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ATTITUDE ESTIMATION USING MEMS IMU AND MAGNETOMETER DATA WITH PI FILTER

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Abstract. *This paper addresses the problem of estimating the attitude of a fixed wing Unmanned Aerial Vehicle (UAV) using a low cost Micro-Electro-Mechanical Sensors (MEMS) inertial measurement unit aided by a three axes magnetometer. The outputs of these devices have inherent noise and additive biases. Using quaternions to conjugate successive small rotations given by the gyroscopes and the other sensors to estimate long term attitude, a filtered observer was applied with spherical linear interpolation (SLERP) to merge these two estimations. This allowed for achieving good performance in both high and low frequencies. The accelerometer data includes a component of the airframe acceleration, which was estimated using a simple centripetal force model. This model can be extended in order to give better results using angular alpha and beta measurements. The magnetometer measurements were parallel to earth's field, which was modeled by the 11th IGRF spherical harmonic expansion. Filter's performance evaluation was done with a flight simulator data stream affected by simulated random walk noise, constant bias, nonlinearities and misalignments. The resulting estimation was compared to the actual values of the simulator. As conclusion, the resulting system is suitable for implementation on embedded hardware and estimates the attitude with good precision, becoming a possible choice for UAVs.*

Keywords: *attitude estimation, MEMS IMU sensors, fixed-wing UAVs, filtered observer*

1. INTRODUCTION

One of the most important tasks in autonomous unmanned aerial vehicles (UAVs) flight is to determine the attitude of the aircraft. Considering the range of applications of UAVs, it is necessary to establish a reasonable, simple and cheap method of attitude estimation. Recently, (Euston et al., 2008) developed a method based on a PI filter, but didn't consider magnetometer data. There are lots of other methods developed to deal with this problem, but many of them require complex models and demands excessive computational processing. The adoption of these methods would raise the final product price and would represent a commercial disadvantage.

The attitude estimation without any filter consists in time integrating the gyroscope data. This method does not give good results because it accumulates the sensor's error as this bias is added at each time step, this evaluation was described by Flenniken (2005). The most common filter used to fix this error accumulation problem is the Kalman filter. However, the Kalman filter represents a difficult method to be implemented in simple systems like a small or medium UAV because it requires matrix inversions and more detailed mathematical steps. Another proposed method is combining the global positioning system (GPS) data among inertial sensors data and apply a complementary Kalman filter (Jung and Tsiotras, 2007).

In this work, a simpler model that applies a filter using only accelerometer, gyroscope and magnetometer data is proposed. The method allows using a low cost Micro-Electro-Mechanical System (MEMS) Inertial Measurement Unit (IMU) combined with simple mathematical model that is based on quaternions theory.

2. METHOD

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The proposed method consists on iterative attitude estimation. The necessary input data are the magnetic field vector, provided by the magnetometers, the acceleration vector measured by the accelerometers, and the rotations provided by the gyroscopes. The steps of attitude estimation are listed below:

First Step - Rotations using Gyroscopes Data and Accelerometers and Magnetometers Attitude Estimation - On this step, the actual estimation of attitude of the aircraft is rotated based on the data received from the gyroscope, reaching updated attitude estimation. At the same step, from the accelerometer data, discounting the centripetal accelerations and from the magnetometer data, other attitude estimation is done.

Second Step - Spherical Linear Interpolation (SLERP) - The proportional-integrative filter is applied to the angular difference between the two estimated attitudes to define the correction parameter *t*.

Third Step - Final Attitude Estimation - The final step consists in applying the SLERP's interpolation between the two attitude estimations of step one using the correction parameter t.

The diagram that represents the method is shown in Fig. 1.

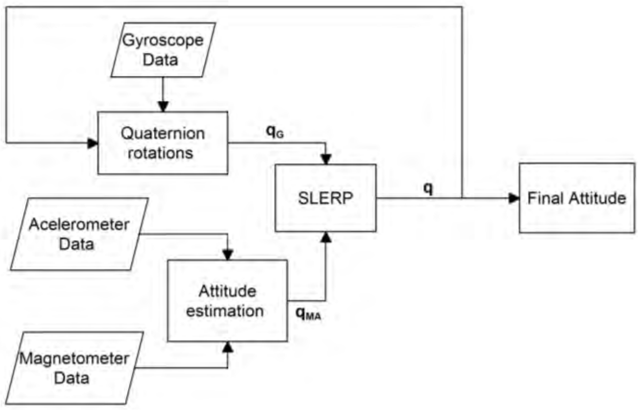

Figure 1. Diagram of the proposed method

3. ROTATIONS USING GYROSCOPE DATA

In order to determine the attitude provided by the MEMS IMU sensor, an iterative method was develop using successive quaternion rotations applied to the sensor's data.

3.1 QUATERNION ROTATIONS

Quaternion is a System of Imaginaries in Algebra first described by (Hamilton, 1844). It is an extension of complex numbers and is a very useful tool for three dimensional dynamics analyses. Quaternions will be used in this method to represent three dimensional rotations. A Euclidian vector **A** = (a_1, a_2, a_3) can be represented as a pure imaginary quaternion that can be written as:

$$a_1\hat{i} \quad a\hat{} \quad a\hat{} \tag{1}$$

An axis vector and an angle can be used to represent any rotation. A rotation quaternion **q** around the vector **B** = (b_1, b_2, b_3) with an angle is defined as:

$$e^{[-(b_1\hat{i} \quad b\hat{} \quad b\hat{})]} \quad cos(\Theta/2) \quad (b_1\hat{i} \quad b\hat{} \quad b\hat{})sen(\Theta/2) \tag{2}$$

As can be seen, the real term of the quaternion represents the rotation angle while the pure imaginary part of the quaternion represents axis vector. Considering a vector as a pure imaginary quaternion, we can express **v'**, the result of the rotation of the vector **B** by the quaternion **q** in Eq. 3.

$$\mathbf{v'} \quad * \mathbf{v} * \tag{3}$$

where $\mathbf{q}^*$ is the complex quaternion conjugate of **q**.

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3.2 GYROSCOPE ROTATIONS

In order to estimate the aircraft attitude using initially the gyroscope data, an algorithm to apply the successive rotations on the previous iteration's estimation using the gyroscope output data in each iteration period was used. The gyroscope outputs are the three angular velocities relative to its axes. The angle of rotation of each axis is determined by multiplying theses velocities by the period of the iteration.

$$_{(x,y,\ )} \qquad _{(x,y,\ )} \cdot \tag{4}$$

Knowing these three angles at each iteration, three rotations are performed, related to each axis of the sensor reference system, to finally determine the resulting attitude provided by the gyroscope.

Considering small rotations, at each iteration:

$$(n \quad 1) \qquad * \quad * \quad * \quad (n) * \quad * \quad * \tag{5}$$

where $\mathbf{q_x}$, $\mathbf{q_y}$, $\mathbf{q_z}$ are the quaternion rotations referent to the angles $_x$, $_y$, $_z$ around the respective axes and , , $_\mathbf{z}$ are the complex conjugates of, respectively, $\mathbf{q_x}$, $\mathbf{q_y}$, $\mathbf{q_z}$.

4. ACCELEROMETER AND MAGNETOMETER'S TTITUDE ESTIMATION

At this step of the method, the attitude is estimated by two successive quaternion rotations. The first rotation is the one between the magnetic field vector measured by the magnetometer in its reference system and the earth's magnetic field vector estimated by a model; this quaternion takes the earth coordinate system to a temporary coordinate system. The second rotation, that leads the temporary coordinate system to the magnetometer's coordinate system, is obtained by discounting the centripetal acceleration from the acceleration measured by the accelerometer. This is done in order to obtain the gravity acceleration in the sensor's reference system from the corresponding data in the earth's reference system, that is constant and known.

4.1 E RTH'S M GNETIC FIELD CORRECTION

As the earth's magnetic field vector is required in the attitude estimation previously detailed, it's necessary to preview the non-uniformity of this vector in relation to the earth's reference system adopted. To deal with these variations the International Geomagnetic Reference Field (IGRF) model, described by Finlay et al. (2010), is used. This model allows calculating the geomagnetic field vector in any position inside or around the earth.

The following formula is used to determine the earth's magnetic scalar potential:

$$(r, \quad , \quad , t) \quad a. \sum_{n \quad 1} \sum_{m \quad 1} \left(\frac{a}{r}\right)^{n \quad 1} \quad {}_n^m(t) \cos(m\emptyset) \quad h_n^m(t) \sin(m\emptyset) \tag{6}$$

By definition, the magnetic field vector **B** is:

$$-\nabla \quad V) \tag{7}$$

where ∇) is the gradient of V. Therefore, using these equations, it is possible to determine the magnetic field vector in any point of earth, neglecting local disturbances.

4.2 CENTRIPETAL ACCELERATION CORRECTION

The implemented filter assumes that the mean acceleration vector points toward the center of the earth in long term analysis, which is correct when we only consider disturbances like wind gusts and small duration maneuvers. However, long duration maneuvers like coordinate turns can generate a long term disturbance, the centripetal force. The solution for this case is to subtract the centripetal force of the acceleration vector measured in the aircraft's coordinate system. Using a simple assumption that any long duration maneuver is a coordinate turn and that any different behavior is not possible to succeed for long time, e.g., a plane cannot make an ascending turn for much time, the following estimative of centripetal acceleration is applied:

$$\tag{8}$$

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where and are the angular and the linear velocity, respectively, measured by the aircraft's sensors.

4.3 ATTITUDE DETERMINATION

During flight, the magnetic field direction is only significantly affected by disturbances that come from the airplane's systems itself. These are consequences of high electrical currents and big ferromagnetic structures near the sensor. Once the magnetometer data can be compensated in order to measure only the earth's magnetic field, it can be used to determine part of the attitude, which needs to be completed by the gravity vector, i.e. the magnetic field can only determine two degrees of freedom.

First, it is determined which quaternion $\mathbf{q_n}$ takes the magnetic field's normalized vector expressed in the earth's coordinate system to the same normalized vector expressed in the sensor's coordinate system ; for that it is used the concept of cross product:

$$\mathbf{v} \tag{9}$$

where $\mathbf{v_n}$ is the normal vector.

The angle of rotation is determined by the scalar product:

$$\mathbf{v} \quad \mathbf{v} \quad \mathbf{v} \quad \mathbf{v} \quad cos \tag{10}$$

And the axis of rotation $\mathbf{v_r}$ is:

$$\mathbf{v} \quad \frac{\mathbf{v}}{\mathbf{v}} \tag{11}$$

This leads to a quaternion $\mathbf{q_1}$ that takes a vector from the earth's coordinate system to a temporary coordinate system where $\mathbf{B_s}$ $\mathbf{B_t}$.

$$\left(cos\left(/ \right),\ {}_{rx}\cdot sin\left(/ \right),\ {}_{ry}\cdot sin\left(/ \right),\ {}_{r}\,.\,sin\left(/ \right)\right) \tag{12}$$

where v_{rx}, v_{ry} and v_{rz} are, respectively, the components of the vector $\mathbf{v_r}$ in the axes x, y and z.

Next, the $\mathbf{g_e}$, the earth's gravity vector in the earth's coordinate system, is rotated by the quaternion $\mathbf{q_1}$, in order to get $\mathbf{g_1}$.

$$* \quad * \tag{13}$$

The purpose is to find the rotation around that minimizes the angle between $\mathbf{g_1}$ and which is the transformation that takes the temporary coordinate system to the sensor's coordinate system. This rotation is obtained finding the angle between the components perpendicular to . It is defined as the component of $\mathbf{g_1}$ that is perpendicular to :

$$- \frac{(\quad)}{(|\quad||\quad|)} \tag{14}$$

The acceleration $'$ is defined in Eq. 15 as the difference between the acceleration measured by the sensor and the centripetal acceleration estimated by the Eq. 8.

$$- \qquad - \tag{15}$$

In order to avoid using too wrong values of in the attitude estimation, it was imposed a condition to its use. If the value of the modulus of is superior to 1.2 g or inferior to 0.8 g, (where g is the gravitational acceleration modulus) it means that is not a good value to the attitude estimation. This way, the estimation only considers the accelerometer and magnetometer's part in iterations where $1.2\,g > |\mathbf{a}'_{\mathbf{s}}| > 0.8\,g$. This consideration was used to avoid deviations in the estimation caused by intense maneuvers or wind gusts that compromises our model of centripetal acceleration.

And is defined as the component of $'$ that is perpendicular to **.** Equation 16 shows the calculation of **.**

$$- \frac{(\quad)}{(|\quad||\quad|)} \tag{16}$$

The angle between and is found by:

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$$sin^{-1} \quad) \tag{17}$$

The quaternion that takes the temporary coordinate system to the sensor coordinate system is then defined in Eq. 18.

$$\left(cos\left(\; / \; \right), \quad _{sx}. sin\left(\; / \; \right), \quad _{sy}. sin\left(\; / \; \right), \quad _{sz}.sin\left(\; / \; \right)\right) \tag{18}$$

Where $_{sx}$, $_{sy}$, and $_{sz}$ are, respectively, the components of $_{\mathbf{s}}$ in the sensor's axes *x*, *y* and *z*.

The final quaternion that represents the estimated attitude of the sensor using only the magnetometer and accelerometer is:

$$* \tag{19}$$

5. FINAL ATTITUDE ESTIMATION

The final attitude estimation consists in merging the attitudes calculated by the two methods. However this merging process includes applying a Proportional-Integrative (PI) filter on the difference of the two calculated attitudes. A spherical linear interpolation is finally done to complete the attitude estimation process.

5.1 THE SPHERICAL LINEAR INTERPOLATION

The spherical linear interpolation is a method used to interpolate linearly two quaternions using the closest path which was described in (Kremer, 2008). It is done by simply using four-dimensional linear algebra.

To interpolate two quaternions, it's necessary to find the shortest angle) between them. This angle is obtained using Eq. 20, as the values of the modules of $\mathbf{q_1}$ and $\mathbf{q_2}$ are one.

$$cos^{-1} \quad) \tag{20}$$

It is important to note that care should be taken in order to avoid getting the second solution of , in the interval *90° < < 270°*, which is not the correct solution.

The next step is to orthonormalise $\mathbf{q_1}$ and $\mathbf{q_2}$ using the Gram-Schmidt process, creating $\mathbf{q_3}$ and consequently an orthonormal basis that consists of $\mathbf{q_1}$ and $\mathbf{q_3}$ that covers the plane of $\mathbf{q_1}$ and $\mathbf{q_2}$.

$$- \frac{(\quad)}{(| \; || \; |)} \; \overline{| \; |} \tag{21}$$

$$\overline{| \; |} \tag{22}$$

Finally, a quaternion $\mathbf{q}$ is written using the produced basis:

$$.cos \;\; t. \;) \qquad .sen \;\; t. \;) \tag{23}$$

where *t* is defined in Eq. 28.

5.2 THE PI FILTER

The PI filter is the core of the final attitude estimation; it is used in order to correct the cumulative errors of the gyroscope.

In order to apply the filter, some quantities were defined. The error quaternion $\mathbf{q_e}$ is defined in Eq. 24.

$$* \tag{24}$$

The angle of the error quaternion is calculated using the definition of a quaternion rotation.

$$. \; cos^{-1} \; Re(\quad) \tag{25}$$

A step rotation quaternion $\mathbf{q_{sr}}$ is defined in order to increment the integrative quaternion $\mathbf{q_{integrative}}$.

$$\left(cos(\ _{i\cdot}\ /\ )\ ,\ x_e.\ sin(\ _{i\cdot}\ /\ ),\ y_e\,.\ sin(\ _{i\cdot}\ /\ ),\ \ _e.sin(\ _{i\cdot}\ /\ )\right) \tag{26}$$

where x_e, y_e and z_e are the components of a unitary vector with the same direction of rotation of the quaternion $\mathbf{q_e}$ and K_i is a real constant. The quaternion that represents the integrative part of the filter is rotated by $\mathbf{q_{sr}}$ at each iteration in order to accumulate the errors quaternions.

$$\mathbf{q_{integrative}}\,(n+1) = \mathbf{q_{sr}} * \mathbf{q_{integrative}}\,(n) \tag{27}$$

The filter is applied using a SLERP between $\mathbf{q_g}$ and $\mathbf{q_{am}}$, with a factor t defined below.

$$t \quad _p(\ ) \qquad _{inte\ \ rati\ \ e} \tag{28}$$

where is defined in Eq. 20 and K_p is a real constant and $_{inte\ \ rati\ \ e}$ is defined below.

$$_{inte\ \ rati\ \ e} \quad .\ cos^{-1}\left[Re\left(\qquad _{\mathbf{v}}\right)\right] \tag{29}$$

The integrative part has the purpose of correcting the constant bias, while the proportional part has the purpose of correcting the scaling factor, the misalignments and the Gaussian noise.

6. RESULTS

Aiming the evaluation of the filter, the creation of a system that could simulate the dynamics of a fixed wing airplane and the sensors errors was necessary. The solution was the integration of the X-Plane® Flight Simulator, software certified by United States Federal Aviation Agency (FAA), and a model for sensors errors, which would give the correct attitude and the simulated sensors outputs, the necessary information to evaluate the filter. The diagram representing the evaluation of the method is shown in Fig. 2.

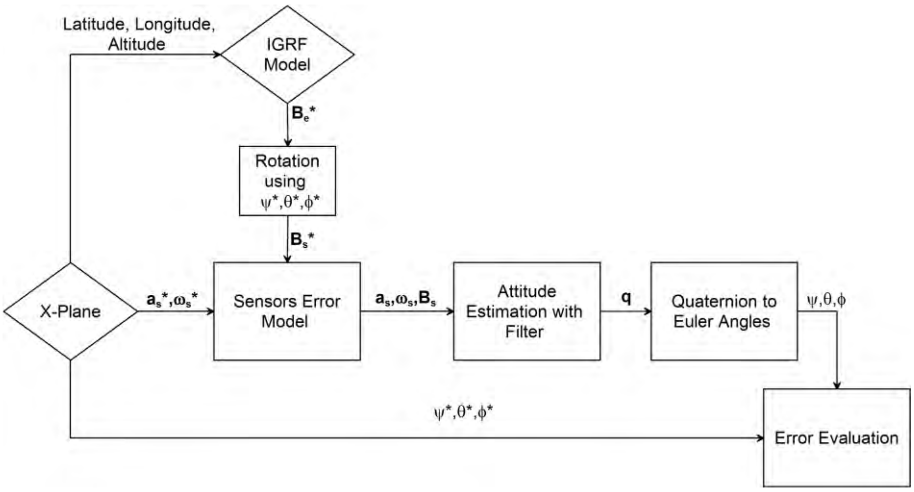

Figure 2. Diagram of the filter's evaluation

6.1 SENSOR'S SIMUL TION

The development of Micro-Electro-Mechanical Systems (MEMS) Inertial Measurement Unit (IMU) sensors has been accelerated by the last few years' improvements in technologies. This led to a deep descent of prices of this kind of sensors, creating new possibilities for the market of small UAVs.

In order to evaluate correctly the developed filter aiming this kind of aircraft, the simulation was based on a model recently cited by Woodman (2007). The model predicts errors caused by four major sources, the non-orthogonality between axes of measurement, scaling factors, constant biases and Gaussian noise. Other more complex approaches are available as those described by Naranjo (2008).

The axes' misalignment is simulated, using a matrix that transforms the value measured in the orthogonal platform axes to the non-orthogonal platform axes. With $_1$, $_2$ and $_3$ being the misalignment angles, the matrix $\mathbf{T}$ becomes:

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$$\mathbf{T} \begin{bmatrix} 1 & 1 & \\ & 1 & \\ & & 1 \end{bmatrix} \tag{30}$$

The scaling factor is caused by a wrong sensitivity of the inner circuitry and mechanism. It is modeled as a linear error and is directly related to the quality of fabrication method. Consequently, the model implements this error as a diagonal matrix K. The scaling error can be seen in Fig 3.

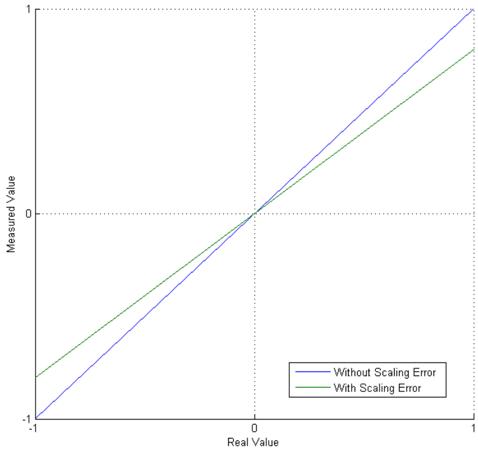

Figure 3. Scaling factor error

$$\begin{bmatrix} 1 \; e_1 & & \\ & 1 \; e & \\ & & 1 \; e \end{bmatrix} \tag{31}$$

The constant bias is also caused by construction errors of both circuitry and mechanism. It is simply modeled as a constant vector **b**.

The Gaussian noise is generated using the method described by Box and Muller (1958) and is directly added to the measurement vector.

The final sensor simulation is done using the Eq. 32.

$$_{\mathbf{M}} \quad \mathbf{T} \quad _{\mathbf{T}} \quad \mathbf{v} \tag{32}$$

where $\mathbf{V_M}$ is the value measured by the sensor, $\mathbf{V_T}$ is the true value, which would be the output if the sensor was perfect. The vector **v** is generated by the Box-Muller Method.

The values used to evaluate the filter were extracted from a commonly used MEMS IMU in small UAVs. The obtained sensor's characteristics are listed in Tab. 1.

Table 1. Errors values for the typical sensor

	Scaling Factor	Misalignment	Standard Deviation of Noise	Constant Bias
Accelerometers	0.10%	0.2°	9.54×10^{-2} m/s^2	1.962×10^{-3} m/s^2
Gyroscopes	0.10%	0.05°	1.657×10^{-2} °/s	1.221×10^{-4} °/s
Magnetometers	0.50%	0.5°	125nT	400nT

6.2 FILTER EVALUATION

The filter evaluation was done using a recorded X-Plane® flight of 50 minutes, with frequency of 60Hz and the error parameters described above.

To calculate the errors, the true attitude of the aircraft provided by the X-Plane® flight data was considered. Two errors are calculated: the difference between the gyroscope pure integrative attitude and the true attitude and the

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difference between the proposed filter attitude and the true attitude. The comparisons of the errors over the time were done for the three angles. The yaw errors are plotted in Fig. 5, the pitch errors in Fig. 6 and the roll errors in Fig. 7.

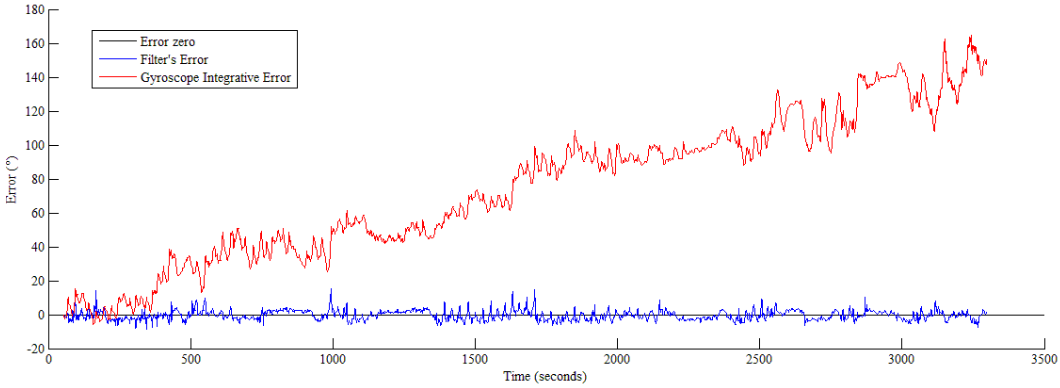

Figure 5. Yaw errors

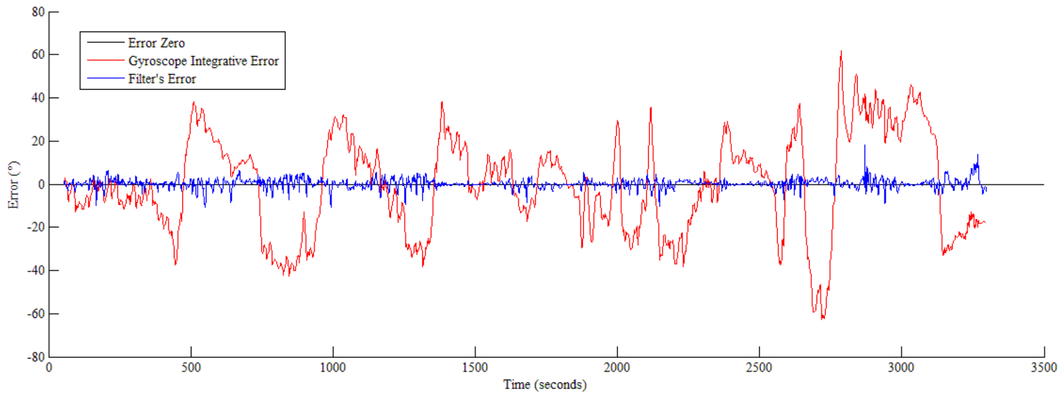

Figure 6. Pitch errors

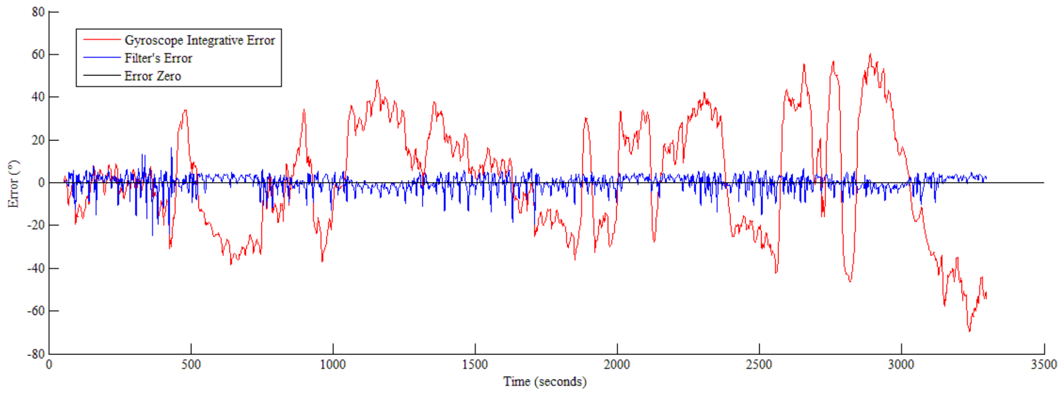

Figure 7. Roll errors

7. CONCLUSION

The developed filter is completely suitable for small UAVs equipped with MEMS IMUs. The estimated angles are sufficiently precise to be used as feedback for control systems. Besides, it proved to be relatively simple in comparison with other methods.

Better results can be obtained using other sensors which could output angle of attack and sideslip angles, as well as relative airspeed; the forces that act in the airframe can be estimated using aerodynamic models. These forces could be

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used to estimate the non-gravitational accelerations to which the airplane is subjected. Subtracting these accelerations from the measured acceleration vector, one could have a better estimation of the gravity vector measured in the aircraft's coordinate system. It is important to point that the centripetal force correction should not be applied along this solution.

Care should be taken in order to minimize and compensate the magnetometer measurement deviations, particularly when the airplane is powered by an electrical motor or when there are other high power embedded circuits. In this work, these variations are not considered in the model.

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