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A CONTRIBUTION TO RECOGNIZE SHAPE AND POSITION OF BASIC FUNCTIONAL GEOMETRIES OBTAINED BY MECHANICAL MANUFACTURING

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Abstract. *The main objective of this study was to establish an algorithm capable to recognize the shape and position of basic functional geometries obtained by mechanical manufacturing. Differential geometry was used to determine Gaussian and mean local curvatures of surfaces described by ordered cloud point representation. The partial derivatives were taken the entire sets of data points and the Gaussian and mean curvatures were then obtained, enabling geometrical shape identification and position. Since the initial data was a set of discrete points, the divided difference method was used in order to determine a numerical approximation to the partial derivatives at each point. The Gaussian and mean curvatures were used to implement the proposed algorithm through a MATLAB application. It was first tested using geometrical data generated on MATLAB and then using a set of points measured from an actual physical shape, using a coordinate measuring machine. The results obtained validated the adequacy and the potential of the proposed algorithm to identify and spatially locate shapes that commonly need to be measured.*

Keywords: shape recognition, shape algorithm, functional geometries, metrology, Gaussian curvatures

1. INTRODUCTION

The actual geometric shape of any body is determined by the surfaces which delimit and separate it from the environment. The surface geometry is defined by the design or manufacturing process, regardless of form deviations. When controlling manufactured parts it is important to consider the effective surface, which is approximately depicted by a set of points actually taken from measures made on the surface of the part [1]. Furthermore, the main functional geometries of most mechanical manufactured components consist of some simple shapes, including the following basic ones: planes, cylinders, spheres and cones. These geometries are the ones demanding most of the measuring effort, for the behavior of mechanisms largely depends on the quality of the surfaces obtained.

The development of automated measurement systems for monitoring these geometries arises as a response to the increasing automation of manufacture. Nowadays the interest on metrology systems supported by computational geometry is expanding, and is leading to the development of work in different areas.

Computational geometry is a branch of computer science that deals with the systematic study of algorithms and data structures for solving computational geometric problems [2]. It appeared by 1970 [3], but only after 1995 precipitated the research interest in computer vision systems [4], due to the cost reduction evolution of computational systems, and to the development of high-resolution digital cameras that was made possible.

The industrial feasibility of metrology based on such systems, for geometric shape recognition, depends on the satisfaction of demanding performance criteria, keeping relative cost competitiveness [5]. Their introduction may allow innovative and specific solutions, oriented to industrial automation, offering reduction in what is considered one of the greatest individual costs of production: the inspection process. In parallel, the errors associated with operator intervention, can be also reduced [6]. When realizing this, the interest from various fields of industrial activities sparked, from electronic to mechanical components manufacturing, among others, with the sectors mainly related to quality control benefiting from its introduction [7]. More recently such systems have also attracted the interest of biomedicine. There are several advantages of using this kind of computer supported systems, where the large volume of data to acquire, the handling speed they allow and the portability offered are key factors contributing to its spreading. However, most systems simply deal with large amount of data, without actually performing any recognition of geometric shapes, which is mandatory in many technological areas, where any advances are not possible without a proper corresponding algorithmic support.

There are already several mathematical tools proposed in the development of the algorithms used. They essentially use differential geometry, often taking advantage of the principal curvatures, the mean curvature and the Gaussian curvature. An example is proposed by Ray and Majumder [8], for the identification of local invariant features of 3D objects partially occluded. For recognition and localization of 2D shapes, R. Ibrayev Yan and Jia-Bin [9] introduced a method based on differential and semi-differential invariants, considering data obtained by contact measurement.

In the field of biomedicine, Giuliani, presented a method based on Gaussian and mean curvatures for shape recognition and growth pattern of a biological organism [10]. Also, Sheng et al proposed a method for geometric modeling based on differential equations, which was validated in terms of facial geometry parameterization using data captured by a 3D laser scanner [11].

The authors do not know a commercial computer vision system applied to metrology that can be referred to as a solution for all industrial applications. There are several systems and algorithms proposed to find the optimal solution for specific cases [6, 12, 13, 14, 15]. Sometimes the routines to support these algorithms work with data obtained by contact measurement, as, for example, those obtained by coordinate measuring machines (CMM). Sometimes, for simplification reasons, the recognition is based on data obtained from 2D curves, and not directly from 3D surfaces data. Many proposals do not come from any engineering field, reason why the main concern is often a qualitative analysis; they are not concerned with the quantitative analysis of the shape and position. However, from the engineering point of view, a desirable capacity for the system is that the shape and position recognition be achieved in real time, to enable, for example, the control of parts during the manufacturing process. This is the main reason why the algorithm should be fast and robust, but, at the same time, the metrological qualities of such a system must be kept in order to minimize the uncertainty of measurement results [16]. Of course, the verification and/or correction of those qualities should be made based on reference surfaces and under specified conditions.

This work then presents a contribution to the computer aided recognition of shape and position of basic functional geometries obtained by mechanical manufacturing. These geometries essentially consist of planes, cylinders, cones or spheres that, alone or in combination, constitute the majority of the functional surfaces of manufactured parts. Because of their functional character, shape recognition is of utmost importance, and the information of spatial position of each shape allows the establishment of the correctness of their relative positions. The performance of mechanisms ultimately depends on both shape and relative position of functional shapes. Experience also shows that those entities actually constitute the essential main entities that have to be considered to perform the dimensional analysis of any mechanical part.

Currently, after data acquisition, the decision on the geometric shape is taken by the operator. Based on the coordinates of the points, the operator decides whether these belong to one or another surface. According to the particular circumstances, for example, three points may be considered to belong to flat surface, or be located on a cylindrical surface with a very large radius directrix, depending on the decision of the CMM operator doing the measurement. The use of a larger set of points reduces the incertitude about the actual surface shape, but is time consuming when manual operation is performed. To deal with a larger set of points, an automatic treatment of data is needed. The algorithm presented in this work uses mathematical tools in the process of recognition and classification of geometric shapes, to allow automatic processing of the data acquired. It was designed to detect itself the geometric shape match and to inform the main data defining its position.

The algorithm starts by reading a set of discrete data, which can be obtained either by contact or non-contact measurement. However, since the main goal of the application is to automate the measurement process, fast data acquisition of a large set of points must be considered in future, such as image acquisition, for example. Since there is no previous knowledge of the function that features the surface represented by the acquired data points, a numerical method must be used to obtain the local approximation of the partial derivatives of first and second order at every point. The method chosen was the divided differences. The recognition of the shape type derives from the partial derivatives so obtained, and, in the case of the a plane geometry, identification results almost immediately from the first order partial derivatives. Regarding the recognition of the rest of the forms mentioned above, the Gaussian and mean curvatures of the surface were evaluated, using the numerical approximation of the local partial derivatives. The algorithm developed was first tested on data generated in MATLAB, based on the analytical equations of the surfaces under study. Subsequently it was applied to a set of points obtained over a real physical shape, using a CMM.

2. NOTATION AND MATHMATICAL DEFINITION OF THE PROBLEM

The proposed algorithm is adaptable to any set of points in a three-dimensional coordinate ordered arrangement. The process starts with data reading, structured as an Nx3 matrix. The data corresponding to the three-dimensional coordinates of the points take the format $\left(x_i, y_j, f(x_i, y_j)\right)$, where $i = 1, 2, \ldots, n$, where $j = 1, 2, \ldots, p$, and where $z_{(i,j)} = f(x_i, y_j)$. Figure 1 shows an illustrative ordered mesh of the data points involved in the process.

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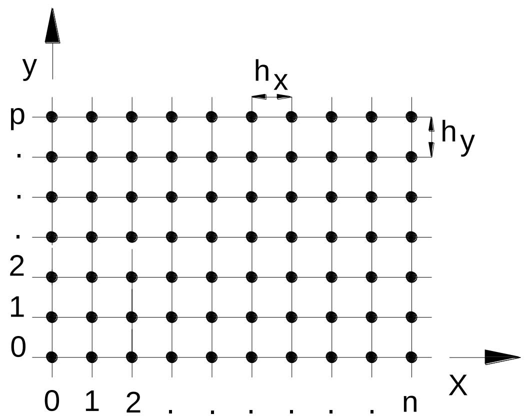

Figure 1. Mesh points projected on the plane XOY.

Since the initial data consist in a set of discrete points, the method of divided differences [17] was used in order to determine a numerical approximation to the partial derivatives at each point. The first order partial derivatives at each point were obtained by Equations (1), where h_x and h_y represent the distance between two consecutive points along the x and y axis, respectively.

$$\frac{\partial z}{\partial x}(i,j)=\frac{z(i+1,j)-z(i-1,j)}{2h_x} \qquad \frac{\partial z}{\partial y}(i,j)=\frac{z(i,j+1)-z(i,j-1)}{2h_y} \tag{1}$$

The numerical approximation to the partial derivatives of second order was obtained by the same method. In this case the equations used are Equations (2).

$$\frac{\partial^2 z}{\partial x^2}(i,j)=\frac{\frac{\partial z}{\partial x}(i+1,j)-\frac{\partial z}{\partial x}(i-1,j)}{2h_x} \qquad \frac{\partial^2 z}{\partial y^2}(i,j)=\frac{\frac{\partial z}{\partial y}(i,j+1)-\frac{\partial z}{\partial y}(i,j-1)}{2h_y} \tag{2}$$

$$\frac{\partial^2 z}{\partial x\partial y}(i,j)=\frac{\frac{\partial z}{\partial x}(i,j+1)-\frac{\partial z}{\partial x}(i,j-1)}{2h_y}$$

The Gaussian and mean curvatures at a given point belonging to the surface were calculated, respectively, by Equations (3) and (4).

$$K(i,j)=\frac{\frac{\partial^2 z}{\partial x^2}(i,j)\frac{\partial^2 z}{\partial y^2}(i,j)-\left(\frac{\partial^2 z}{\partial x\partial y}(i,j)\right)^2}{\left(1+\left(\frac{\partial z}{\partial x}(i,j)\right)^2+\left(\frac{\partial z}{\partial y}(i,j)\right)^2\right)^2} \tag{3}$$

$$H(i,j)=\frac{\left(1+\left(\frac{\partial z}{\partial y}(i,j)\right)^2\right)\frac{\partial^2 z}{\partial x^2}(i,j)-2\frac{\partial z}{\partial x}(i,j)\frac{\partial z}{\partial y}(i,j)\frac{\partial^2 z}{\partial x\partial y}(i,j)+\left(1+\left(\frac{\partial z}{\partial x}(i,j)\right)^2\right)\frac{\partial^2 z}{\partial y^2}(i,j)}{2\left(1+\left(\frac{\partial z}{\partial x}(i,j)\right)^2+\left(\frac{\partial z}{\partial y}(i,j)\right)^2\right)^{3/2}} \tag{4}$$

The recognition of the different geometric shapes was made based upon the satisfaction of the decision conditions presented in Table 1, where *r* is the radius of the considered shape.

Table 1. Decision conditions for different geometric shapes.

	$\frac{\partial z}{\partial x}(i,j) / \frac{\partial z}{\partial y}(i,j)$	$K(i,j)$	$H(i,j)$
Plane	Constant1/Constant2		
Sphere		$1/r^2$	
Cylinder		0	$1/2r$
Cone		0	$1/2r_i$

3. ESTABLISHMENT OF THE ALGORITHM

A major advantage of the application of computer vision systems to geometrical metrology, along with the absence of contact, is the high speed of measurement, which provides the acquisition of a large volume of data in a short period of time. The three dimensional coordinates represented by this large volume of data constitute a cloud of points. The treatment of such an amount of data demands a suitable algorithm that will be of great importance in the recognition of the functional geometric shapes obtained by mechanical manufacturing. Figure 2 presents the algorithm in the form of a flow chart defining the logical sequence of steps needed to solve the problem.

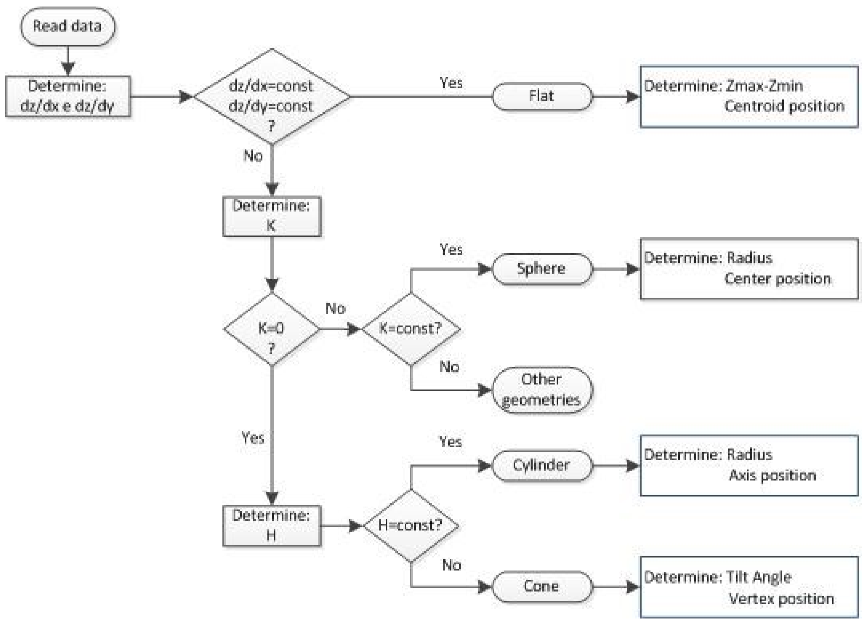

Figure 2. Flowchart of the algorithm.

3.1 Flat surfaces recognition

Almost all mechanical components have nominally flat surfaces. These surfaces are always characterized by deviations from the theoretical geometric plane, or mathematical plane. There are several factors that contribute to these deviations. Considering that to obtain the better quality planar surfaces machining processes are usually needed, most of those factors are related with the cutting forces and temperature changes involved in the machining process, although other origins may be encountered when different technological processes were involved in part manufacturing.

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The flat surfaces obtained by mechanical manufacturing, in addition to the microgeometric irregularities, which often characterize the manufacturing processes, present also macrogeometric irregularities which are generally considered form deviations. According to metrology jargon, when a planar surface is considered, these deviations are designated deviations from flatness. The interpretation of these deviations, according to ISO 1101, suggests that the degree of approximation or separation of a real surface, in relation to a nominally flat surface, determines the degree of flatness of that surface.

The recognition algorithm used for the planar form, as shown in the flowchart of Figure 2, follows the sequence:

1. Point cloud data reading that, as mentioned above, must be structured in a matrix Nx3;
2. First order partial derivatives determination at each point according to Eqs. (1). If these derivatives turn to be constants, the program stops obeying the criterion, and then the cloud of data points relate to a flat surface.

3.2 Spherical surfaces recognition

The sphere, as mentioned above, is also a functional basic geometry in mechanical manufacturing. Obtaining spherical surfaces with high accuracy, due to the increasing development of machining processes, is assuming increasing importance. In industry, the deviation from the spherical shape, or sphericity, has an important effect on the circular motion of components in various machines. Therefore, defects such as roughness, curling or shape can result in the generation of a large amount of heat, causing a rise in the surface temperature of the components involved, resulting in wear and life reduction. Thus, recognition of the spherical shape and the control of its deviation becomes of paramount importance in mechanical manufacturing [18]. Since international standards, including ISO 1101, do not characterize this deviation explicitly, various contributions have been proposed, and some may be found in the references [19, 20, 21, 22, 23]. In this context, the algorithm proposed in this paper also makes the recognition of spherical shape and position, using the Gaussian curvature of the cloud of points acquired on an actual surface. Continuing to follow the flow chart of Figure 2, if the first order partial derivatives are not constant, the determination of second order partial derivatives and the Gaussian curvature follows. If the Gaussian curvature returns a constant value different from zero, then the test stops and the point cloud data refers to a spherical surface.

3.3 Ccylindrical and conical surfaces recognition

Surfaces of revolution, especially the cylindrical ones, are very common in mechanical construction, either as shafts or as holes. From the geometric point of view, these surfaces can be considered as being generated by a straight line (generatrix), moving parallel to another line (axis of the cylinder), and constantly leaning on a circumference (directrix) concentric with the cylinder axis in a plane normal to it.

There are several factors contributing to the surfaces, generated by mechanical manufacturing, be not perfect. It is often necessary to evaluate the deviation between the actual surface and the mathematically perfect one. ISO 1101 defines the cylindrical shape deviation, or cylindricity, as the tolerance zone between two coaxial cylinders, inside which shall be contained the real surface. The same rule sets in a similar manner to the conical deviation, or taper, as the tolerance zone between two coaxial cones. This means that all data in the cloud of points should be contained within these tolerance zones.

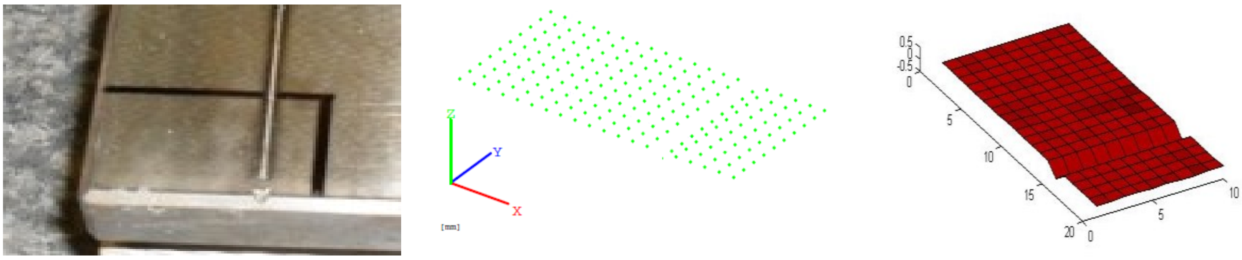
Thus, continuing to follow the flowchart of Figure 2, if the Gaussian curvature is zero, the calculation of the mean curvature follows. If this one is constant, then the criterion makes the program to stop, and the cloud data points refer to a cylindrical surface. If it is variable, the program also stops, but the cloud of data points relates to a conical surface.

4. APPLICATION OF THE ALGORITHM

4.1 Shape and position recognition of flat surfaces

Initially, the algorithm was tested on analytical data generated in MATLAB, in order to get confidence in the procedure. Afterwards the algorithm was applied to simple shapes, whose three-dimensional coordinate data of the cloud points correspond to the intersection points of the lines of an ordered regular grid. Therefore, following the flowchart in Figure 2, if the determination of the first order partial derivative returns only constant values the identification of a flat surface is immediate. This determination was made based on a subroutine that satisfies the equations (1).

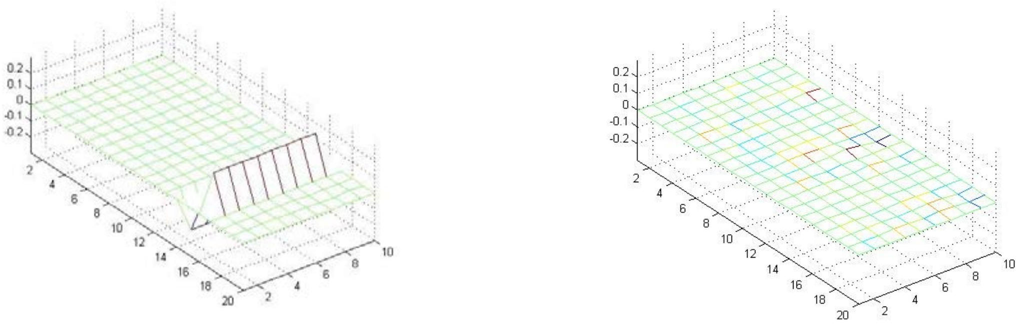
A simple sub-routine was created to convert and order these data obtained from different devices and formats into three-dimensional coordinates. Figure 3 depicts an example of application using a flat surface of a part shown in Figure 3a, in which the area subjected to measurement using a CMM is marked. Figure 3b shows the cloud of points associated to this measurement, as the output of the CMM, which is composed by ten lines of points parallel to the X axis and 20 lines parallel to the Y axis providing three-dimensional coordinate data for 200 points. Figure 3c shows the three-dimensional surface acquired by the model out of these data points.



a. b. c

Figure 3. Planar surface handling
a - Actual surface area subjected to measurement (bordered in black).
b - Cloud points obtained with a CMM.
c - Surface generated by the model.

Intentionally, the ten points belonging to line 15, are located in a "V" slot in the measurement area and are below the plane taken as reference, which, in this case, is coincident with the measured surface. The tolerance specified, determining the total variability to the surface, is then also a deciding factor of the geometric shape. In this particular case, the surface is considered flat when the specified tolerance limits are greater than the distance between points at levels Z_{max} and Z_{min} , in a direction perpendicular to the reference plane. As shown in the flowchart of Figure 2, the decision condition on the flat surface establishes that the first order partial derivative must be constant. Figure 4 shows that these derivatives are constant, both along the axis X and Y, except for the groove where dz/dx expectedly changes.



a. b.

Figure 4. Plane decision conditions:
a - dz/dx = constant.
b – dz/dy = constant.

The spatial position of the flat shape is sufficiently defined by a plane parallel to the data set, containing the centroid of the elegible data set (Eq. (5)), the plane versor (Eq. (6)) and by the directions relative to the axes X, Y and Z (Eq. (7)).

$$\overline{X} = \frac{1}{N}\left(\sum x_i, \sum y_i, \sum z_i\right) \quad (5)$$

$$u = \left(-\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1\right) \quad (6)$$

$$\alpha = ar\cos\left(-\frac{\partial z/\partial x}{\|u\|}\right) \qquad \beta = ar\cos\left(-\frac{\partial z/\partial y}{\|u\|}\right) \qquad \gamma = ar\cos\left(-\frac{1}{\|u\|}\right) \quad (7)$$

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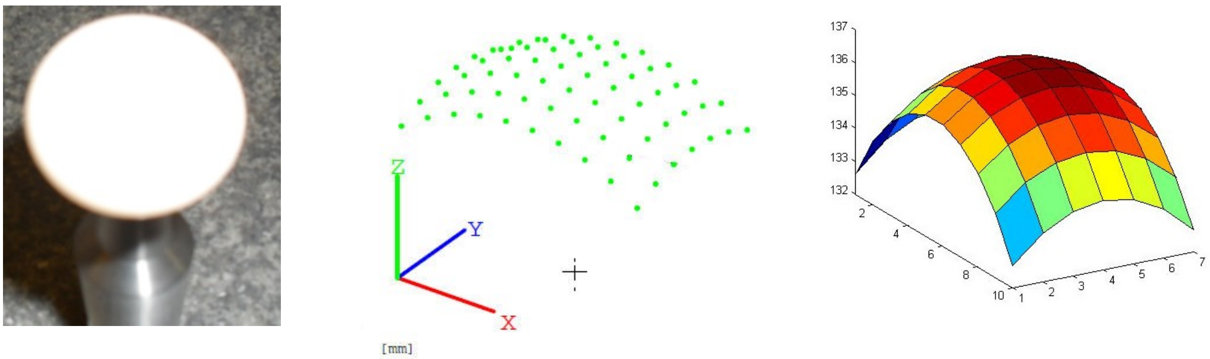
4.2 Shape and position recognition of spherical surfaces

When constant values for the first order partial derivatives are not exclusively returned, the decision on the flat surface is denied, ant starts the determination of the second order partial derivatives using equations (2) and then the Gaussian curvature (K) at each point is evaluated using Equation (3).

When determining the Gaussian curvature, if a constant value different from zero is returned for all points in the cloud, one can conclude to be in presence of a spherical surface. The radius of the spherical surface, having the same value at any given point, can be easily obtained from the Gaussian and mean curvatures.

Figure 5a shows the CMM standard ball, in which the acquisition of the point cloud shown in Figure 5b was performed. In this case, a mesh with seven lines of points parallel to the X axis and ten lines of points parallel to the Y axis was established, allowing to obtain the three-dimensional coordinate data on 70 points. Figure 5c shows the surface generated by the model from these three-dimensional data points.



a. b. c..

Figure 5. A spherical surface subject measurement.
a - Spherical surface measured (CMM standard ball)
b - Cloud points obtained with the CMM.
c - Surface generated in the model.

The decision condition for spherical shapes states that, if the partial derivatives of the first order are not constant, the Gaussian curvature must be constant and different from zero (Fig. 6).

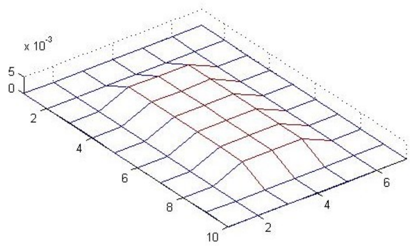


Figure 6. Spherical shape decision condition:
K = nonzero constant.

The spatial position of a spherical shape is sufficiently defined by the coordinates of its center and the value of its radius. So the position problem can be solved by determining the average center position using the curvature and local normal vector at each of the cloud points belonging to the surface.

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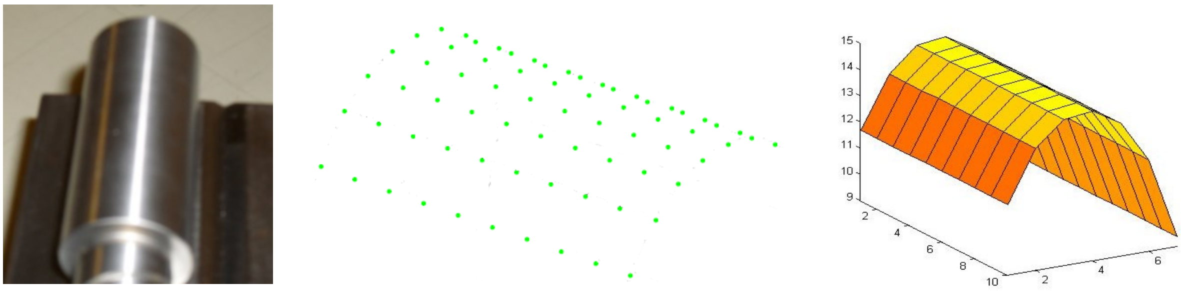
4.3 Shape and position recognition of cylindrical and conical surfaces

The interest in the calculation of K lies in the fact that it expresses an invariant feature of the surface at each point. That means its value does not depend on the position that surface occupies in three the dimensional space, but it only depends on the geometry itself. Then, if K has a zero value, the next step is the calculation of the mean curvature (H) using equation (4). If H is constant, then the surface is cylindrical; otherwise, the surface is conical.

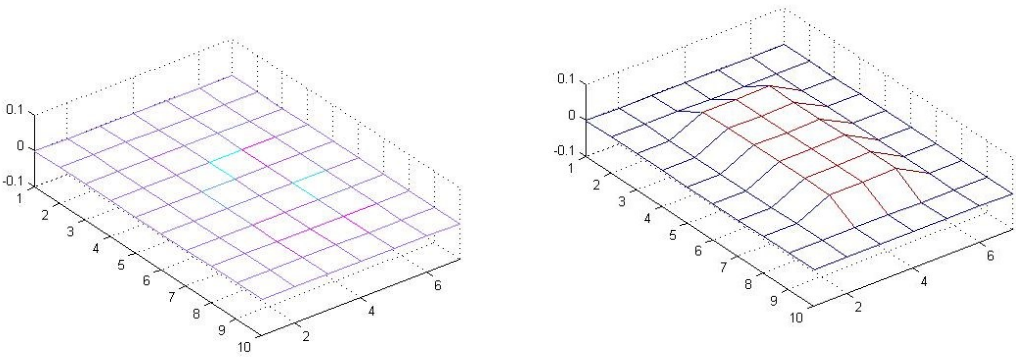
As for the decision about the flat surface, the specified cylindricity tolerance will also be a deciding factor on the geometrical shape. Figure 7a shows the cylindrical part on which the data acquisition was made, and the cloud point is shown in Figure 7b. In that case, a constitution having seven rows of dots parallel to the X axis and ten dot rows parallel to the Y axis, allowed to obtain the three-dimensional coordinate data on 70 points. Figure 7c shows the surface generated in the model.



a. b. c.

Figure 7. Cylindrical surface measurement.
a. - Cylindrical surface measured.
b. - Cloud points obtained with a CMM.
c. - Surface generated in the model.

The decision condition on the cylindrical surface states that, the Gaussian curvature being null (Fig. 8a), the mean curvature is constant and different from zero (Fig. 8b). The effectiveness of the proposed model in recognizing the cylindrical surface, when applied to the cloud points obtained with a CMM is shown in Figure 8.



a. b.

Figure 8. Cylindrical shape decision condition:
a: K = *0* in the cylindrical surface
b: H = *nonzero constant* in the cylindrical surface

Thus, the radius was determined based on the mean curvature using the equation $H = {}^{1}/_{2r}$.

The position of the cylindrical shape is sufficiently defined by a point on its axis and by the axis angles it forms with the coordinate axes. The point chosen was the midpoint of the segment corresponding to the data axis.

Figure 9a shows the conical part on which the point cloud shown in Figure 9b was acquired. Figure 9c shows the surface generated by the model.

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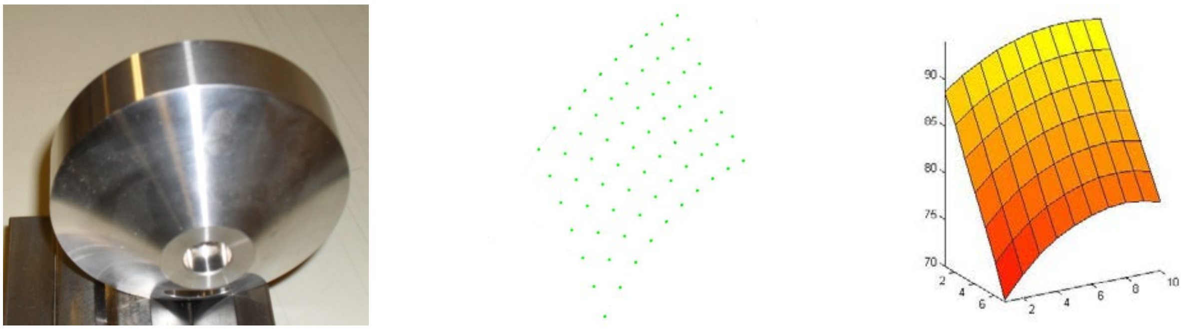
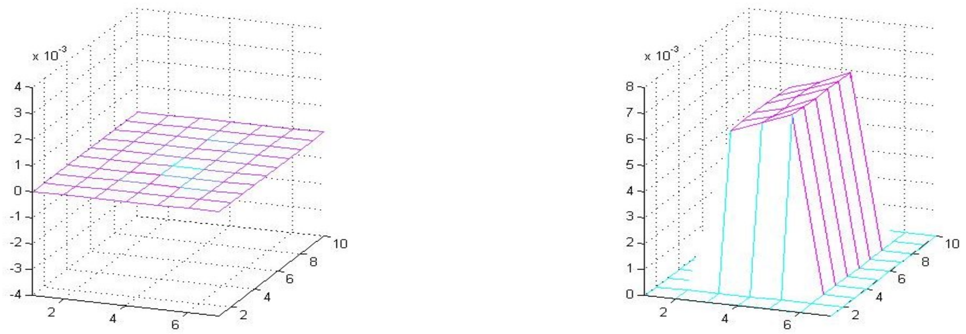


a. b. c..

Figure 9a. Conical surface measurement.
a. - Conical surface measured.
b - Cloud points obtained with a CMM.
c - Surface generated in the model.

The decision condition on the conical surfaces states that, the Gaussian curvature being null (Fig. 10a), the mean curvature is variable (Fig. 10b).



a. b.

Figure 10. Conical shape decision conditions:
a: K = *0* in the conical surface;
b: H = *variable* in the conical surface

The position of the conical shape is sufficiently defined by the coordinates of the vertex and the angles and that its axis forms relatively to the coordinate axes.

5. ANALYSIS AND DISCUSSION OF RESULTS

The cloud data points were generated in MMC, resulting from actual touching on real surfaces with the geometric shapes desired in this study. Initially, these geometric attributes were obtained by the MMC software and subsequently by the model developed. At this stage, for the sake of simplicity, the results obtained with the MMC are conventionally considered correct. The recognition of different forms was tested by generating the respective surfaces (Figs. 3c, 5c, 7c and 9c) and by the decision conditions checking (4a, 4b, 6, 8a, 8b, 10a, 10b). The model demonstrated good robustness in the recognition of all geometric shapes treated. However, there was some limitation on the recognition of the position of cylindrical and conical surfaces. In fact, Shakarji [12] had already referenced them as the most difficult to treat. Thus, determination of the attributes relating to the position, at this stage, was obtained considering the axes of the cylinder and cone, parallel to an axis of the coordinate system. Currently, this difficulty is being studied, yearning that this subject can be a forthcoming publication.

The results obtained are presented in Table 2, and validate the suitability and potential of the algorithm proposed for the identification of the shape and spatial position of the geometric surfaces studied.

Table 2. Comparison of results between the two measurement systems.

		MMC	**Model**
Plane	Zmax-Zmin [mm]	0,4799	0,4760
	Centroid [mm]		14,4998 11,4991 -0,0219
Sphere	Radius [mm]	15,9834	15,6848
	Center [mm]	0,0035 -0,0003 120,8925	0,0044 0,0000 120,8923
Cylinder	Radius [mm]	10.9919	10,8787
	Axis angles [◦:′:′′]	179:58:59; 89:59:01; 89:59:44	179:58:25; 89:59:39; 90:01:32
	Axis (Point) [mm]	-36.5005	-36.5000
		30.0021	30.0017
		2.7612	2.7523
Cone	Tilt Angle [◦:′:′′]	59◦ 57′ 13′′	60:08:19
	Vertex [mm]	51.7798	51,7802
		-30.0026	-30,0272
		45.8811	45,9983

6. CONCLUSIONS

The algorithm proposed in this paper was developed intending to be flexible and adaptable to different data acquisition systems. The data simply must be converted to three-dimensional coordinates and structured in the form of an Nx3 matrix. The use of Gaussian and mean curvatures proved very effective in decision-making algorithm. These values are intrinsic to each geometry and are invariant to the position it occupies in space. The advantage of the proposed algorithm in the treatment of acquired data is to be able to recognize and to classify the shape and position of basic functional geometries without the use of the operator, i.e., in an automatic mode. The verification of the conditions of the decision and the results validated the suitability and potential of the proposed algorithm.

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