that can operate. Due to the proximity of the bottom, velocity flow around the hull is increased, thus generating a region of low pressure causing the vessel to increase its draft. Consequently problems emerge, like the collision with the bottom of the hull and the propeller objects, excessive vibration of the propulsion system and the structure as a whole.

As a result, in navigation in shallow waters, there is a considerable increase of added mass compared with navigation in deep water, due to the fact that the first ship vibration natural frequency decreases and thus can be close to any excitation frequency (machinery onboard propulsion system, etc.) and, therefore, the ship can find a resonance condition.

Structural resonance (local or global) is a condition that must be avoided, because high vibration levels can produce discomfort in the crew (accommodation), but can also lead to the structure to collapse, causing huge human and economic losses.

2. VIBRATION ANALYSIS

Any move that repeats after a time interval is called vibration or oscillation. The most human activities involve vibration, since any system possessing mass and stiffness is subject to vibration.

A vibration system has, in general, a medium to store potential, kinetic and a gradual loss of energy. The vibration of a system involves alternating transfer of its potential energy to kinetic energy and vice versa. However, if the system is damped, certain amount energy is dissipated in each vibration cycle.

2.1 Discrete System Analysis

According to RAO (2011), most of the vibration systems are continuous with an infinite number of degrees of freedom. Vibration analysis of continuous systems requires the solution of partial differential equations, which is very complicated. In fact, for some partial differential equations, there is no analytical solution. On the other hand, the analysis system with multi degrees of freedom requires solving a system of ordinary differential equations, which is relatively simple. Then, to simplify the analysis, the system is approximated as a discrete system with a large number of degrees of freedom.

To discrete the system, it must be taken into account that the discrete model has to accurately represent the continuous model. In this context, the discrete model is constructed from the use of lumped masses, springs and dampers, all with properties equivalent to the continuous system. Thus, the system of "n" degrees of freedom has "n" natural frequencies that correspond to its own mode of vibration. Figure 1 shows the discrete model of a supported beam, using lumped masses.

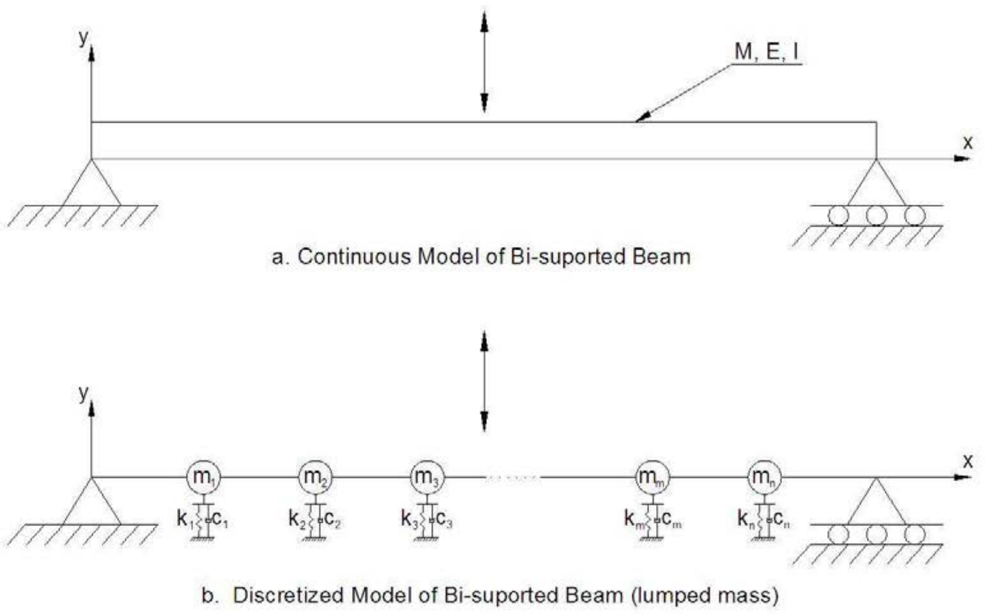

Figure 1. Discrete Model of a Supported Beam.

Having discrete vibratory system, we must obtain the motion equations which are obtained from the application of Newton's second law. However, it is customary to derive the motions equations for systems with "n" degrees of freedom from the Lagrange equations.

From Newton's second law we have:

$$m_i \ddot{x}_i = \sum_j F_{ij} \qquad \text{(for mass } m_i\text{)} \tag{1}$$

e,

$$J_i \ddot{\theta}_i = \sum_j M_{ij} \qquad \text{(for rigid body os inertia } J_i\text{)} \tag{2}$$

Where:
x_i: linear displacement (mm);
m_i: concentrated massas values (kg);
F_{ij}: external force (N);
θ_i: angular displacement (rad);
J_i: mass moment of inertia (kg.m^2);
M_{ij}: external moments (N.m);

Or, using Lagrange's equations:

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$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = Q_j^{(n)} \ , \quad j = 1, 2, \ldots, n \tag{3}$$

Where:
T: potencial energy (J);
V: kinetic energy (J);
Q_j: generalized forces and moments (N, N.m);
q_j: generalized coordinates (mm, rad);
n: number of degrees of freedom;

Using Eq. (1) and Eq. (2) or Eq. (3) the discrete system motion equations can be obtained, which can be used to determine the displacements of each model element.

2.2 Timoshenko's Beam Definition

For a ship vibration analysis, it is necessary to represent the ship structure composed by basic structural elements, and it is customary to use the term hull girder, since the ship's motion in a fluid can be compared with the movement of a beam.

Thus, the ship, due to their large size (depth, breadth and length) can be analyzed as a beam, but due to the difficult estimation of deflections in its movement, one has to choose a beam model that allows us to obtain reliable results and is in this context Timoshenko beam (Timoshenko, 1937) is chosen.

The Timoshenko beam theory considers that in its deflection, the cross sections remain plane, but not orthogonal to the neutral axis, due to the effect of shear force, causing the section to rotate around its center of shear. In the case of a ship, where the dimensions of the width, compared to the length, can not be ignored, it makes big difference in the calculation when shear distortion is not considered.

The effect of rotation in cross section is presented in a beam whose cross-section is not negligible in respect to its length, and the bending moment produces a section rotation, which has different slopes for the position and length of the beam whose axis of rotation is the horizontal neutral axis. This effect can be expressed in terms of angular acceleration and the rotational inertia of the section.

Equation (4) represents the behavior of the beam due to bending and Equation (5) represents the behavior of the beam due to shear (Timoshenko, 1937).

$$M(x,t) = EI(x)\frac{\partial \gamma(x,t)}{\partial x} \tag{4}$$

$$V(x,t) = k'GA(x)\beta(x,t) \tag{5}$$

where:
$M(x,t)$: bending moment (N-m);
$V(x,t)$: shear force (N);
$\gamma\ (x,t)$: transversal section slope due pure bending (rad);
$I(x)$: area moment of inertia of transversal section (m^4);
$\beta(x,t)$: shear strain (rad);
$k'GA(x)$: shear strength;
E: Young's modulus (Pa);

2.3 Eigenvalue Problem

From Newton's second law, Eq. (1) and Eq. (2), or from Lagrange equation, Eq. (4), it is obtained the motion equation of each discrete system element that can be rearranged in matrix form, thus, the general motion equation has the following form:

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{f(t)\} \tag{6}$$

where:
$[M]$: mass matrix;
$[C]$: damping matrix;
$[K]$: stiffness matrix;
$\{f(t)\}$: external forces vector;
$\{x(t)\}$: displacements vector;

In the case of frequencies and natural modes of vibration analysis, the damping matrix and the external loads vectors will be discarded, thus, the equation (6) will be as follows:

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = 0 \tag{7}$$

In order to facilitate the solution of Equation (7), the variables separation method is used, namely:

$$\{x(t)\} = \{X(x)\}T(t) \tag{8}$$

Where:
$\{X(x)\}$: space dependent factor of $x(t)$;
$T(t)$: time dependent factor of $x(t)$;

From Eq. (8), making derivatives and substituting in, we have:

$$[M]\{X(x)\}\ddot{T}(t) + [K]\{X(x)\}T(t) = 0 \tag{9}$$

Equation (9) can be re-written as scalar in its "n" degrees of freedom:

$$\left(\sum_{j=1}^{n} m_{ij}X_j\right)\ddot{T}(t) + \left(\sum_{j=1}^{n} k_{ij}X_j\right)T(t) = 0\,, \qquad i = 1, 2, \dots, n \tag{10}$$

Equation (10) can be obtained from the following relationship:

$$\frac{\left(\sum_{j=1}^{n} k_{ij}X_j\right)}{\left(\sum_{j=1}^{n} m_{ij}X_j\right)} = -\frac{\ddot{T}(t)}{T(t)} = cte = \omega^2 \tag{11}$$

Thus, one obtains the following expressions:

$$T(t) = C_1 \cos(\omega t + \phi) \tag{12}$$

$$\big[[K] - \omega^2[M]\big]\{X(x)\} = 0 \tag{13}$$

Equation (12) shows that all coordinates have a harmonic motion with the same frequency ω and the same phase angle. However, the frequency ω can not arbitrary, it must satisfy Eq (13). In Eq (13) we can see that there is a trivial solution {X (x)} = 0. For nontrivial solutions of Equation (13), the determinant of the matrix Δ coefficient should be zero. This is:

$$\Delta = |[K] - \omega^2\,[M]| = 0 \tag{14}$$

Equation (14) is known as the eigenvalue problem and ω is the natural frequency of the system, and there is a ω for each displacements set. Thus, replacing ω in Eq. (13), the displacements can be obtained and are also known as vibration modes. Figure 2 presents the first 5 natural vibration modes expected in the analysis of a supported beam.

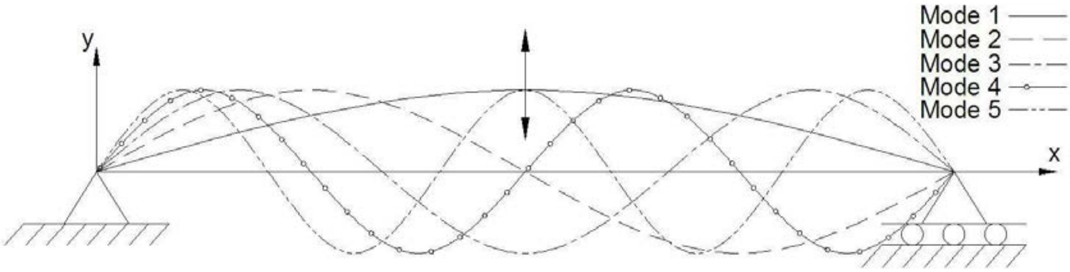

Figure 2. Five first natural vibration modes of a supported beam.

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3. ADDED MASS

The hull girder differs from a simple beam of civil structure for being partially submerged. This consideration directly involves the concept of added mass.

When a body vibrates or moves in accelerated motion in its six degrees of freedom in a fluid medium, it generates the movement of fluid particles. The reactions of these particles motion on the body modify its structural behavior, which requires the consideration of added mass that depends on the depth of the fluid medium and the shape of the body under examination.

It will be considered the cross section of a ship. From the fluid flow around the hull, it is possible to calculate the added mass by calculating the kinetic energy of the system, considering the floating body and the fluid motions, according to Figure 3:

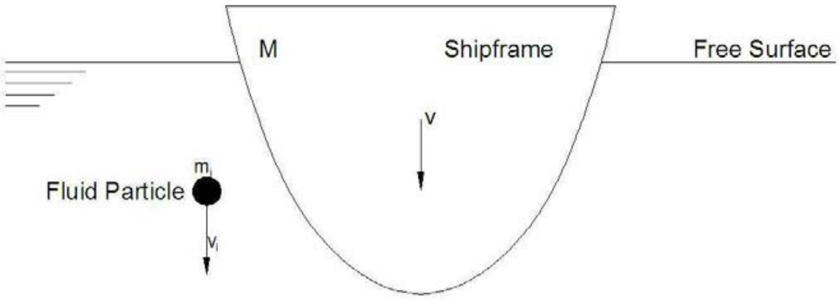

Figure 3. Analysis of ship's motion in a fluid.

$$E_c = \frac{1}{2} M v^2 + \frac{1}{2} \sum_{i=1}^{\infty} m_i v_i^2 = \frac{1}{2} M v^2 + \frac{1}{2} M' v^2 \qquad (15)$$

Onde
E_c: kinetic energy (J);
M: body mass (kg);
v: vertical velocity, heave motion (m/s);
m_i: fluid particle i mass (kg);
v_i: fluid particle velocity i (m/s);
M': hydrodynamic added mass (Kg);

The calculation has been developed using the analogy with a cross section of a cylinder, due to the simplicity of its form and considering the body floating in an infinite fluid without interference with others surfaces. After that, transformations are made to ship sections by the use of conformal mapping method. Thus, the calculation of the added mass (per unit length) λ_{33} is made by the following expression:

$$\lambda_{33} = \rho \frac{\pi B^2}{8} k_{33} \qquad (16)$$

where:
ρ: fluid mass density (kg/m^3);
B: section ship breadth (m);
k_{33}: vertical added mass transformation coefficient by conformal mapping;

The coefficient k_{33}, which transforms the added mass calculated for cylindrical sections into ship sections, depends on the type of ship and also the navigation area, either in deep or shallow water. As this paper studies the dynamic behavior of a ship in shallow water, it is necessary to calculate the transformation coefficients of added mass for navigation in shallow waters.

The coefficients for vertical added mass in shallow water take into account the area coefficient β and the ratio between the depth where the vessel is operating and its draft.

Based on the experimental data, Prohasky (Korotkin, 2009) developed by experimental analysis and considering small oscillation amplitudes, the curves of coefficient k_{33} for different shapes sections, according to the area coefficient β of each section and the ratio between depth h and draft T. The data were plotted as seen in Figure 4.

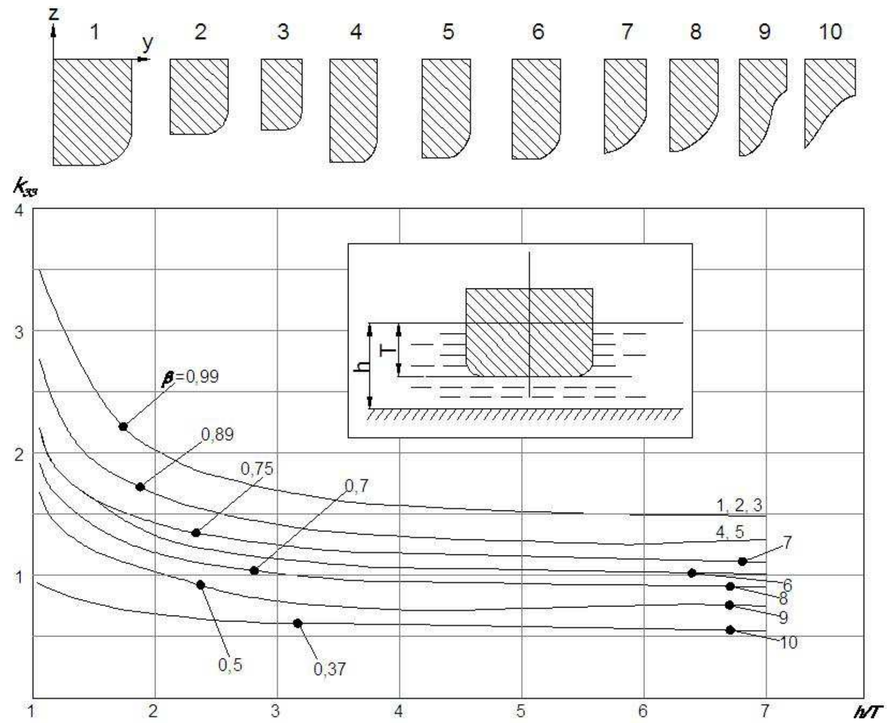

Figura 4. Curvas de k_{33} para navegação em para águas rasas, Prohasky (Korotkin, 2009).

After finding the coefficients of mass correction for each additional section, we take into account that the real flow around the hull is not two-dimensional as calculated, but three-dimensional. Townsin (1969) proposed a correction using a formulation that utilizes the parameter J_n, shown below:

$$J_n = 1{,}02 - 3(1{,}2 - \frac{1}{n})\frac{B}{L} \tag{17}$$

Where:
n: vibration mode number of nodes;
B: section breadth (m);
L: ship length between perpendiculars (m);

In Eq. 17 it is noted that the larger the number of nodes in the vibration mode n, the smaller will be the J_n values.

4. CASE STUDY

4.1 Ship Main Particulars

The ship chosen for this study, which belongs to the database of laboratory LEDAV at UFRJ, was a bulk carrier type of river navigation, and its operation is, in most of the navigation, held in shallow water. Figure 5 shows a view of the navigation of the ship.

The main particulars of the ship studied are presented in Table 1:

Table 1. Ship Main Particulars.

Total length	95.36 m
Length between perpendiculars	91.0 m
Moulded breadth	15.5 m
Moulded depth	4.75 m
Project draft	4.1 m
Maximum draft	4.23 m
Displacement	5272.0 t

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Figure 5. Bulk Carrier type of river navigation.

In order to study the ship, finite elements models were used. The Finite Element Method has shown great reliability in the results, as long as the input data are chosen with care.

Two approaches were considered to develop the model ship. The first one is a beam *1D* model by the use of equivalent beams representing aft, parallel body and ship's stern. The second approach was to develop a *3D* model and in that form accurately represent the structure and geometry of the ship. Even for the two models is calculated the hydrodynamic added mass which is distributed throughout the body in order to make the analysis.

4.2 *1D* Model

The *1D* model is based on the use of beams equivalent to represent the structure of the ship (Brasil, 2011). For this, it is necessary to know the properties of the cross sections of the ship. Thus, we selected some typical sections of different ship parts and modeled using the software PROSEC (Troyman *et. al.* 1987). This software is based on the theory of flow shear stress in thin-walled sections.

The software is used to model the sections from the forms and structural elements that directly influence ship longitudinal stiffness. Figure 4 shows the output and the input data necessary for the *1D* model development.

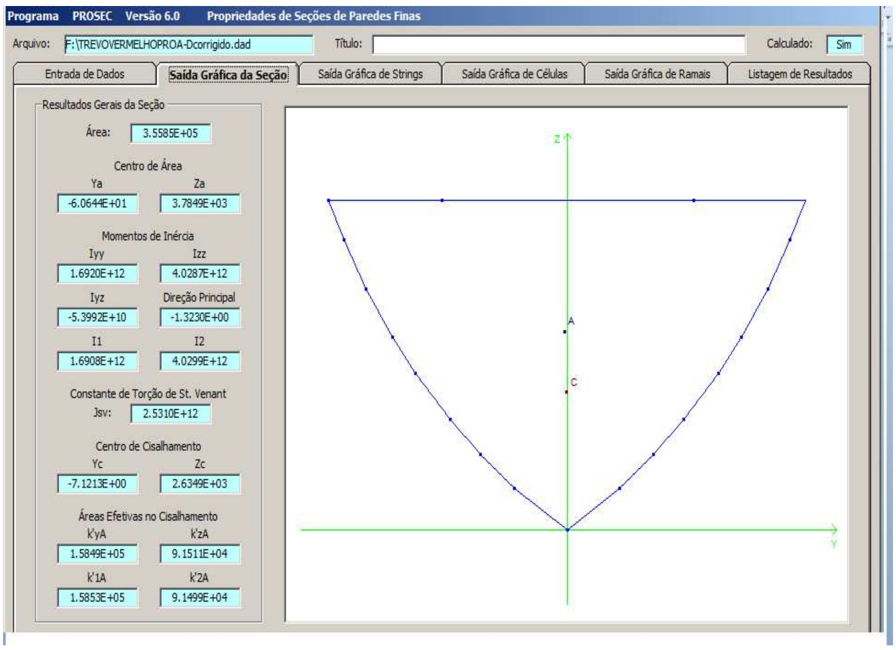

Figure 4. Output and results calculated by the software PROSEC.

Where:
A; Ya e Za: steel area [mm²], and transversal and vertical area center (mm);
Iyy: moment of inertia related to transversal axis (Y) (mm^4);
Izz: moment of inertia related to vertical axis (Z) (mm^4);
Iyz: moment of inertia related to 45° axis (mm^4);
J: torsion constant (mm^4);
$k'yA$ e $k'zA$: transversal and vertical effective shear area (mm²);
Yc e Zc: transversal and vertical shear center (mm);

The importance of using the software PROSEC is to calculate the moments of inertia of all sections and the effective shear area. Once done the calculation of the sections properties the *1D* model is developed and showed in Fig. 5.

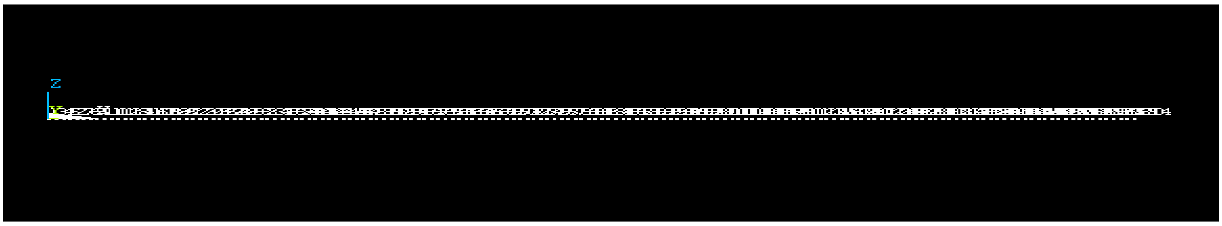

Figure 5. *1D* Model developed by a finite element software.

4.3 *3D* Model

The *3D* model was carried out taking into consideration the ship shapes, and all the structural elements that directly influence the structural rigidity of the ship are considered (Mello, 2011, Moreira, 2013). The *3D* model was composed by shell elements, which allow general strain calculation. Figure 6 shows a perspective view of the developed *3D* model.

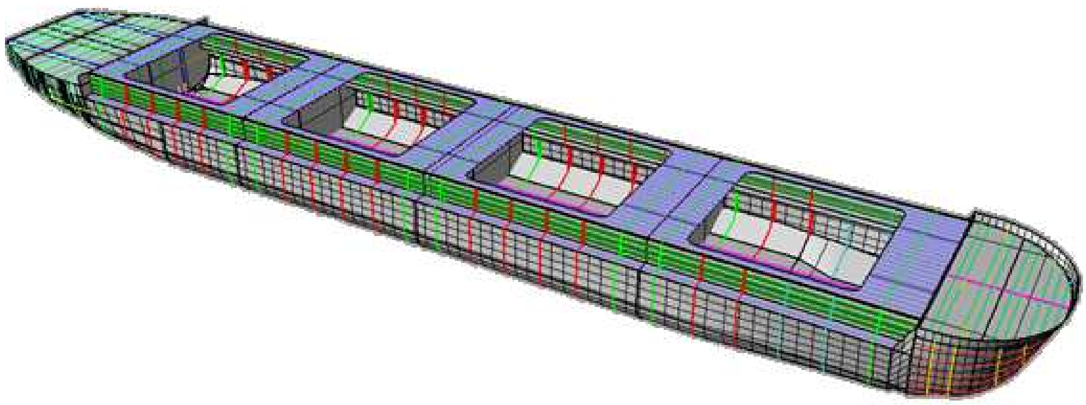

Figure 6. *3D* Model developed by a finite element software.

The advantage of a *3D* model is that it is not necessary to calculate the properties of beam sections, since the finite element software makes the calculation taking into account the model geometry itself.

For both models, the material used is the structural steel that has the following properties:

- Young modulus (E) – 210 *GPa*;
- Poisson coefficient (ν) – 0,3;
- Mass density (ρ) – 7850 kg/m³.

4.4 Added Mass Calculation

The calculation of the added mass was done in the design draft of 4.1 m, with a depth of 5 m, and the displacement was obtained from the hydrostatic curves, and has the value of 5272 t.

In order to apply the added mass to the model, it was necessary to make the product between the average mass per unit length, from Eq. (16), and the longitudinal length between two consecutive frames. These calculations were made for all the intervals between the frames along the entire length of the ship waterline.

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The added mass is distributed along the model, proportionally to the submerged area of each section. This will be done by the product between the total vertical added mass and the area percentage relative to the sum of all the submerged areas. Equation (18) shows the performed calculation:

$$M'(x) = \left(\frac{Area\_sub(x)}{\sum Area\_sub}\right) \cdot M' \tag{18}$$

Where:
$M'(x)$: distributed added mass at each frame (kg);
$Area\_sub(x)$: submerged área at analysed section (m^2);
$\Sigma Area\_sub$: sum of submerged areas of all sections (m^2);

We must also remember that this *2D* mass should be corrected by considerations of the actual shape of the vessel (three-dimensional), according to each vibration mode to be analyzed. This correction is made by the product between the vertical additional mass M' and the coefficient J_n calculated by the Townsin formula (1969), where n represents the number of nodes in each mode.

The same way as the added mass was distributed along the ship length, an estimate was made, using the submerged areas, for the distribution of the ship displacement. Equation (19) shows the procedure of performed calculation:

$$M(x,n) = M'(x).J_n + \left(\frac{Area\_Sub(x)}{\sum Area\_Sub}\right).\Delta \tag{19}$$

Where:
$M(x,n)$: total distributed mass at each frame (kg);
Δ: displacement at design draft (kg);

5. RESULTS ANALYSIS

From the models developed in items 4.2 and 4.3, and making added mass distribution, we used the finite element software to make the calculation of frequencies and natural vibration modes. The input parameters to calculate modes started with a scan frequency of 0.01 Hz to maximum 100 Hz (in order to avoid the rigid body modes). Table 2 presents the first 5 ship natural frequencies, both for the *1D* model as for the *3D* model. Figures 7, 8, 9, 10 and 11 show the mode shapes for each of the developed models.

Table 2. Results for the 5 first frequencies of ship natural vibration.

Mode	Frequency (Hz)		Difference
	1D	*3D*	(%)
1	0,540	0,746	27,61
2	1,209	1,758	31,23
3	2,277	2,929	22,26
4	3.608	4.066	11,26
5	5.598	5.150	8,70

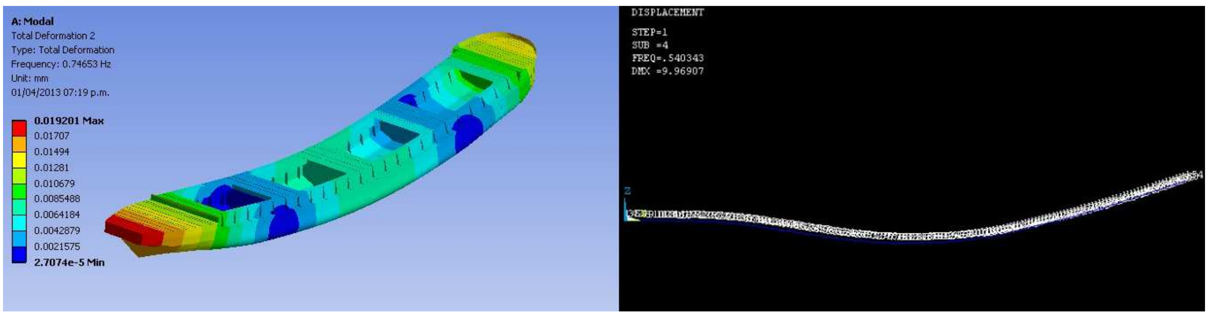

Figure 7 - First ship natural vibration mode.

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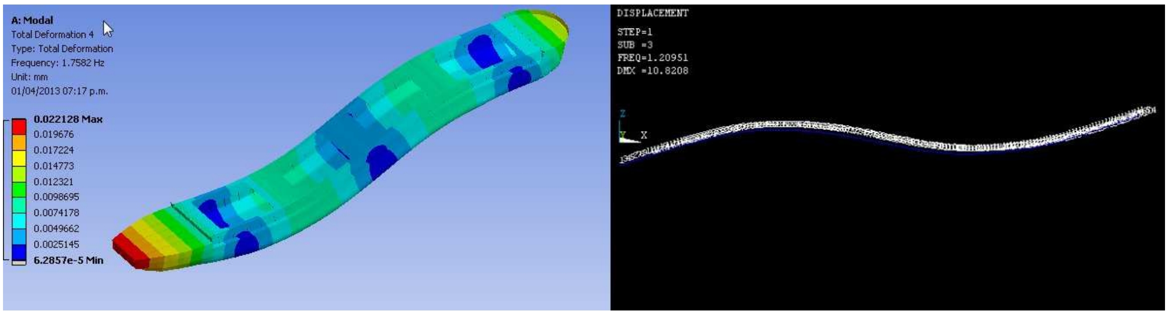

Figure 8 - Second ship natural vibration mode.

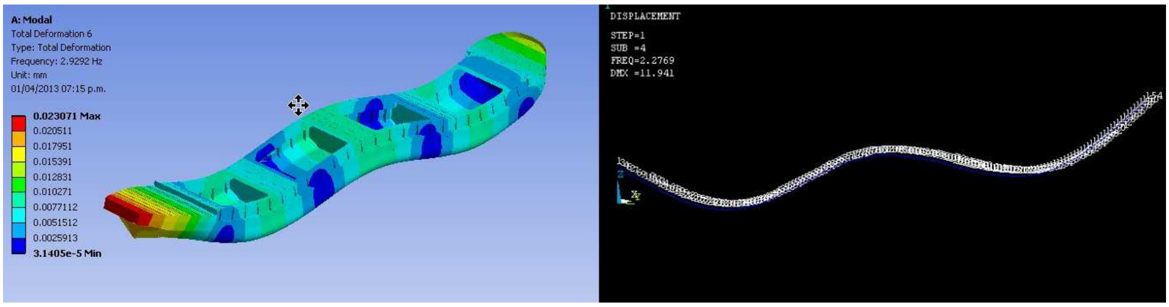

Figure 9 - Third ship natural vibration mode.

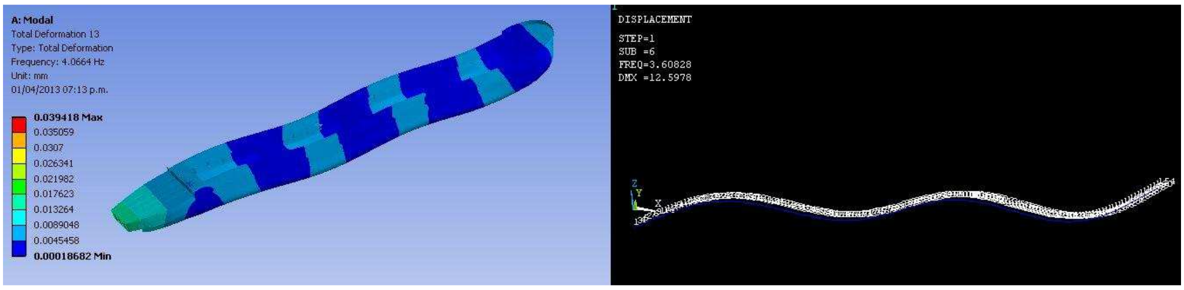

Figure 10 - Fourth ship natural vibration mode.

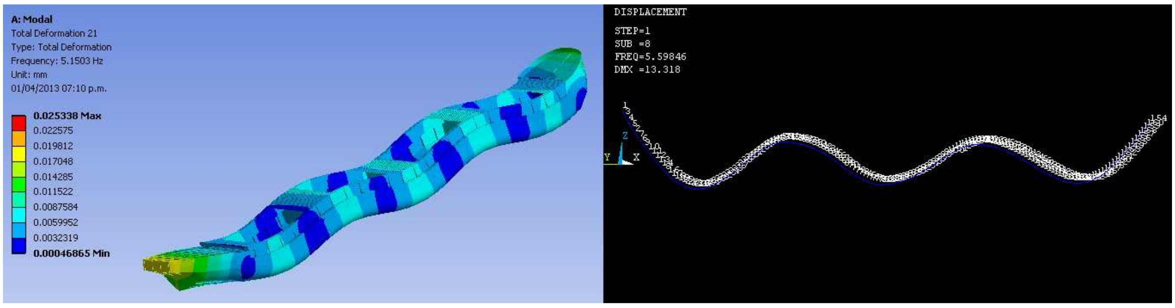

Figure 11 - Fifth ship natural vibration mode.

6. CONCLUSIONS

In the present work it was found that the interference of the bottom of the flow around the hull of the vessel causes further increase in mass and thus reduces the vertical frequency of the first natural vibration modes of the vessel.

In order to make a complete study, it would be necessary that the numeric results were compared with the experimental results of the vessel used as a case study. With this, one could prove the efficiency of the method used to find the values of the added masses, and consequently the corresponding natural frequencies.

When comparing the results from the one-dimensional models and three-dimensional values were significantly distanced at most 31.23% in the second vertical vibration mode. This may be explained by the complexity of the analysis in the model. That is, in the *1D* model several simplifications are made, especially in the calculation of the

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properties of sections and particularly in the calculation of the effective area in shear. Subsequent studies are necessary to solve this question.

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